1 Infiniteness of delay-1, arity-2 unary deterministic oritatami system

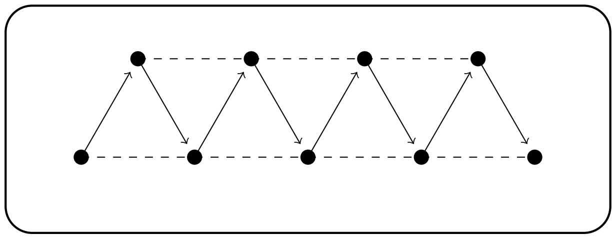


Fig. 1. zig-zag conformation

1.1 Introduction

In this section, we prove that unary oritatami system can form infinitely at delay 1 and arity 1 deterministically moreover the infinite terminal which its oritatami system can yield is only zig-zag conformations shown in Fig1.

Let Ξ be a deterministic oritatami system of delay 1 and arity 2. Assume its seed σ consists of n beads. Let us denote its transcript by $w = w_1 w_2 w_3 \cdots$ for some $w_1, w_2, w_3 \in \Sigma = \{e\}$. For $i \geq 0$ let C_i be the unique elongation of σ by $w[1..i]$ that is foldable by Ξ. Hence $C_0 = \sigma$.

Let us consider the stabilization of the i-th bead a_i upon C_{i-1}. The bead cannot collaborate with any succeeding bead $w_{i+1}, w_{i+2}, \cdots$ at delay 1. There are just two ways to get stabilized at delay 1. One way is to be bound to another bead. The other way is through a *tunnel section*. A tunnel section consists of four beads that occupt four neighbors of a point(Fig.2). Assume that four of the six neighbors of a point p are occupied by beads $a_{j_1}, a_{j_2}, a_{j_3}, a_{j_4}$ while the other two are not occupied. We call such beads as p *inside of a tunnel* and such beads as p' *entrance of a tunnel* without a case that p' is inside of a tunnel. If the beads a_{i-2} and a_{i-1} are stabilized respectively at one of the two free neighbors and at p one after another, then the next bead a_i cannot help but be stabilized at the other free neighbor. In this way, a_i can get stabilized without being bound.

If a bead is stabilized through a tunnel section, then it can provide two binging capabilities and create tunnel sections.

Theorem 1 (Tunnel Troll Theorem). *Let Ξ be an unary oritatami system of delay 1 and arity 2. If a bead is stabilized through a tunnel section, then it consume some binging capabilities.*

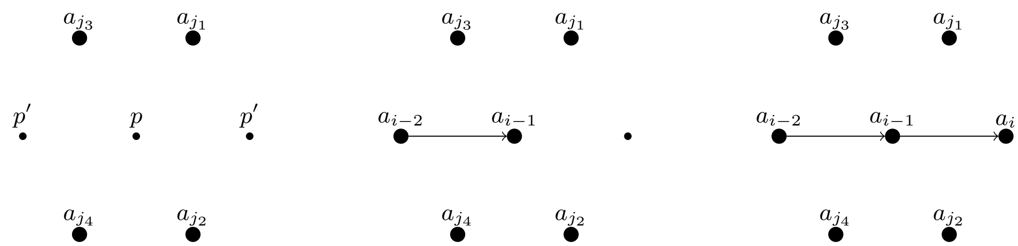


Fig. 2. Through a tunnel section

1.2 Proof of Tunnel Troll Theorem

Assume Ξ is deterministic. Let us represent its transcript w as $w = w_1w_2w_3\cdots$ for beads $w_1, w_2, w_3, \cdots \in \Sigma = \{e\}$. Each of these beads is stabilized either by being bound or through a tunnel section (or by both). How they are stabilized can be described by a binary sequence S of b's (bound) and t's (tunnel section); priority is given to t, that is, $S[i] = t$ if the i-th bead w_i is stabilized not only by being bound but also through a tunnel section. On the other hand, each of beads are bound either inside of tunnel or outside (Fig. 5). If a bead is stabilized at inside of tunnel, then its successor is already decided the position either inside of a tunnel or outside. Moreover, If a bead is stabilized at outside of a tunnel, then its position is an entrance of a tunnel or otherwise.

Tunnel sections have three possible shape with considering symmetry such as acute turn, straight and obtuse turn (Figure 3). Let us focus entrances of a tunnel such as p', then Entrances have two possible shape (Figure 4).

Let us consider tunnel sections only tunnel A and B. See Fig. 6. From appendix (Entrance of Tunnel A, B and Exit of Tunnel), we add maximum number which each transitions provide binding capabilities for each edge where a is number of consuming binding capabilities when the bead is stabilized at position of *successer in outside*.

