On the power of oritatami cotranscriptional folding with unary bead sequence [0]

Szilárd Zsolt Fazekas[1], Kohei Maruyama[2], Reoto Morita[2], Shinnosuke Seki[2]

[1]Akita University, [2]University of Electro-Communications

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What is oritatami system?

Oritatami system is a mathematical model for co-transcriptional folding(CF). (Geary, Meunier, Schabanel and Seki. MFCS 2016.)

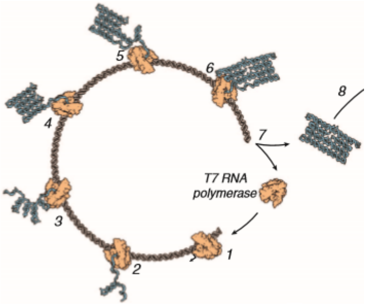


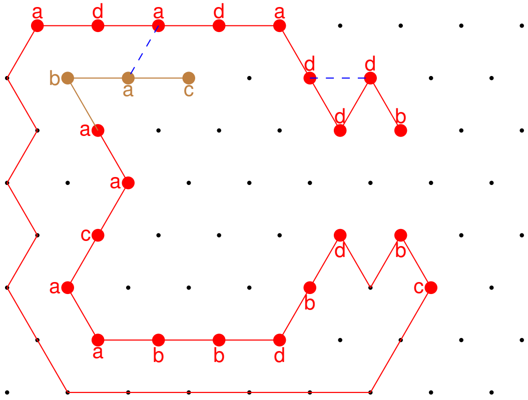
Figure: RNA tile is self-assembled (RNA Origami)

(Geary, Rothemund and Andersen. Science 345(6198), 2014)

How oritatami system works?

An example

$\Sigma = \{a, b, c, d\}$, $R = \{(a,a), (b,b), (c,c), (d,d)\}$,
arity $\alpha = 2$, *delay* $\delta = 3$, $w = \underline{bac}bcadbcbab$



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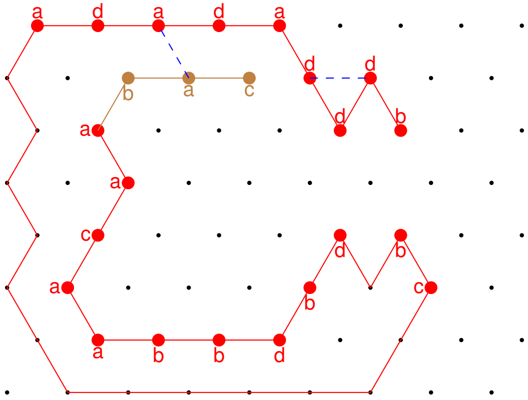

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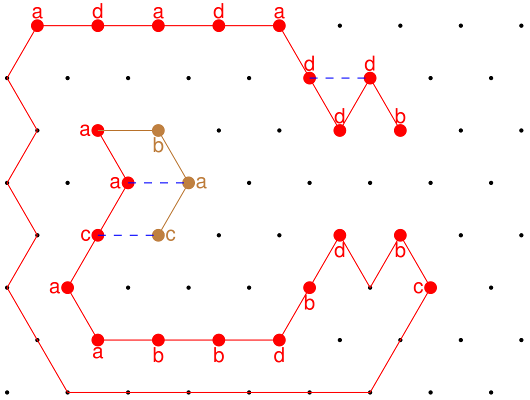

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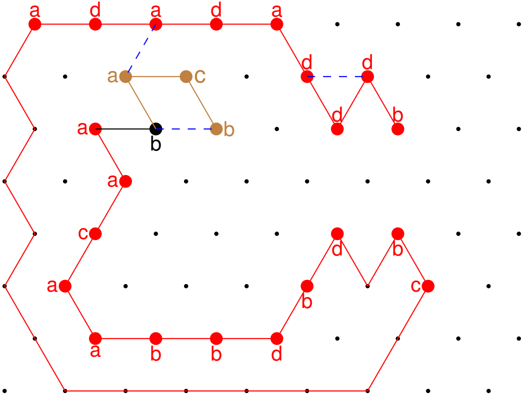

