

# On the power of oritatami cotranscriptional folding with unary bead sequence<sup>0</sup>

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科学技術振興機構  
Japan Science and Technology Agency

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# What is oritatami system?

Oritatami system is a mathematical model for cotranscriptional folding(CF). (Geary, Meunier, Schabanel and Seki.)

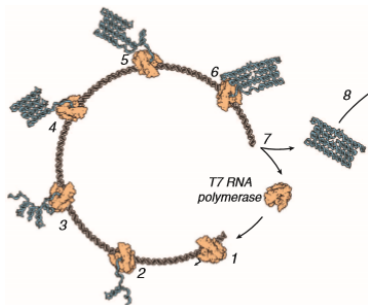


Figure: RNA tile is self-assembled (RNA Origami)

(C. Geary, P.W.K. Rothmund and E.S. Andersen. Science 345(6198), 2014)



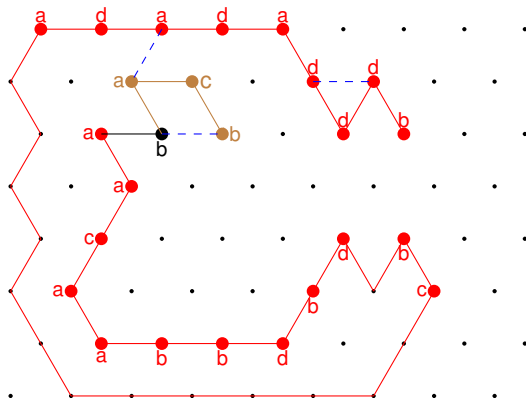




# How oritatami system works?

## An example

$\Sigma = \{a, b, c, d\}$ ,  $R = \{(a, a), (b, b), (c, c), (d, d)\}$ ,  
arity  $\alpha = 2$ , delay  $\delta = 3$ ,  $w = \underline{bacb}cadbcbab$





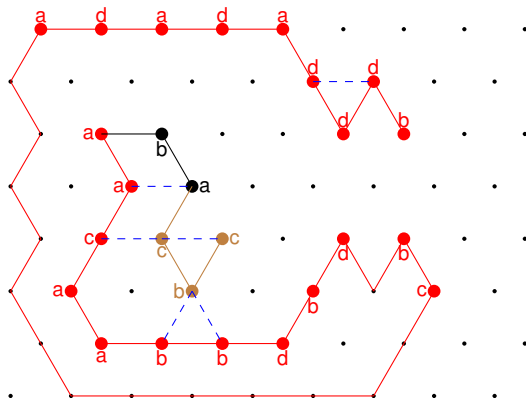




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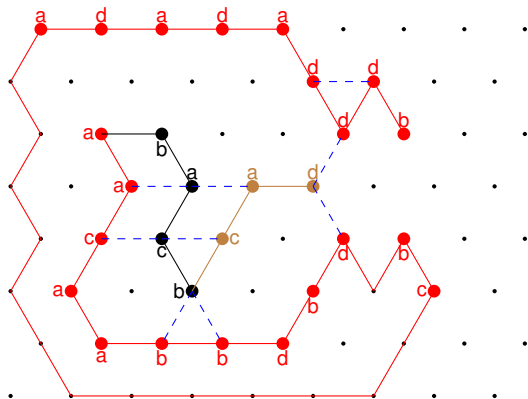




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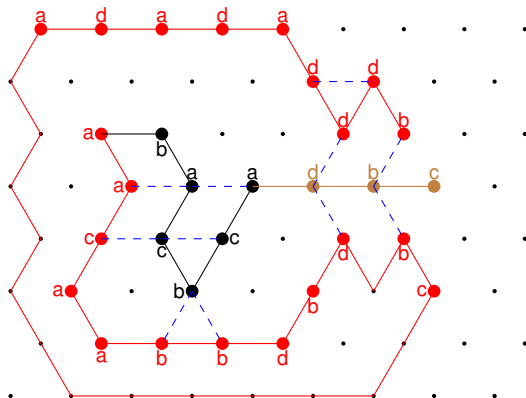




