On the power of oritatami cotranscriptional folding with unary bead sequence [0]

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What is oritatami system?

Oritatami system is a mathematical model for co-transcriptional folding(CF). (Geary, Meunier, Schabanel and Seki. MFCS 2016.)

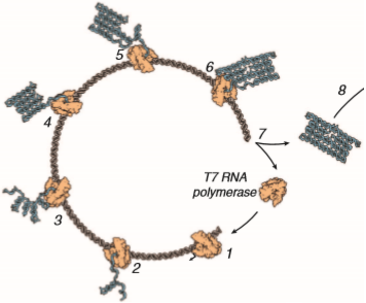
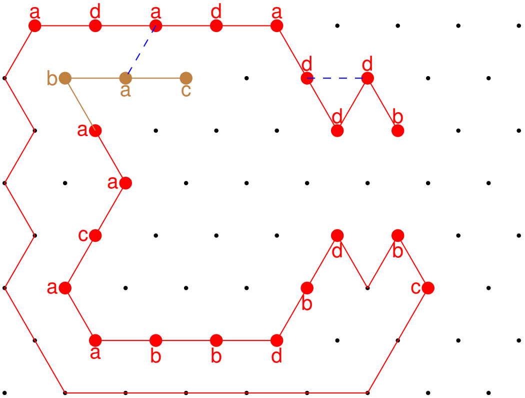


Figure: RNA tile is self-assembled (RNA Origami)

(Geary, Rothemund and Andersen. Science 345(6198), 2014)

An example

$\Sigma = \{a, b, c, d\}$, $R = \{(a, a), (b, b), (c, c), (d, d)\}$,
arity $\alpha = 2$, *delay* $\delta = 3$, $w = \underline{bac}bcadbcbab$



How oritatami system works?

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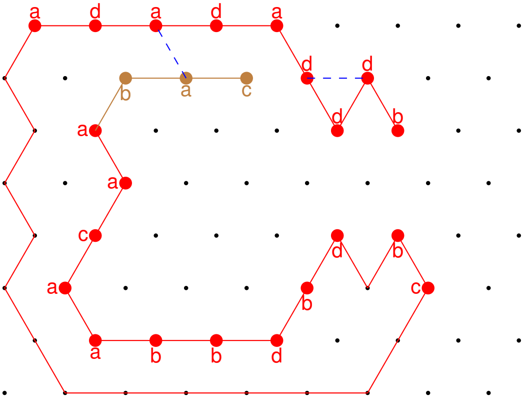

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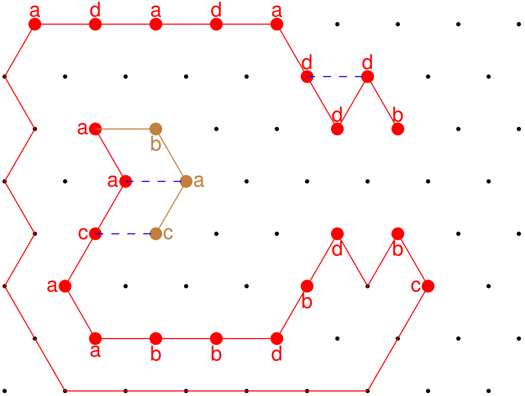

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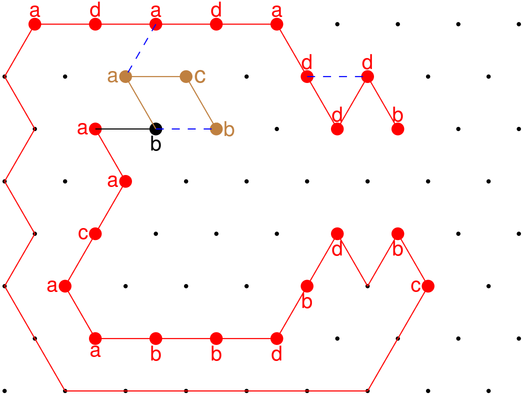

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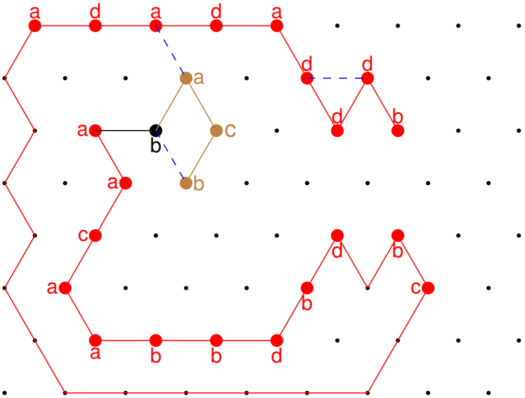