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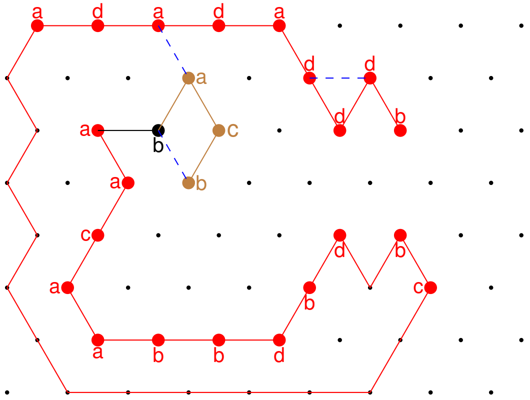

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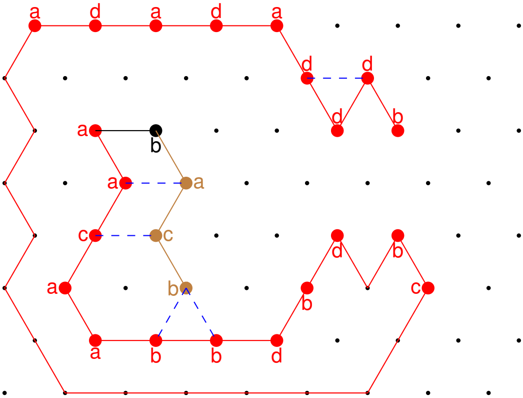

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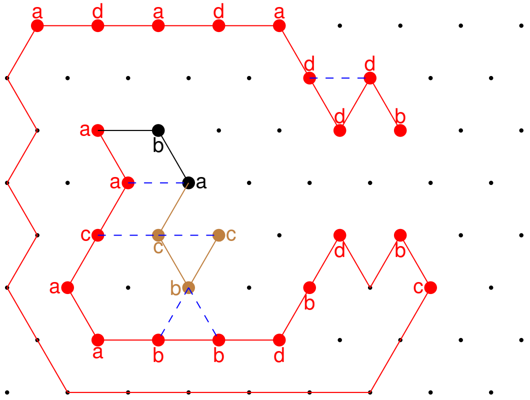

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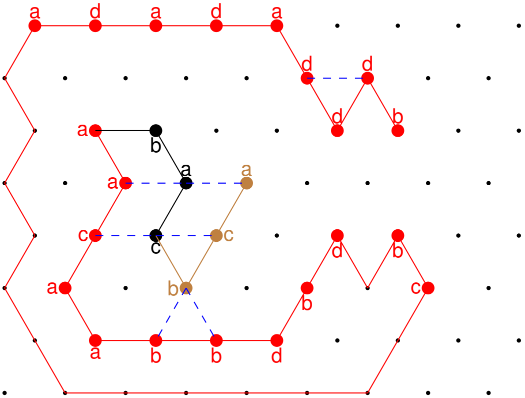

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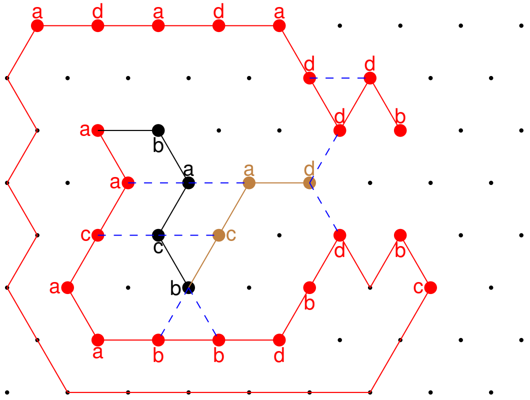

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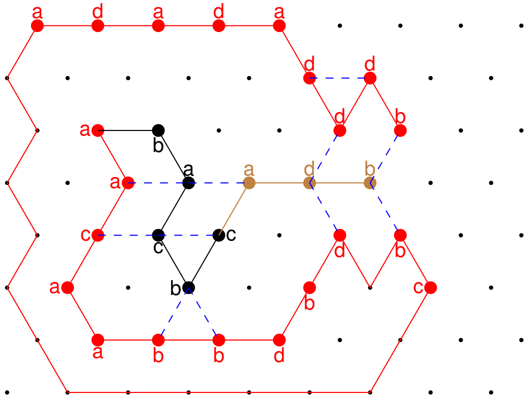

