Robust Optimizations of Structural and Aerodynamic Designs

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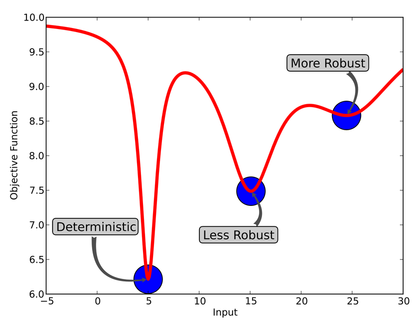
Why Uncertainty Quantification?

- Design variables and input parameters are always subject to random variations
  - → Uncertain operating conditions (weather, ice accumulation on wing), boundary conditions
  - → Uncertainties from lack of knowledge about a quantity (manufacturing tolerances)

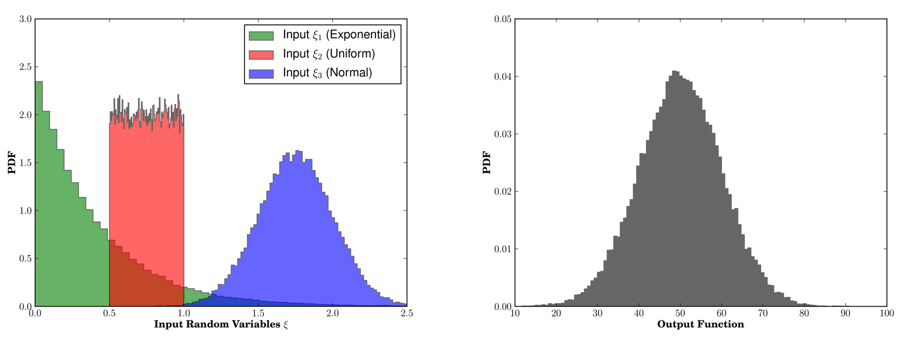


- Allowances must be made to accommodate likely variations/uncertainties
  - → Determine the real effects of uncertainties on the design (robust or vulnerable)
  - → Obtain confidence intervals for results (range of possible outcomes)
    - ∗ e.g. 95% probability (confidence) that the target C_L is achieved
    - ∗ e.g. 1% probability of violation of constraint
  - → Identify the limitations of the design (and improve)
  - → Reliability analysis for certification and quality assurance purposes

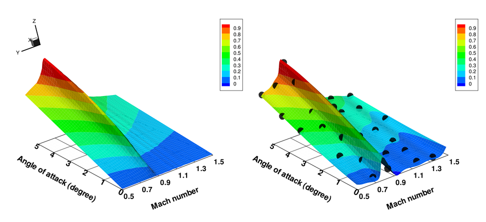
Types of Uncertainties

Aleatory / Irreducible / Type A

- Inherent randomness or variations:
  - → input parameters (Young's modulus, shear force)
  - → design variables
  - → operating environment (cruise settings, temperature)
- Input probability distributions are known (sometimes assumed)
- Goal is to determine the output distribution



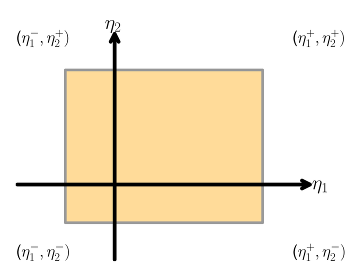
- Monte Carlo sampling
- Tens of thousands of function evaluations
- Surrogate models to approximate the simulation output (kriging, polynomial chaos)



Epistemic / Reducible / Type B

- Lack of knowledge about the appropriate value
- Bounds can be specified $I(\eta) = [\eta^-, \eta^+] = [\bar{\eta} - \tau, \bar{\eta} + \tau]$
- Goal: determine the worst and best scenarios within the interval $I(\eta)$

⋆ Extensive sampling
  - → $10^3 - 10^6$ evaluations
  - → Prohibitively expensive



⋆ Bound Constrained Optimization

$$\begin{aligned} \underset{\beta}{\text{minimize/maximize}} \quad & f = f(\eta), \\ \text{subject to} \quad & \beta \in I(\eta) = [\bar{\eta} - \tau, \bar{\eta} + \tau]. \end{aligned}$$

→ Attractive even for bigger problems (scales linearly)

