

POLYNOMIAL IDENTITY INVOLVING BINOMIAL THEOREM AND FAULHABER'S FORMULA

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CONTENTS

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1. BINOMIAL POWER SUMS

$$\sum_{t=0}^1 (-1)^t \binom{1}{t} n^{1-t} \sum_{k=1}^n k^{t+1} = \frac{1}{6}(-n + n^3)$$

$$\sum_{t=0}^2 (-1)^t \binom{2}{t} n^{2-t} \sum_{k=1}^n k^{t+2} = \frac{1}{30}(-n + n^5)$$

$$\sum_{t=0}^3 (-1)^t \binom{3}{t} n^{3-t} \sum_{k=1}^n k^{t+3} = \frac{1}{420}(-10n + 7n^3 + 3n^7)$$

$$\sum_{t=0}^4 (-1)^t \binom{4}{t} n^{4-t} \sum_{k=1}^n k^{t+4} = \frac{1}{630}(-21n + 20n^3 + n^9)$$

$$\sum_{t=0}^5 (-1)^t \binom{5}{t} n^{5-t} \sum_{k=1}^n k^{t+5} = \frac{1}{2772}(-210n + 231n^3 - 22n^5 + n^{11})$$

$$\sum_{t=0}^6 (-1)^t \binom{6}{t} n^{6-t} \sum_{k=1}^n k^{t+6} = \frac{1}{60060}(-15202n + 18200n^3 - 3003n^5 + 5n^{13})$$

$$\sum_{t=0}^7 (-1)^t \binom{7}{t} n^{7-t} \sum_{k=1}^n k^{t+7} = \frac{1}{51480}(-60060n + 76010n^3 - 16380n^5 + 429n^7 + n^{15})$$

$$\sum_{t=0}^8 (-1)^t \binom{8}{t} n^{8-t} \sum_{k=1}^n k^{t+8} = \frac{1}{218790}(-1551693n + 2042040n^3 - 516868n^5 + 26520n^7 + n^{17})$$

2. FAULHABER'S FORMULAE

$$\sum_{k=1}^n k^1 = \frac{1}{2}(-n + n^2)$$

$$\sum_{k=1}^n k^2 = \frac{1}{6}(n - 3n^2 + 2n^3)$$

$$\sum_{k=1}^n k^3 = \frac{1}{4}(n^2 - 2n^3 + n^4)$$

$$\sum_{k=1}^n k^4 = \frac{1}{30}(-n + 10n^3 - 15n^4 + 6n^5)$$

$$\sum_{k=1}^n k^5 = \frac{1}{12}(-n^2 + 5n^4 - 6n^5 + 2n^6)$$

$$\sum_{k=1}^n k^6 = \frac{1}{42}(n - 7n^3 + 21n^5 - 21n^6 + 6n^7)$$

$$\sum_{k=1}^n k^7 = \frac{1}{24}(2n^2 - 7n^4 + 14n^6 - 12n^7 + 3n^8)$$

$$\sum_{k=1}^n k^8 = \frac{1}{90}(-3n + 20n^3 - 42n^5 + 60n^7 - 45n^8 + 10n^9)$$

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