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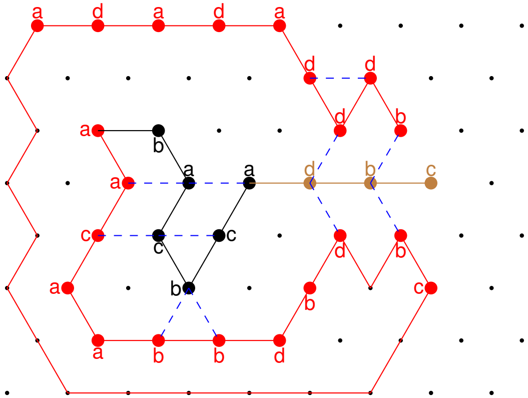

Oritatami system and determinism

An example

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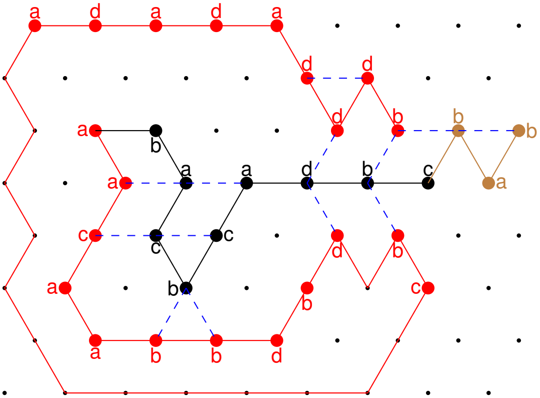

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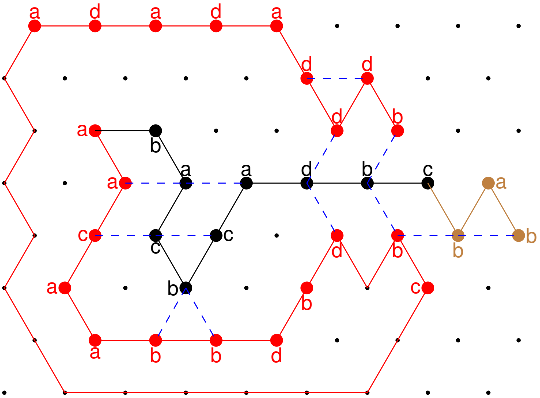


Turing universality

Theorem (Geary, Meunier, Schabanel and Seki. ISAAC 2018.)

Deterministic oritatami system at *delay* $\delta = 3$ are Turing complete. (the simulation needs 542 bead types)

- A smaller Turing universal system
- The characterization of non-Turing-universal systems

Theorem

Polynomial upper bounds on the size of structures
$\longrightarrow$ Not Turing universal

The system at *delay* $\delta = 1$ and *arity* $\alpha = 1$ is not Turing universal. (Demaine et al. DNA24, 2018.)

Problem

Problem

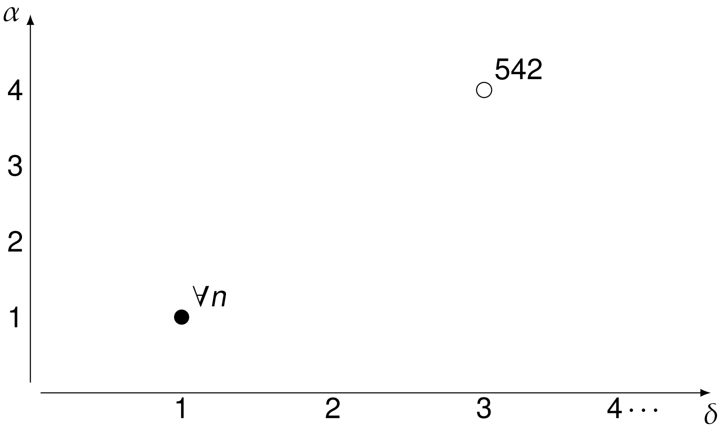
Give an upper bound on the length of a transcript of a *delay* δ, *arity* α deterministic oritatami system by a function in δ, α, and seed length *n*.

- input : *delay* δ, *arity* α and seed of length *n*
- output : an upper bound

Unary variant

Let's focus on unary.