## How oritatami system works?

## An example

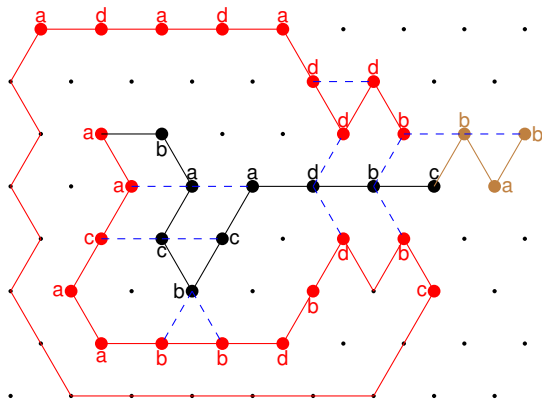
$\Sigma = \{a, b, c, d\}$ ,  $R = \{(a, a), (b, b), (c, c), (d, d)\}$ ,  
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# A deterministic oritatami system

## An example

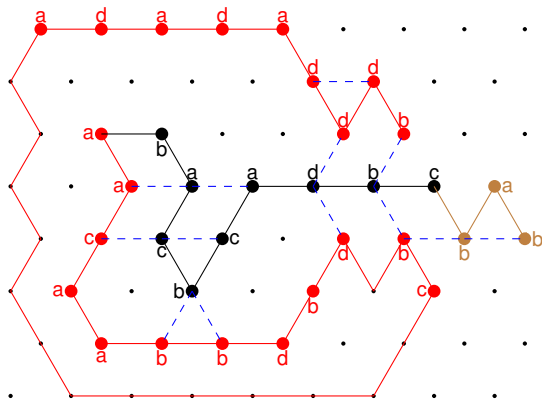
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# A deterministic oritatami system

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$\Sigma = \{a, b, c, d\}$ ,  $R = \{(a, a), (b, b), (c, c), (d, d)\}$ ,  
arity  $\alpha = 2$ , delay  $\delta = 3$ ,  $w = \text{bacbcadbcbab}$







# Turing universal

Theorem (C. Geary *et al.* ISAAC, 2018)

Oritatami system at *delay*  $\delta = 3$  which employs 542 types of beads is Turing universal.

Theorem

Polynomial length of conformations  $\longrightarrow$  Non-Turing-universal

# Problem

## Problem

Give an upper bound on the length of a transcript of a *delay*  $\delta$ , *arity*  $\alpha$  deterministic oritatami system by a function in  $\delta$ ,  $\alpha$ , and seed  $n$ .

## Oritatami System

input : *delay*  $\delta$ , *arity*  $\alpha$ , seed, rule, transcript  
output : conformation

# Why unary?

Because we considered the unary oritatami system is good for a first step towards the characterization of non-Turing-universal oritatami systems.

