UNIT-I
MATRICES

Short Answer Questions:-

1. If A is orthogonal matrix, prove that A^T and A^{-1} are also orthogonal.

2. Express the matrix A as sum of symmetric and Skew-symmetric matrices where

A= $\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$

3. If A, B are orthogonal matrices, each of order ‘n’ then AB and BA are orthogonal matrices.

4. Define rank of a matrix and find the value of k such that rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & K & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is ‘2’

5. Prove that $\frac{1}{2}\begin{pmatrix} \boldsymbol{1+i} & \boldsymbol{-1+i} \\ \boldsymbol{1+i} & \boldsymbol{1-i} \end{pmatrix}$ is a unitary matrix.

6. If ‘A’ is Hermitian matrix prove that ‘iA’ is a Skew- Hermitian matrix.

7. State the conditions when the system of non-homogenous equations AX=B will have

i) unique solution ii) Infinite no of solutions iii) No solution.

Long Answer Questions:

1. Prove that every square matrix can be uniquely expressed as sum of symmetric and skew- symmetric matrices.

2. Express the matrix $\begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$ as the sum of Hermitian and Skew-Hermitian Matrices.

3. Show that A= $\begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary if and only if $a^2+b^2+c^2=1$

4. Reduce the following matrices into normal form and hence find its rank $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -1 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & 7 \end{bmatrix}$

5. Reduce the following matrices into echelon form and hence find its rank $\begin{bmatrix} 2 & -2 & 0 & 4 \\ 2 & 0 & 1 & -1 \\ 0 & 6 & 2 & 3 \\ 1 & -1 & 1 & -2 \end{bmatrix}$

6. Find the inverse of a matrix by using Gauss-Jordan method $\begin{bmatrix} -1 & -3 & 3 & 1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

7.Discuss for what values of μ, λ the simultaneous equations

$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ have (i). No solution (ii) A unique solution (iii)An infinite number of solutions.

8.Show that the only real number λ for which the system

$x+2y+3z = \lambda x$, $3x+y+2z= \lambda y$, $2x+3y+z = \lambda z$, has non-zero solution is 6 and solve them.

9. Solve the equations $x + y + z = 6$, $3x + 3y +4 z =20$, $2x + y + 3z = 13$

using Gauss elimination method.

10.Solve the following system of equations by Gauss-Seidel method

$25x + 2y + 2z = 69$, $2x + 10y + z = 63$, $x + y +z = 43$.

11. Determine b such that the system of homogeneous equations $2x+y+2z=0$, $x+y+3z=0$, $4x+3y+bz=0$ has trivial and non trivial solutions. Find the non trivial solutions.

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UNIT-II

EIGEN VALUES AND EIGEN VECTORS

Short Answer Questions:-

1. Define modal and spectral matrices.

2. Prove that Eigen values of a real symmetric matrix are real.

3. Prove that if ‘λ’ is an Eigen value of a non-singular matrix A, then $\frac{|A|}{\lambda}$ IS an Eigen value of matrix Adj A.

4. Show that the Eigen values of a unitary matrix are of unit modulus.

5. Prove that a square matrix ‘A’ and its transpose A^T have the same Eigen values.

6. Prove that if $\lambda_1, \lambda_2, \lambda_3 \ldots\ldots \lambda_n$ are the latent roots of ‘A’ then $\lambda_1^n, \lambda_2^n, \lambda_3^n \ldots\ldots \lambda_n^n$ are the the latent roots of A^n

7. State Cayley- Hamilton theorem.

8. Find the sum and product of the Eigen values of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

9. Define Quadratic form.

Long Answer Questions:-

1. Determine the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

2. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and hence find A^{-1} and A^4.

3. Determine the eigen values and eigen vectors of B = $2A^2 - \frac{1}{2}A + 3I$ where A = $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$

4. Reduce the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to diagonal form.

5. Find the nature of the quadratic form, index, signature and rank of $10x^2+2y^2+5z^2-4xy-10xz+6yz$.

6. Determine the modal matrix for A= $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ and hence diagonalizable the matrix.

7. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 2 & -1 & 3 \end{bmatrix}$ by and hence find A^4 .

8. Reduce the Quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canonical form by orthogonal transformation and hence find the nature, index, signature and rank.

IMPORTANT QUESTIONS OF SQUENCE AND SERIES:

SAQ:

1.State Cauchy's root test.

2.Test for convergence$\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$

3.State Leibnitz's test

4.State D'Alembert's Ratio test.

5. Show that 1- $\frac{1}{2^2}$ + $\frac{1}{3^2}$ -$\frac{1}{4^2}$ +….. is absolutely convergent.

6.Show that $\frac{2}{1^p}$ +$\frac{3}{2^p}$ +$\frac{4}{3^p}$ +……… is convergent for $p > 2$ and divergent for $p \leq 2$

7.State Raabe's test.

8.Test for the convergence of $\sum(1+\frac{1}{n})^{-n^2}$

9. State Geometric series.

LAQ:

1. Test whether the series $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$is convergent.

2.Find the nature of the series$\sum\frac{(n!)^2}{(2n)!}.x^{2n}$ (x>0)

3. Examine the convergence of the series 1+ $\frac{3}{7}x$ +$\frac{3.6}{7.10}x^2$ + $\frac{3.6.9}{7.10.13}x^3$ +……:x>0

4. Test for convergence $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n(n+1)(n+2)}}$

5. Discuss the convergence of the series $\frac{1}{2\sqrt{1}}$ + $\frac{x^2}{3\sqrt{2}}$ + $\frac{x^4}{4\sqrt{3}}$ + $\frac{x^6}{5\sqrt{4}}$ + …….