Optimization Problem

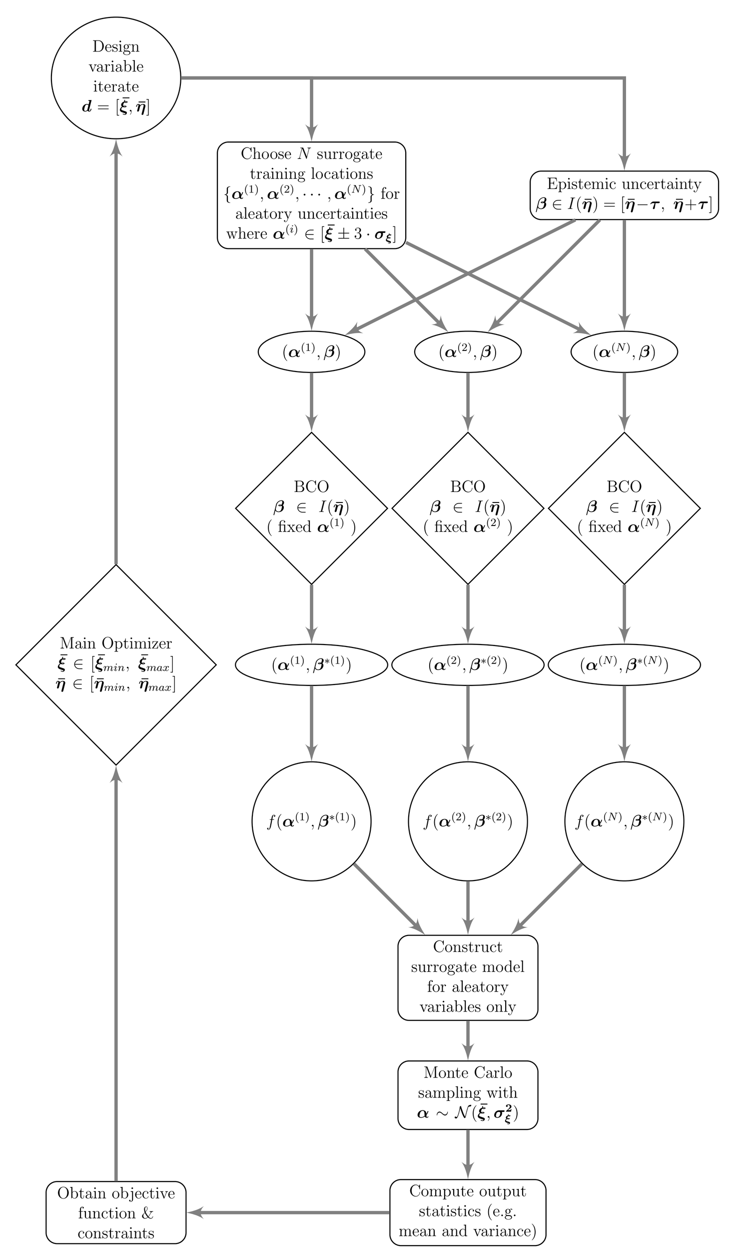
Determinisic Optimization

$$\begin{aligned} \underset{\mathbf{d}}{\text{minimize}} \quad & J = J(f, \mathbf{q}, \mathbf{d}), \\ \text{subject to} \quad & R(\mathbf{q}, \mathbf{d}) = \mathbf{0}, \\ & g(f, \mathbf{q}, \mathbf{d}) \leq \mathbf{0}. \end{aligned}$$