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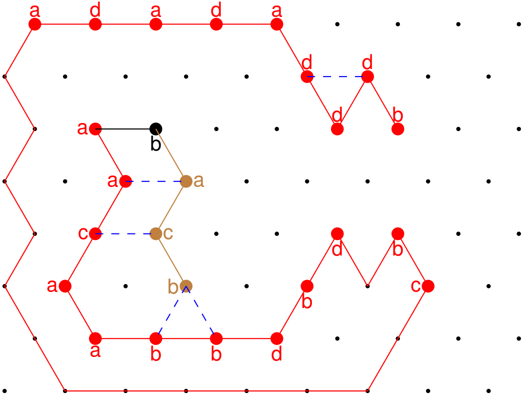

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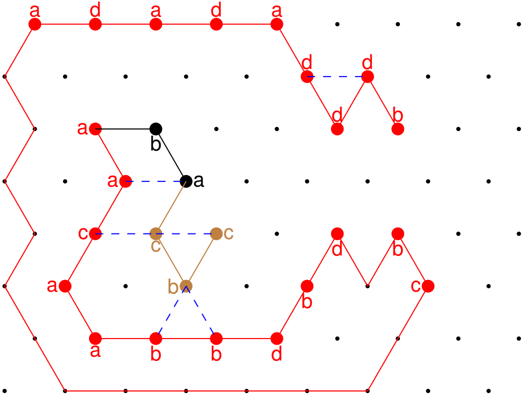

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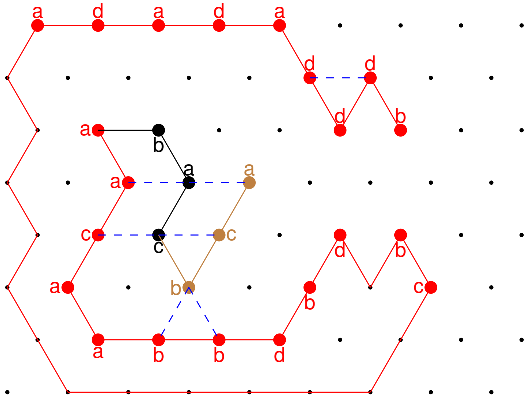

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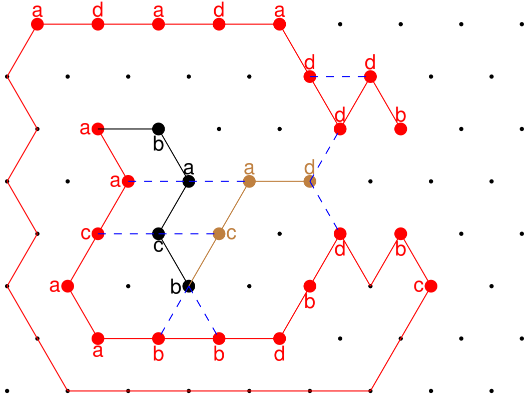

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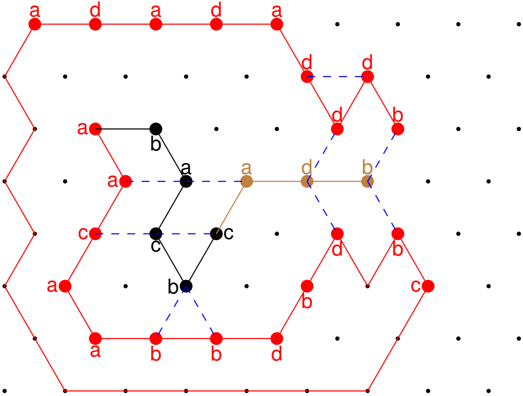

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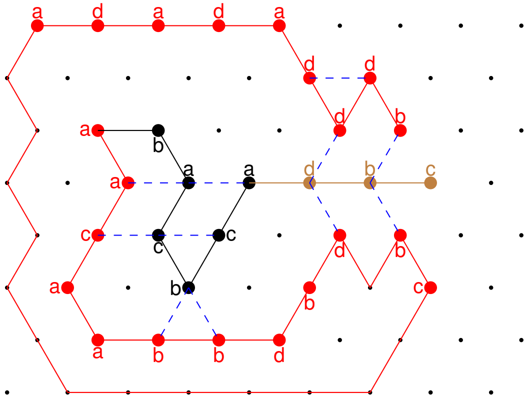