# Cases of non-Turing-universal oritatami systems

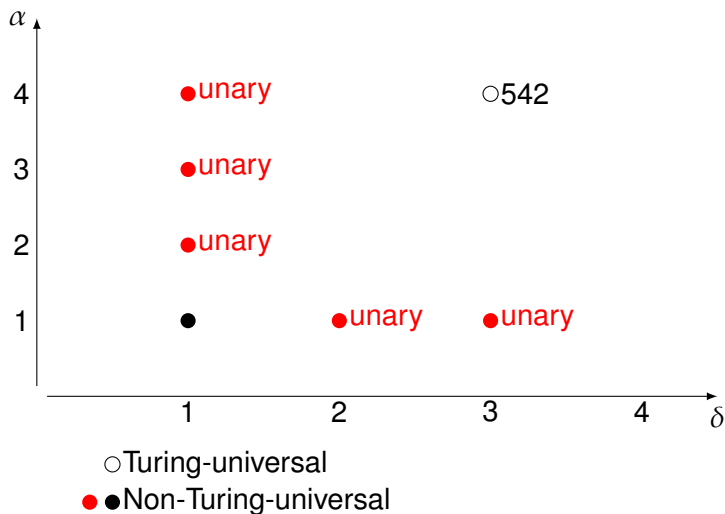


Figure: Cases of non-Turing-universal oritatami systems

# Cases of non-Turing-universal oritatami systems

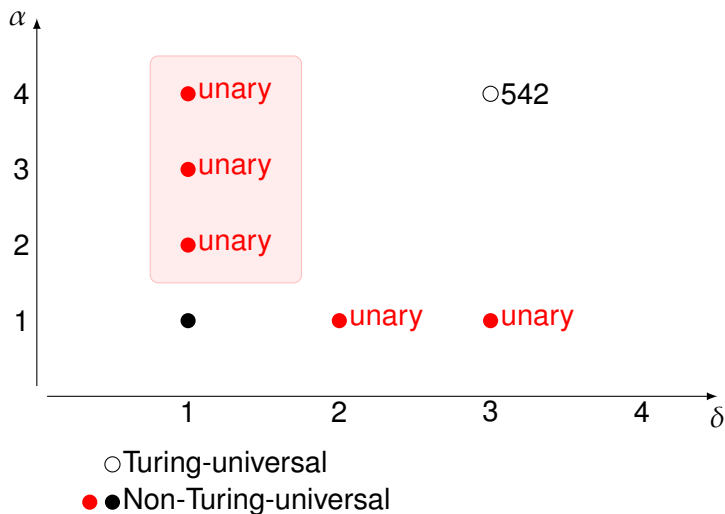
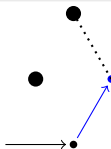


Figure: Cases of non-Turing-universal oritatami systems

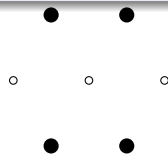
# Deterministic unary oritatami systems at delay 1

## Two ways for a bead stabilization

- To be bound to another bead.
- Through a 1-in-1-out structure called the tunnel section.



Bond

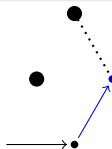


Tunnel section

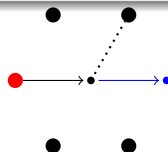
# Deterministic unary oritatami systems at delay 1

## Two ways for a bead stabilization

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Bond



Tunnel section

# Deterministic unary oritatami systems at delay 1

## Results ( $\delta = 1$ )

$$\alpha = 4 \quad 3n^2 + 3n + 1$$

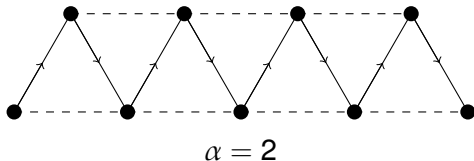
$$\alpha = 3 \quad 4n + 14$$

$$\alpha = 2 \quad \infty \text{ but zigzag after } (27n^2 + 9n + 1)$$

<sup>a</sup> $a$

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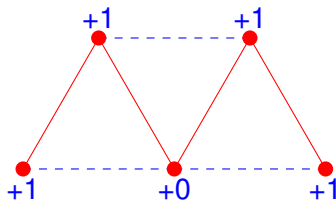
<sup>a</sup>c.f.  $\alpha = 1$ :  $10n$  (Demaine et al. 2018 DNA24)