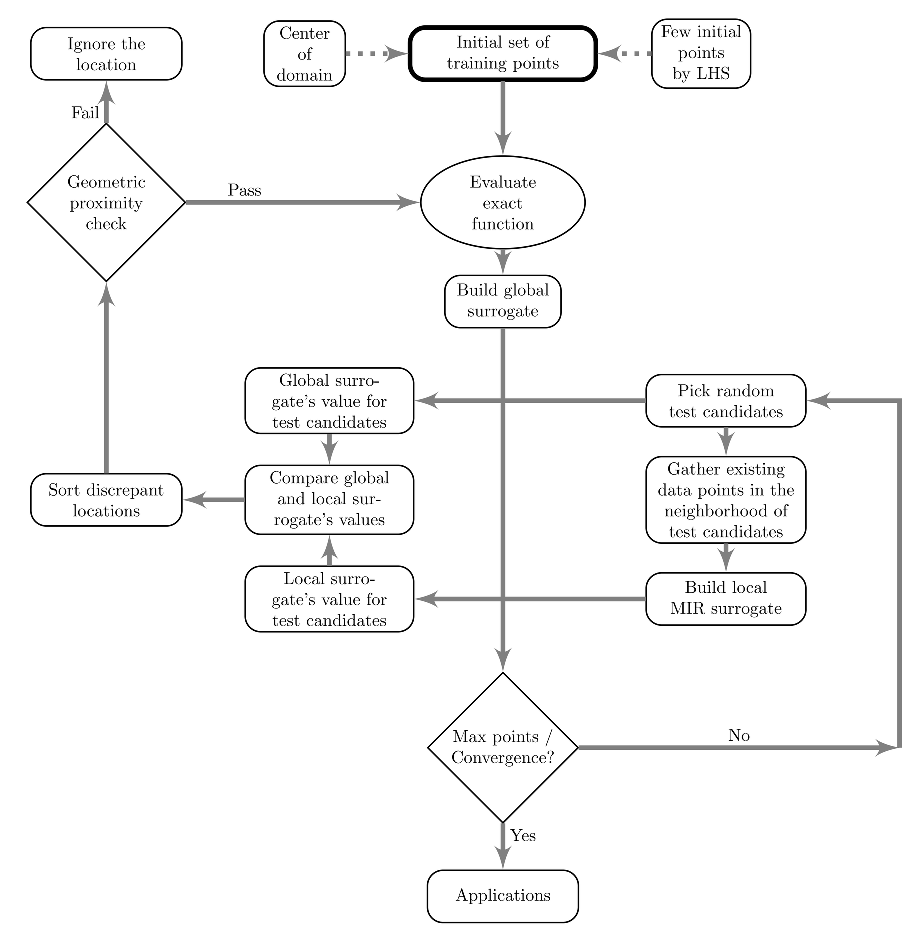
Robust Optimization

$$\begin{aligned} \underset{\xi, \eta}{\text{minimize}} \quad & \mathcal{J} = \mathcal{J}(\mu_{f*}, \sigma^2_{f*}, q, \xi, \eta), \\ \text{subject to} \quad & R(q, \xi, \eta) = 0, \\ & g^r = g(\mu_{f*}, q, \xi, \eta) + k\sigma_{f*} \leq 0. \end{aligned}$$

Mixed OUU Framework

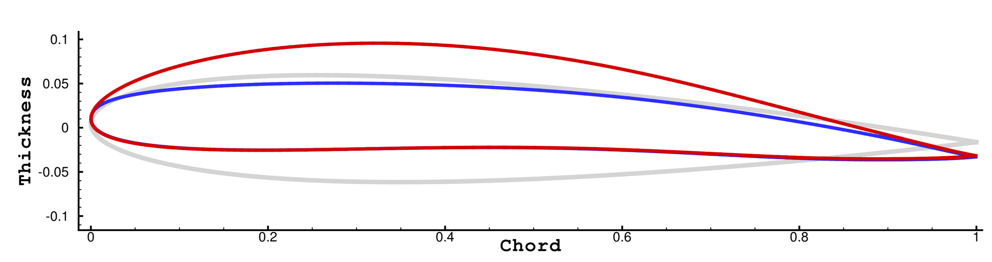


Dynamic Training Framework



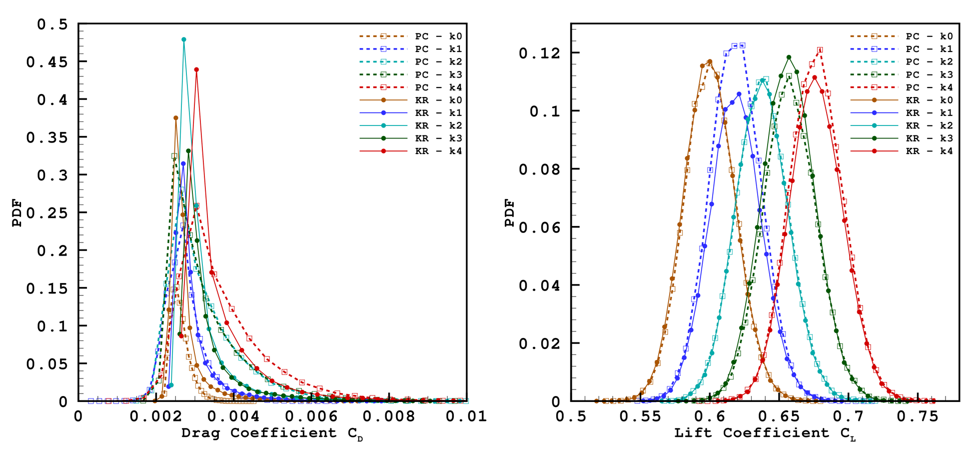
Airfoil Optimization

Random Variable	Description	Uncertainty Type	τ_{min}	τ_{max}	Standard Deviation
$\eta_{1,2,13,14}$	Shape design variables	Epistemic	-0.00125	0.00125	-
η_{3-12}	Shape design variables	Epistemic	-0.01	0.01	-
ξ_α	Angle of attack	Aleatory	-	-	0.1°
ξ_M	Mach number	Aleatory	-	-	0.01

