

COMMENTS ON CONCRETE MATHEMATICS (2E) BINOMIAL COEFFICIENTS

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1. CONVENTIONS

- Use variable z that indicates complex value in generating functions.
- Give particular names to binomial identities, for example *absorption identity*
- Give particular names to generating functions to remember them easily
- Use subscript indices for generating functions that are powers of some value t , for clarity. Example: $A_t(z) = (1 + z)^t$ for binomial coefficients.

2. IMPORTANT BINOMIAL IDENTITIES

3. IMPORTANT GENERATING FUNCTIONS

Identity 3.1. *Cauchy product rule of two generating functions $A(z)$, $B(z)$*

$$A(z) \cdot B(z) = \left(\sum_{n=0}^{\infty} a_n z^n \right) \left(\sum_{n=0}^{\infty} b_n z^n \right) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^n a_k b_{n-k} \right) z^n$$

Identity 3.2. *Cauchy product rule for $(1 + z)^{r+s}$*

$$(1 + z)^{r+s} = \sum_{n=0}^{\infty} \left(\sum_{k=0}^n \binom{r}{k} \binom{s}{n-k} \right) z^n$$

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Identity 3.3. *Shift selected coefficient of generating function*

$$[z^{p-q}]A(z) = [z^p]z^q A(z)$$

$$[z^{p+q}]A(z) = [z^p]\frac{1}{z^q}A(z)$$

Identity 3.4. *Binomial coefficient, fixed r*

$$\binom{r}{n} = [z]^n (1+z)^r$$

Identity 3.5. *Shifted binomial coefficient, fixed m, r*

$$\binom{r}{m+n} = [z]^n \frac{(1+z)^r}{z^m}$$

Identity 3.6. *Binomial coefficient of multiset [1, eq. 8], fixed k*

$$A_k(z) = \sum_{n=0}^{\infty} \binom{n}{k} z^n = \frac{z^k}{(1-z)^{k+1}}$$

Then

$$\binom{t}{k} = [z]^t \frac{z^k}{(1-z)^{k+1}}$$

So that iteration goes over upper index of binomial coefficient.

Identity 3.7. *Shifted Binomial coefficient of multiset, fixed k*

$$\binom{t}{k+r} = [z]^t \frac{z^{k+r}}{(1-z)^{k+r+1}}$$

REFERENCES

- [1] Faris, William G. Generating Functions Notes for Math 447, 2011. <https://math.arizona.edu/~faris/combinatoricsweb/generate.pdf>.

Version: Local-0.1.0