# zig-zag conformation

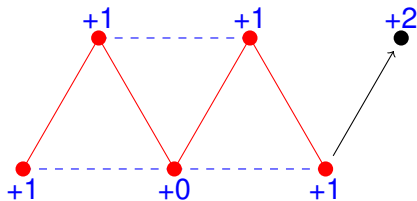
Deterministic oritatami system at  $\delta = 1$  and at  $\alpha = 2$  can make zig-zag conformation



$$\alpha = 2$$

# zig-zag conformation

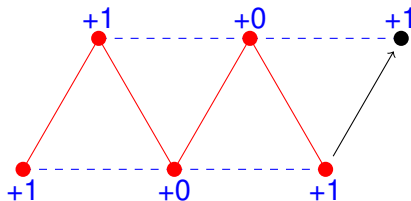
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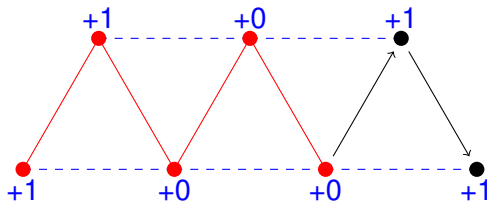
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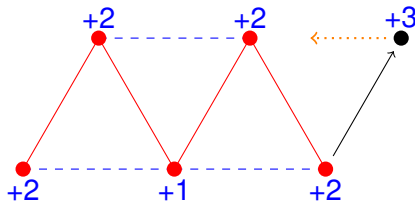
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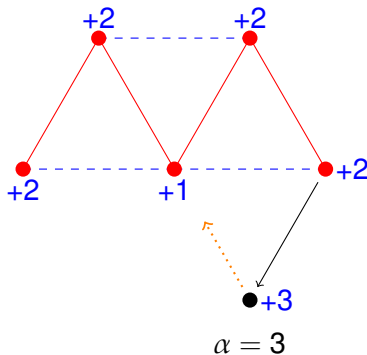
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$$\alpha = 3$$

# zig-zag conformation

Deterministic oritatami system at  $\delta = 1$  and at  $\alpha = 2$  can make zig-zag conformation



# Deterministic unary oritatami systems at delay 1

## Results ( $\delta = 1$ )

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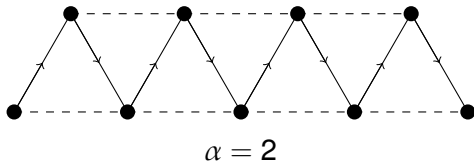
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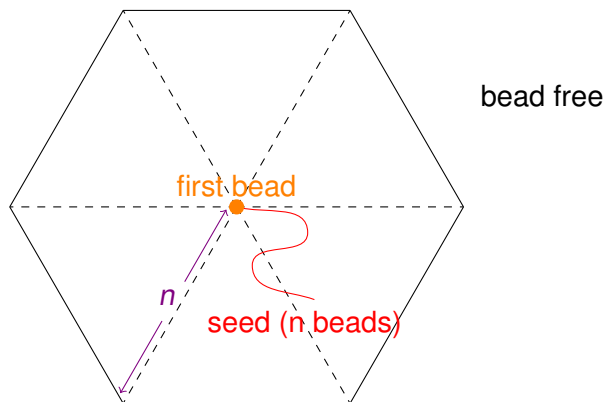
<sup>a</sup>c.f.  $\alpha = 1$ :  $10n$  (Demaine et al. 2018 DNA24)



# Deterministic unary oritatami systems at delay 1

$$\alpha = 4$$

The terminal conformation at  $\alpha = 4$  is of length at most  $3n^2 + 3n + 1(\odot_O^n)$ .