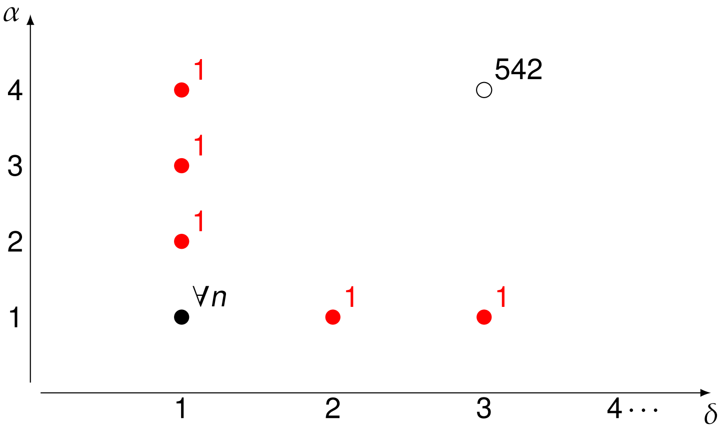
Turing universality and oritatami systems



○Turing-universal
●Not Turing universal

Superscript specifies
the alphabet size

Turing universality and oritatami systems



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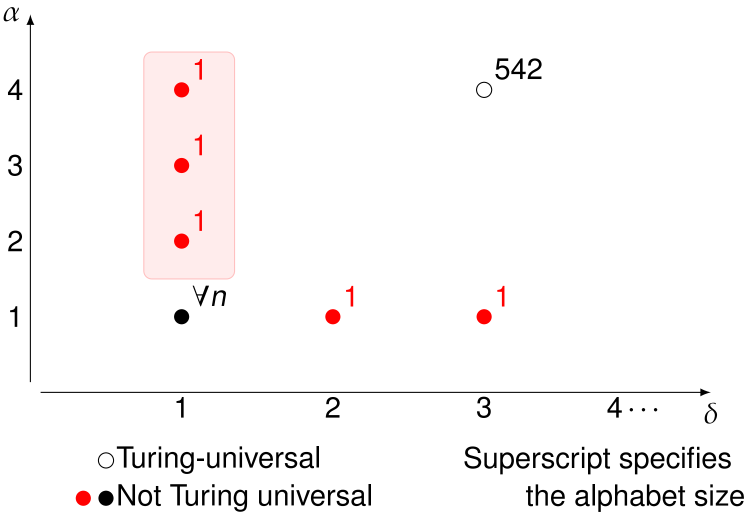
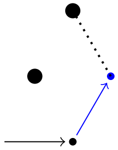


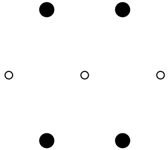
Oritatami systems at delay 1

The two ways to stabilize a bead at delay 1

- To be bound to another bead.
- Through a 1-in-1-out structure called the tunnel section.



Bond



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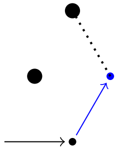

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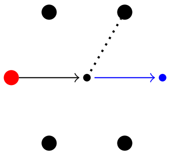


Tunnel section

Deterministic unary oritatami systems at delay 1

Results ($\delta = 1$)

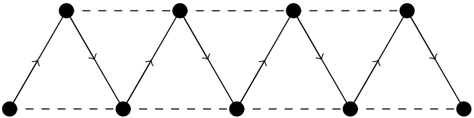
$\alpha = 4$ $3n^2+3n+1$

$\alpha = 3$ $4n+14$

$\alpha = 2$ ∞ but zigzag after $(27n^2+9n+1)$

$*$[a]

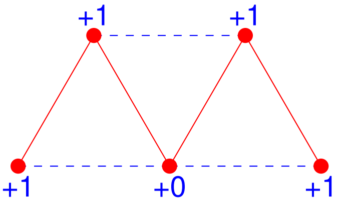
[a] c.f. $\alpha = 1$: $10n$ (Demaine et al. 2018 DNA24)



$\alpha = 2$

zig-zag

Deterministic unary oritatami system at $\delta = 1$ and at $\alpha = 2$ can make zig-zag but cannot at larger arity



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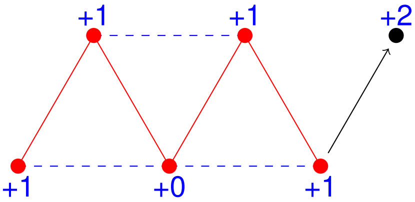

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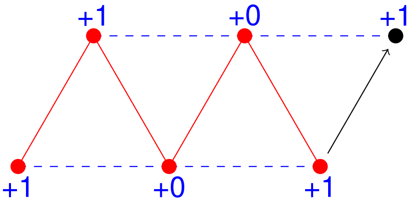

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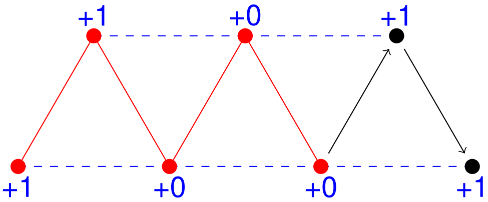