6.Does the series$\sum_{0}^{\infty}\frac{(-1)^n}{\sqrt{n^2+1}}$ converge absolutely, conditionally or diverge?

7.Examine for absolute convergence the series x-$\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+$………

8.Test the convergence of the series: $\sum_{n=1}^{\infty}\frac{1.3.5\ldots(2n+1)}{2.5.8\ldots\ldots(3n+2)}$

9.Examine the following series for absolute and conditional convergence of

$$\frac{1}{5\sqrt{2}}-\frac{1}{5\sqrt{3}}+\frac{1}{5\sqrt{4}}-\cdots\ldots\ldots$$

UNIT-IV: CALCULUS
SAQ:

1. Verify Rolle's theorem for f(x) = $2x^3+x^2-4x-2$ in $[-\sqrt{3},\sqrt{3}]$
2. State Lagrange's Mean value theorem and its geometrical interpretation.
3. State Cauchy's Mean value theorem.
4. Prove that B (m, n) = B (n, m).
5. B (m, n) = 2 $\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta\, cos^{2n-1} d\theta$
6. Evaluate $\int_0^{\infty} e^{x^3}\, x^7$ dx.
7. Evaluate $\int_0^1 x^5(1-x)^3 dx$
8. Evaluate the improper integral $\int_0^{\infty} \sqrt{x} e^{-x^2} dx$ using Gamma function.
9. Write the formula to find the surface area of revolution about x-axis.

LAQ

1. Verify the Rolle's theorem for the function *f(x)*=$\frac{sinx}{e^x}$in *(0,π)*

2. Show that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem.

3. Verify Cauchy's mean value theorem for the function e^x and e^{-x}in the interval (a.b).

4. Find the value of $\Gamma(\frac{1}{2})$.

5. Find the surface area of the solid generated by revolving the loop of the curve $9y^2=x(x-3)^2$

6.If a<b,Prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ using legrange's mean value theorem and hence deduce that (i) $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$

6. Find the volume of the solid that results when the region enclosed by the curves xy=1 x-axis and x=1 rotated about x-axis.

7. Expand $\tan^{-1} x$ in powers of (x-1) using Maclaurin's theorem

8. Show that $|\cos b - \cos a| \leq |b-a|$.

9. Using Beta and Gamma functions, evaluate the integral $\oint_0^{\infty} x^4\, e^{-x^2}$ dx.

10. If m and n are positive integers then prove that $(m,n) = \frac{(m-1)!(n-1)!}{(m+n-1)!}$.

11. Evaluate $\int_0^{\pi} \sin^7 x\, cos^4 x dx$ using Beta and Gamma functions.

UNIT-V

MULTIVARIABLE CALCULUS

SAQ

1. State Euler's theorem for function of two variables.

2. Verify Euler's theorem for the function $z = ax^2 + 2hxy + by^2$

3. If z=f(x+ay) +g(x-ay), prove that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$

4. $u = x^2+y^2$, $x=at^2$, $y=2at$ then find $\frac{du}{dt}$.

5. Find first and second order partial derivatives of $tan^{-1}(\frac{x^2+y^2}{x+y})$

6. $u = f(x-y, y-z, z-x)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

7. If $x=u(1+v)$, $y=v(1+u)$ then prove that $\frac{\partial(x,y)}{\partial(u,v)} = 1+u+v$.

8. If f(x,y)=xy+(x-y) then find the stationary points.

9. Discuss the maximum and minimum of $x^2+y^2+6x+12$

10. Verify $u=2x-y+3z$, $v=2x-y-z$, $w=2x-y+z$ are functionally dependent and if so, find the relation between them.

LAQ

1. If $u = \log(\frac{x^2+y^2}{x+y})$ then prove that $xu_x + yu_y = 1$.

2. If $\sin^{-1}\frac{x}{y} + \cos^{-1}\frac{y}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\delta u}{\delta y} = 0$.

4. if $x = rcos\theta, y = rsin\theta\ find\ \frac{\partial(x,y)}{\partial(r,\theta)}\ and\ \frac{\partial(r,\theta)}{\partial(x,y)}$. Also show that $\frac{\partial(x,y)}{\partial(r,\theta)}.\frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

5. Show that the following functions are functionally dependent and hence find the relation between them $u = \sin^{-1}x + \sin^{-1}y, v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$.

6. Examine for minimum and maximum values of sinx + siny + sin(x+y).

7. A rectangular box open at the top is to have volume of 32cubic ft. Find the dimensions of the box requiring least material for its construction.

8. Find the maximum value of $u = x^2y^3z^4\ if\ 2x + 3y + 4z = a$.

10. Show that functions $u = xy+yz+zx$, $v = x^2+y^2+z^2$ and $w=x+y+z$ are functionally related. Find the relation between them.

12. Examine the function $f(x,y) = x^4+y^4-2x^2+4xy-2y^2$ for extreme values $(x>0, y>0)$

13. Find the maximum and minimum values of the function $f(x,y) = x^3y^2(1-x-y)$

14. Find the maximum and minimum values of $x+y+z$ subject to $\frac{1}{x}+\frac{1}{y}+\frac{1}{y} = 1$.

15. The temperature T at any point (x, y, z) in space is $T=400xyz^2$ Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.