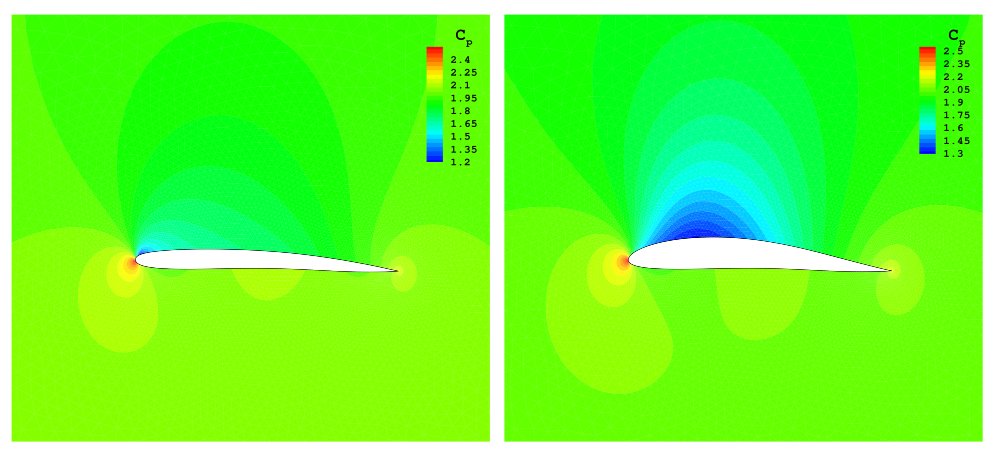


NACA 0012, Deterministic, Robust Airfoils ($k = 4$)

Type	k	P_k	$\mu_{c_{dmax}}$	$\sigma^2_{c_{dmax}}$	$\mu_{c_{l_{min}}}$	$\sigma_{c_{l_{min}}}$	α	M	No. of F/FG Evals. & Iterations
Initial	-	-	$4.72 \cdot 10^{-4}$	-	0.335	-	2.000°	0.650	
Deterministic	-	-	$1.17 \cdot 10^{-3}$	-	0.600	-	2.510°	0.600	49/49 – 24
Robust-KR	0	0.5000	$2.72 \cdot 10^{-3}$	$2.03 \cdot 10^{-7}$	0.600	$1.84 \cdot 10^{-2}$	2.013°	0.600	844/844-23
Robust-PC	0	0.5000	$2.62 \cdot 10^{-3}$	$5.80 \cdot 10^{-8}$	0.600	$1.82 \cdot 10^{-2}$	2.389°	0.600	675/6751-16
Robust-KR	1	0.8413	$2.93 \cdot 10^{-3}$	$3.07 \cdot 10^{-7}$	0.619	$1.86 \cdot 10^{-2}$	2.065°	0.600	434/434-13
Robust-PC	1	0.8413	$2.73 \cdot 10^{-3}$	$2.50 \cdot 10^{-7}$	0.618	$1.84 \cdot 10^{-2}$	3.058°	0.600	434/434-15
Robust-KR	2	0.9772	$3.10 \cdot 10^{-3}$	$4.46 \cdot 10^{-7}$	0.637	$1.88 \cdot 10^{-2}$	2.179°	0.600	831/831-19
Robust-PC	2	0.9772	$3.20 \cdot 10^{-3}$	$8.58 \cdot 10^{-7}$	0.637	$1.89 \cdot 10^{-2}$	2.193°	0.600	710/710-22
Robust-KR	3	0.9986	$3.28 \cdot 10^{-3}$	$6.23 \cdot 10^{-7}$	0.657	$1.90 \cdot 10^{-2}$	2.301°	0.600	650/650-21
Robust-PC	3	0.9986	$3.25 \cdot 10^{-3}$	$9.83 \cdot 10^{-7}$	0.658	$1.92 \cdot 10^{-2}$	2.352°	0.600	1145/1145-21
Robust-KR	4	0.9999	$3.56 \cdot 10^{-3}$	$9.50 \cdot 10^{-7}$	0.677	$1.93 \cdot 10^{-2}$	2.421°	0.600	620/620-15
Robust-PC	4	0.9999	$3.65 \cdot 10^{-3}$	$1.25 \cdot 10^{-6}$	0.677	$1.93 \cdot 10^{-2}$	2.427°	0.600	2104/2104-36



