Robust Optimization of a Wing Under Structural and Material Uncertainties

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Outline

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   - Why Uncertainty Quantification?
   - Types of Uncertainties
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     - Epistemic Uncertainties
     - Mixed Uncertainties
2. Robust Design Optimization Under Uncertainty
   - Mixed OUU Framework
3. Wing Optimization
   - Model Geometry and Analysis
   - Problem Formulation
   - Results
4. Conclusion

Why Uncertainty Quantification? I

- Design variables and input parameters are always subject to variations
  - Uncertain operating conditions (weather, ice accumulation on wing)
  - Uncertainties in boundary conditions/problem parameters
  - Uncertainties from lack of knowledge about a quantity (manufacturing tolerances)
  - Modeling inaccuracies (Navier-Stokes/Euler)
  - Random elements in a simulation
- Allowances must be made to accommodate likely variations/uncertainties

Why Uncertainty Quantification? II

- Traditionally we use **factor of safety** based on heuristics/expert opinion

A Typical Stress Constraint

$$g(\boldsymbol{d}) = \frac{\sigma}{\sigma_{max}} - 1 \leq 0 \Longrightarrow g(\boldsymbol{d}) = F_s \cdot \frac{\sigma}{\sigma_{max}} - 1 \leq 0$$

- What is an adequate or good factor of safety?
- Assumed **Factor of Safety** can be:
    - Adequate as well as over-conservative
    - Inadequate and prone to failure
- Increasingly difficult to come up with a factor of safety for radically new designs

Why Uncertainty Quantification? III

Why Quantify Uncertainties?

- Determine the real effects of uncertainties on the design (robust or vulnerable)
- Obtain confidence intervals for results (range of possible outcomes)
  - 95% probability (confidence) that the target C_L is achieved
  - 1% probability of violation of constraint #10
- Identify the limitations of the design (and improve)
- Reliability analysis for certification and quality assurance purposes

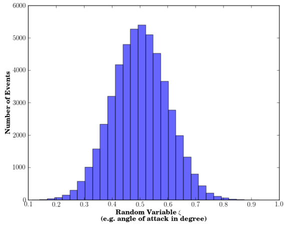
Uncertainty Types

- Aleatory / Irreducible / Type A
- Epistemic / Reducible / Type B
- Mixed

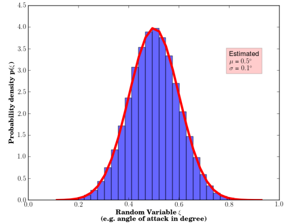
Aleatory Uncertainties I

Characteristics

- Inherent randomness or variations:
  - input parameters (Youngs modulus, shear force)
  - design variables
  - operating environment (cruise settings, temperature)
- Input probability distributions are known (sometimes assumed)
- Goal is to determine the output distribution



Available data

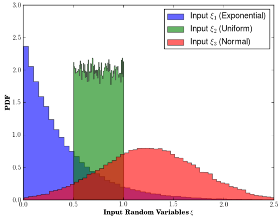


Fitted/Assumed distribution

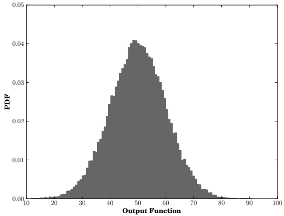
Aleatory Uncertainties II

Quantifying Aleatory Uncertainties

- Input data is available (mean, standard dev., distribution type)
- Need to know the input–output relationship of uncertainties
- Use Monte Carlo Sampling (MCS)
- Need thousands of simulations
- Use **surrogate models** to approximate the simulation output (kriging, polynomial chaos)