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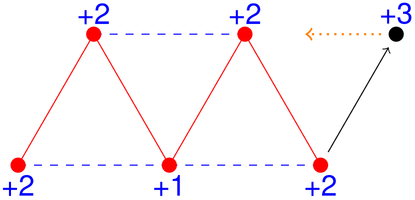

$\alpha = 3$

zig-zag

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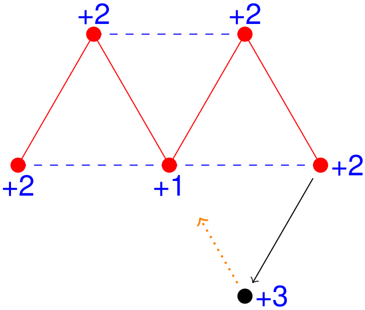

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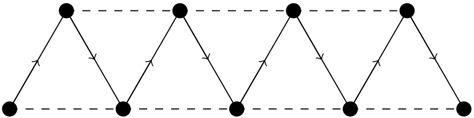

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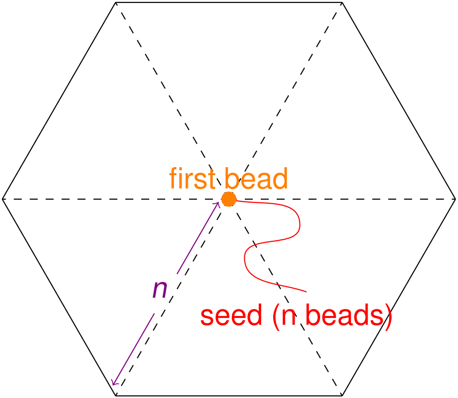
Deterministic unary oritatami systems at delay 1

$\alpha = 4$

The terminal conformation at $\alpha = 4$ is of length at most $3n^2 + 3n + 1(\hexagon_O^n)$.



bead free

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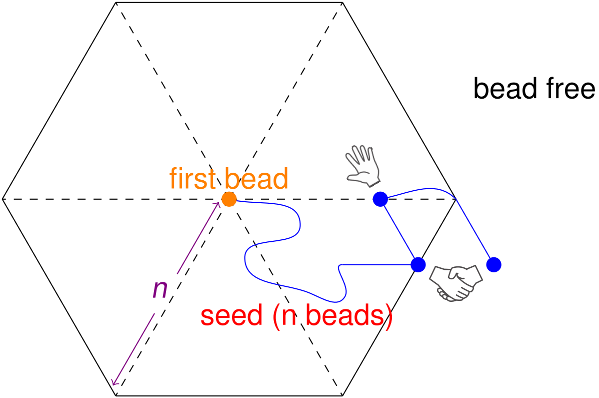

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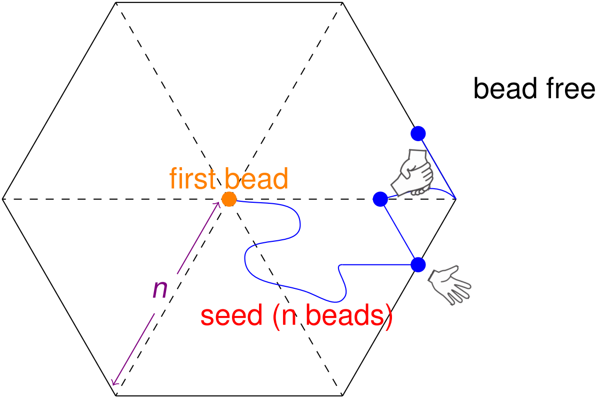
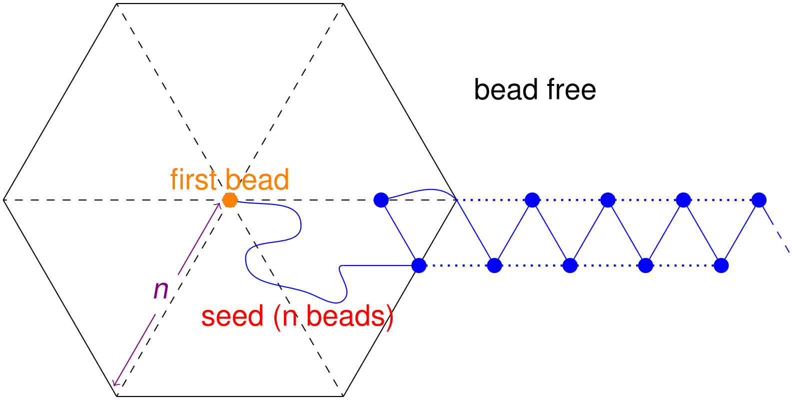


Deterministic unary oritatami systems at delay 1

$\alpha = 2$ ($\delta = 1$)

A transcript folds into the zig-zag conformation after its $(27n^2 + 9n + 1)$-th bead ($\hexagon_O^{3n}$).



