Student Information

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Answer 1

a)

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Table 1: The truth table of the given compound proposition.

b)

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$	$((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

Table 2: The truth table for the given formula, where the expression on the last column holds true for any value of x, y, and z, showing us that the formula is a **tautology**.

Answer 2

$$
\begin{aligned}
\neg p \to (q \to r) &\equiv \neg(\neg p) \lor (q \to r) && \textit{Table 7, Line 1}\\
&\equiv p \lor (q \to r) && \textit{Double negation law}\\
&\equiv p \lor (\neg q \lor r) && \textit{Table 7, Line 1}\\
&\equiv (p \lor \neg q) \lor r && \textit{Associative law}\\
&\equiv (\neg q \lor p) \lor r && \textit{Commutative law}\\
&\equiv \neg q \lor (p \lor r) && \textit{Associative law}\\
&\equiv \neg(\neg q) \to (p \lor r) && \textit{Table 7, Line 3}\\
&\equiv q \to (p \lor r) && \textit{Double negation law}
\end{aligned}
$$

Hence; $\neg p \to (q \to r)$ is logically equivalent to $q \to (p \lor r)$.

Answer 3

a) $\forall x L(x, Burak)$

b) $\forall x L(Hazal, x)$

c) $\forall x \exists y L(x, y)$

d) $\neg \exists x \forall y (L(x, y))$

e) $\forall x \exists y L(y, x)$

f) $\forall x (\neg L(x, Mustafa) \land \neg L(x, Burak))$

g) $\exists x \exists y \forall z (L(Ceren, x) \land (L(Ceren, y) \land x \neq y) \land (L(Ceren, z) \to (z = x \lor z = y)))$

h) $\forall x \exists y \forall z (L(x, y) \land (L(x, z) \to y = z))$

i) $\forall x \forall y (L(x, y) \to x \neq y)$

j) $\exists x \exists y \forall z (L(x, x) \land L(x, y) \land x \neq y) \land (L(x, z) \to (z = x \lor z = y))$

Answer 4

Table 3: Proof of $p \to q$, $\neg q \vdash \neg p$. This proof will be refered as *Modus Tollens* or *M.T.* for short.

1	$p \to q$	*premise*
2	$\neg q$	*premise*
3	p	*assumption*
4	q	$\to_e, 1, 3$
5	$\perp$	$\neg_e, 2, 4$
6	$\neg p$	$\neg_i, 3-5$

Table 4: Proving that $p, p \to (r \to q), q \to s \vdash \neg q \to (s \vee \neg r)$

1	p	*premise*
2	$p \to (r \to q)$	*premise*
3	$q \to s$	*premise*
4	$r \to q$	$\to_e, 1, 2$
5	$\neg q$	*assumption*
6	$\neg r$	*M.T.*,4,5
7	$s \vee \neg r$	$\vee_i, 6$
8	$\neg q \to (s \vee \neg r)$	$\to_i, 5-7$

Answer 5

In this question, we are going to use several theorems that will be proven first, like in the previous question.

1	$\neg p \wedge \neg q$	*premise*
2	$\neg p$	$\wedge_e, 1$
3	$\neg q$	$\wedge_e, 1$
4	$p \vee q$	*assumption*
5	p	*assumption*
6	$\bot$	$\neg_e, 2, 5$
7	$p \to \bot$	$\to_i, 5-6$
8	q	*assumption*
9	$\bot$	$\neg_e, 3, 8$
10	$q \to \bot$	$\to_i, 5-6$
11	$\bot$	$\vee_e, 4, 7, 10$
12	$\neg(p \vee q)$	$\neg_i, 4-11$

Table 5: De Morgan's Law: $\neg p \wedge \neg q \vdash \neg(p \vee q)$. Note that we have used $\bot$ to represent the Boolean value *False* in line 7 and 10 to be able to use conjunction elimination in line 11.. This theorem will be refered as $DeM._\vee$.

1	$p \vee q$	*premise*
2	$\neg q$	*premise*
3	$\neg p$	*assumption*
4	$\neg p \wedge \neg q$	$\wedge_i, 2, 3$
5	$\neg(p \vee q)$	$DeM._\vee, 4$
6	$\bot$	$\neg_e, 1, 5$
7	$\neg\neg p$	$\neg_i, 3-6$
8	p	$\neg\neg_e, 7$

Table 6: Lemma: $p \vee q, \neg q \vdash p$. This lemma will be refered as Koluacik's rule.

1	$\neg\exists x p(x)$	
2	x_0	*constant*
3	$p(x_0)$	*assumption*
4	$\exists x p(x)$	$\exists_i, 3$
5	$\bot$	$\neg_e, 1, 4$
6	$\neg p(x_0)$	$\neg_i, 3-5$
7	$\forall x \neg p(x)$	$\forall_i, 2-7$

Table 7: De Morgan's Law for quantifiers. This law will be refered as $DeM._\exists$.

Table 8: Proof of $\forall x(p(x) \to q(x)), \neg\exists z r(z), \exists y p(y) \vee r(a) \vdash \exists z q(z)$.

1		$\forall x(p(x) \to q(x))$	*premise*
2		$\neg\exists z r(z)$	*premise*
3		$\exists y p(y) \vee r(a)$	*premise*
4		$\forall z \neg r(z)$	$DeM._{\exists z}, 2$
5		$\neg r(a)$	$\forall_{e, z \leftarrow a}, 4$
6		$\exists y p(y)$	*Koluacik's rule*,$3, 5$
7	b	$p(b)$	*assumption*
8		$p(b) \to q(b)$	$\forall_{e, x \leftarrow b}, 1$
9		$q(b)$	$\to_e, 7, 8$
10		$\exists z q(z)$	$\exists_i, 9$
11		$\exists z q(z)$	$\exists_e, 6, 7-10$