Linear Algebraic System

define $\mathcal{L}(\circ, \lambda(\xi)) = R(\circ) + \lambda(\xi) F(\circ)$

where $\circ = \xi, u(\xi)$

$\underset{\lambda(\xi)}{\text{solve}}$ $\dfrac{\partial \mathcal{L}(\circ, \lambda(\xi))}{\partial u} = 0$

evaluate $\dfrac{dF(\circ, \lambda(\xi))}{d\xi} = \dfrac{\partial F(\circ)}{\partial \xi} + \lambda(\xi) \dfrac{\partial R(\circ)}{\partial \xi}$

Linear ODE System

define $\mathcal{L}(\circ, \lambda(t, \xi)) = R(\circ) + \lambda(t, \xi) F(\circ)$

where $\circ = t, \xi, u(t, \xi), \dot{u}(t, \xi), \ddot{u}(t, \xi)$

$\underset{\lambda(t,\xi)}{\text{solve}}$ $\dfrac{\partial \mathcal{L}(\circ, \lambda(t, \xi))}{\partial u} - \dfrac{d}{dt}\left(\dfrac{\partial \mathcal{L}(\circ, \lambda(t, \xi))}{\partial \dot{u}}\right) + \dfrac{d^2}{dt^2}\left(\dfrac{\partial \mathcal{L}(\circ, \lambda(t, \xi))}{\partial \ddot{u}}\right) = 0$

evaluate $\dfrac{dF(\circ, \lambda(t, \xi))}{d\xi} = \dfrac{\partial F(\circ)}{\partial \xi} + \lambda(t, \xi) \dfrac{\partial R(\circ)}{\partial \xi}$

Linear Stochastic ODE System

define $\mathcal{L}(\circ, \lambda(t, y(\xi))) = R(\circ) + \lambda(t, y(\xi)) F(\circ)$

where $\circ = t, y(\xi), u(t, y(\xi)), \dot{u}(t, y(\xi)), \ddot{u}(t, y(\xi))$

$\underset{\lambda(t,y(\xi))}{\text{solve}}$ $\dfrac{\partial \mathcal{L}(\circ, \lambda(t, y(\xi)))}{\partial u} - \dfrac{d}{dt}\left(\dfrac{\partial \mathcal{L}(\circ, \lambda(t, y(\xi)))}{\partial \dot{u}}\right) + \dfrac{d^2}{dt^2}\left(\dfrac{\partial \mathcal{L}(\circ, \lambda(t, y(\xi)))}{\partial \ddot{u}}\right) = 0$

evaluate $\dfrac{d\mathbb{E}\left[F(\circ, \lambda(t, y(\xi)))\right]}{d\xi}, \dfrac{d\mathbb{V}\left[F(\circ, \lambda(t, y(\xi)))\right]}{d\xi}, \dfrac{d\mathbb{S}\left[F(\circ, \lambda(t, y(\xi)))\right]}{d\xi}$