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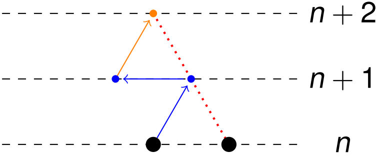

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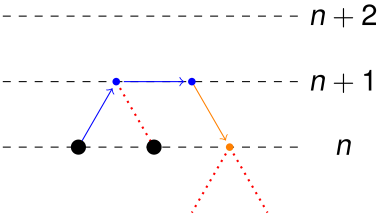

free hands: −2

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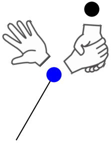
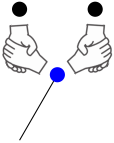
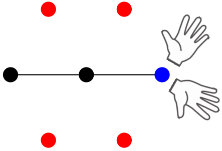


free hands: ± 0



free hands: -2



free hands: $\leq +2$

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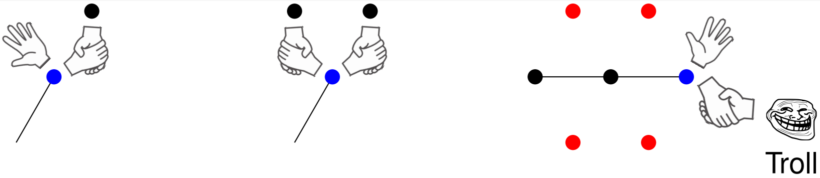


free hands: ± 0 free hands: -2 free hands: ± 0

Tunnel Troll Theorem

Illustrated by Gido

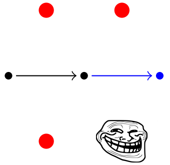
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

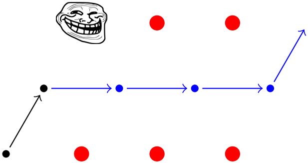
$\alpha \geq 4$ # of free hands does not increase / tunnel section.

$\alpha = 3$ Troll consumes bonds / tunnel section.

$\alpha = 2$ Troll consumes bonds / tunnel.



Tunnel section



Tunnel

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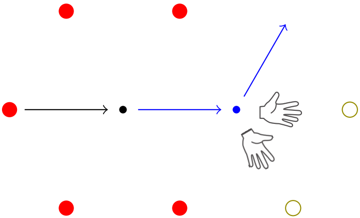
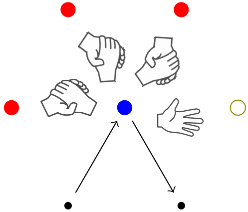
$\alpha \geq 4$ # of free hands does not increase when using a tunnel section.





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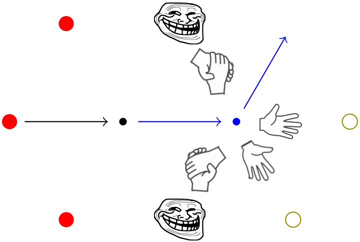
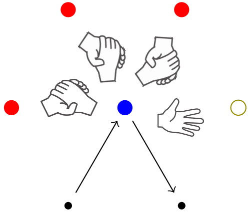
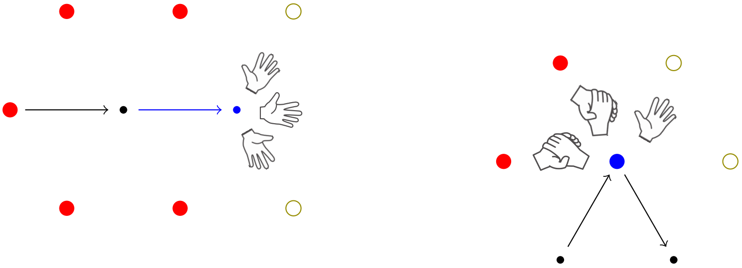




Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

$\alpha = 3$ At least one free hand is consumed / tunnel section.



