Student Information

Full Name : Deniz Koluaçık
Id Number : 2310274

Answer 1

$x^{p-1} \equiv 1 \pmod p$ By using Fermat's Little Theorem *(see p. 281, theorem 3)*. Then $y \leq (p-1)$. By *The Division Theorem II (see p. 239)* There exists a unique pair of (q, r), where $0 \leq r < y$, such that $(p-1) = yq + r$.

We can rewrite the given congruence as the following:

$$\begin{aligned} 1 \equiv a^{yq+r} &\equiv (a^y)^q a^r && \pmod p \\ &\equiv 1^q a^r && \pmod p \\ &\equiv a^r && \pmod p \end{aligned}$$

We have found that a^r must be congruent to 1 modulo p. If a were greater than 0. Then since y is the smallest positive integer such that $a^y \equiv 1 \pmod p$, $r \geq y$ would have to be true, which is a contradiction $(0 \leq r < y)$. Then $r = 0$.

Take another look at the equation $(p-1) = yq + r$. $r = 0$, then $r \mid (p-1)$.

Answer 2

For some $a \in \mathbb{Z}^+$

$$\begin{aligned} 2(a+169)^2 + 10(a+169) - 7 &\equiv 2a^2 + 4(169a) + 169^2 + 10a + 10(169) - 7 \pmod{169} \\ &\equiv 2a^2 + 10a - 7 \pmod{169} \end{aligned}$$

Appearently for $n = a$, and $n = a + 169$, the remainder when $(2n^2 + 10n - 7)$ is divided by 169 is same. Therefore, if for $n = a$, $169 \nmid (2n^2 + 10n - 7)$, then for $n = a + 169$, $169 \nmid (2n^2 + 10n - 7)$, as well. Based on this logic, if we can show that for all $n = a$ such that $1 \leq a \leq 169$, $169 \nmid (2a^2 + 10n - 7)$, then we can safely say that $\forall n (169 \nmid (2n^2 + 10n - 7))$.

In the following few pages there is a complete table consisting of lines showing that for each value of n in the aforementioned domain, $169 \nmid (2n^2 + 10n - 7)$ is true, one by one.

for $n = 1$	$(2n^2 + 10n - 7) = 5$	Since $5 \equiv 5 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 2$	$(2n^2 + 10n - 7) = 21$	Since $21 \equiv 21 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 3$	$(2n^2 + 10n - 7) = 41$	Since $41 \equiv 41 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 4$	$(2n^2 + 10n - 7) = 65$	Since $65 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 5$	$(2n^2 + 10n - 7) = 93$	Since $93 \equiv 93 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 6$	$(2n^2 + 10n - 7) = 125$	Since $125 \equiv 125 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 7$	$(2n^2 + 10n - 7) = 161$	Since $161 \equiv 161 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 8$	$(2n^2 + 10n - 7) = 201$	Since $201 \equiv 32 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 9$	$(2n^2 + 10n - 7) = 245$	Since $245 \equiv 76 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 10$	$(2n^2 + 10n - 7) = 293$	Since $293 \equiv 124 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 11$	$(2n^2 + 10n - 7) = 345$	Since $345 \equiv 7 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 12$	$(2n^2 + 10n - 7) = 401$	Since $401 \equiv 63 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 13$	$(2n^2 + 10n - 7) = 461$	Since $461 \equiv 123 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 14$	$(2n^2 + 10n - 7) = 525$	Since $525 \equiv 18 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 15$	$(2n^2 + 10n - 7) = 593$	Since $593 \equiv 86 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 16$	$(2n^2 + 10n - 7) = 665$	Since $665 \equiv 158 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 17$	$(2n^2 + 10n - 7) = 741$	Since $741 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 18$	$(2n^2 + 10n - 7) = 821$	Since $821 \equiv 145 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 19$	$(2n^2 + 10n - 7) = 905$	Since $905 \equiv 60 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 20$	$(2n^2 + 10n - 7) = 993$	Since $993 \equiv 148 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 21$	$(2n^2 + 10n - 7) = 1085$	Since $1085 \equiv 71 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 22$	$(2n^2 + 10n - 7) = 1181$	Since $1181 \equiv 167 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 23$	$(2n^2 + 10n - 7) = 1281$	Since $1281 \equiv 98 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 24$	$(2n^2 + 10n - 7) = 1385$	Since $1385 \equiv 33 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 25$	$(2n^2 + 10n - 7) = 1493$	Since $1493 \equiv 141 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 26$	$(2n^2 + 10n - 7) = 1605$	Since $1605 \equiv 84 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 27$	$(2n^2 + 10n - 7) = 1721$	Since $1721 \equiv 31 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 28$	$(2n^2 + 10n - 7) = 1841$	Since $1841 \equiv 151 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 29$	$(2n^2 + 10n - 7) = 1965$	Since $1965 \equiv 106 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 30$	$(2n^2 + 10n - 7) = 2093$	Since $2093 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 31$	$(2n^2 + 10n - 7) = 2225$	Since $2225 \equiv 28 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 32$	$(2n^2 + 10n - 7) = 2361$	Since $2361 \equiv 164 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 33$	$(2n^2 + 10n - 7) = 2501$	Since $2501 \equiv 135 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 34$	$(2n^2 + 10n - 7) = 2645$	Since $2645 \equiv 110 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 35$	$(2n^2 + 10n - 7) = 2793$	Since $2793 \equiv 89 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$