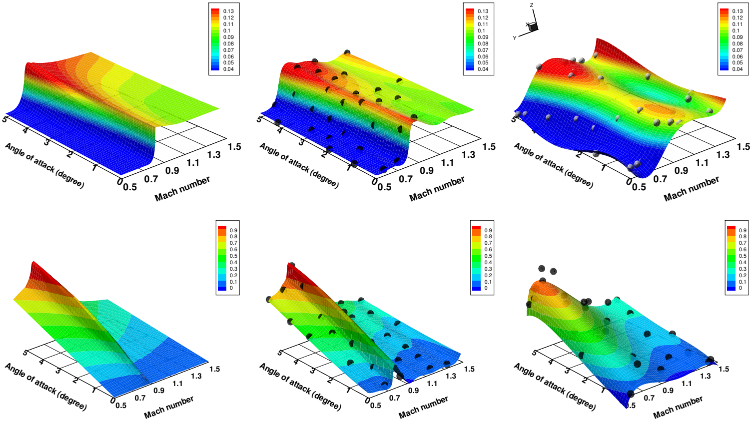


Input distributions



Output distribution

Aleatory Uncertainties III

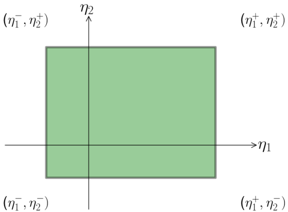


: Contours of exact database (left), kriging (middle) and PCE (right) for drag (top) and lift coefficients (bottom) with 30 training points.

Epistemic Uncertainties I

Characteristics

- Lack of knowledge about the appropriate value
- Only bounds can be specified $I(\boldsymbol{\eta}) = [\boldsymbol{\eta}^-, \boldsymbol{\eta}^+] = [\bar{\boldsymbol{\eta}} - \boldsymbol{\tau}, \bar{\boldsymbol{\eta}} + \boldsymbol{\tau}]$
- Goal: determine the worst and best scenarios within the interval $I(\boldsymbol{\eta})$



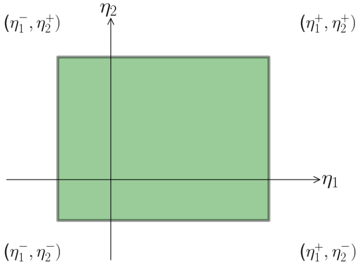
: Bounds on epistemic variables

Epistemic Uncertainties II

Goal: determine the worst and best scenarios within the bounds

1. Extensive Sampling

- Need $10^3 - 10^6$ simulations
- Prohibitively expensive for bigger problems



: Bounds on epistemic variables

Epistemic Uncertainties III

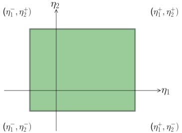
Goal: determine the worst and best scenarios within the bounds

2. Bound Constrained Optimization

- Optimization problem:

$$\begin{aligned} \underset{\boldsymbol{\beta}}{\text{minimize/maximize}} \quad & f = f(\boldsymbol{\eta}), \\ \text{subject to} \quad & \boldsymbol{\beta} \in I(\boldsymbol{\eta}) = [\bar{\boldsymbol{\eta}} - \boldsymbol{\tau}, \bar{\boldsymbol{\eta}} + \boldsymbol{\tau}]. \end{aligned}$$

- L-BFGS optimizer (needs gradients)
- Attractive even for bigger problems (scales linearly)



: Bounds on epistemic variables

Mixed Aleatory & Epistemic Uncertainties

Quantifying Mixed Uncertainties

- Comprise of both aleatory ξ and epistemic uncertainties η
  - Naive approach: Nested Sampling
    - Very expensive (millions of function evaluations)
    - Not computationally affordable
  - Our approach: IMCS+BCO
    - Surrogate models for aleatory uncertainties
    - Bound constrained optimization for epistemic uncertainties
    - Few hundred (or thousand) function evaluations (manageable)

Optimization Problem Formulation I

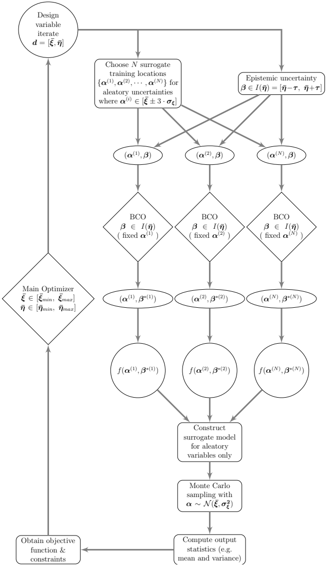
Deterministic Optimization

$$\begin{aligned} \underset{\boldsymbol{d}}{\text{minimize}} \quad & J = J(f, \boldsymbol{q}, \boldsymbol{d}), \\ \text{subject to} \quad & R(\boldsymbol{q}, \boldsymbol{d}) = 0, \\ & g(f, \boldsymbol{q}, \boldsymbol{d}) \leq 0. \end{aligned}$$