Next, we consider on tunnel C section. If $w[i]$ is stabilized by tunnel C and $S[i+1]$ is t, then $w[i+1]$ is stabilized by tunnel A or B because if $w[i+1]$ is stabilized by tunnel C, then C_{i+1} is a terminal. Hence, tunnel C section is devided cases sush as Figure 7. Cases of $S[i...] = bt^l (l \geq 2)$ are already considered (Upper). According to appendix (Tunnel C), cases of $S[i..i+2] = btb$ also consume some binding capabilities (Lower).

Thus, if a bead is stabilized through a tunnel section, then it consume some binging capabilities.

1.3 Appendix of Tunnel Troll Theorem

Entrance of TunnelA,B Fig.8 exhibits all the three kinds of entrance of tunnel A, B. Any cases in $\delta = 1, \alpha = 2$ consume some binding capabilities into the follows.

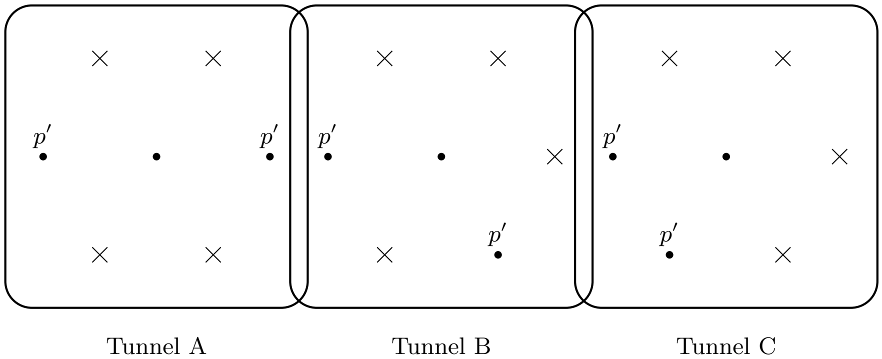


Fig. 3. All possible tunnel sections

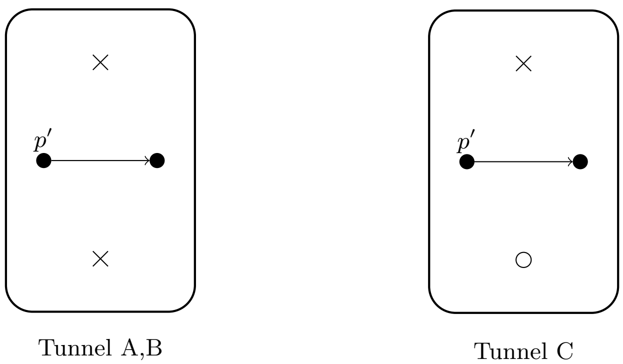


Fig. 4. Entrance of a tunnel

- Case of t_0
  Let us consider points of c, d either occupied or not. A point c or d is free because if both of them are occupied, p' is inside of tunnel. If c is free, then p' has to be bound to a bead except A due to deterministically stabilize. In this situation, at least three neighbors of A are free that is at most three neighbors of A are occupied. A leave at least one binging capability because beads of beighbors are predecessor and successor in addition A is able to consume itself binging capabilities only one-time. Hence, p' must be bound to A. Thus, a case of t_0 consumes some binding capabilities.
- Case of $t_{\pm 60}$
  In this case, a point c or d is free, too. If c is free, p' has to be bound to A or B. If c is occupied, then d is free. This time, B has some binding capabilitie so p' has to be bound to B.

  In this situation, p' is able to supply a binding capability. if this capability is active, p' bind a bead into c or d. However, B and P are exist in back