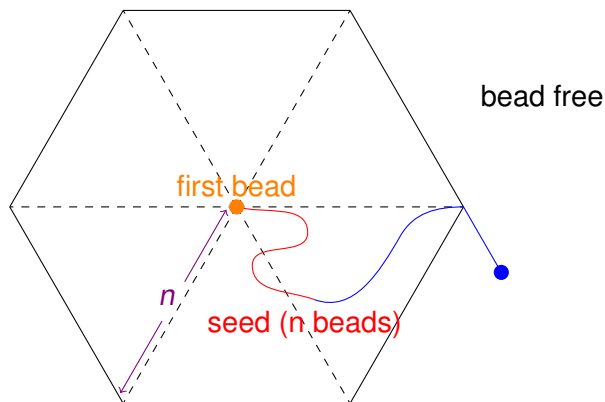




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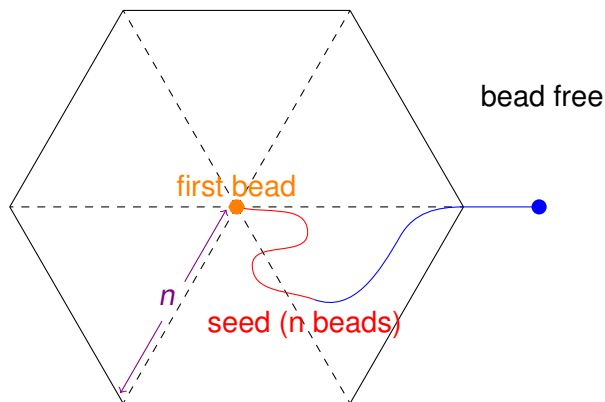
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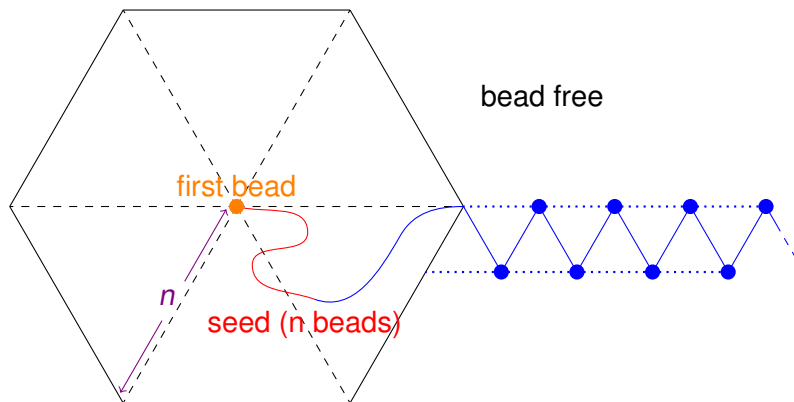
The terminal conformation at  $\alpha = 4$  is of length at most  $3n^2 + 3n + 1(\odot_O^n)$ .



# Deterministic unary oritatami systems at delay 1

$$\alpha = 2 \ (\delta = 1)$$

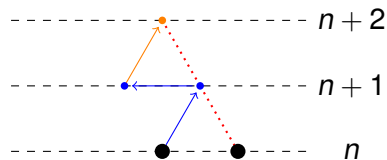
A transcript folds into the zig-zag conformation after its  $(27n^2 + 9n + 1)$ -th bead ( $\odot_{\mathcal{O}}^{3n}$ ).



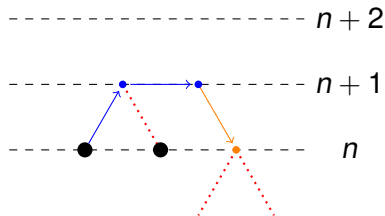
# Deterministic unary oritatami systems at delay 1

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A transcript folds into the zig-zag conformation after its  $(27n^2 + 9n + 1)$ -th bead ( $\square_O^{3n}$ ).



zig-zag conformation

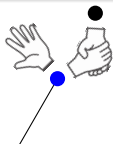


free hands  $\leq 2$

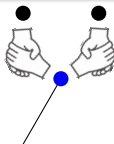
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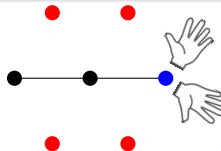
A transcript folds into the zig-zag conformation after its  $(27n^2 + 9n + 1)$ -th bead ( $\square_O^{3n}$ ).



free hands =  $\pm 0$



free hands =  $-2$

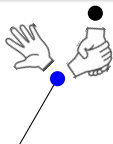


free hands  $\leq +2$

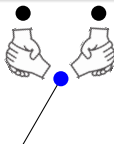
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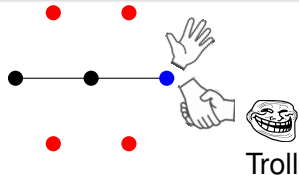
A transcript folds into the zig-zag conformation after its  $(27n^2 + 9n + 1)$ -th bead ( $\hexagon_O^{3n}$ ).



