POLYNOMIAL IDENTITY INVOLVING BINOMIAL THEOREM AND FAULHABER'S FORMULA

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Contents

1. Conclusions

In this manuscript we have shown that for every $n \geq 1,\ n, m \in \mathbb{N}$ there are coefficients $\mathbf{A}_{m,0}, \mathbf{A}_{m,1}, \dots, \mathbf{A}_{m,m}$ such that the polynomial identity holds

$$n^{2m+1} = \sum_{k=1}^{n} \mathbf{A}_{m,0} k^0 (n-k)^0 + \mathbf{A}_{m,1} (n-k)^1 + \cdots + \mathbf{A}_{m,m} k^m (n-k)^m$$

In particular, the coefficients $\mathbf{A}_{m,r}$ may be evaluated both ways, by constructing and solving a system of linear equations or applying recurence relation, all these approaches are explained in section (1,2 refs), respectively, including detailed examples.

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