BINOMIAL IDENTITIES

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1. Binomial identities

Identity for negative r in binomial coefficients

$$\binom{r}{k} = (-1)^k \binom{k-r-1}{k}$$
$$\binom{-r}{k} = (-1)^k \binom{k+r-1}{k}$$

Thus, the generating function follows

$$\frac{1}{(1+z)^r} = \sum_k (-1)^k \binom{k+r-1}{k} z^k = 1 - \binom{r}{1} x + \binom{r+1}{2} x^2 - \binom{r+2}{3} x^3 \cdots$$
$$\frac{1}{(1-z)^r} = \sum_k \binom{k+r-1}{k} z^k = 1 + \binom{r}{1} x + \binom{r+1}{2} x^2 + \binom{r+2}{3} x^3 \cdots$$

Thus

$$[z^n] \frac{1}{(1-z)^r} = \binom{n+r-1}{n}$$

Cauchy product of two generating functions

$$A(z) \cdot B(z) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} a_k b_{n-k} \right) z^n$$