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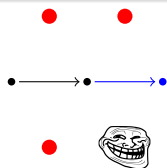
# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

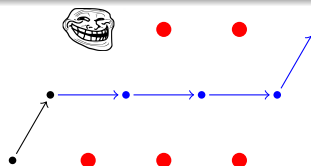
$\alpha \geq 4$  No free hand supplies / tunnel section.

$\alpha = 3$  Troll consumes bonds / tunnel section.

$\alpha = 2$  Troll consumes bonds / tunnel.



Tunnel section

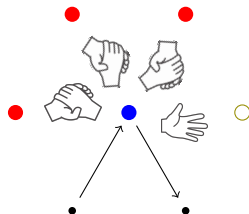
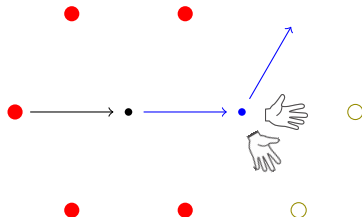


Tunnel

# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

$\alpha \geq 4$  Any hands are not supplied with using a tunnel section.

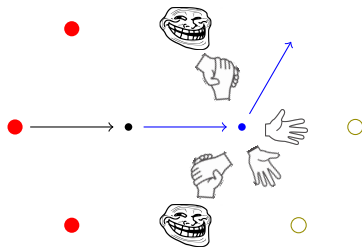




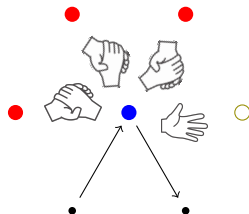
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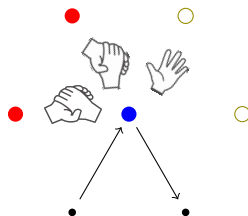
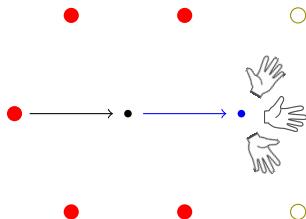
free hands  $\leq 0$



# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

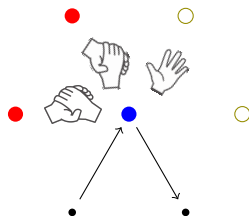
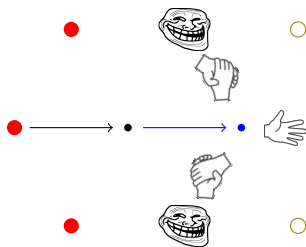
$\alpha = 3$  At least one free hand is decreased / tunnel section.



# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

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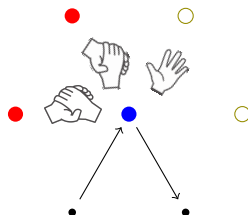
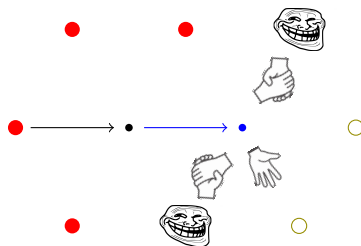


free hands  $\leq -1$

# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

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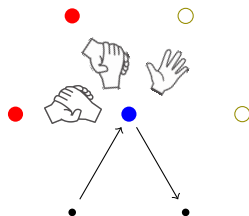
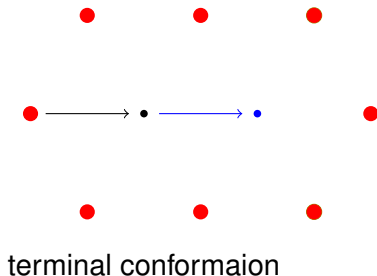