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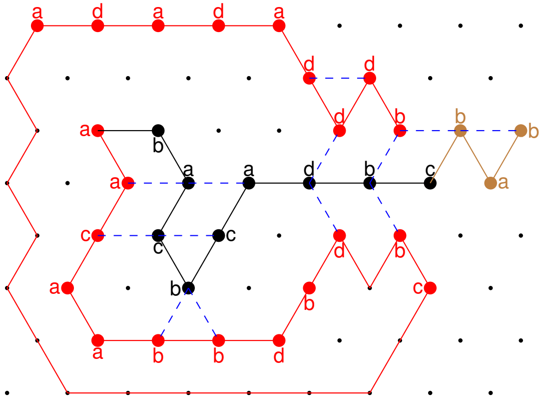

Oritatami system and determinism

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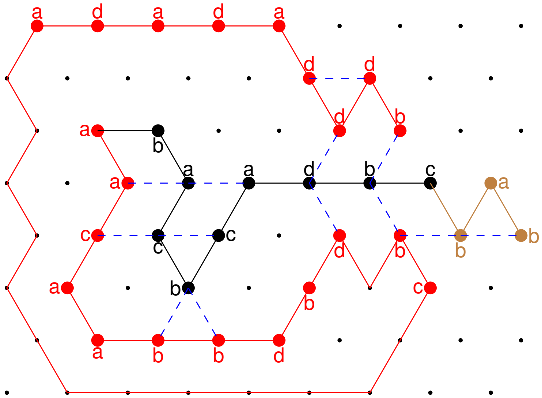


Turing universality

Theorem (Geary, Meunier, Schabanel and Seki. ISAAC 2018.)

The deterministic oritatami system at *delay* $\delta = 3$ with 542 types of beads is Turing universal.

- A smaller Turing universal system
- The characterization of non-Turing-universal systems

Theorem

Polynomial upper bounds on the size of structures
$\longrightarrow$ Not Turing universal

The system at *delay* $\delta = 1$ and *arity* $\alpha = 1$ is not Turing universal. (Demaine et al. DNA24, 2018.)

Problem

Problem

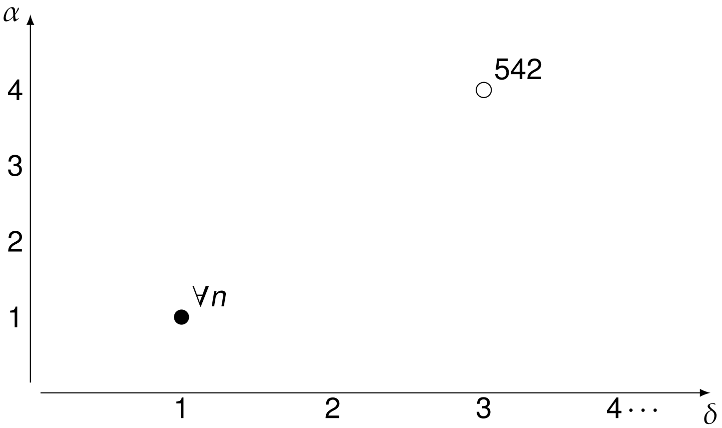
Give an upper bound on the length of a transcript of a *delay* δ, *arity* α deterministic oritatami system by a function in δ, α, and seed *n*.

- input : *delay* δ, *arity* α and seed *n*
- output : an upper bound

Unary variant

Let's focus on unary.