for $n = 36$	$(2n^2 + 10n - 7) = 2945$	Since $2945 \equiv 72 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 37$	$(2n^2 + 10n - 7) = 3101$	Since $3101 \equiv 59 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 38$	$(2n^2 + 10n - 7) = 3261$	Since $3261 \equiv 50 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 39$	$(2n^2 + 10n - 7) = 3425$	Since $3425 \equiv 45 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 40$	$(2n^2 + 10n - 7) = 3593$	Since $3593 \equiv 44 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 41$	$(2n^2 + 10n - 7) = 3765$	Since $3765 \equiv 47 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 42$	$(2n^2 + 10n - 7) = 3941$	Since $3941 \equiv 54 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 43$	$(2n^2 + 10n - 7) = 4121$	Since $4121 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 44$	$(2n^2 + 10n - 7) = 4305$	Since $4305 \equiv 80 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 45$	$(2n^2 + 10n - 7) = 4493$	Since $4493 \equiv 99 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 46$	$(2n^2 + 10n - 7) = 4685$	Since $4685 \equiv 122 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 47$	$(2n^2 + 10n - 7) = 4881$	Since $4881 \equiv 149 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 48$	$(2n^2 + 10n - 7) = 5081$	Since $5081 \equiv 11 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 49$	$(2n^2 + 10n - 7) = 5285$	Since $5285 \equiv 46 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 50$	$(2n^2 + 10n - 7) = 5493$	Since $5493 \equiv 85 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 51$	$(2n^2 + 10n - 7) = 5705$	Since $5705 \equiv 128 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 52$	$(2n^2 + 10n - 7) = 5921$	Since $5921 \equiv 6 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 53$	$(2n^2 + 10n - 7) = 6141$	Since $6141 \equiv 57 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 54$	$(2n^2 + 10n - 7) = 6365$	Since $6365 \equiv 112 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 55$	$(2n^2 + 10n - 7) = 6593$	Since $6593 \equiv 2 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 56$	$(2n^2 + 10n - 7) = 6825$	Since $6825 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 57$	$(2n^2 + 10n - 7) = 7061$	Since $7061 \equiv 132 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 58$	$(2n^2 + 10n - 7) = 7301$	Since $7301 \equiv 34 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 59$	$(2n^2 + 10n - 7) = 7545$	Since $7545 \equiv 109 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 60$	$(2n^2 + 10n - 7) = 7793$	Since $7793 \equiv 19 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 61$	$(2n^2 + 10n - 7) = 8045$	Since $8045 \equiv 102 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 62$	$(2n^2 + 10n - 7) = 8301$	Since $8301 \equiv 20 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 63$	$(2n^2 + 10n - 7) = 8561$	Since $8561 \equiv 111 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 64$	$(2n^2 + 10n - 7) = 8825$	Since $8825 \equiv 37 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 65$	$(2n^2 + 10n - 7) = 9093$	Since $9093 \equiv 136 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 66$	$(2n^2 + 10n - 7) = 9365$	Since $9365 \equiv 70 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 67$	$(2n^2 + 10n - 7) = 9641$	Since $9641 \equiv 8 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 68$	$(2n^2 + 10n - 7) = 9921$	Since $9921 \equiv 119 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 69$	$(2n^2 + 10n - 7) = 10205$	Since $10205 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 70$	$(2n^2 + 10n - 7) = 10493$	Since $10493 \equiv 15 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 71$	$(2n^2 + 10n - 7) = 10785$	Since $10785 \equiv 138 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$