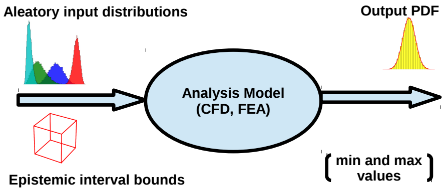
Robust Optimization

$$\begin{aligned} \underset{\boldsymbol{\xi}, \boldsymbol{\eta}}{\text{minimize}} \quad & \mathcal{J} = \mathcal{J}(\mu_{f*}, \sigma^2_{f*}, \boldsymbol{q}, \boldsymbol{\xi}, \boldsymbol{\eta}), \\ \text{subject to} \quad & R(\boldsymbol{q}, \boldsymbol{\xi}, \boldsymbol{\eta}) = 0, \\ & g^r = g(\mu_{f*}, \boldsymbol{q}, \boldsymbol{\xi}, \boldsymbol{\eta}) + k\sigma_{f*} \leq 0. \end{aligned}$$

Mixed OUU Framework: IMCS+BCO I

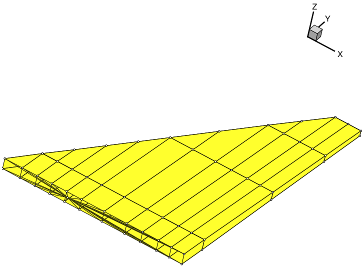


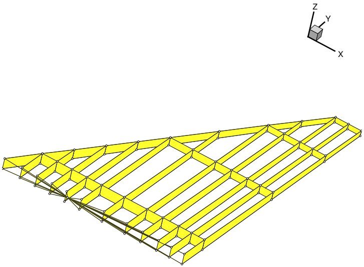
Mixed OUU Framework: IMCS+BCO II



: Figure illustrating the propagation of aleatory and epistemic uncertainties.

Model Geometry





Flight Condition

- $220^\circ/s$ roll maneuver
- Mach number of 0.7
- Dynamic pressure of 5.86 *psi*

Finite Element Analysis

Table: Components of the wing analysis model with corresponding element types.

Wing component	Element Type	Design Variable ID	Lower Limit	Upper Limit
Connection Rods for Shear Elements	PROD	1	0.10 in^2	10.0 in^2
Spars	PSHEAR	2, 3, 4, 5, 19, 20, 21, 22	0.25 in	1.50 in
Spar Caps	PROD	6, 7, 8, 9, 23, 24, 25, 26	0.10 in^2	1.25 in^2
Ribs	PSHEAR	10	0.25 in	1.50 in
Skins	PQDMEM1/ PTRMEM1	11, 12, 13, 14, 15, 16, 17, 18	0.10 in	1.50 in

Finite Element Analysis

- ASTROS (Automated Structural Optimization System) for a finite element analysis
- Fortran-Python interface for fetching function values and sensitivies

Problem Formulation

Deterministic

$$\begin{aligned} \underset{\boldsymbol{d}}{\text{minimize}} \quad & W = W(\boldsymbol{d}) \\ \text{subject to} \quad & g_{disp} = \frac{Z}{Z_{max}} - 1 \leq 0 \\ & g_{stress} = \frac{\Sigma}{\Sigma_{max}} - 1 \leq 0 \\ & \boldsymbol{d}_{lb} \leq \ \boldsymbol{d}_{1-26} \ \leq \boldsymbol{d}_{ub} \end{aligned} \tag{1}$$

Robust

$$\begin{aligned} \underset{\boldsymbol{d}}{\text{minimize}} \quad & \mathcal{J} = \mu_W + \sigma_W^2 \\ \text{subject to} \quad & g_i^r = \mu_{g_i} + k\sigma_{g_i} \leq 0 \end{aligned} \tag{2}$$

- Σ refers to the von Mises stresses
- Z refers to the vertical nodal displacements at the aft end of the wing

Constraints

Table: List of constraints in the optimization.