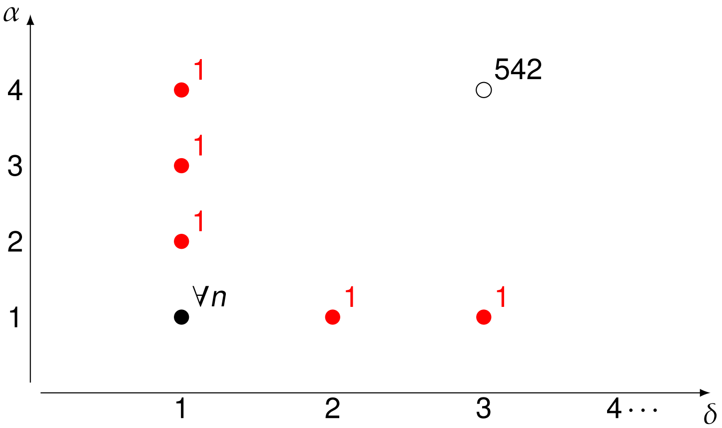
Turing-universal oritatami systems



○Turing-universal
●Not Turing universal

Superscript specifies the alphabet size

Turing-universal oritatami systems



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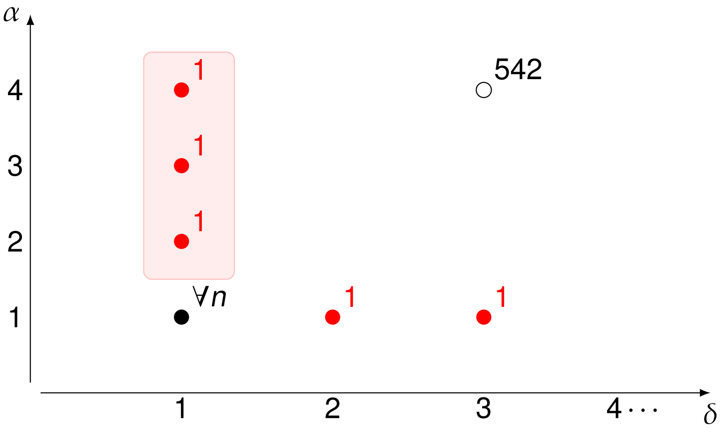

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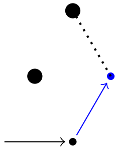
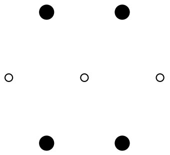
The two ways to stabilize a bead at delay 1

- To be bound to another bead.
- Through a 1-in-1-out structure called the tunnel section.



Bond



Tunnel section

Oritatami systems at delay 1

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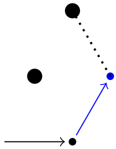

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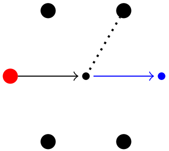


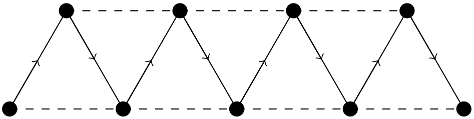
Tunnel section

Deterministic unary oritatami systems at delay 1

Results ($\delta = 1$)

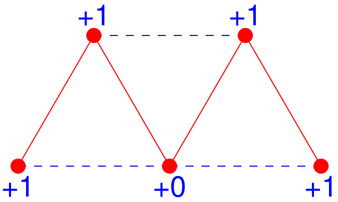
$\alpha = 4$ $3n^2+3n+1$
$\alpha = 3$ $4n+14$
$\alpha = 2$ ∞ but zigzag after $(27n^2+9n+1)$
$*$[a]

[a] c.f. $\alpha = 1$: $10n$ (Demaine et al. 2018 DNA24)



$\alpha = 2$

Deterministic unary oritatami system at $\delta = 1$ and at $\alpha = 2$ can make zig-zag but cannot at larger arity



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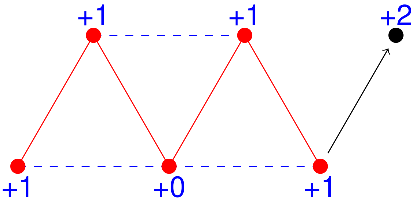

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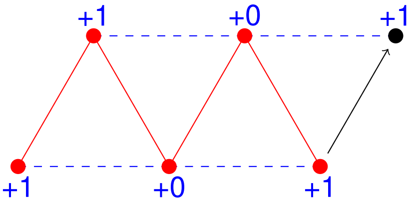

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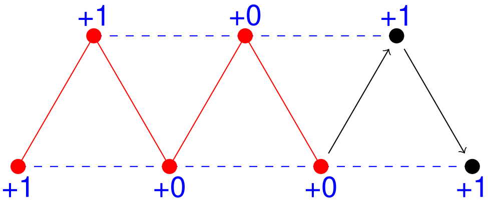

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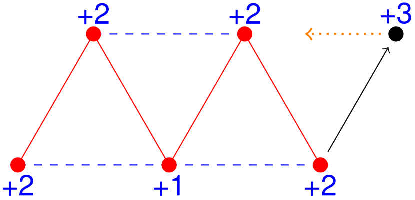

