

On the power of oritatami cotranscriptional folding with unary bead sequence ¹

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What is oritatami system?

Oritatami system is a mathematical model for cotranscriptional folding(CF). (Geary, Meunier, Schabanel and Seki.)

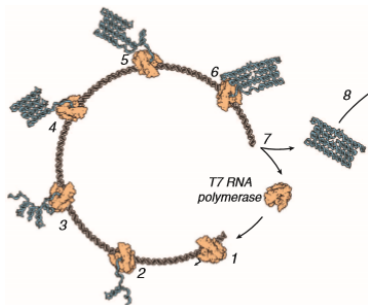


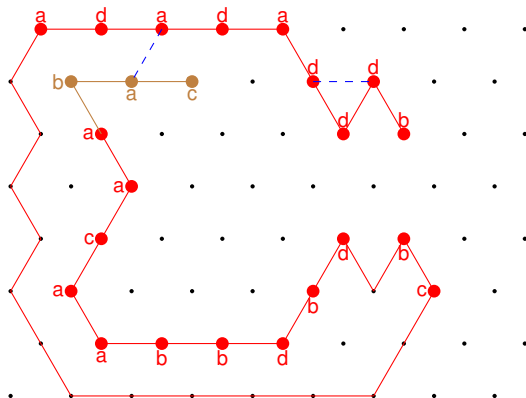
Figure: RNA tile is self-assembled (RNA Origami)

(C. Geary, P.W.K. Rothmund and E.S. Andersen. Science 345(6198), 2014)

How oritatami system works?

An example

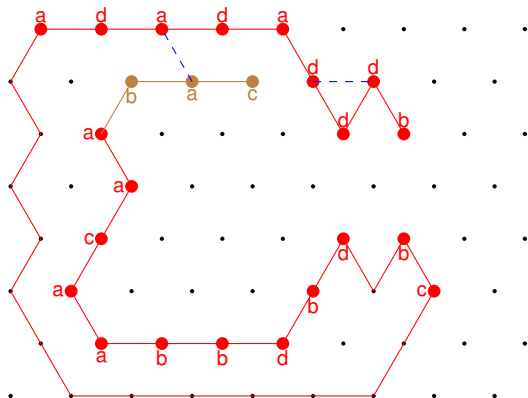
$\Sigma = \{a, b, c, d\}$, $R = \{(a, a), (b, b), (c, c), (d, d)\}$,
arity $\alpha = 2$, delay $\delta = 3$, $w = \underline{bac}bcadbcbab$



How oritatami system works?

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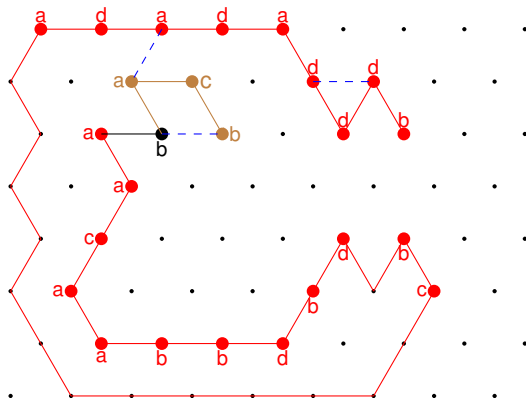
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How oritatami system works?

An example

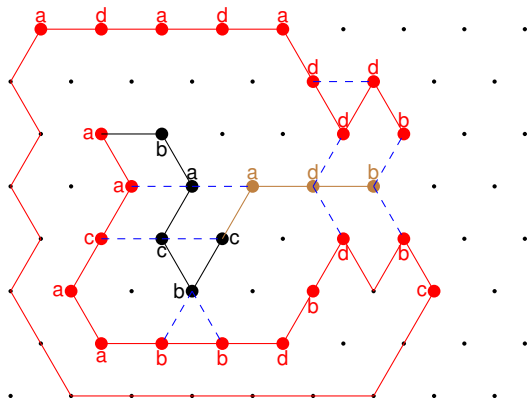
$\Sigma = \{a, b, c, d\}$, $R = \{(a, a), (b, b), (c, c), (d, d)\}$,
arity $\alpha = 2$, delay $\delta = 3$, $w = \underline{bacb}cadbcbab$



How oritatami system works?