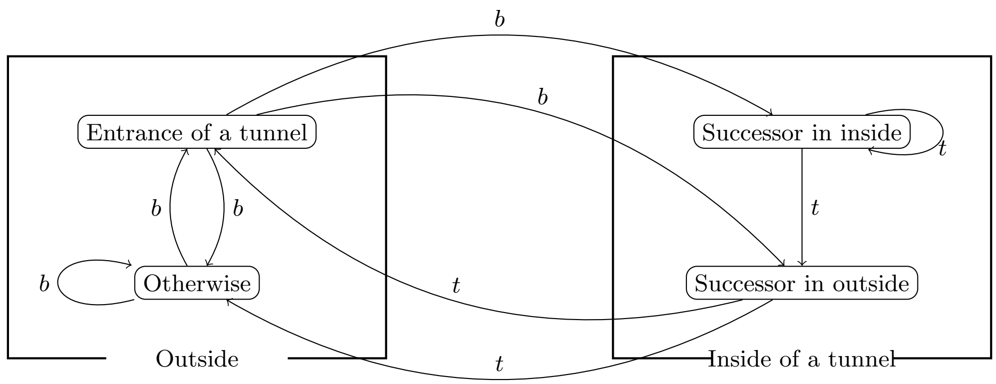


Fig. 5. Cases on position of a bead

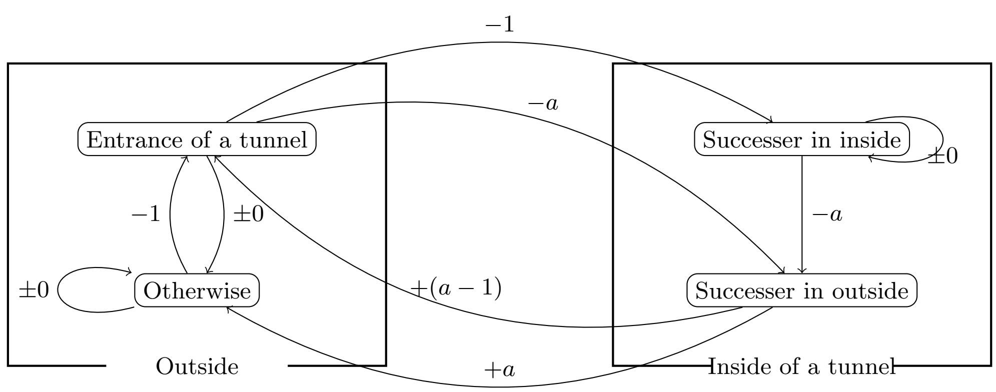


Fig. 6. Tunnel A,B における bond の増加量

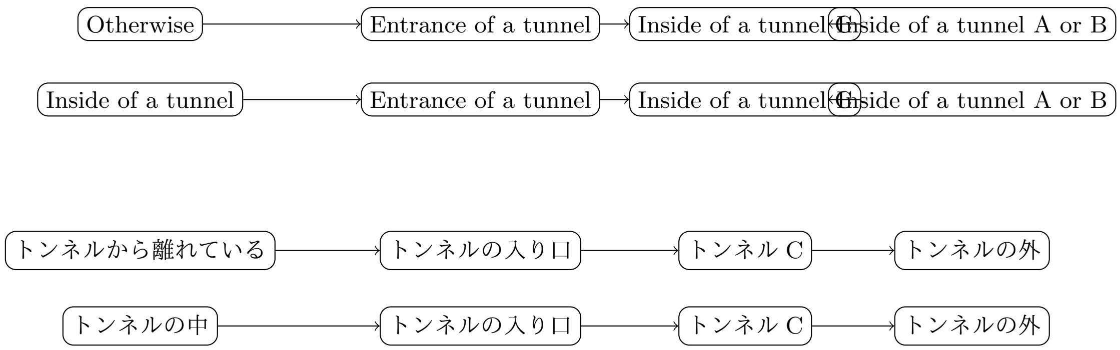


Fig. 7. Tunnel C の場合分け

bone. According to Jordan curve theorem, any successors of p' cannot reach a point c or d so this capability is inactive. Thus, a case of $t_{\pm 60}$ consumes some binding capabilities.

– Case of $t_{\pm 120}$
  Binding capabilities that p' supply are inactive according to Jordan curve theorem on A and P. Moreover, p' has to be bound to one of c, d,e in order to deterministically stabilize. Thus, a case of $t_{\pm 120}$ consumes some binging capabilities.

