Homework 1 for Math 6644 : Iterative Methods for Systems of Equations

(Due on Jan. 25)

Please write your answers to each question neatly.

(1) Find an orthonormal basis for the column space of matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

(2) Consider a matrix

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 6 & 0 \\ -2 & 0 & 7 \end{bmatrix}.$$

Please answer the following questions without computer help.

(a) Is this matrix diagonalizable? Why or why not?

(b) Compute the 1-norm, 2-norm, Frobenius norm and infinite-norm.

(c) Compute the spectral radius of A. Compare and comments on the your values from (b) and (c).

(3) Construct a 3×3 dense matrix G with $\rho(G) > 1$. Define an iteration,

$$\vec{x}_{k+1} = G\vec{x}_k + \vec{c}.$$

Find an example for $\vec{x}_0$ and $\vec{c}$ such that the iteration does not converge.

(4) Give the matrix expression for the symmetric Gauss-Seidel iterations.

(5) Discreteize the following differential equation

$$\begin{cases} -u'' + 4u = 0 & x \in [0, 1] \\ u(0) = -1, \quad u(1) = 2 \end{cases}$$

by the central difference scheme. Write your linear system of equations (you must give the matrix A and $\vec{b}$ explicitly). Solve the system by using classical iteration such as Jacobi, Gauss-Seidel and SOR with $n = 1000$. Test you relaxation parameter in SOR for several values and decide which one is better. You need to discuss your results.