PDF of drag and lift coefficients at the optimum design

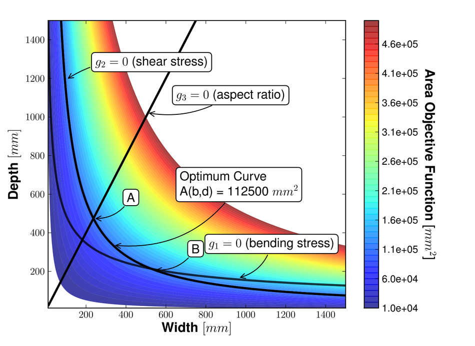


Pressure coefficient contours at different optimum designs (deterministic and robust).

Type	$\mu_{c_{d_{max}}}$	$\sigma^2_{c_{d_{max}}}$	$\mu_{c_{l_{min}}}$	$\sigma_{c_{l_{min}}}$	No. of Function/ Gradient Evals.
IMCS-BCO (Kriging)	$8.85 \cdot 10^{-4}$	$4.32 \cdot 10^{-9}$	0.186	$1.72 \cdot 10^{-2}$	38/38
IMCS-BCO (PC)	$9.27 \cdot 10^{-4}$	$3.97 \cdot 10^{-8}$	0.186	$1.72 \cdot 10^{-2}$	36/36
MCS-BCO	$8.98 \cdot 10^{-4}$	$2.98 \cdot 10^{-8}$	0.186	$1.72 \cdot 10^{-2}$	6153/6153

Cantilever Beam Design

$$\begin{aligned} \underset{b,d}{\text{minimize}} \quad & A(b,d) = bd, \\ \text{subject to} \quad & g_1(b,d,\mathcal{M}) = \frac{6\mathcal{M}}{bd^2\sigma_{allow}} - 1 \leq 0, \\ & g_2(b,d,\mathcal{V}) = \frac{3\mathcal{V}}{2bd\tau_{allow}} - 1 \leq 0, \\ & g_3(b,d) = \frac{d}{2b} - 1 \leq 0, \\ \text{bounds} \quad & 100\ mm \ \leq \ b, d \ \leq \ 600\ mm. \end{aligned}$$



Random Variable	Description	Uncertainty Type	τ_{min}	τ_{max}	Mean	Standard Deviation	Unit
b	Breadth	Epistemic	-10	10	-	-	mm
d	Width	Epistemic	-10	10	-	-	mm
$\mathcal{M}$	Bending Moment	Aleatory	-	-	$40 \cdot 10^6$	40000	$N \cdot mm$
$\mathcal{V}$	Shear Force	Aleatory	-	-	$150 \cdot 10^3$	1500	N

Type	k	P_k	Width b mm	Depth d mm	Area A $\cdot 10^3\ mm^2$	No. of F/FG Evals. & Iterations
Initial Design	-	-	300	300	90.0	-
Det ($F_s = 1.0$)	-	-	335.5	335.4	112.5	33/33-7
Det ($F_s = 1.5$)	-	-	595.5	283.4	168.7	45/45-8
Robust-KR	0	0.5000	347.4	343.4	126.3	7046/3523-7
Robust-PC	0	0.5000	347.4	343.4	126.3	7917/7917-8
Robust-KR	1	0.8413	349.7	344.5	127.5	7146/3573-7
Robust-PC	1	0.8413	349.7	344.5	127.5	8037/8037-8
Robust-KR	2	0.9772	398.5	305.4	128.8	7686/3843-7
Robust-PC	2	0.9772	398.5	305.4	128.8	9661/9661-9
Robust-KR	3	0.9986	386.5	317.8	130.0	8694/4347-8
Robust-PC	3	0.9986	386.5	317.8	130.0	11669/11669-10
Robust-KR	4	0.9999	356.6	347.5	131.1	7286/3643-7
Robust-PC	4	0.9999	356.6	347.5	131.1	8196/8196-8

Bibliography

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