Constraint Type	Description	Symbol	Quantity	Value
Displacement	Wing tip (6 nodes)	g_{1-6}	Upper limit	$+3.0$ *in*
		g_{7-12}	Lower limit	-3.0 *in*
von Mises Stress	Top skins (28)	g_{13-40}	Tensile limit(13-21)	$+1.0 \cdot 10^4$ *psi*
			Compression limit(22-30)	$-1.0 \cdot 10^4$ *psi*
			Shear limit(32-40)	$+5.0 \cdot 10^3$ *psi*
von Mises Stress	Bottom skins (28)	g_{41-68}	Tensile limit(41-49)	$+1.0 \cdot 10^4$ *psi*
			Compression limit(50-59)	$-1.0 \cdot 10^4$ *psi*
			Shear limit(60-68)	$+5.0 \cdot 10^3$ *psi*

Uncertainty Modeling

Table: Assumed input uncertainties for the wing optimization under uncertainty problem.

Random Variable	Symbol	Uncertainty Type	Distribution Type	Lower Bound	Upper Bound	Mean	Std. Dev.	Unit
Skins, spars, spar caps ribs, posts	$\boldsymbol{d}_{1-26}$	Epistemic	–	-0.025	0.025	–	–	in
Young's modulus	E	Aleatory	Normal	–	–	10^7	$2.5 \cdot 10^4$	psi
Poisson ratio	ν	Aleatory	Normal	–	–	0.33	0.033	–
Weight density	ρ	Aleatory	Normal	–	–	0.10	0.003	lb/in^3

Comparison of Designs

Table: The design variable values at the initial and optimum designs.

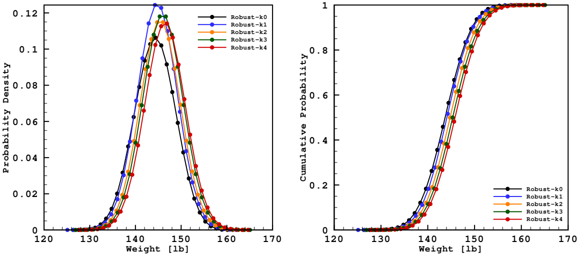
DV	Initial	Deterministic	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
1	5.050	0.617	0.699	0.701	0.704	0.710	0.711
2	0.875	0.252	0.254	0.254	0.254	0.255	0.255
3	0.875	0.250	0.251	0.251	0.251	0.251	0.251
4	0.875	0.252	0.251	0.251	0.251	0.251	0.251
5	0.875	0.260	0.260	0.259	0.258	0.258	0.257
6	0.675	0.104	0.108	0.107	0.109	0.108	0.108
7	0.675	0.100	0.100	0.100	0.100	0.100	0.100
8	0.675	0.102	0.102	0.102	0.102	0.102	0.102
9	0.675	0.104	0.104	0.104	0.104	0.104	0.104
10	0.875	0.354	0.409	0.412	0.413	0.416	0.422
11	0.800	0.111	0.111	0.111	0.111	0.111	0.111
12	0.800	0.128	0.133	0.134	0.134	0.132	0.133
13	0.800	0.342	0.39 5	0.397	0.397	0.405	0.406
14	0.800	0.38	0.428	0.421	0.413	0.435	0.436
15	0.800	0.166	0.210	0.212	0.213	0.217	0.219
16	0.800	0.265	0.324	0.324	0.325	0.325	0.325
17	0.800	0.519	0.581	0.588	0.594	0.600	0.605
18	0.800	0.405	0.443	0.449	0.458	0.456	0.463
19	0.875	0.276	0.297	0.300	0.304	0.311	0.319
20	0.875	0.257	0.256	0.255	0.254	0.254	0.253
21	0.875	0.324	0.360	0.365	0.366	0.372	0.376
22	0.875	0.316	0.347	0.347	0.349	0.354	0.361
23	0.675	0.332	0.434	0.440	0.451	0.455	0.458
24	0.675	0.101	0.101	0.101	0.101	0.101	0.101
25	0.675	0.104	0.107	0.106	0.106	0.106	0.106
26	0.675	0.113	0.116	0.117	0.118	0.119	0.121