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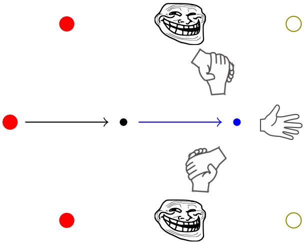
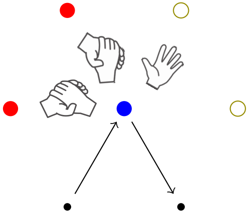


free hands ≤ -1



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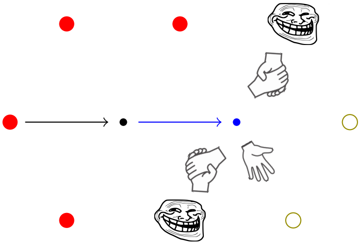
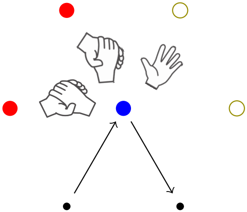


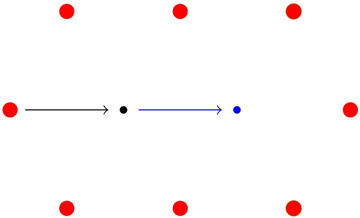


free hands ≤ -1

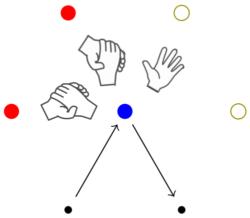
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

$\alpha = 3$ At least one free hand is consumed / tunnel section.

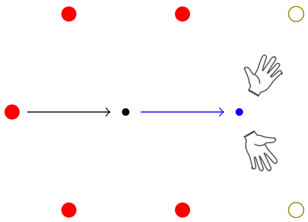


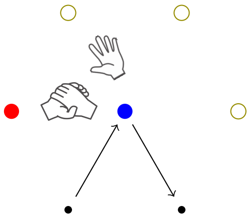
terminal conformation



Tunnel Troll Theorem

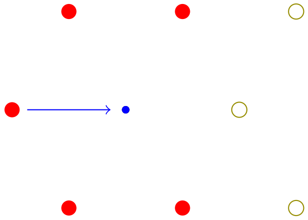
$\alpha = 2$ At least one free hand is consumed / tunnel.

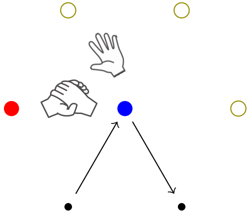




Tunnel Troll Theorem

$\alpha = 2$ At least one free hand is consumed / tunnel.





Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

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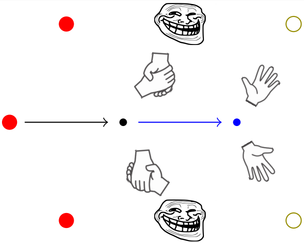
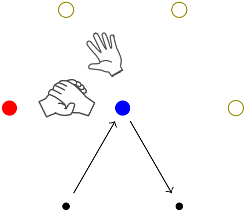


free hands ≤ 0



Tunnel Troll Theorem

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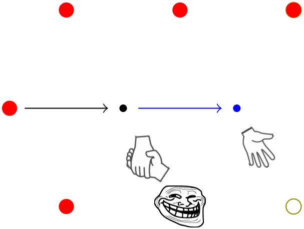
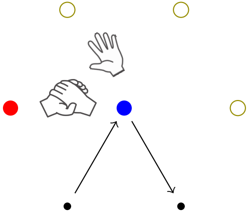


free hands ≤ 0



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

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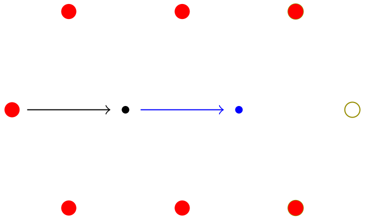
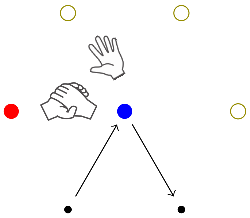
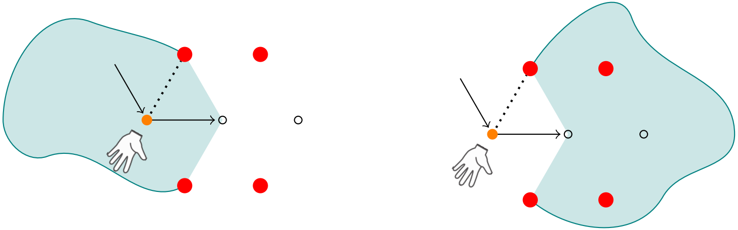


inside tunnel



Jordan curve theorem

A closed curve which is non-self-intersecting, divides the plane into inside and outside.



