POLYNOMIAL IDENTITY INVOLVING BINOMIAL THEOREM AND FAULHABER'S FORMULA

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Contents

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1. Binomial power sums

$$\sum_{t=0}^{1}(-1)^t\binom{1}{t}n^{1-t}\sum_{k=1}^{n}k^{t+1}=\frac{1}{6}(-n+n^3)$$

$$\sum_{t=0}^{2}(-1)^t\binom{2}{t}n^{2-t}\sum_{k=1}^{n}k^{t+2}=\frac{1}{30}(-n+n^5)$$

$$\sum_{t=0}^{3}(-1)^t\binom{3}{t}n^{3-t}\sum_{k=1}^{n}k^{t+3}=\frac{1}{420}(-10n+7n^3+3n^7)$$

$$\sum_{t=0}^{4}(-1)^t\binom{4}{t}n^{4-t}\sum_{k=1}^{n}k^{t+4}=\frac{1}{630}(-21n+20n^3+n^9)$$

$$\sum_{t=0}^{5}(-1)^t\binom{5}{t}n^{5-t}\sum_{k=1}^{n}k^{t+5}=\frac{1}{2772}(-210n+231n^3-22n^5+n^{11})$$

$$\sum_{t=0}^{6}(-1)^t\binom{6}{t}n^{6-t}\sum_{k=1}^{n}k^{t+6}=\frac{1}{60060}(-15202n+18200n^3-3003n^5+5n^{13})$$

$$\sum_{t=0}^{7}(-1)^t\binom{7}{t}n^{7-t}\sum_{k=1}^{n}k^{t+7}=\frac{1}{51480}(-60060n+76010n^3-16380n^5+429n^7+n^{15})$$

$$\sum_{t=0}^{8}(-1)^t\binom{8}{t}n^{8-t}\sum_{k=1}^{n}k^{t+8}=\frac{1}{218790}(-1551693n+2042040n^3-516868n^5+26520n^7+n^{17})$$

2. Faulhaber's formulae

$$\sum_{k=1}^{n} k^1 = \frac{1}{2}(-n + n^2)$$
$$\sum_{k=1}^{n} k^2 = \frac{1}{6}(n - 3n^2 + 2n^3)$$
$$\sum_{k=1}^{n} k^3 = \frac{1}{4}(n^2 - 2n^3 + n^4)$$
$$\sum_{k=1}^{n} k^4 = \frac{1}{30}(-n + 10n^3 - 15n^4 + 6n^5)$$
$$\sum_{k=1}^{n} k^5 = \frac{1}{12}(-n^2 + 5n^4 - 6n^5 + 2n^6)$$
$$\sum_{k=1}^{n} k^6 = \frac{1}{42}(n - 7n^3 + 21n^5 - 21n^6 + 6n^7)$$
$$\sum_{k=1}^{n} k^7 = \frac{1}{24}(2n^2 - 7n^4 + 14n^6 - 12n^7 + 3n^8)$$
$$\sum_{k=1}^{n} k^8 = \frac{1}{90}(-3n + 20n^3 - 42n^5 + 60n^7 - 45n^8 + 10n^9)$$

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