Objective Function

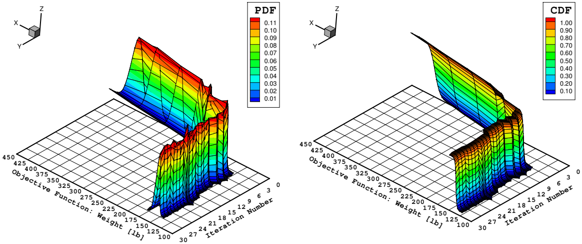
Table: Objective function values for deterministic and robust optima.

Type	k	P_k	μ_W *lb*	σ_W^2 *lb*	$\mathcal{J}$ *lb*	Total Structural Weight *lb*	% Increase in Cost Function	% Increase Total Weight
Deterministic	-	-	103.7	-	103.7	24463.7	-	-
Robust	0	0.5000	144.3	18.7	163.0	24504.3	39.2	0.166
Robust	1	0.8413	144.8	18.9	163.7	24504.8	39.6	0.168
Robust	2	0.9772	145.4	19.0	164.4	24505.4	40.2	0.170
Robust	3	0.9986	146.0	19.2	165.2	24506.0	40.8	0.173
Robust	4	0.9999	146.5	19.3	165.8	24506.5	41.3	0.175

PDFs and CDFs of Objective Function



: PDFs (left) and CDFs (right) of the objective function for different robust cases.



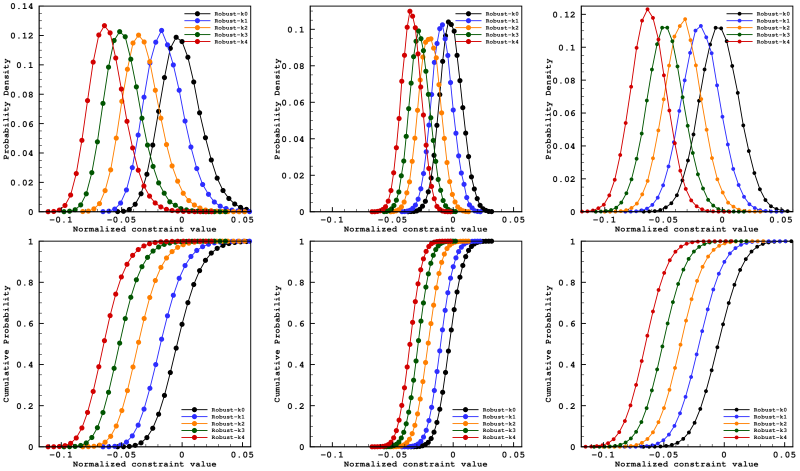
: PDF (left) and CDF (right) of the objective function with the number of optimizer iterations ($k = 4$).

Active Constraints

Table: List of constraints that are active at the optimum solution: $|g_i| < 10^{-2}$.

Opt. Case	Deterministic	Robust-k0	Robust-k1	Robust-k2	Robust-k3	Robust-k4
# of active constraints	10	1	4	2	1	1
List	31,32 47,48,49,50 55,56,57,58	57	47,48 57, 58	32 57	57	57

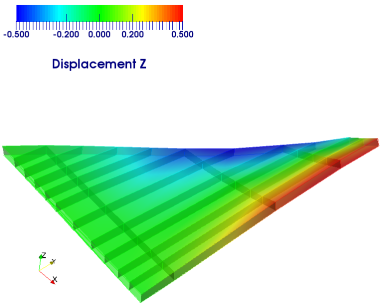
PDFs and CDFs of Constraints



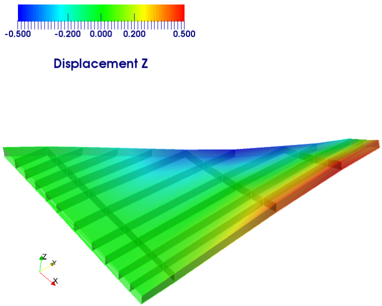
: g_{32} : g_{47} : g_{58}

: Comparison of PDFs (top) and CDFs (bottom) for selected constraints.