An example

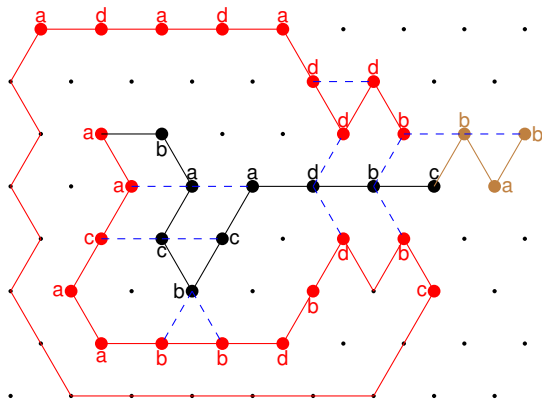
$\Sigma = \{a, b, c, d\}$, $R = \{(a, a), (b, b), (c, c), (d, d)\}$,
arity $\alpha = 2$, delay $\delta = 3$, $w = bacbcadbcbab$



A deterministic oritatami system

An example

$\Sigma = \{a, b, c, d\}$, $R = \{(a, a), (b, b), (c, c), (d, d)\}$,
arity $\alpha = 2$, delay $\delta = 3$, $w = \text{bacbcadbcbab}$



Turing universal

Theorem (C. Geary *et al.* ISAAC, 2018)

Oritatami system at *delay* $\delta = 3$ which employs 542 types of beads is Turing universal.

Theorem

Polynomial length of conformations \longrightarrow Non-Turing-universal

Problem

Problem

Give an upper bound on the length of a transcript of a *delay* δ , *arity* α deterministic oritatami system by a function in δ , α , and seed n .

Oritatami System

input : *delay* δ , *arity* α , seed, rule, transcript
output : conformation

Why unary?

Because we considered the unary oritatami system is good for a first step towards the characterization of non-Turing-universal oritatami systems.

Cases of non-Turing-universal oritatami systems

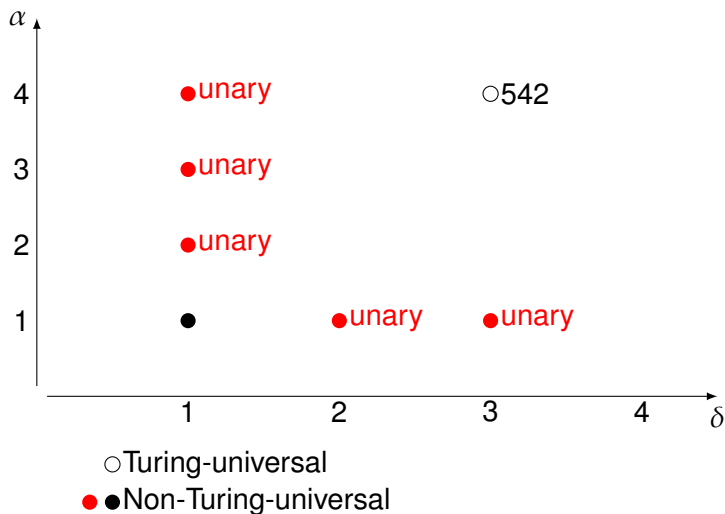


Figure: Cases of non-Turing-universal oritatami systems

Cases of non-Turing-universal oritatami systems

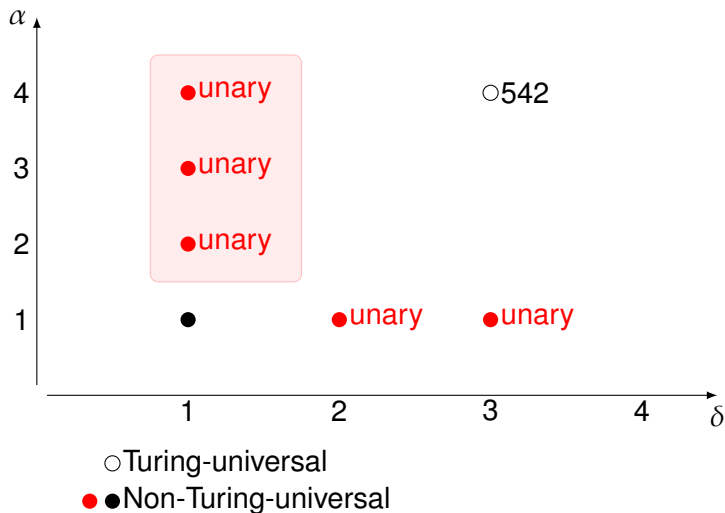
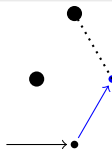


Figure: Cases of non-Turing-universal oritatami systems

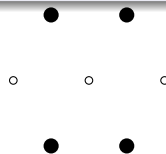
Deterministic unary oritatami systems at delay 1

Two ways for a bead stabilization

- To be bound to another bead.
- Through a 1-in-1-out structure called the tunnel section.