$\alpha = 3$

zig-zag

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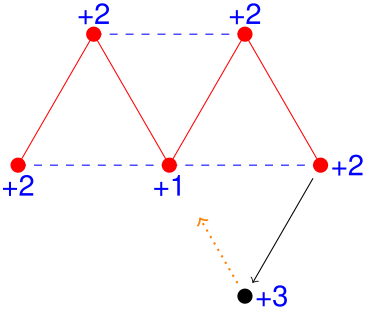

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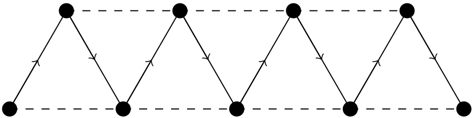
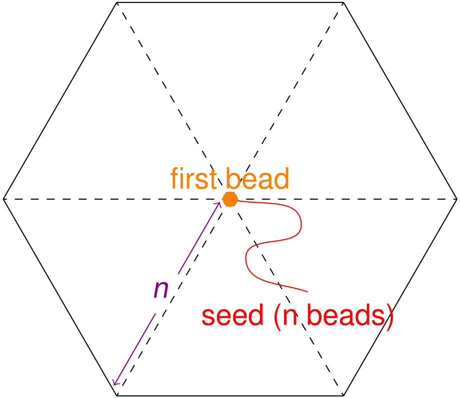


$\alpha = 2$

Deterministic unary oritatami systems at delay 1

$\alpha = 4$

The terminal conformation at $\alpha = 4$ is of length at most $3n^2 + 3n + 1 (\hexagon_O^n)$.



bead free

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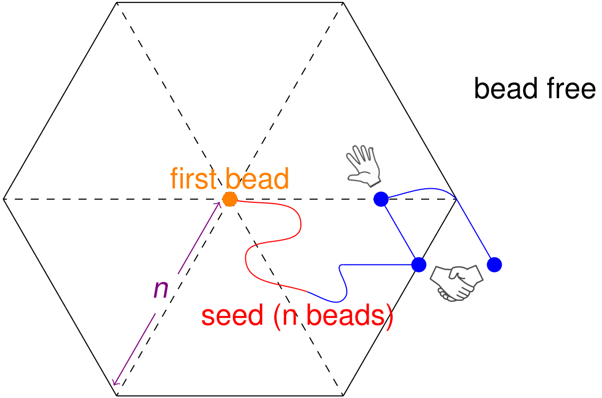

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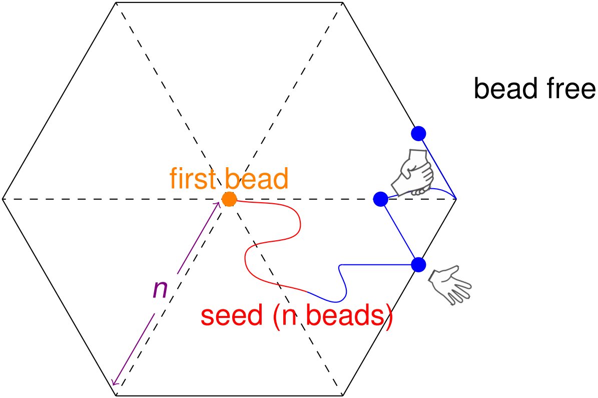
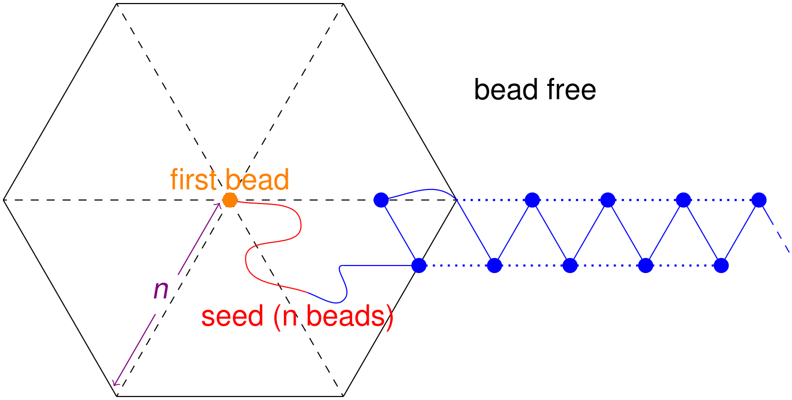


Deterministic unary oritatami systems at delay 1

$\alpha = 2$ ($\delta = 1$)

A transcript folds into the zig-zag conformation after its $(27n^2 + 9n + 1)$-th bead ($\hexagon_O^{3n}$).