Displacement Comparisons I



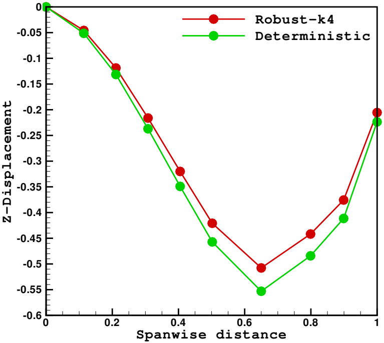
: Deterministic



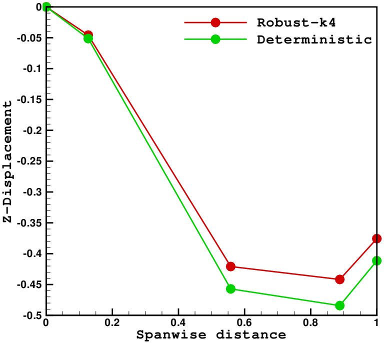
: Robust ($k = 4$)

: Nodal displacements in vertical direction.

Displacement Comparisons II



: Leading edge



: Trailing edge

: Plot of spanwise nodal displacements.

Verification using Monte-Carlo Sampling

Table: Comparison of IMCS-BCO with MCS-BCO for mixed OUU propagation.

Function	Simulation Type	μ_W	σ_W^2	No. of ASTROS calls
Weight	IMCS-BCO	405.2625	147.8309	61
	MCS-BCO	405.4212	148.1853	31322

Computational Cost

Table: A comparison of computational cost for robust and deterministic optimizations.

Opt. Case	Deterministic	Robust-k0	Robust-k1	Robust-k2	Robust-k3	Robust-k4
CPU Hours	0.15	353.3	394.1	343.9	343.5	402.0
Avg. # F/FG per surrogate (including BCOs)	-	189	190	189	189	189
Avg. # F/FG per OUU iteration	69	13010	13073	13020	12998	13011
No. of optimizer iterations	26	27	29	26	26	30
Total # of F/FG Evals.	1794	351270	379110	338504	337941	390327

Conclusion

- Developed a robust optimization framework:
  - Aleatory uncertainties are propagated using MCS of surrogate models
  - Epistemic uncertainties are propagated using BCOs
  - Mixed uncertainties are propagated using IMCS+BCO
- Applied framework to a robust structural wing optimization:
  - Comparison of robust and deterministic designs
  - Robustness studies in terms of PDFs and CDFs
  - Demonstrated computational savings with IMCS-BCO

Acknowledgments

1. Ohio Supercomputing Center
2. Wataru Yamazaki for his kriging surrogate model

References

1. Zhu, C., Byrd, R. H., Lu, P., and Nocedal, J., "L-BFGS-B: A Limited Memory FORTRAN Code for Solving Bound Constrained Optimization Problems", Tech. Rep. NAM-11, Department of Electrical Engineering and Computer Science, North western University, Evanston, Illinois, USA, 1994.
2. Helton, J. C., Johnson, J. D., Oberkampf, W. L., and Storlie, C. B., "A sampling-based computational strategy for the representation of epistemic uncertainty in model predictions with evidence theory" Tech. Rep. SAND2006-5557, Sandia National Laboratories, 2006.
3. Diegert, K., Klenke, S., Novotny, G., Paulsen, R., Pilch, M., and Trucano, T., "Toward a More Rigorous Application of Margins and Uncertainties within the Nuclear Weapons Life Cycle - A Sandia Perspective," Tech. Rep. SAND2007-6219, Sandia National Laboratories, 2007.
4. Helton, J. C., Oberkampf, J. D. J. W. L., and Sallaberry, C. J., "Representation of Analysis Results Involving Aleatory and Epistemic Uncertainty" Tech. Rep. SAND2008-4379, Sandia National Laboratories, 2008.
5. B.A. Lockwood, M. P. Rumpfkeil, W. Yamazaki and D. J. Mavriplis, "Uncertainty Quantification in Viscous Hypersonic Flows using Gradient Information and Surrogate Modeling". AIAA paper 2011-885
6. K. Boopathy and M.P. Rumpfkeil, "Unified Framework for Training Point Selection and Error Estimation for Surrogate Models", AIAA Journal, Volume 53(1), pp. 215–234.