Bond

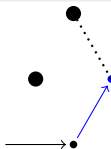


Tunnel section

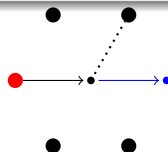
Deterministic unary oritatami systems at delay 1

Two ways for a bead stabilization

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Tunnel section

Deterministic unary oritatami systems at delay 1

Results ($\delta = 1$)

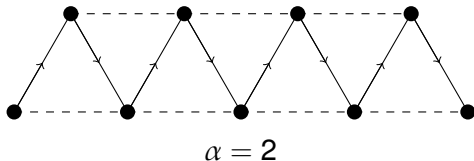
$$\alpha = 4 \quad 3n^2 + 3n + 1$$

$$\alpha = 3 \quad 4n + 14$$

$$\alpha = 2 \quad \infty \text{ but zigzag after } (27n^2 + 9n + 1)$$

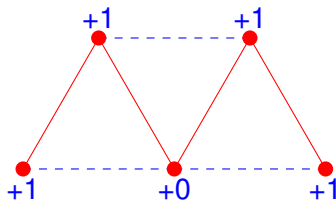
^a a

^ac.f. $\alpha = 1$: $10n$ (Demaine et al. 2018 DNA24)



zig-zag conformation

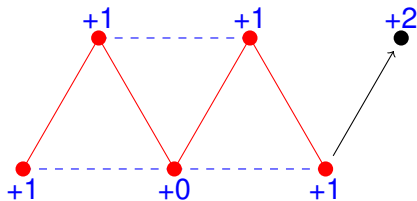
Deterministic oritatami system at $\delta = 1$ and at $\alpha = 2$ can make zig-zag conformation



$$\alpha = 2$$

zig-zag conformation

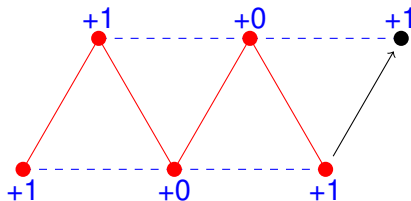
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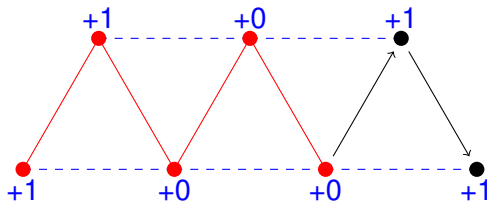
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zig-zag conformation

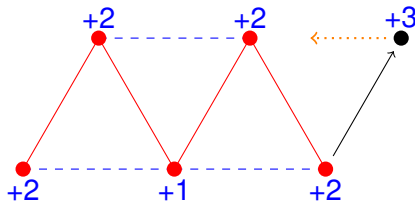
Deterministic oritatami system at $\delta = 1$ and at $\alpha = 2$ can make zig-zag conformation



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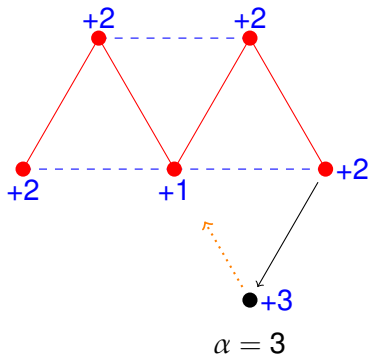
Deterministic oritatami system at $\delta = 1$ and at $\alpha = 2$ can make zig-zag conformation



$$\alpha = 3$$

zig-zag conformation

Deterministic oritatami system at $\delta = 1$ and at $\alpha = 2$ can make zig-zag conformation



Deterministic unary oritatami systems at delay 1

Results ($\delta = 1$)