for $n = 72$	$(2n^2 + 10n - 7) = 11081$	Since $11081 \equiv 96 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 73$	$(2n^2 + 10n - 7) = 11381$	Since $11381 \equiv 58 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 74$	$(2n^2 + 10n - 7) = 11685$	Since $11685 \equiv 24 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 75$	$(2n^2 + 10n - 7) = 11993$	Since $11993 \equiv 163 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 76$	$(2n^2 + 10n - 7) = 12305$	Since $12305 \equiv 137 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 77$	$(2n^2 + 10n - 7) = 12621$	Since $12621 \equiv 115 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 78$	$(2n^2 + 10n - 7) = 12941$	Since $12941 \equiv 97 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 79$	$(2n^2 + 10n - 7) = 13265$	Since $13265 \equiv 83 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 80$	$(2n^2 + 10n - 7) = 13593$	Since $13593 \equiv 73 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 81$	$(2n^2 + 10n - 7) = 13925$	Since $13925 \equiv 67 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 82$	$(2n^2 + 10n - 7) = 14261$	Since $14261 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 83$	$(2n^2 + 10n - 7) = 14601$	Since $14601 \equiv 67 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 84$	$(2n^2 + 10n - 7) = 14945$	Since $14945 \equiv 73 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 85$	$(2n^2 + 10n - 7) = 15293$	Since $15293 \equiv 83 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 86$	$(2n^2 + 10n - 7) = 15645$	Since $15645 \equiv 97 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 87$	$(2n^2 + 10n - 7) = 16001$	Since $16001 \equiv 115 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 88$	$(2n^2 + 10n - 7) = 16361$	Since $16361 \equiv 137 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 89$	$(2n^2 + 10n - 7) = 16725$	Since $16725 \equiv 163 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 90$	$(2n^2 + 10n - 7) = 17093$	Since $17093 \equiv 24 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 91$	$(2n^2 + 10n - 7) = 17465$	Since $17465 \equiv 58 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 92$	$(2n^2 + 10n - 7) = 17841$	Since $17841 \equiv 96 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 93$	$(2n^2 + 10n - 7) = 18221$	Since $18221 \equiv 138 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 94$	$(2n^2 + 10n - 7) = 18605$	Since $18605 \equiv 15 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 95$	$(2n^2 + 10n - 7) = 18993$	Since $18993 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 96$	$(2n^2 + 10n - 7) = 19385$	Since $19385 \equiv 119 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 97$	$(2n^2 + 10n - 7) = 19781$	Since $19781 \equiv 8 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 98$	$(2n^2 + 10n - 7) = 20181$	Since $20181 \equiv 70 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 99$	$(2n^2 + 10n - 7) = 20585$	Since $20585 \equiv 136 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 100$	$(2n^2 + 10n - 7) = 20993$	Since $20993 \equiv 37 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 101$	$(2n^2 + 10n - 7) = 21405$	Since $21405 \equiv 111 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 102$	$(2n^2 + 10n - 7) = 21821$	Since $21821 \equiv 20 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 103$	$(2n^2 + 10n - 7) = 22241$	Since $22241 \equiv 102 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 104$	$(2n^2 + 10n - 7) = 22665$	Since $22665 \equiv 19 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 105$	$(2n^2 + 10n - 7) = 23093$	Since $23093 \equiv 109 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 106$	$(2n^2 + 10n - 7) = 23525$	Since $23525 \equiv 34 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 107$	$(2n^2 + 10n - 7) = 23961$	Since $23961 \equiv 132 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$