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# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

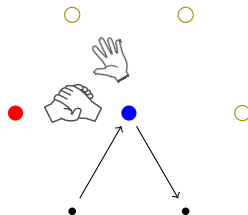
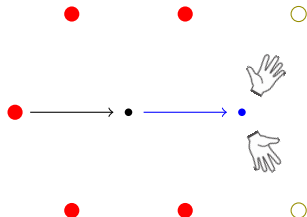
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# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

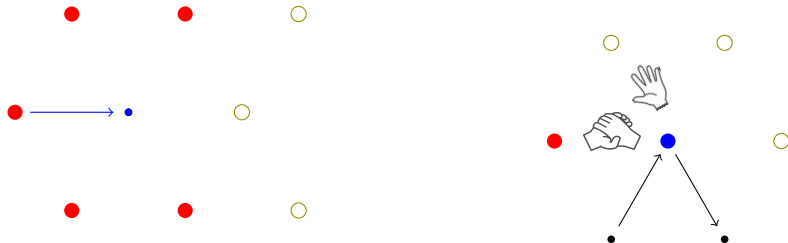
$\alpha = 2$  At least one free hand is decreased / tunnel.



# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

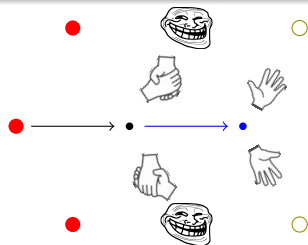
$\alpha = 2$  At least one free hand is decreased / tunnel.



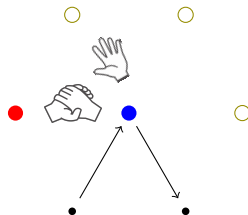
# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

$\alpha = 2$  At least one free hand is decreased / tunnel.



free hands  $\leq 0$

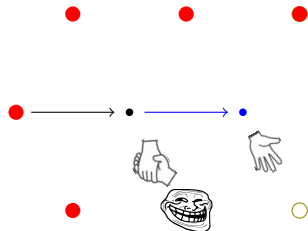




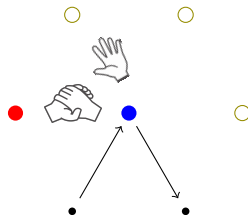
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## Tunnel Troll Theorem

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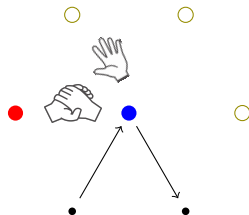
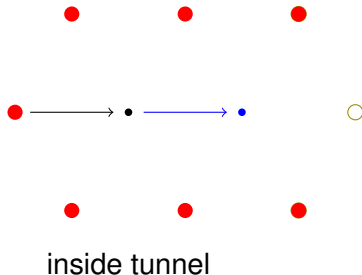
free hands  $\leq 0$



# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem

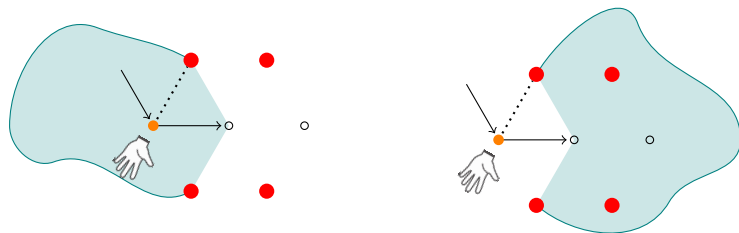
$\alpha = 2$  At least one free hand is decreased / tunnel.



# Deterministic unary oritatami systems at delay 1

## Jordan curve theorem (Hales 2007)

A closed curve which is a non-self-intersecting divides into inside and outside.