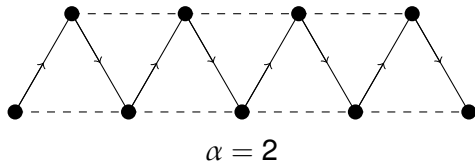
$$\alpha = 4 \quad 3n^2 + 3n + 1$$

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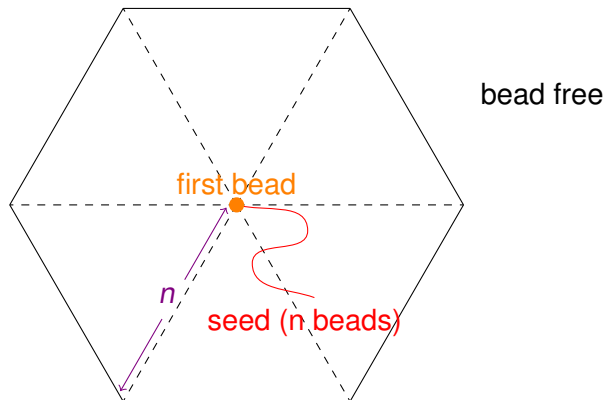
^ac.f. $\alpha = 1$: $10n$ (Demaine et al. 2018 DNA24)



Deterministic unary oritatami systems at delay 1

$$\alpha = 4$$

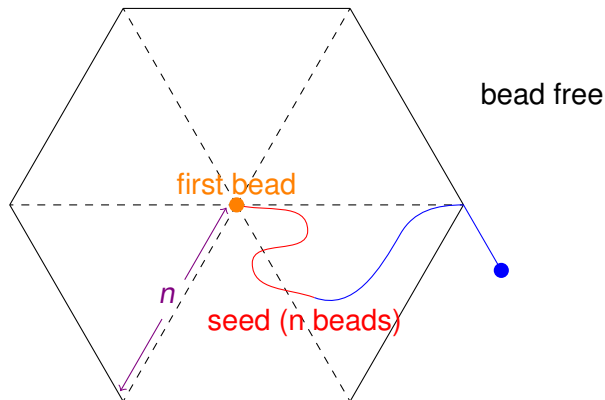
The terminal conformation at $\alpha = 4$ is of length at most $3n^2 + 3n + 1(\odot_O^n)$.



Deterministic unary oritatami systems at delay 1

$$\alpha = 4$$

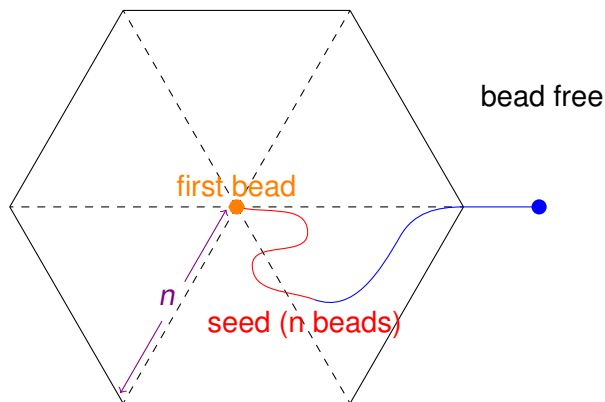
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Deterministic unary oritatami systems at delay 1

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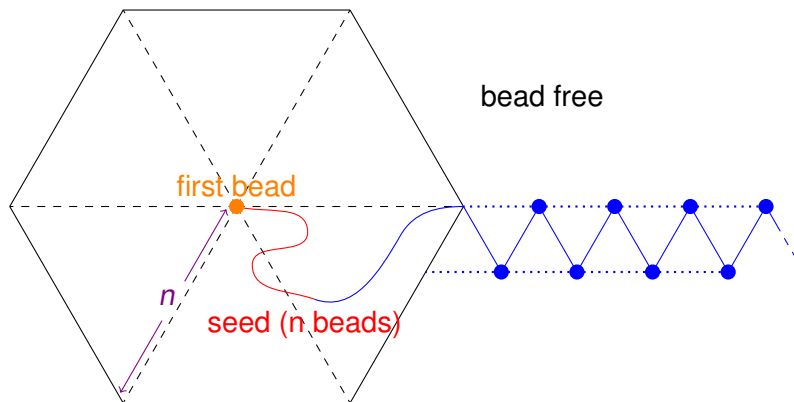
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Deterministic unary oritatami systems at delay 1

$$\alpha = 2 \ (\delta = 1)$$

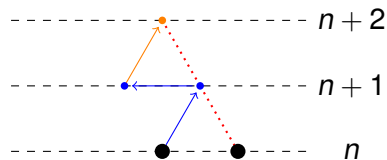
A transcript folds into the zig-zag conformation after its $(27n^2 + 9n + 1)$ -th bead ($\odot_{\mathcal{O}}^{3n}$).