At $\alpha = 2$, Troll consumes free hands at an entrance of tunnel, too.

ご清聴ありがとうございました。

Szilárd Zsolt Fazekas

Shinnosuke Seki

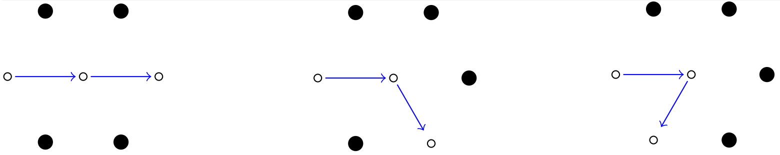
Collaborators

Future works

- Case of $\delta \geq 4$
- Non unary case
- Other ways of simulating Turing machine

Tunnel Troll Theorem ($\alpha = 2$)

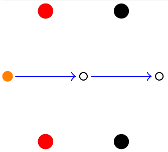
Type of tunnel sections



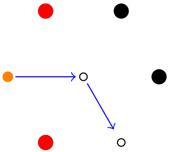
Straight Obtuse Accute

Tunnel Troll Theorem ($\alpha = 2$)

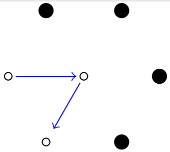
Type of tunnel sections



Straight



Obtuse

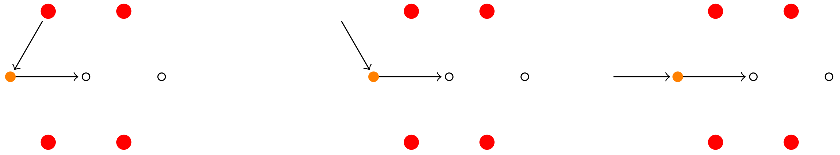


Accute

Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

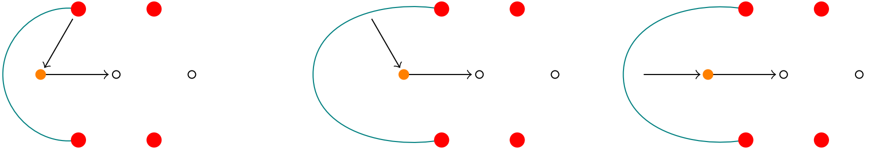
Directions to enter a tunnel



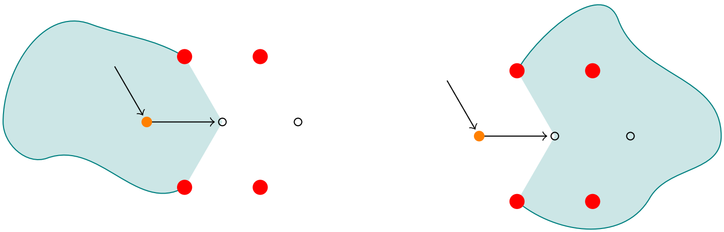
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

Directions to enter a tunnel



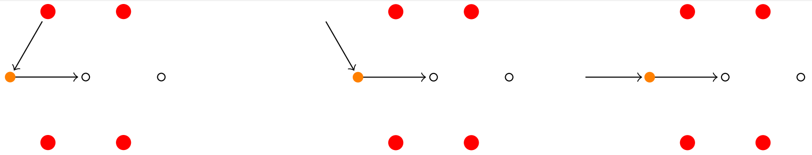
freehands $\leq \pm 0$



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

If this orange bead is stabilized by bonds, total bonds decrease.



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

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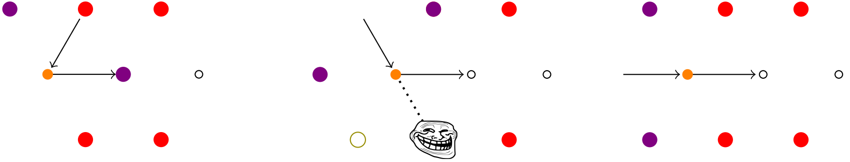

cannot follow

Troll...

inside tunnel

freehands ≤ -1

References I