Exercise 3

For each of the following statements, either prove it is true or give a counterexample.

a. If $P(a \mid b, c) = P(b \mid a, c)$, then $P(a \mid c) = P(b \mid c)$

b. If $P(a \mid b, c) = P(a)$, then $P(b \mid c) = P(b)$

c. If $P(a \mid b) = P(a)$, then $P(a \mid b, c) = P(a \mid c)$

a. 正确，证明为真：

由条件概率公式：P(a | b, c) = P(a, b, c)/P(b, c)　P(b | a, c) = P(a, b, c)/P(a, c)

又由于 P(a | b, c) = P(b | a, c)，则 P(a, b, c)/P(b, c)= P(a, b, c)/P(a, c)，

整理后得到：　P(a, c) = P(b, c)

b. 错误，反例：式(a | b, c)=P(a) 说明 a 与 b, c 无关。设 a 为明天的天气变量，b 为一个人牙疼与否变量，c 为同一个人是否有蛀牙的变量，那么显然不会有 P(b | c) = P(b)，即一个人有蛀牙时的牙疼概率显然比平时会遇到牙疼的概率要大

c. 错误，反例：假设同时抛掷两枚硬币，正面结果记为 0，反面结果记为 1。设 a 为第一枚硬币的结果，b 为第二枚硬币的结果，c 为两者结果的异或。那么 a 与 b 是相互独立的，但在给定 c 的情况下，可从 b 的结果直接推出确定的 a 的结果，即在给定 c 的条件下 a、b 并不是独立的。

Exercise 8

Given the full joint distribution shown in Figure 12.3, calculate the following:

a. $\mathbf{P}(toothache)$.

b. $\mathbf{P}(Cavity)$.

c. $\mathbf{P}(Toothache \mid cavity)$.

d. $\mathbf{P}(Cavity \mid toothache \lor catch)$.

Figure 12.3

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

a. *P* (toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

P(toothache) = <0.2, 0.8>

b. *P* (Cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2

P(Cavity) = <0.2, 0.8>

c. **P**(Toothache | cavity) = <(0.108 + 0.012)/0.2, (0.072 + 0.008)/0.2> = <0.6, 0.4>

d. *P* (toothache ∨ catch) = 0.108+0.016+0.012+0.064+0.072+0.144=0.416

P(Cavity|toothache ∨ catch) = <(0.108 + 0.012 + 0.072)/0.416, (0.016 + 0.064 + 0.144)/0.416> = <0.4615, 0.5384>

Exercise 23 (normalization-exercise)

In this exercise, you will complete the normalization calculation for the meningitis example. First, make up a suitable value for $P(s \mid \neg m)$, and use it to calculate unnormalized values for $P(m \mid s)$ and $P(\neg m \mid s)$ (i.e., ignoring the $P(s)$ term in the Bayes' rule expression, Equation (12.14)). Now normalize these values so that they add to 1.

(12.14)

$$\begin{aligned} P(s \mid m) &= 0.7 \\ P(m) &= 1/50000 \\ P(s) &= 0.01 \\ P(m \mid s) &= \frac{P(s \mid m)P(m)}{P(s)} = \frac{0.7 \times 1/50000}{0.01} = 0.0014. \end{aligned}$$

1. 设定合理的 P(s |¬m):

P(s |¬m)= P(¬m | s)P(s)/P(¬m) = (1-P(m | s))×P(s) / (1-P(m))

=(1-0.0014)×0.01 / (49999/50000)

≈0.009986

2. 计算非归一化值(忽略分母上的 P(s))

P(m | s) = P(s | m)P(m) = 0.7×1/50000 ≈ 0.000014

P(¬m | s) = P(s | ¬m)P(¬m) = 0.009986×49999/50000 ≈ 0.009986

3. 归一化处理

归一化因子 α = 0.000014 + 0.009986 = 0.01

P(m | s) = 0.000014/α = 0.0014

P(¬m | s) = 0.009986/α = 0.9986

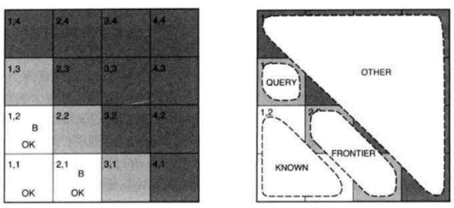
Exercise 29

In our analysis of the wumpus world, we used the fact that each square contains a pit with probability 0.2, independently of the contents of the other squares. Suppose instead that exactly $N/5$ pits are scattered at random among the N squares other than [1,1]. Are the variables $P_{i,j}$ and $P_{k,l}$ still independent? What is the joint distribution $\mathbf{P}(P_{1,1}, \ldots, P_{4,4})$ now? Redo the calculation for the probabilities of pits in [1,3] and [2,2].

（1）如果陷阱总数是确定的，那么 $P_{i,j}$ $P_{k,l}$ 不再是相互独立的。因为知晓了一个陷阱在 k,l 存在将减小 i,j 处出现陷阱的概率，即 P($P_{i,j}$ = true|$P_{k,l}$ = true) < P($P_{i,j}$ = true|$P_{k,l}$ = false)。

（2）联合分布 **P**($P_{1,1}$, …, $P_{4,4}$)是一个均匀分布：向量的各个分量有着相同的概率值 $1/C_{15}^{3}$=1/455。

（3）15 个格子中共有 15/5=3 个陷阱，已知 known 区域无陷阱，$b_{1,2}$、$b_{2,1}$ 有微风



(a) (b)

按照此图，对 $P_{1,3}$=true 的情况下进行全局赋值：

①frontier 都是陷阱：1 种

②frontier 中有一个陷阱，other 中有一个陷阱：2×10=20 种

$P_{1,3}$=true 时共有 1+20=21 种赋值方式

对 $P_{1,3}$=false 的情况下进行全局赋值：

①frontier 都是陷阱：10 种

②frontier 中有一个陷阱 $b_{2,2}$，other 中有 2 个陷阱：C_{10}^{2}=45 种

$P_{1,3}$=true 时共有 10+45=55 种赋值方式

则 $\mathbf{P}(P_{1,3}) = \alpha\langle 21, 55\rangle = \langle 0.276, 0.724\rangle$

同理，对 $P_{2,2}$=true 有 1+2×10+C_{10}^{2}=66 种，对 $P_{2,2}$=false 有 10 种

则 $\mathbf{P}(P_{2,2}) = \alpha\langle 66, 10\rangle = \langle 0.868, 0.132\rangle$