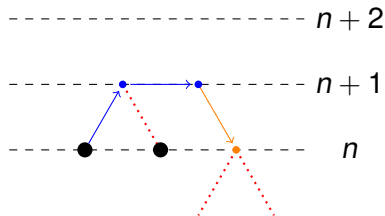
Deterministic unary oritatami systems at delay 1

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A transcript folds into the zig-zag conformation after its $(27n^2 + 9n + 1)$ -th bead (\square_O^{3n}).



zig-zag conformation

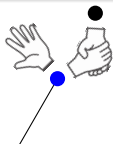


free hands ≤ 2

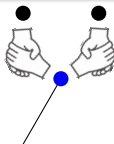
Deterministic unary oritatami systems at delay 1

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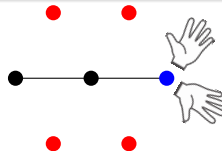
A transcript folds into the zig-zag conformation after its $(27n^2 + 9n + 1)$ -th bead (\square_O^{3n}).



free hands = ± 0



free hands = -2

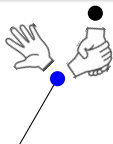


free hands $\leq +2$

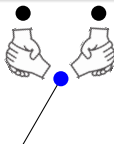
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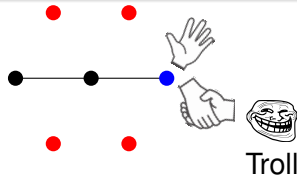
A transcript folds into the zig-zag conformation after its $(27n^2 + 9n + 1)$ -th bead (\hexagon_O^{3n}).



free hands = ± 0



free hands = -2



free hands $\leq +2$

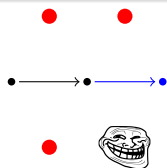
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

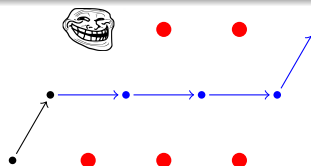
$\alpha \geq 4$ No free hand supplies / tunnel section.

$\alpha = 3$ Troll consumes bonds / tunnel section.

$\alpha = 2$ Troll consumes bonds / tunnel.



Tunnel section

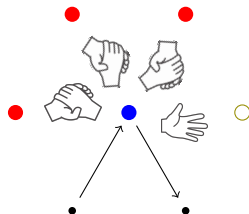
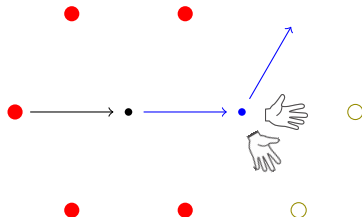


Tunnel

Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

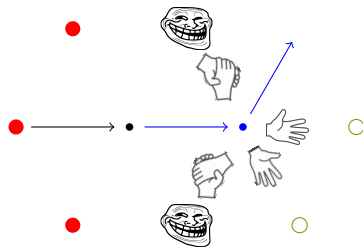
$\alpha \geq 4$ Any hands are not supplied with using a tunnel section.



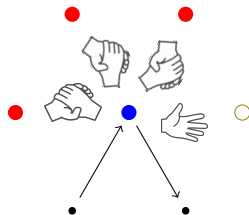
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

$\alpha \geq 4$ Any hands are not supplied with using a tunnel section.



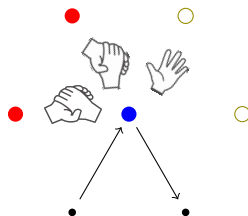
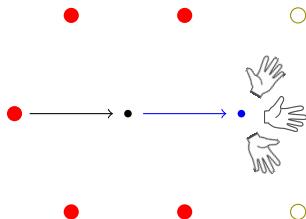
free hands ≤ 0



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

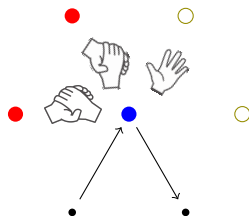
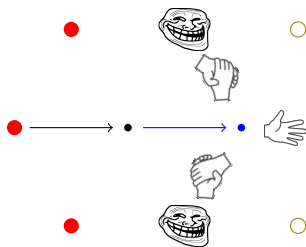
$\alpha = 3$ At least one free hand is decreased / tunnel section.



