**Topic in Smart Health Info – Assignment 1**

**Question 1:**

|  |  |  |
| --- | --- | --- |
| Doctors | Patient1 | Patient2 |
| Doctor4 | 130 | 95 |
| Doctor5 | 118 | 83 |

Given Table shows the number of minutes taken by each doctor to address each patient. So, we must formulate an assignment of patients to doctors in such a way that total hours spent by the doctors is minimized.

If we look at the optimal equation, it can be written something like this:

**Z = 130 X41 + 95X42 + 118X51 + 83X52**  ………..…………………………………………………………………………. (\*)

whereas is nothing but whether that doctor-patient assignment is true or false meaning can be either 1 or 0.

And we must assign one doctor to only one patient. So, equation constraints can be defined as below:

X41 + X42 = 1, either one of the variables can be true at a time.

X41 + X51 = 1, either one of the variables can be true at a time.

X51 + X52 = 1, either one of the variables can be true at a time.

X42 + X52 = 1, either one of the variables can be true at a time. ………………………………………… (1)

If we try to represent these variables on 2D graph (Figure 1) we can draw one line at a time because of 4 different variables. Figure 1 represents first equation with X41 and X42.

A picture containing flock, bird, text, outdoor

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Figure 1: Graphical representation with given constraints in (1)

There can be only two possible scenarios:

* Doctor4 will be assigned to patient1 and doctor5 will be assigned to patient2.

In that case, X41 and X52 must be true which means X41 = 1, X52 = 1, X42 = 0, X51 = 0.

So, Min Z = 130 X41 + 95X42 + 118X51 + 83X52

Such that, X41 = 1 and X52 = 1 …………………………………………………………………………………. (2)

If we try to visualize these constraints through graphical representation, it looks something like Figure 2.

A map of a wire fence

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Figure 2: Graphical representation for given constraints in (2)

Min Z = 130 + 0 + 0 + 83 = 213 minutes

* Doctor5 will be assigned to patient1 and doctor4 will be assigned to patient2.

In that case, X42 = 1 and X51 = 1. If we try to put these values in (\*),

Z = 0 + 118 + 95 + 0 = 213 minutes.

So, if we consider any assignment of doctor to patient, it will be always our optimal solution. We were looking for one optimal solution, whereas we got multiple optimal solutions for this given table.

We can assign any patient to any doctor, either way we can expect minimum optimal time i.e. 213 minutes to address both the patients.

**Question 2:**

After assigning random integer values, time requirements table looks something like this:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Doctors | Patient1 | Patient2 | Patient3 | Patient4 | Patient5 | Patient6 |
| Doctor1 | 65 | 120 | 68 | 62 | 149 | 109 |
| Dcotor2 | 93 | 118 | 148 | 102 | 108 | 75 |
| Doctor3 | 137 | 101 | 71 | 120 | 69 | 136 |
| Doctor4 | 130 | 95 | 142 | 58 | 115 | 148 |
| Doctor5 | 118 | 83 | 147 | 116 | 83 | 136 |
| Doctor6 | 82 | 50 | 50 | 118 | 55 | 120 |

So, consider variable which is defined by

1 if the ith doctor is assigned to jth patient.

0 if the ith doctor is not assigned to jth patient.

Now, as the problem says one doctor is to be assigned to one patient.

Therefore, Doctor and patient assignments will be

, respectively.

The total problem equation will be given by

Determine > 0 (i, j = 1,2,3,4,5,6) in order to

**Minimize**

Subjected to above constraints,

when I = 1,2,3,…6

when j = 1,2,3,…6

And is either zero or one and is the time required to do the patient work.

If we try to solve this by using algebraic equations, it will be difficult to work on. So, let us solve this problem by using **Hungarian technique**:

1. Locate the smallest cost element in each row in the table and subtract it from each element of that row. This way you will get zero in each row. Reduced table will look something like this:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Doctors | Patient1 | Patient2 | Patient3 | Patient4 | Patient5 | Patient6 |
| Doctor1 | 3 | 58 | 6 | 0 | 87 | 47 |
| Dcotor2 | 18 | 43 | 73 | 27 | 33 | 0 |
| Doctor3 | 68 | 31 | 2 | 51 | 0 | 67 |
| Doctor4 | 72 | 37 | 84 | 0 | 57 | 90 |
| Doctor5 | 35 | 0 | 64 | 34 | 0 | 53 |
| Doctor6 | 32 | 0 | 0 | 68 | 5 | 70 |

1. Now in the constructed table, starting from first column, locate the smallest cost element in each column and subtract this smallest element from each element of that column. This way you will get zero in each column.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Doctors | Patient1 | Patient2 | Patient3 | Patient4 | Patient5 | Patient6 |
| Doctor1 | 0 | 58 | 6 | 0 | 87 | 47 |
| Dcotor2 | 15 | 43 | 73 | 27 | 33 | 0 |
| Doctor3 | 65 | 31 | 2 | 51 | 0 | 67 |
| Doctor4 | 69 | 37 | 84 | 0 | 57 | 90 |
| Doctor5 | 32 | 0 | 64 | 34 | 0 | 53 |
| Doctor6 | 29 | 0 | 0 | 68 | 5 | 70 |

1. Now assignments are made for the reduced table:

Put a Yellow mark on the single zero that you see first in a row for assignment(Because that is the lowest time taken by that doctor on a patient) and all other zeros in the same row will be strike out.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Doctors | Patient1 | Patient2 | Patient3 | Patient4 | Patient5 | Patient6 |
| Doctor1 | 0 | 58 | 6 | ~~0~~ | 87 | 47 |
| Dcotor2 | 15 | 43 | 73 | 27 | 33 | 0 |
| Doctor3 | 65 | 31 | 2 | 51 | 0 | 67 |
| Doctor4 | 69 | 37 | 84 | 0 | 57 | 90 |
| Doctor5 | 32 | 0 | 64 | 34 | ~~0~~ | 53 |
| Doctor6 | 29 | 0 | ~~0~~ | 68 | 5 | 70 |

1. Now perform the same operation column wise. Put a yellow mark on single zero and strike out the other zero that you have in the same column.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Doctors | Patient1 | Patient2 | Patient3 | Patient4 | Patient5 | Patient6 |
| Doctor1 | 0 | 58 | 6 | ~~0~~ | 87 | 47 |
| Dcotor2 | 15 | 43 | 73 | 27 | 33 | 0 |
| Doctor3 | 65 | 31 | 2 | 51 | 0 | 67 |
| Doctor4 | 69 | 37 | 84 | 0 | 57 | 90 |
| Doctor5 | 32 | 0 | 64 | 34 | ~~0~~ | 53 |
| Doctor6 | 29 | ~~0~~ | 0 | 68 | 5 | 70 |

1. As you can see every single doctor has been assigned to every other patient. So, we can say that optimal solution has been achieved.

1. So, the final assignment looks something like this:

Doctor1 -> Patient1,

Doctor2 -> Patient6,

Doctor3 -> Patient5,

Doctor4 -> Patient4,

Doctor5 -> Patient2,

Doctor6 -> Patient3.

And total time expended by the ER will be: 65 + 75 + 69 + 58 + 83 + 50 = 400 minutes.

**Question 3:**

As you can see from Screenshot1, optimal solution achieved through Gurobi script is 213.0 minutes which is same to what we got in question 1 solution.

A screenshot of text

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Screenshot 1: Solution for part 1

from Screenshot2, optimal solution achieved with Gurobi script is 400.0 minutes which is same to what we got in question 2 solution with the same assignment of patients to the doctors.

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Screenshot 2: Solution for part 2