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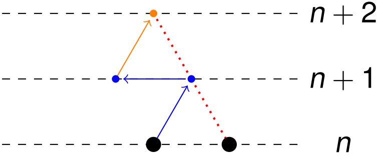

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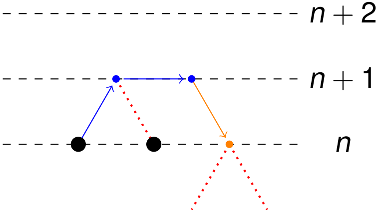

free hands: -2

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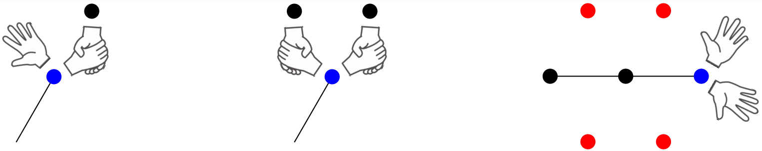

free hands: ± 0 free hands: -2 free hands: $\leq +2$

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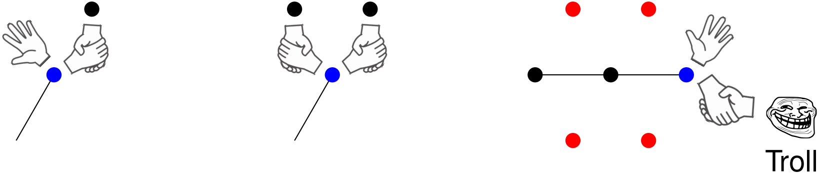


free hands: ± 0 free hands: -2 free hands: ± 0

Tunnel Troll Theorem

Illustrated by Gido

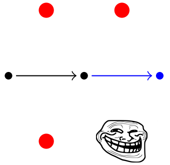
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

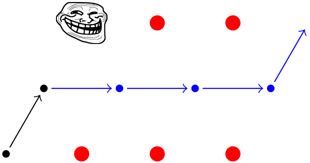
$\alpha \geq 4$ # of free hands does not increase / tunnel section.

$\alpha = 3$ Troll consumes bonds / tunnel section.

$\alpha = 2$ Troll consumes bonds / tunnel.



Tunnel section

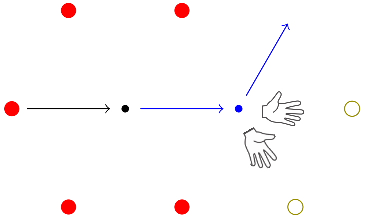


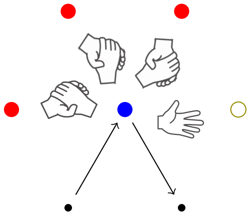
Tunnel

Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

$\alpha \geq 4$ Any hands are not supplied with using a tunnel section.





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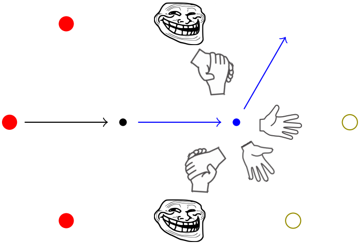
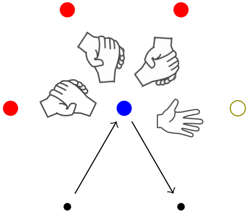
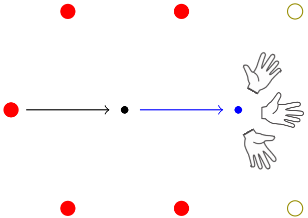


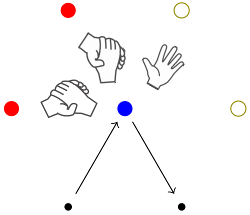
free hands ≤ 0



Tunnel Troll Theorem

$\alpha = 3$ At least one free hand is decreased / tunnel section.





