Prove that

$$\sum_{k=0}^{t} \binom{t-k}{r} \binom{k}{s} = \binom{t+1}{r+s+1}$$

This approach is based on generating functions.

By `http://math.arizona.edu/~faris/combinatoricsweb/generate.pdf`

We know that

$$\sum_{n=0}^{\infty} \binom{n}{k} y^n = \frac{y^k}{(1-y)^{k+1}}$$

Then

$$x \cdot \sum_{l=0}^{\infty} \binom{l}{r} x^l \cdot \sum_{k=0}^{\infty} \binom{k}{s} x^k = x \cdot \frac{x^r}{(1-x)^{r+1}} \cdot \frac{x^s}{(1-x)^{s+1}} = \frac{x^{r+s+1}}{(1-x)^{r+s+2}}$$
$$= \sum_{n=0}^{\infty} \binom{n}{r+s+1} x^n$$

The coefficient of x^{t+1} of the $x \cdot \sum_{l=0}^{\infty} \binom{l}{r} x^l \cdot \sum_{k=0}^{\infty} \binom{k}{s} x^k$ is

$$[z^{t+1}]A(z)B(z) = \sum_{k=0}^{t} a_k b_{t-k} = \sum_{k=0}^{t} \binom{t-k}{r} \binom{k}{s}$$

Note that upper summation bound is t while coefficient is $[z^{t+1}]$ it is because of the x factor in the generating function. While

$$[z^{t+1}] \sum_{n=0}^{\infty} \binom{n}{r+s+1} x^n = \binom{t+1}{r+s+1}$$

Thus

$$\binom{t+1}{r+s+1} = \sum_{k=0}^{t} \binom{t-k}{r} \binom{k}{s}$$