Exercise 15.15

A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep, with no observations, is 0.7.
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.

（1）形式化为 DBN：

该动态贝叶斯网络有三个变量：

S_t ： 学生在第 t 天的晚上有充足睡眠

R_t ： 学生第 t 天眼睛是红的

C_t：学生第 t 天在课上睡觉

其中，S_t 是 S_{t+1}、E_tC_t的父节点

由题目描述可知：$P(S_0)=0.7$

$P(S_{t+1} | S_t) = 0.8$　$P(S_{t+1} | \neg S_t) = 0.3$

$P(E_t | S_t) = 0.2$　$P(E_t | \neg S_t) = 0.7$

$PC_t | S_t) = 0.1$　$P(C_t | \neg S_t) = 0.3$

（2）重构为单变量的 HMM：

只需将 E_t 和 C_t 两个双值变量组合起来成为一个四值变量 EC_t，可取值为（0, 0）, (0, 1), (1, 0), (1, 1)，其中第一位表示学生第 t 天眼睛是否是红的，第二位表示学生第 t 天是否在课上睡觉。

（3）给出该 HMM 的 CPT：

S_t	$P(S_{t+1} \mid S_t)$
t	0.8
f	0.3

	$P(EC_t \mid S_t)$			
S_t	(0,0)	(0,1)	(1,0)	(1,1)
t	(1-0.2)*(1-0.1)=0.72	(1-0.2)*0.1=0.08	0.2*(1-0.1)=0.18	0.2*0.1=0.02
f	(1-0.7)*(1-0.3) =0.21	(1-0.7)*0.3=0.09	0.7*(1-0.3)=0.49	0.7*0.3=0.21

Exercise 15.17

For the DBN specified in Exercise 15.15 and for the evidence values

$\mathbf{e}_1 = not\ red\ eyes,\ not\ sleeping\ in\ class$

$\mathbf{e}_2 = red\ eyes,\ not\ sleeping\ in\ class$

$\mathbf{e}_3 = red\ eyes,\ sleeping\ in\ class$

perform the following computations:

1. State estimation: Compute $P(EnoughSleep_t|\mathbf{e}_{1:t})$ for each of $t = 1, 2, 3$.
2. Smoothing: Compute $P(EnoughSleep_t|\mathbf{e}_{1:3})$ for each of $t = 1, 2, 3$.
3. Compare the filtered and smoothed probabilities for $t = 1$ and $t = 2$.

1. 利用 5.15 的 CPT：

S_t	$P(S_{t+1} \mid S_t)$
t	0.8
f	0.3

	$P(EC_t \mid S_t)$			
S_t	e1=(0,0)	(0,1)	e2=(1,0)	e3=(1,1)
t	0.72	0.08	0.18	0.02
f	0.21	0.09	0.49	0.21

$P(S_0)$ = <0.7, 0.3>

$P(S_1)$ = $\sum_{s0} P(S1|s0)P(s0)$

= <0.8, 0.2>0.7 + <0.3, 0.7>0.3

= <0.65, 0.35>

$P(S_1 \mid e_1) = \alpha P(e_1 \mid S_1)P(S_1)$

$= \alpha\langle 0.72, 0.21\rangle\langle 0.65, 0.35\rangle$

$= \alpha\langle 0.468, 0.0735\rangle$

$= \langle 0.8643, 0.1357\rangle$

$P(S_2 \mid e_1) = \sum_{s1} P(S2|s1)P(s1|e1)$

$= \langle 0.8, 0.2\rangle 0.8643 + \langle 0.7, 0.3\rangle 0.1357$

$= \langle 0.7864, 0.2136\rangle$

$P(S_2 \mid e_{1:2}) = \alpha P(e_2 \mid S_2)P(S_2 \mid e_1)$

$= \alpha\langle 0.18, 0.49\rangle\langle 0.7864, 0.2136\rangle$

$= \alpha\langle 0.141552, 0.104664\rangle$

$= \langle 0.5749, 0.4251\rangle$

$P(S_3 \mid e_{1:2}) = \sum_{s2} P(S3|s2)P(s2|e1:2)$

$= \langle 0.8, 0.2\rangle 0.5749 + \langle 0.3, 0.7\rangle 0.4251$

$= \langle 0.58745, 0.41255\rangle$

$P(S_3 \mid e_{1:3}) = \alpha P(e_3 \mid S_3)P(S_3 \mid e_{1:2})$

$= \alpha\langle 0.02, 0.21\rangle\langle 0.58745, 0.41255\rangle$

$= \alpha\langle 0.011749, 0.0866355\rangle$

$= \langle 0.1194, 0.8806\rangle$

	$P(EC_t \mid S_t)$			
S_t	e1=(0,0)	(0,1)	e2=(1,0)	e3=(1,1)
t	0.72	0.08	0.18	0.02
f	0.21	0.09	0.49	0.21

