SUMS OF POWERS VIA CENTRAL FINITE DIFFERENCES AND NEWTON'S FORMULA

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Abstract. Your abstract here.

1. Introduction

Your introduction here. Include some references [1, 2, 3]. Lorem Ipsum is simply dummy text of the printing and typesetting industry. Lorem Ipsum has been the industry's standard dummy text ever since the 1500s, when an unknown printer took a galley of type and scrambled it to make a type specimen book. It has survived not only five centuries, but also the leap into electronic typesetting, remaining essentially unchanged. It was popularised in the 1960s with the release of Letraset sheets containing Lorem Ipsum passages, and more recently with desktop publishing software like Aldus PageMaker including versions of Lorem Ipsum.

Image example

Date: December 29, 2025.

2010 *Mathematics Subject Classification.* 05A19, 05A10, 41A15, 11B68, 11B73, 11B83.

Key words and phrases. Sums of powers, Newton's interpolation formula, Finite differences, Binomial coefficients, Faulhaber's formula, Bernoulli numbers, Bernoulli polynomials, Interpolation, Discrete convolution, Combinatorics, Polynomial identities, Central factorial numbers, Stirling numbers, Eulerian numbers, Worpitzky identity, Pascal's triangle, OEIS.

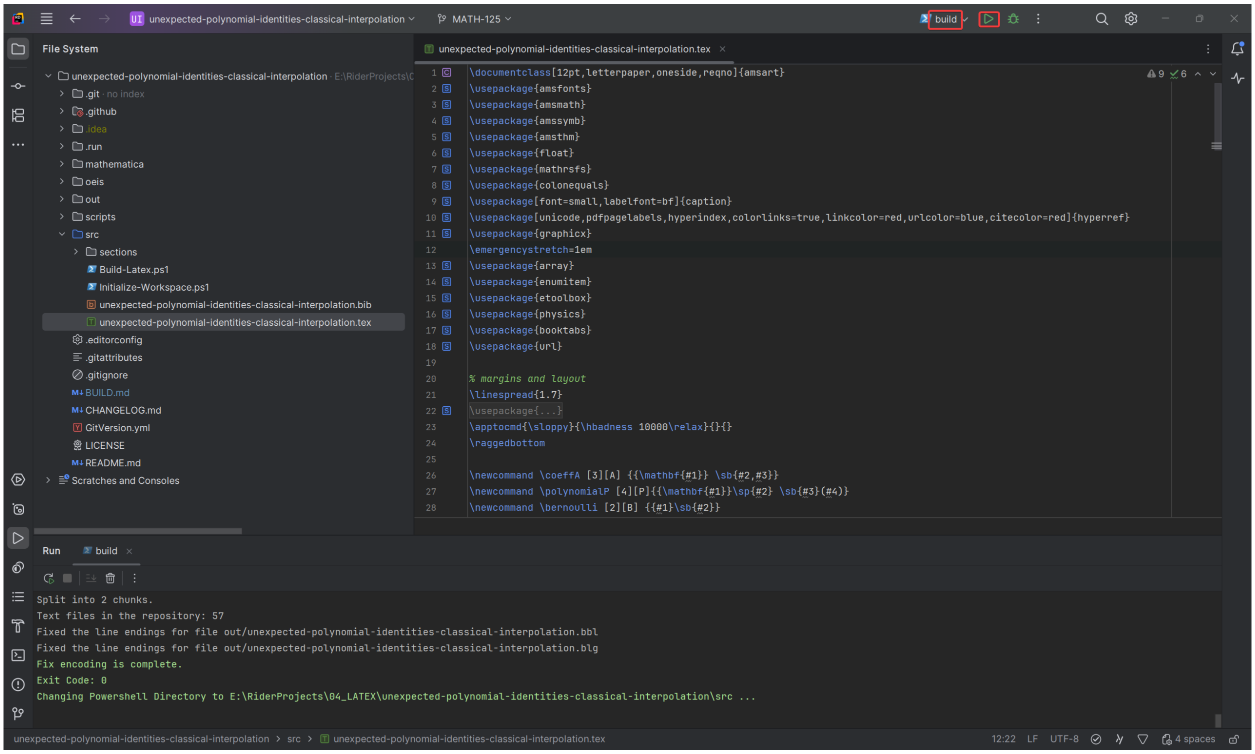

Figure 1. Image example.

m/r	0	1	2	3	4	5	6	7
0	1							
1	1	6						
2	1	0	30					
3	1	-14	0	140				
4	1	-120	0	0	630			
5	1	-1386	660	0	0	2772		
6	1	-21840	18018	0	0	0	12012	
7	1	-450054	491400	-60060	0	0	0	51480

Table 1. Coefficients $\mathbf{A}_{m,r}$. See OEIS sequences [4, 5].

$$\left[\!\!\left[\begin{matrix} a \\ b \end{matrix} \right]\!\!\right]_m$$

$$\left[\!\!\left[\begin{matrix} a \\ b \end{matrix} \right]\!\!\right]_m$$

And for any natural m we have polynomial identity

$$x^m = \sum_{k=1}^{m} T(m,k) x^{[k]} \tag{1}$$

where $x^{[k]}$ denotes central factorial defined by

$$x^{[n]} = x\left(x + \frac{n}{2} - 1\right)^{\underline{n-1}}$$

where $(n)^{\underline{k}} = n(n-1)(n-2)\cdots(n-k+1)$ denotes falling factorial in Knuth's notation. In particular,

$$x^{[n]} = x\left(x + \frac{n}{2} - 1\right)\left(x + \frac{n}{2} - 1\right)\cdots\left(x + \frac{n}{2} - n - 1\right) = x\prod_{k=1}^{n-1}\left(x + \frac{n}{2} - k\right) \tag{2}$$

This is an equation reference (1).

Continuing similarly, we are able to derive the formula for multifold sums of powers, which is

Theorem 1.1 (Multifold sums of powers via Newton's series)**.** *For non-negative integers r, n, m and an arbitrary integer t*

$$\Sigma^r\, n^m = \sum_{j=0}^{m} \Delta^j t^m \left[\left(\sum_{s=1}^{r} (-1)^{j+s-1}\binom{j+t-1}{j+s}\Sigma^{r-s}\, n^0\right) + \binom{n-t+r}{j+r}\right]$$

Proof. By Newton's series for power and repeated applications of the segmented hockey stick identity. □

2. Conclusions

Conclusions of your manuscript.

Here is an itemize list with adjusted margins

- Conclusion 1 ....
- Conclusion 2 ....
- Conclusion 3 ....

Total derivative: $\frac{\mathrm{d}y}{\mathrm{d}x}$

$$\frac{\mathrm{d}y}{\mathrm{d}x}$$

Partial derivative: $\frac{\partial f}{\partial x}$

Second total derivative: $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

Mixed partial: $\frac{\partial^2 f}{\partial x \partial y}$

3. Acknowledgements

The author is grateful to [Full Name] for his valuable contribution [contribution] about the fact that [interesting claim].

References

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[4] Petro Kolosov. Entry A302971 in The On-Line Encyclopedia of Integer Sequences, 2018. https://oeis.org/A302971.

[5] Petro Kolosov. Entry A304042 in The On-Line Encyclopedia of Integer Sequences, 2018. https://oeis.org/A304042.

Version: `Local-0.1.0`

Sources: github.com/kolosovpetro/SumsOfPowersViaCentralFiniteDifferencesAndNewtonFormula

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