ROW SUMS CONJECTURE IN ITERATED RASCAL TRIANGLES

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ABSTRACT. In [1], Gregory et al. provide the following conjecture for row sums of iterated rascal triangles. For every i

$$\sum_{k=0}^{4i+3} \binom{4i+3}{k}_i = 2^{4i+2}$$

where $\binom{n}{k}_i$ is an iterated rascal number.

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1. INTRODUCTION

Rascal triangle is Pascal-like numeric triangle developed in 2010 by three middle school students, Alif Anggoro, Eddy Liu, and Angus Tulloch [?]. During math classes they were

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Sources: https://github.com/kolosovpetro/RowSumsConjectureInRascalTriangle challenged to provide the next row for the following number triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ & & & \cdots & & & \end{array}$$

Teacher's expected answer was the one that matches Pascal's triangle, e.g "1 4 6 4 1". However, Anggoro, Liu, and Tulloch suggested "1 4 5 4 1" instead. They devised this new row via what they called diamond formula

$$\mathbf{South} = \frac{\mathbf{East} \cdot \mathbf{West} + 1}{\mathbf{North}}$$

So they obtained the following triangle

n/k	0	1	2	3	4	5	6	7
0	1							
1	1	1						
2	1	2	1					
3	1	3	3	1				
4	1	4	5	4	1			
5	1	5	7	7	5	1		
6	1	6	9	10	9	6	1	
7	1	7	11	13	13	11	7	1

Table 1. Rascal triangle.

Indeed, the forth row is "1 4 5 4 1" because $4 = \frac{1 \cdot 3+1}{1}$ and $5 = \frac{3 \cdot 3+1}{2}$. Since then, a lot of work has been done over the topic of rascal triangles. In this article we stick our attention to the one of rascal triangles generalizations, namely iterated rascal triangles [1]. Iterated rascal number is defined via a sum of binomial coefficients multiplication

Definition 1.1. *Iterated rascal number*

$$\binom{n}{k}_i = \sum_{m=0}^{i} \binom{n-k}{m} \binom{k}{m} \tag{1.1}$$

Thus, the rascal triangle (1) is the triangle generated by $\binom{n}{k}_1$.

2. Row sums conjecture

In [1], Gregory et al. provide the following conjecture for row sums of iterated rascal triangles.

Conjecture 2.1. *For every i*

$$\sum_{k=0}^{4i+3} \binom{4i+3}{k}_i = 2^{4i+2}$$

where $\binom{n}{k}_i$ is an iterated rascal number.

References

[1] Gregory, Jena and Kronholm, Brandt and White, Jacob. Iterated rascal triangles. *Aequationes mathematicae*, pages 1–18, 2023. https://doi.org/10.1007/s00010-023-00987-6.

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