for $n = 108$ $(2n^2 + 10n - 7) = 24401$ Since $24401 \equiv 65 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 109$ $(2n^2 + 10n - 7) = 24845$ Since $24845 \equiv 2 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 110$ $(2n^2 + 10n - 7) = 25293$ Since $25293 \equiv 112 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 111$ $(2n^2 + 10n - 7) = 25745$ Since $25745 \equiv 57 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 112$ $(2n^2 + 10n - 7) = 26201$ Since $26201 \equiv 6 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 113$ $(2n^2 + 10n - 7) = 26661$ Since $26661 \equiv 128 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 114$ $(2n^2 + 10n - 7) = 27125$ Since $27125 \equiv 85 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 115$ $(2n^2 + 10n - 7) = 27593$ Since $27593 \equiv 46 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 116$ $(2n^2 + 10n - 7) = 28065$ Since $28065 \equiv 11 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 117$ $(2n^2 + 10n - 7) = 28541$ Since $28541 \equiv 149 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 118$ $(2n^2 + 10n - 7) = 29021$ Since $29021 \equiv 122 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 119$ $(2n^2 + 10n - 7) = 29505$ Since $29505 \equiv 99 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 120$ $(2n^2 + 10n - 7) = 29993$ Since $29993 \equiv 80 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 121$ $(2n^2 + 10n - 7) = 30485$ Since $30485 \equiv 65 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 122$ $(2n^2 + 10n - 7) = 30981$ Since $30981 \equiv 54 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 123$ $(2n^2 + 10n - 7) = 31481$ Since $31481 \equiv 47 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 124$ $(2n^2 + 10n - 7) = 31985$ Since $31985 \equiv 44 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 125$ $(2n^2 + 10n - 7) = 32493$ Since $32493 \equiv 45 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 126$ $(2n^2 + 10n - 7) = 33005$ Since $33005 \equiv 50 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 127$ $(2n^2 + 10n - 7) = 33521$ Since $33521 \equiv 59 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 128$ $(2n^2 + 10n - 7) = 34041$ Since $34041 \equiv 72 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 129$ $(2n^2 + 10n - 7) = 34565$ Since $34565 \equiv 89 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 130$ $(2n^2 + 10n - 7) = 35093$ Since $35093 \equiv 110 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 131$ $(2n^2 + 10n - 7) = 35625$ Since $35625 \equiv 135 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 132$ $(2n^2 + 10n - 7) = 36161$ Since $36161 \equiv 164 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 133$ $(2n^2 + 10n - 7) = 36701$ Since $36701 \equiv 28 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 134$ $(2n^2 + 10n - 7) = 37245$ Since $37245 \equiv 65 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 135$ $(2n^2 + 10n - 7) = 37793$ Since $37793 \equiv 106 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 136$ $(2n^2 + 10n - 7) = 38345$ Since $38345 \equiv 151 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 137$ $(2n^2 + 10n - 7) = 38901$ Since $38901 \equiv 31 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 138$ $(2n^2 + 10n - 7) = 39461$ Since $39461 \equiv 84 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 139$ $(2n^2 + 10n - 7) = 40025$ Since $40025 \equiv 141 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 140$ $(2n^2 + 10n - 7) = 40593$ Since $40593 \equiv 33 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 141$ $(2n^2 + 10n - 7) = 41165$ Since $41165 \equiv 98 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 142$ $(2n^2 + 10n - 7) = 41741$ Since $41741 \equiv 167 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$
for $n = 143$ $(2n^2 + 10n - 7) = 42321$ Since $42321 \equiv 71 \pmod{169}$ $169 \nmid (2n^2 + 10n - 7)$

for $n = 144$	$(2n^2 + 10n - 7) = 42905$	Since $42905 \equiv 148 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 145$	$(2n^2 + 10n - 7) = 43493$	Since $43493 \equiv 60 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 146$	$(2n^2 + 10n - 7) = 44085$	Since $44085 \equiv 145 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 147$	$(2n^2 + 10n - 7) = 44681$	Since $44681 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 148$	$(2n^2 + 10n - 7) = 45281$	Since $45281 \equiv 158 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 149$	$(2n^2 + 10n - 7) = 45885$	Since $45885 \equiv 86 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 150$	$(2n^2 + 10n - 7) = 46493$	Since $46493 \equiv 18 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 151$	$(2n^2 + 10n - 7) = 47105$	Since $47105 \equiv 123 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 152$	$(2n^2 + 10n - 7) = 47721$	Since $47721 \equiv 63 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 153$	$(2n^2 + 10n - 7) = 48341$	Since $48341 \equiv 7 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 154$	$(2n^2 + 10n - 7) = 48965$	Since $48965 \equiv 124 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 155$	$(2n^2 + 10n - 7) = 49593$	Since $49593 \equiv 76 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 156$	$(2n^2 + 10n - 7) = 50225$	Since $50225 \equiv 32 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 157$	$(2n^2 + 10n - 7) = 50861$	Since $50861 \equiv 161 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 158$	$(2n^2 + 10n - 7) = 51501$	Since $51501 \equiv 125 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 159$	$(2n^2 + 10n - 7) = 52145$	Since $52145 \equiv 93 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 160$	$(2n^2 + 10n - 7) = 52793$	Since $52793 \equiv 65 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 161$	$(2n^2 + 10n - 7) = 53445$	Since $53445 \equiv 41 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 162$	$(2n^2 + 10n - 7) = 54101$	Since $54101 \equiv 21 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 163$	$(2n^2 + 10n - 7) = 54761$	Since $54761 \equiv 5 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 164$	$(2n^2 + 10n - 7) = 55425$	Since $55425 \equiv 162 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 165$	$(2n^2 + 10n - 7) = 56093$	Since $56093 \equiv 154 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 166$	$(2n^2 + 10n - 7) = 56765$	Since $56765 \equiv 150 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 167$	$(2n^2 + 10n - 7) = 57441$	Since $57441 \equiv 150 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 168$	$(2n^2 + 10n - 7) = 58121$	Since $58121 \equiv 154 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$
for $n = 169$	$(2n^2 + 10n - 7) = 58805$	Since $58805 \equiv 162 \pmod{169}$	$169 \nmid (2n^2 + 10n - 7)$