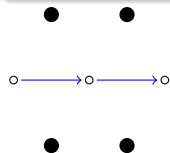
At  $\alpha = 2$ , Troll consumes free hands an entrance of tunnel, too.

Thank you for listening!!

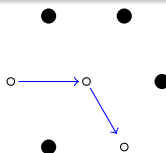
# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem ( $\alpha = 2$ )

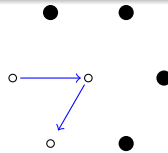
Type of tunnel sections



Straight



Obtuse

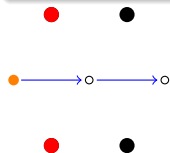


Accute

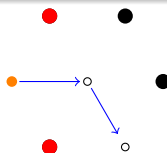
# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem ( $\alpha = 2$ )

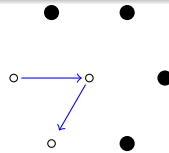
Type of tunnel sections



Straight



Obtuse

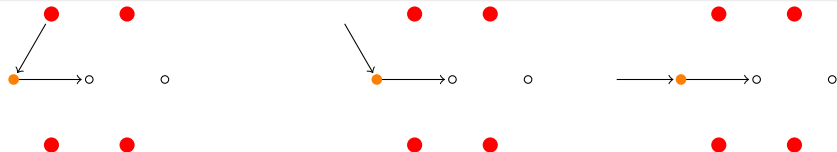


Accute

# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem ( $\alpha = 2$ )

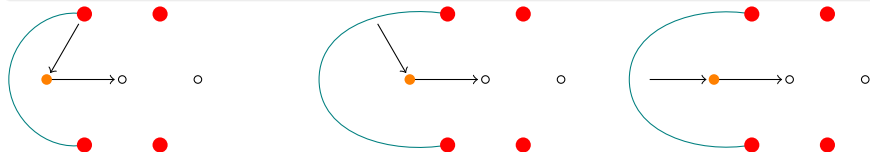
Directions to enter a tunnel



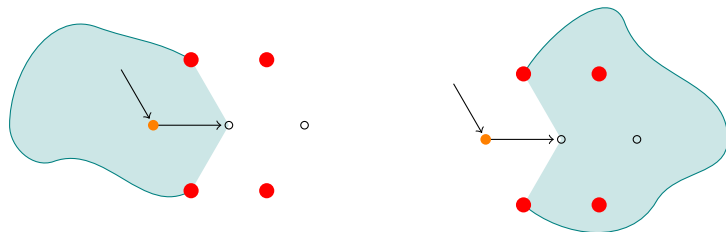
# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem ( $\alpha = 2$ )

Directions to enter a tunnel



freehands  $\leq \pm 0$

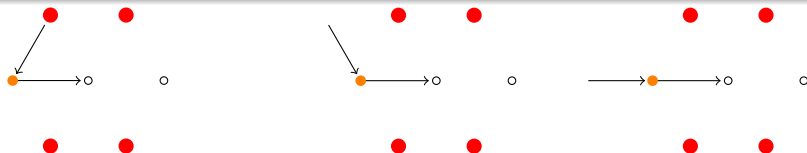




# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem ( $\alpha = 2$ )

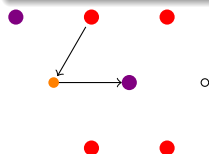
If this orange bead is stabilized by bonds, total bonds decrease.



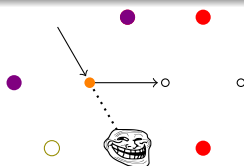
# Deterministic unary oritatami systems at delay 1

## Tunnel Troll Theorem ( $\alpha = 2$ )

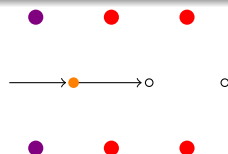
If this orange bead is stabilized by bonds, total bonds decrease.



cannot follow



Troll...



inside tunnel

freehands  $\leq -1$

