1 The problem

Use generating functions to prove that

$$\sum_{k} \binom{l}{m+k} \binom{s+k}{n} (-1)^k = (-1)^{l+m} \binom{s-m}{n-l} \tag{1}$$

First, get rid of n in the problem itself for the sake of clarity. Let be n=t

$$\sum_{k} {l \choose m+k} {s+k \choose t} (-1)^k = (-1)^{l+m} {s-m \choose t-l}$$

Let's use the identity

$$\binom{m}{n} = [x^n](1+x)^m$$