We have shown that for all values of n from 1 to 169, $169 \nmid (2n^2 + 10n - 7)$. We can conclude by stating that $160 \nmid (2n^2 + 10n - 7), \forall n \in \mathbb{Z}^+$.

Answer 3

$$(a - b) \equiv 0 \pmod{n} \tag{1}$$
$$(a - b) \equiv 0 \pmod{m} \tag{2}$$

Let the prime factorization of n and m be $\{n_1 \times n_2 \times \ldots \times n_i\}$ and $\{m_1 \times m_2 \times \ldots \times m_i\}$, respectively, where for all x and y, $n_x \neq m_y$, since $\gcd(n, m) = 1$. By (1), and (2), we can say that $n \mid (a - b)$, and $m \mid (a - b)$. Therefore, the prime factorization of $(a - b)$ includes all the elements from the prime factorizations of both m and n. Hence, $(m \times n) \mid (a - b)$, and finally,

$$(a - b) \equiv 0 \pmod{m \times n}$$
$$a \equiv b \pmod{m \times n}$$

Answer 4

We will use induction for n:

1. Base case ($n = 1$):

$$\sum_{j=1}^{1} j(j+1)\ldots(j+k-1) = 1 \times 2 \times \ldots \times k = \frac{1 \times 2 \times \ldots \times k \times (k+1)}{k+1}$$

2. Inductive Hypothesis:

$$\sum_{j=1}^{n} j(j+1)\ldots(j+k-1) = \frac{n(n+1)\ldots(n+k)}{k+1}$$

3. Show that the following is true:

$$\sum_{j=1}^{n+1} j(j+1)\ldots(j+k-1) = \frac{(n+1)(n+2)\ldots(n+k+1)}{k+1}$$

$$\begin{aligned}
\sum_{j=1}^{n+1} j(j+1)\ldots(j+k-1) &= \sum_{j=1}^{n} j(j+1)\ldots(j+k-1) + (n+1)(n+2)\ldots(n+k) \\
&= \frac{n(n+1)\ldots(n+k)}{k+1} + (n+1)(n+2)\ldots(n+k) \\
&= \left(\frac{n}{k+1} + 1\right)(n+1)(n+2)\ldots(n+k) \\
&= \frac{(n+1)(n+2)\ldots(n+k)(n+k+1)}{k+1}
\end{aligned}$$

Hence, by mathematical induction

$$\sum_{j=1}^{n} j(j+1)\ldots(j+k-1) = \frac{n(n+1)\ldots(n+k)}{k+1}$$

Answer 5

1. Show that $H_n \leq 7^n$ for $n = 0, n = 1, n = 2, n = 3$.
   - $H_0 = 1 \leq 7^0$
   - $H_1 = 3 \leq 7^1$
   - $H_2 = 5 \leq 7^2$
   - $H_3 = 103 \leq 7^3$
2. We construct our inductive hypothesis, assume that $H_i \leq 7^i$ is true for $3 \leq i \leq n$.
3. Now, we need to show that $H_{n+1} \leq 7^{n+1}$ where $H_{n+1} = 5H_{n-1} + 5H_{n-2} + 63H_{n-3}$.

$$\begin{aligned} H_{n+1} &= 5H_{n-1} + 5H_{n-2} + 63H_{n-3} \\ &\leq 5 \times 7^n + \frac{5}{7} \times 7^n + \frac{9}{7} \times 7^n \\ &= 7 \times 7^n = 7^{n+1} \end{aligned}$$

We have shown that $H_{n+1} \leq 7^{n+1}$. Hence, by strong induction, $H_n \leq 7^n$ for all $n \geq 0$.