Any Questions?

Mean and Variance

Mean and variance from surrogate

$$\mathcal{J} = w_1 \, \mu_{f*} + w_2 \, \vartheta_{f*} \tag{3}$$

$$\mu_{f*} \approx \frac{1}{\widetilde{N}} \sum_{k=1}^{\widetilde{N}} \widehat{f^*}(\boldsymbol{\alpha}^k) \tag{4}$$

$$\vartheta_{f*} \approx \left(\frac{1}{\widetilde{N}} \sum_{k=1}^{\widetilde{N}} \widehat{f^*}^2(\boldsymbol{\alpha}^k) \right) - \mu_{f*}^2 \tag{5}$$

- w_1 and w_2 are user specified weights
- The Monte Carlo samples $\boldsymbol{\alpha}^{(k)}$, $k = 1, \ldots, \widetilde{N}$ are chosen based on their underlying probability distribution
- $\widehat{f}^*$ represents the surrogate approximated value of exact function f^*

Aleatory Gradients

Aleatory gradients

$$\frac{d\mathcal{J}}{d\boldsymbol{\xi}} = \frac{\partial \mathcal{J}}{\partial \mu_{f*}} \frac{d\mu_{f*}}{d\boldsymbol{\xi}} + \frac{\partial \mathcal{J}}{\partial \vartheta_{f*}} \frac{d\vartheta_{f*}}{d\boldsymbol{\xi}} = w_1 \boxed{\frac{d\mu_{f*}}{d\boldsymbol{\xi}}} + w_2 \boxed{\frac{d\vartheta_{f*}}{d\boldsymbol{\xi}}} \tag{6}$$

$$\boxed{\frac{d\mu_{f*}}{d\boldsymbol{\xi}}} \approx \frac{1}{\widetilde{N}} \sum_{k=1}^{\widetilde{N}} \frac{d\widehat{f^*}(\boldsymbol{\alpha}^k)}{d\boldsymbol{\alpha}^k} \frac{d\boldsymbol{\alpha}^k}{d\boldsymbol{\xi}} = \frac{1}{\widetilde{N}} \sum_{k=1}^{\widetilde{N}} \frac{d\widehat{f^*}(\boldsymbol{\alpha}^k)}{d\boldsymbol{\alpha}^k} \tag{7}$$

$$\boxed{\frac{d\vartheta_{f*}}{d\boldsymbol{\xi}}} \approx \left(\frac{2}{\widetilde{N}} \sum_{k=1}^{\widetilde{N}} \widehat{f^*}(\boldsymbol{\alpha}^k) \frac{d\widehat{f^*}(\boldsymbol{\alpha}^k)}{d\boldsymbol{\alpha}^k} \right) - 2\mu_{f*} \frac{d\mu_{f*}}{d\boldsymbol{\xi}} \tag{8}$$

Epistemic Gradients I

Epistemic gradients

$$\frac{d\mathcal{J}}{d\boldsymbol{\eta}} = \frac{\partial \mathcal{J}}{\partial \mu_{f*}} \frac{d\mu_{f*}}{d\boldsymbol{\eta}} + \frac{\partial \mathcal{J}}{\partial \vartheta_{f*}} \frac{d\vartheta_{f*}}{d\boldsymbol{\eta}} = w_1 \frac{d\mu_{f*}}{d\boldsymbol{\eta}} + w_2 \frac{d\vartheta_{f*}}{d\boldsymbol{\eta}} \tag{9}$$

Approximations

$$\frac{d\mu_{f*}}{d\boldsymbol{\eta}} \approx \left.\frac{df^*}{d\boldsymbol{\eta}}\right|_{(\boldsymbol{\xi}=\bar{\boldsymbol{\xi}},\boldsymbol{\eta}=\bar{\boldsymbol{\eta}})} \quad \text{and} \quad \frac{d\vartheta_{f*}}{d\boldsymbol{\eta}} \approx 0 \tag{10}$$

Dynamic Training Framework