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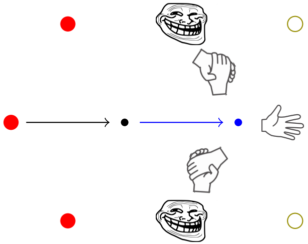
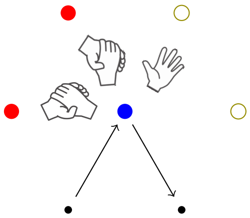


free hands ≤ -1



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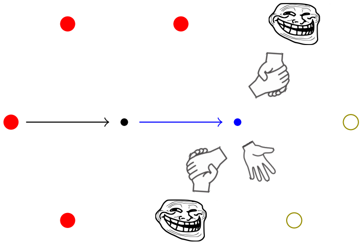

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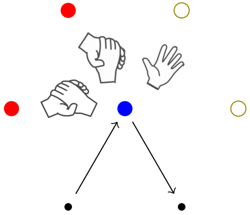

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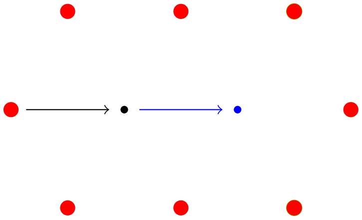
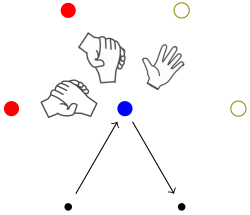
$\alpha = 3$ At least one free hand is decreased / tunnel section.

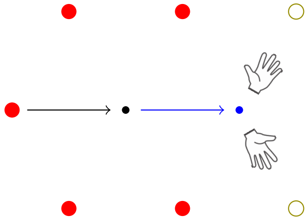


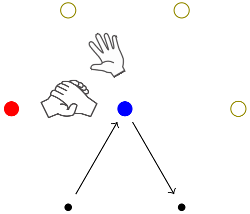
terminal conformaion



Tunnel Troll Theorem

$\alpha = 2$ At least one free hand is decreased / tunnel.

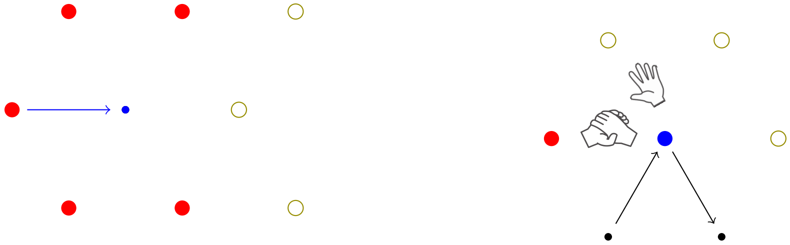




Deterministic unary oritatami systems at delay 1

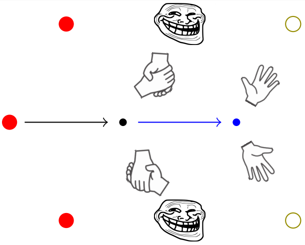
Tunnel Troll Theorem

$\alpha = 2$ At least one free hand is decreased / tunnel.

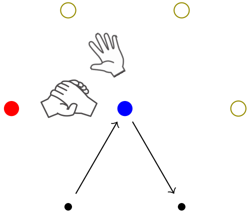


Tunnel Troll Theorem

$\alpha = 2$ At least one free hand is decreased / tunnel.

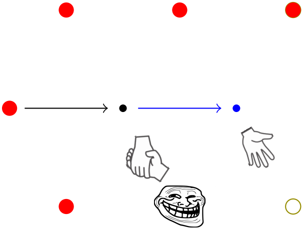


free hands ≤ 0

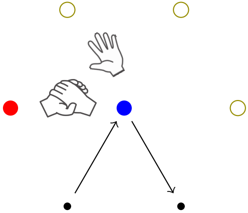


Tunnel Troll Theorem

$\alpha = 2$ At least one free hand is decreased / tunnel.



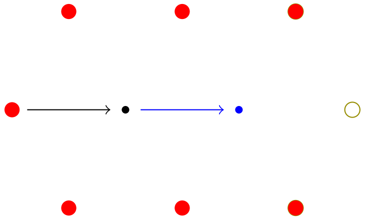
free hands ≤ 0



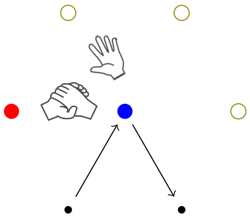
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem

$\alpha = 2$ At least one free hand is decreased / tunnel.



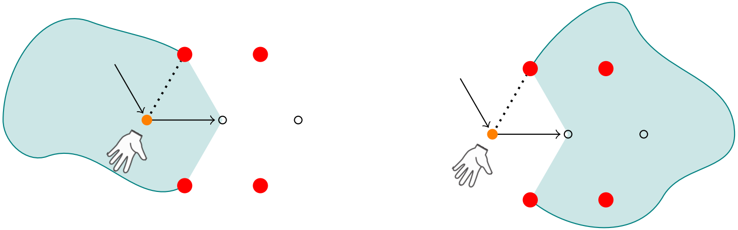
inside tunnel



Deterministic unary oritatami systems at delay 1

Jordan curve theorem

A closed curve which is a non-self-intersecting divides the plane into inside and outside.