S_t	$P(S_{t+1} \mid S_t)$
t	0.8
f	0.3

2. 先计算反向信息：

$P(e_3 \mid S_3) = <0.02, 0.21>$

$P(e_3 \mid S_2) = \sum_{s3} P(e3|s3)P(s3|S2)$ （注意这里的变量 S_2 是条件）

$= 0.02<0.8, 0.3>+0.21<0.2, 0.7>$

$= <0.058, 0.153>$

$P(e_{2:3} \mid S_1) = \sum_{s2} P(e2|s2)P(e3|s2)P(s2|S1)$ （给定 s2 后 e2,e3 条件独立）

$= 0.18*0.02<0.8, 0.3>+0.49*0.21<0.2, 0.7>$

$= <0.02346, 0.07311>$

再计算过去状态的后验分布：

$P(S_1 \mid e_{1:3}) = \alpha P(S_1 \mid e_1)P(e_{2:3} \mid S_1)$

$= \alpha<0.72, 0.21><0.02346, 0.07311>$

$= \alpha<0.0168912, 0.0153531>0.03224$

$= <0.5239, 0.4761>$

$P(S_2 \mid e_{1:3}) = \alpha P(S_2 \mid e_{1:2})P(e_3 \mid S_1)$

$= \alpha<0.72*0.18, 0.21*0.49><0.02, 0.21>$

$= \alpha<0.002592, 0.021609>$

$= <0.1071, 0.8929>$

$P(S_3 \mid e_{1:3}) = <0.1194, 0.8806>$ （由第一小问计算结果直接获得）

3. t=1, 2 时的过滤分析和平滑分析结果如下：

$P(S_1 \mid e_1) = <0.8643, 0.1357>$　$P(S_2 \mid e_{1:2}) = <0.5749, 0.4251>$

$P(S_1 \mid e_{1:3}) = <0.5239, 0.4761>$　$P(S_2 \mid e_{1:3}) = <0.1071, 0.8929>$

可看出：平滑分析在最后整合了以往所有的观测证据，发现学生意思睡眠不佳的表现越来越严重，并且由于该模型中睡眠不佳更倾向于连续，于是将学生较早时间开始睡眠不佳的概率提高了。

Exercise 15.19

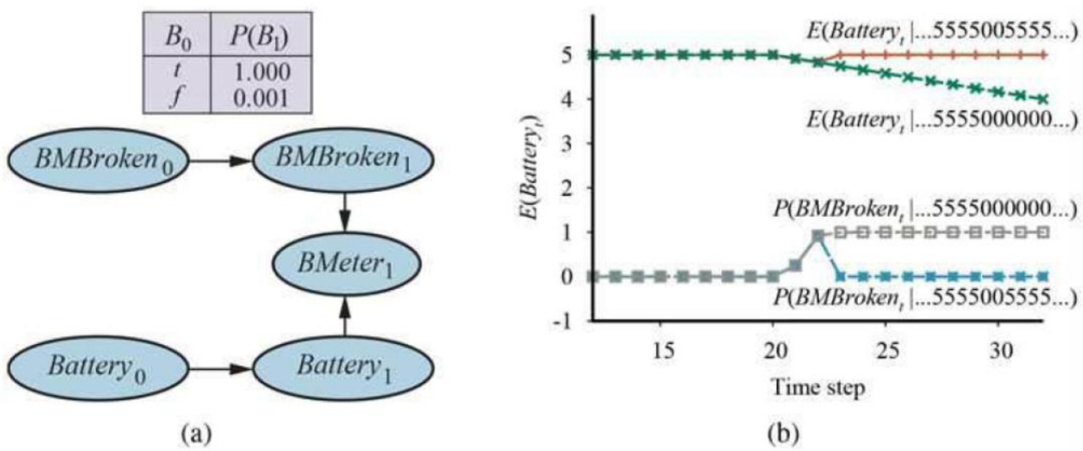
This exercise analyzes in more detail the persistent-failure model for the battery sensor in Figure 14.15(a) (page 489).

a. Figure 14.15(b) stops at $t = 32$. Describe qualitatively what should happen as $t \to \infty$ if the sensor continues to read 0.

b. Suppose that the external temperature affects the battery sensor in such a way that transient failures become more likely as temperature increases. Show how to augment the DBN structure in Figure 14.15(a), and explain any required changes to the CPTs.

c. Given the new network structure, can battery readings be used by the robot to infer the current temperature?

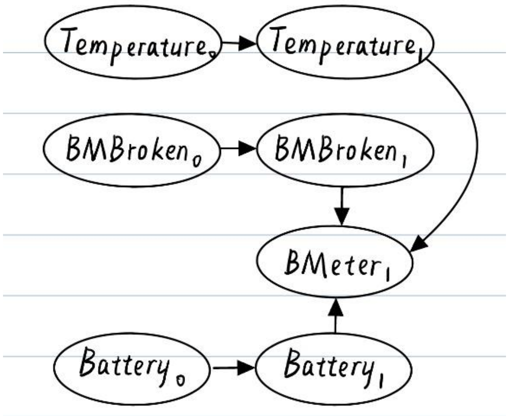
Figure 14.15



a. $t \to \infty$时，电池电量会耗尽，传感器持续读 0 的电池电量期望曲线 $E(Battery_t$ $|...5555000000...)$会趋无限近于 0；

传感器损坏的概率曲线 $P(BMBroken_t$ $|...5555000000...)$ 会无限趋近于 1。

b. 新的 DBN 结构如图：



CPTs 需要的改动：需考虑温度因素，当温度越高时，传感器发生瞬时故障的概率越高

c. 可以。此时以温度为查询变量，传感器读数为证据变量，通过过滤分析实时计算出当前温度的后验概率分布。