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

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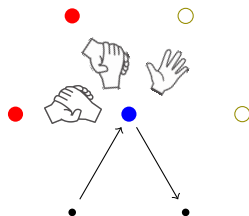
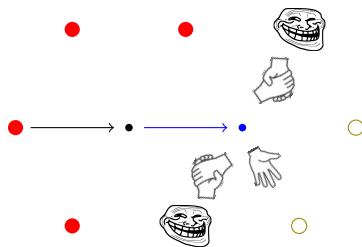


free hands ≤ -1

Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

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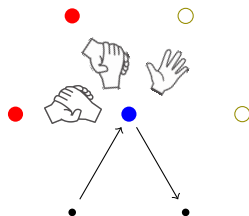
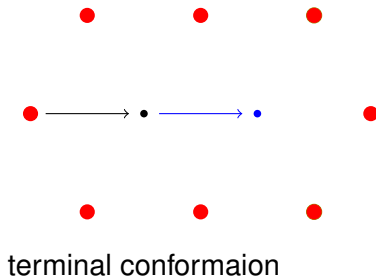


free hands ≤ -1

Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

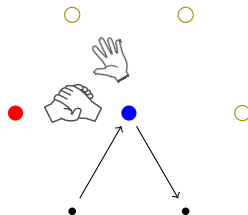
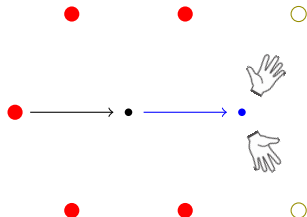
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Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

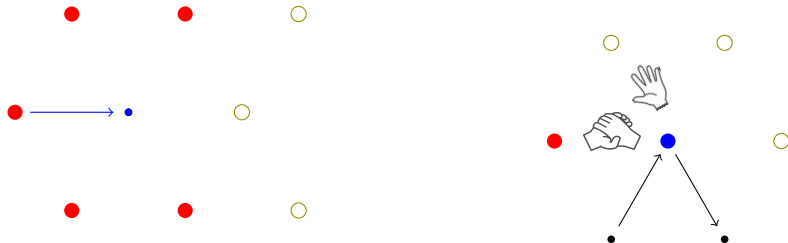
$\alpha = 2$ At least one free hand is decreased / tunnel.



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

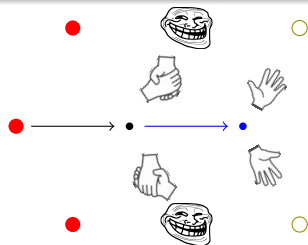
$\alpha = 2$ At least one free hand is decreased / tunnel.



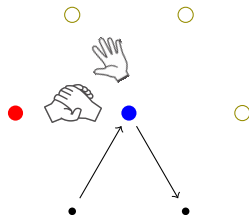
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

$\alpha = 2$ At least one free hand is decreased / tunnel.



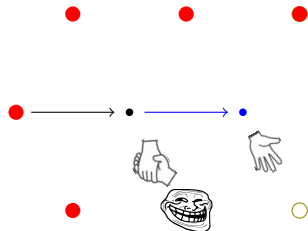
free hands ≤ 0



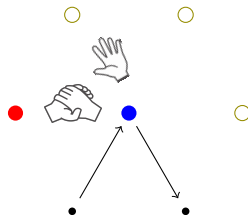
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

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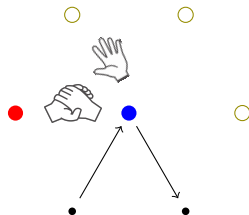
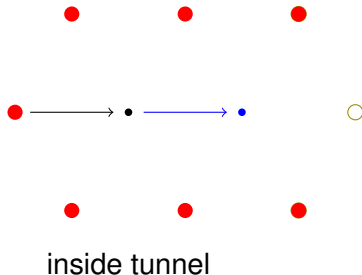
free hands ≤ 0



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

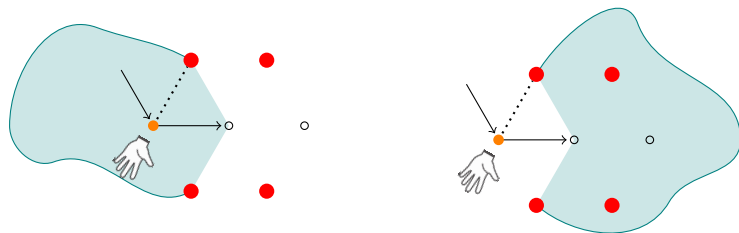
$\alpha = 2$ At least one free hand is decreased / tunnel.



