1. Introduction

Artificial Neural Networks (ANNs) are systems of interconnected processing units whose behaviour is governed by two fundamental mechanisms: **activation dynamics** and **synaptic dynamics**.

- **Activation dynamics** describe how neuron states (activations) evolve with time for a fixed input.
- **Synaptic dynamics** describe how connection weights change during learning.

Mathematically, both are represented using **first-order differential equations**, which model the rate of change of activation or weight with respect to time.

2. Passive Decay Model

$$\frac{dx_i(t)}{dt} = -A_i x_i(t)$$

Symbols

- $x_i(t)$: activation (state) of neuron indexed by $i$at time t
- i: index identifying the neuron
- t: time variable
- $\frac{dx_i(t)}{dt}$: derivative of $x_i(t)$with respect to time (rate of change)
- A_i: positive decay (leakage) constant for neuron i
- $-$: indicates reduction (decay) of activation

3. Modified Passive Decay Model

$$\frac{dx_i(t)}{dt} = -\frac{A_i}{C_i} x_i(t)$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time

- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- C_i: membrane capacitance of neuron i
- $\frac{A_i}{C_i}$: effective decay rate
- $-$: decay effect

4. Non-Zero Resting Potential Model

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + P_i$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- P_i: constant input (resting potential or bias)
- $+$: additive effect

5. External Input Activation Model

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + B_i I_i$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation

- A_i: decay constant
- B_i: input gain (scaling factor)
- I_i: external input signal applied to neuron i
- $+$: additive contribution

6. Additive Autoassociative Model

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + B_i I_i + \sum_{j=1}^{N} W_{ij} f_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change of activation
- A_i: decay constant
- B_i: input gain
- I_i: external input to neuron i
- $\sum_{j=1}^{N}$: summation over index $j$from 1 to N
- j: index of presynaptic neuron
- N: total number of neurons
- W_{ij}: synaptic weight from neuron $j$to neuron i
- $f_j(\cdot)$: output function of neuron j
- $x_j(t)$: activation of neuron $j$at time t

7. Inhibitory Feedback Model

$$\frac{dx_i(t)}{dt} =- A_i x_i(t) - B_i I_i - \sum_{j=1}^{N} W_{ij} f_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron $i$at time t
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: scaling constant
- I_i: external input
- $\sum_{j=1}^{N}$: summation over neurons
- j: neuron index
- N: total neurons
- W_{ij}: weight from neuron $j$to neuron i
- $f_j(x_j(t))$: output of neuron j

8. Perkel's Model

$$\frac{dx_i(t)}{dt} = \frac{1}{R_i} x_i(t) + \sum_{j=1}^{N} \frac{1}{R_{ij}} \phi_j(x_j(t))$$

Symbols

- $x_i(t)$: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- R_i: resistance of neuron i

- $\frac{1}{R_i}$: conductance
- $\sum_{j=1}^{N}$: summation over neurons
- j: neuron index
- N: total neurons
- R_{ij}: resistance between neurons $i$and j
- $\phi_j(\cdot)$: output function
- $x_j(t)$: activation of neuron j

9. Hopfield Model

$$\frac{dx_i(t)}{dt} =- A_i x_i(t) + \sum_{j=1}^{N} W_{ij} f(x_j(t)) + I_i$$

Symbols

- $x_i(t)$: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- $\sum_{j=1}^{N}$: summation
- j: neuron index
- N: total neurons
- W_{ij}: symmetric connection weight
- $f(\cdot)$: bounded activation function

- $x_j(t)$: activation of neuron j
- I_i: external input

10. Heteroassociative Network

Layer-1

$$\frac{dx_i(t)}{dt} = -A_i x_i(t) + \sum_{j=1}^{M} f_j(y_j(t))V_{ji} + I_i$$

Symbols

- $x_i(t)$: activation of neuron $i$in layer-1
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- $\sum_{j=1}^{M}$: summation over layer-2 neurons
- j: index of neuron in layer-2
- M: number of neurons in layer-2
- $f_j(\cdot)$: output function of neuron j
- $y_j(t)$: activation of neuron $j$in layer-2
- V_{ji}: weight from neuron $j$to neuron i
- I_i: external input

Layer-2

$$\frac{dy_j(t)}{dt} = -B_j y_j(t) + \sum_{i=1}^{N} g_i(x_i(t))W_{ij} + J_j$$

Symbols

- $y_j(t)$: activation of neuron $j$in layer-2
- t: time

- $\frac{dy_j(t)}{dt}$: rate of change
- B_j: decay constant
- $\sum_{i=1}^{N}$: summation over layer-1 neurons
- i: neuron index in layer-1
- N: number of neurons in layer-1
- $g_i(\cdot)$: output function of neuron i
- $x_i(t)$: activation of neuron i
- W_{ij}: weight from neuron $i$to neuron j
- J_j: external input

11. Bidirectional Associative Memory (BAM)

Same equations as heteroassociative network with

- $V = W^T$: weight matrix Vequals transpose of weight matrix W

12. Basic Shunting Model

$$\frac{dx_i(t)}{dt} =- A_i x_i(t) + (B_i - x_i(t))I_i$$

Symbols

- $x_i(t)$: activation
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper saturation limit

- I_i: external input
- $\left(B_i - x_i(t)\right)$: remaining activation capacity

13. On-Center Off-Surround Model

$$\frac{dx_i(t)}{dt} =- A_i x_i(t) + (B_i - x_i(t))I_i - x_i(t) \sum_{j \neq i} I_j$$

Symbols

- $x_i(t)$: activation
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- I_i: excitatory input
- $\sum_{j \neq i}$: summation over all neurons except i
- $j \neq i$: index condition
- I_j: inhibitory inputs

14. Modified Shunting Model

$$\frac{dx_i(t)}{dt} =- A_i x_i(t) + (B_i - x_i(t))I_i - (E_i + x_i(t)) \sum_{j \neq i} I_j$$

Symbols

- $x_i(t)$: activation
- t: time

- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- E_i: lower bound parameter
- I_i: excitatory input
- $\sum_{j \neq i}$: summation excluding neuron i
- I_j: inhibitory inputs

15. Shunting Model with Feedback

$$\frac{dx_i(t)}{dt} = -A_i x_i + (B_i - C_i x_i)[I_i + f_i(x_i)] - (E_i + D_i x_i)[J_i + \sum_{j \neq i} f_j(x_j$$

Symbols

- x_i: activation of neuron i
- t: time
- $\frac{dx_i(t)}{dt}$: rate of change
- A_i: decay constant
- B_i: upper bound
- C_i: scaling constant for excitatory term
- I_i: external excitatory input
- $f_i(x_i)$: feedback function of neuron i
- E_i: lower bound parameter
- D_i: scaling constant for inhibitory term
- J_i: external inhibitory input

- $\sum_{j \neq i}$: summation over neurons except i
- $f_j(x_j)$: output of neuron *j*
- w_{ji}: feedback weight from neuron $j$to neuron i

16. Synaptic Dynamics Model

$$\frac{dw_{ij}(t)}{dt} =- w_{ij}(t) + f_i(x_i(t))f_j(x_j(t))$$

Symbols

- $w_{ij}(t)$: synaptic weight from neuron $j$to neuron $i$at time t
- $\frac{dw_{ij}(t)}{dt}$: rate of change of weight
- t: time
- $f_i(x_i(t))$: output of neuron i
- $x_i(t)$: activation of neuron i
- $f_j(x_j(t))$: output of neuron *j*
- $x_j(t)$: activation of neuron *j*