

COMMENTS ON CONCRETE MATHEMATICS (2E) BINOMIAL COEFFICIENTS

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1. INTRODUCTION

1.1. Generating functions. Generating function is a power series that generates an infinite sequence of numbers $\{a_0, a_1, a_2, a_3, \dots\}$

$$A(z) = a_0 + a_1z + a_2z^2 + a_3z^3 + \dots = \sum_{k=0}^{\infty} a_k z^k$$

Coefficient of z^n in $A(z)$ denoted as

$$[z^n]A(z) = a_n$$

is the n -th term of the sequence. For example, generating function for the sequence of binomial coefficients is

$$(1+z)^r = \sum_{k=0}^{\infty} \binom{r}{k} z^k$$

Let be a product of two generating functions $A(z)$ and $B(z)$, then c_n in such sequence is a sum

$$c_n = \sum_{k=0}^n a_k b_{n-k}$$

Above sum is called the convolution of two sequences $\{a_0, a_1, a_2, a_3, \dots\}$ and $\{b_0, b_1, b_2, b_3, \dots\}$.

So that

$$[z^n]A(z)B(z) = c_n$$

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Example for Vandermonde convolution, let be $A(z) = (1 + z)^r$ and $B(z) = (1 + z)^s$, then multiplying them

$$A(z)B(z) = (1 + z)^r(1 + z)^s = (1 + z)^{r+s}$$

Then the coefficient of z^n in $(1 + z)^{r+s}$ is

$$[z^n]A(z)B(z) = [z^n](1 + z)^{r+s} = \sum_{k=0}^n a_k b_{n-k} = \sum_{k=0}^n \binom{r}{k} \binom{s}{n-k} = \binom{r+s}{n}$$

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