Deterministic unary oritatami systems at delay 1

Jordan curve theorem (Hales 2007)

A closed curve which is a non-self-intersecting divides into inside and outside.



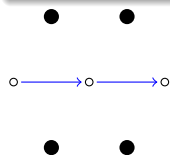
At $\alpha = 2$, Troll consumes free hands an entrance of tunnel, too.

Thank you for listening!!

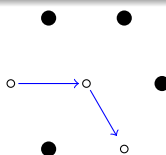
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

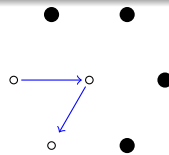
Type of tunnel sections



Straight



Obtuse

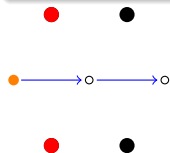


Accute

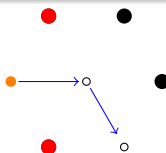
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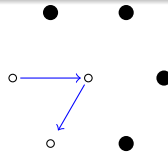
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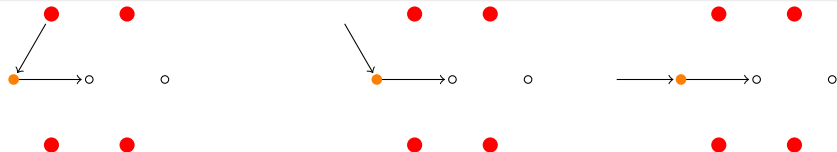


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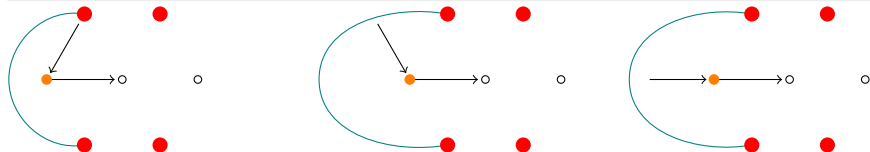
Directions to enter a tunnel



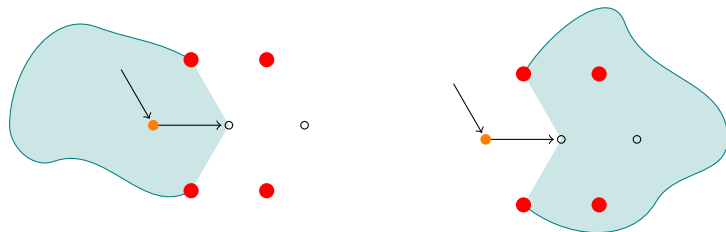
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Tunnel Troll Theorem ($\alpha = 2$)

Directions to enter a tunnel



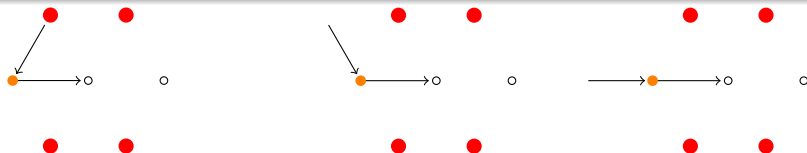
freehands $\leq \pm 0$



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

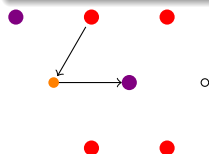
If this orange bead is stabilized by bonds, total bonds decrease.



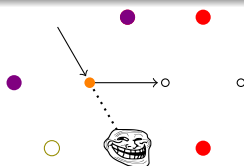
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

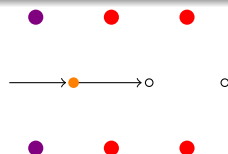
If this orange bead is stabilized by bonds, total bonds decrease.



cannot follow



Troll...



inside tunnel

freehands ≤ -1

