

Exercise 7.13

1. $\text{Smoke} \Rightarrow \text{Smoke}$

$$\equiv \neg \text{Smoke} \vee \text{Smoke} \quad \equiv \text{True}$$

Valid

2. $\text{Smoke} \Rightarrow \text{Fire}$

$$\equiv \neg \text{Smoke} \vee \text{Fire}$$

Neither

3. $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg \text{Smoke} \Rightarrow \neg \text{Fire})$

$$\equiv (\neg \text{Smoke} \vee \text{Fire}) \Rightarrow (\neg \neg \text{Smoke} \vee \neg \text{Fire})$$

$$\equiv (\neg \text{Smoke} \vee \text{Fire}) \Rightarrow (\text{Smoke} \vee \neg \text{Fire})$$

$$\equiv \neg(\neg \text{Smoke} \vee \text{Fire}) \vee (\text{Smoke} \vee \neg \text{Fire})$$

$$\equiv (\text{Smoke} \wedge \neg \text{Fire}) \vee \text{Smoke} \vee \neg \text{Fire}$$

$$\equiv (\text{Smoke} \vee \text{Smoke} \vee \neg \text{Fire}) \wedge (\neg \text{Fire} \vee \text{Smoke} \vee \neg \text{Fire})$$

$$\equiv (\text{Smoke} \vee \neg \text{Fire}) \wedge (\neg \text{Fire} \vee \text{Smoke})$$

$$\equiv \neg \text{Fire} \vee \text{Smoke}$$

Neither

4. $\text{Smoke} \vee \text{Fire} \vee \neg \text{Fire}$

$$\equiv \text{Smoke} \vee \text{True} \quad \equiv \text{True}$$

Valid

$$\begin{aligned}
5. & \left((Smoke \wedge Heat) \Rightarrow Fire \right) \Leftrightarrow \left((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire) \right) \\
& \equiv (\neg Smoke \vee \neg Heat \vee Fire) \Leftrightarrow (\neg Smoke \vee Fire \vee \neg Heat \vee Fire) \\
& \equiv (\neg Smoke \vee \neg Heat \vee Fire) \Leftrightarrow (\neg Smoke \vee \neg Heat \vee Fire) \\
& \text{(设 } A = \neg Smoke \vee \neg Heat \vee Fire \text{)} \\
& \equiv A \Leftrightarrow A \equiv (A \Rightarrow A) \wedge (A \Rightarrow A) \equiv (\neg A \vee A) \wedge (\neg A \vee A) \\
& \equiv True \wedge True \equiv True
\end{aligned}$$

Valid

$$\begin{aligned}
6. & Big \vee Dump \vee (Big \Rightarrow Dump) \\
& \equiv Big \vee Dump \vee \neg Big \vee Dump \\
& \equiv True \vee Dump \equiv True
\end{aligned}$$

Valid

$$\begin{aligned}
7. & (Big \wedge Dump) \vee \neg Dump \\
& \equiv (Big \vee \neg Dump) \wedge (Dump \vee \neg Dump) \\
& \equiv (Big \vee \neg Dump) \wedge True \\
& \equiv (Big \vee \neg Dump)
\end{aligned}$$

Neither

Exercise 7.23

1.真值表如下：用 1 表示 True，用 0 表示 False

设 $A = [(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party]$

Food	Drinks	Party	$(Food \Rightarrow Party)$	$(Drinks \Rightarrow Party)$	$(Food \wedge Drinks) \Rightarrow Party$	A
0	0	0	1	1	1	1
0	0	1	1	1	1	1

0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	0	0	1
1	1	1	1	1	1	1

故，该语句是**有效的**。

2. 左右两边分别转换为 CNF

左边: $(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party})$

$\equiv (\neg \text{Food} \vee \text{Party}) \vee (\neg \text{Drinks} \vee \text{Party})$

$\equiv (\neg \text{Food} \vee \text{Party} \vee \neg \text{Drinks} \vee \text{Party})$

$\equiv (\neg \text{Food} \vee \neg \text{Drinks} \vee \text{Party})$

右边: $(\text{Food} \wedge \text{Drinks}) \Rightarrow \text{Party}$

$\equiv \neg(\text{Food} \wedge \text{Drinks}) \vee \text{Party}$

$\equiv (\neg \text{Food} \vee \neg \text{Drinks}) \vee \text{Party}$

$\equiv (\neg \text{Food} \vee \neg \text{Drinks} \vee \text{Party})$

解释：左右两边化为 CNF 后是相同的语句，设 $B = (\neg \text{Food} \vee \neg \text{Drinks} \vee \text{Party})$ ，所以原语

句相当于 $B \Rightarrow B$ ，对于任意的 B 都是有效的

3. 证明：

需证明 $(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party}) \vee \neg [(\text{Food} \wedge \text{Drinks}) \Rightarrow \text{Party}]$ 是不可满足的

(1) 化为 CNF

$(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party}) \vee \neg [(\text{Food} \wedge \text{Drinks}) \Rightarrow \text{Party}]$

$\equiv (\neg \text{Food} \vee \neg \text{Drinks} \vee \text{Party}) \wedge \neg [\neg \text{Food} \vee \neg \text{Drinks} \vee \text{Party}]$

$\equiv (\neg \text{Food} \vee \neg \text{Drinks} \vee \text{Party}) \wedge \text{Food} \wedge \text{Drinks} \wedge \neg \text{Party}$

(2) 归结操作

子句集合为 $\text{clauses} = \{\neg \text{Food} \vee \neg \text{Drinks} \vee \text{Party}, \text{Food}, \text{Drinks}, \neg \text{Party}\}$

子句归结后形成新子句集合 $\text{new} = \{\neg \text{Drinks} \vee \text{Party}, \neg \text{Food} \vee \text{Party}, \neg \text{Food} \vee \neg \text{Drinks}\}$

更新子句集合 $\text{clauses} = \{\neg \text{Drinks} \vee \text{Party}, \neg \text{Food} \vee \text{Party}, \neg \text{Food} \vee \neg \text{Drinks}\} \cup \text{clauses}$

子句归结后形成新子句集合 $\text{new} = \{\text{Food}, \text{Drinks}, \neg \text{Party}, \neg \text{Food}, \neg \text{Drinks}, \text{Party}, \dots\}$

更新子句集合 $\text{clauses} = \text{new} \cup \text{clauses}$

子句归结过程中有： Food ， $\neg \text{Food}$ 归结为空子句

所以语句 $(\text{Food} \Rightarrow \text{Party}) \vee (\text{Drinks} \Rightarrow \text{Party}) \vee \neg [(\text{Food} \wedge \text{Drinks}) \Rightarrow \text{Party}]$ 是不可满足的，

即原语句是有效的。