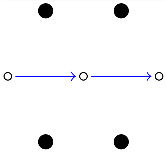


At $\alpha = 2$, Troll consumes free hands at an entrance of tunnel, too.

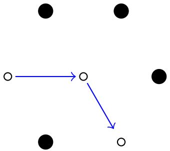
Thank you for listening!!

Tunnel Troll Theorem ($\alpha = 2$)

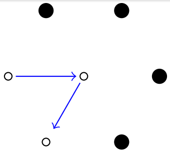
Type of tunnel sections



Straight



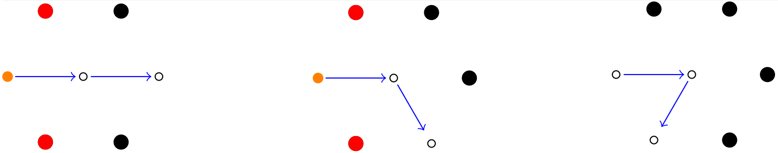
Obtuse



Accute

Tunnel Troll Theorem ($\alpha = 2$)

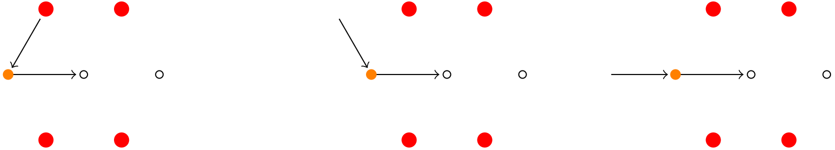
Type of tunnel sections



Straight Obtuse Accute

Tunnel Troll Theorem ($\alpha = 2$)

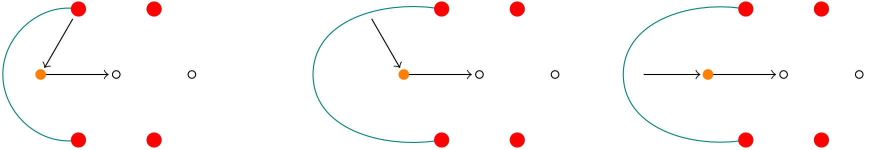
Directions to enter a tunnel



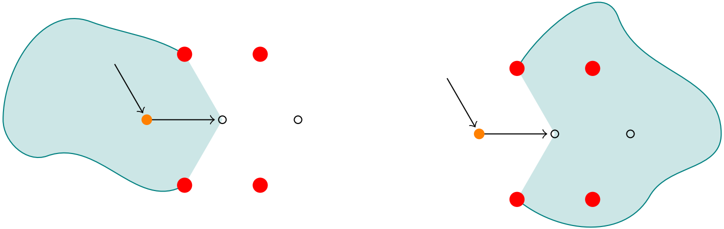
Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

Directions to enter a tunnel

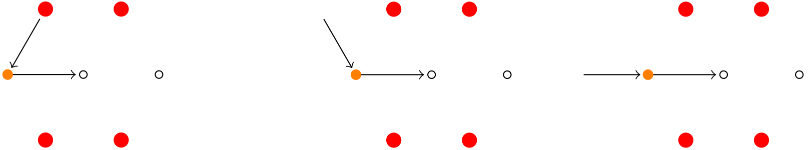


freehands $\leq \pm 0$



Tunnel Troll Theorem ($\alpha = 2$)

If this orange bead is stabilized by bonds, total bonds decrease.



Deterministic unary oritatami systems at delay 1

Tunnel Troll Theorem ($\alpha = 2$)

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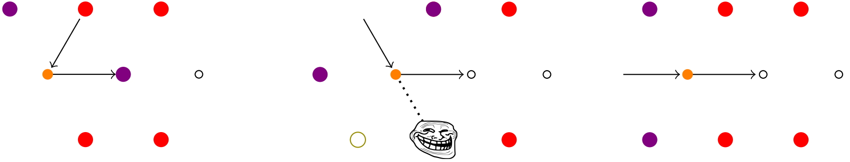

cannot follow　　Troll...　　inside tunnel

freehands ≤ -1

References I