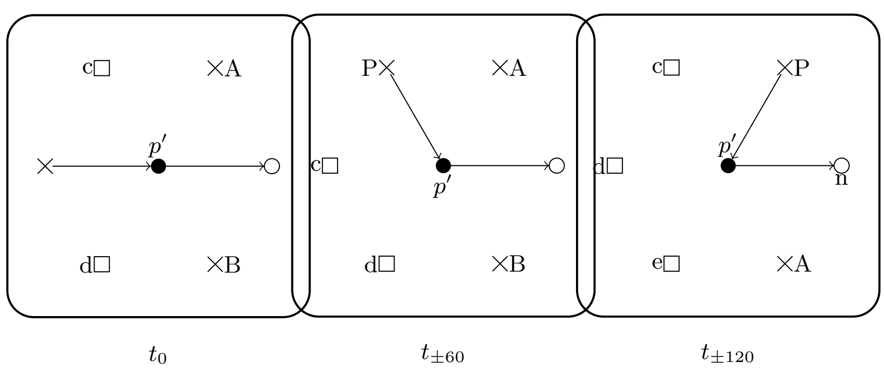


Fig. 8. Direction into a entrance

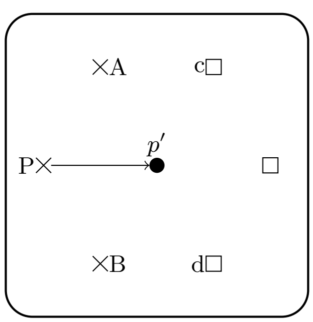
Exit of Tunnel Fig.9 exhibits all the two kings of exit of tunnel. At least one of point c or d is free because if both of them are occupied, p' is inside of tunnel.

$\delta = 1, \alpha = 2$ Any cases of $\delta = 1, \alpha = 2$ supply at most a binding capabilities into follows where a is number of predecessor of p' consumes binging capabilities.

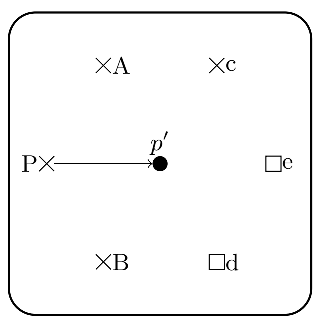
- Case of c and d are free
  This case can be regarded same situation as entrance. See Fig.9 (Left). Predecessor P has to be bound A and B because each of A and B leave binding capabilities. Hence, at least $a = 2$. This time, $\alpha = 2$ that is this case supply at most only a binding capabilities.
- Case of c is occupied
  See Fig.9 (Right). If c is pccupied, then d is free so that P has to be bound B. Hence, at least $a = 1$. This case can supply two binding capabilities but p' can bind to only one of e or d because e or d will be occupied a successor of p'. Therefore, this case supply at most only $a = 1$ active binding capabiliry.

$\delta = 1, \alpha \geq 3$ Any cases of $\delta = 1, \alpha \geq 3$ consume some binding capabilities into follows.

- Case of c and d are free
  In $\alpha \geq 3$, if three neighbors of a bead leave, then it can supply two binding capabilities. Therefore Predecessor P has to be bound A and B, and p', too. In this case, at least consumes four bindings and supplies at most two bindings. Thus, it consumes some binding capabilities, totally.
- Case of c is occupied
  In this case, B leave at least two bindings and A, c also leave at least one binding. Therefore P has to be bound A and B, and p' also has to be bound B and c. In this case, at least consumes four bindings and supplies at most one binding. Thus, it consumes some binding capabiliries, totally.



Point c and d are free



Point c is occupied

Fig. 9. Exit of Tunnel

Tunnel C Assume $w[i]$ is a bead which stabilized by tunnel C. Let us consider kinds of stabilization $S[i-2..i] = tbt$ or $S[i-2..i] = bbt$ except cases of $w[i]$ is inside of tunnel A, B.

Case of $S[i-2..i] = tbt$ Fig.10 exhibits all the two kinds of stabilization depending on structures of tunnel C.

– Left of Fig.10
  In this figure, Bead A has at least one binding so that w_{i-2} has to bound A. Moreover, w_{i-1} has to bound one of B, C, D in order to stabilize deterministically. On the other hand, w_i can supply two bindings but free neighbors of w_i are two points. One of them is occupied a successor. Therefore w_i can only bind one of e, f that is w_i supplies at most one binding. Thus, this case consumes some binding capabilities.
– Right of Fig.10
  This cases are divided on number of capabilities that w_{i-2} consumes.
  - w_i does not consume any bindings
    w_{i-1} has to bound one of B, C, D in order to stabilize deterministically. w_i has to be bound to w_{i-2} because w_{i-2} has bindings. This time, let us consider either e is occupied or not. If e is occupied, then w_{i-2} has no active bindings that is this situation consumes some binding capabilities. If e is free and w_{i+1} is stabilized in e, then w_{i-2} has to bind w_{i+1}. Therefore, In this case, stabilization of $w[i-2..i+1]$ consumes some bindings. If e is free and w_{i+1} is stabilized except e, then this oritatami system has to use two binding capabilities in order to bind w_{i+1}. Therefore, in this case consumes some bindings. Thus in this cases consume some binding capabilities.
  - w_i consumes one binding
    In this case, w_{i-1} has to be bound one of B, C, D. In addition, w_{i-2} and w_i are not supply any bindings. Thus, in this cases consume some binding capabilities.。
  - w_i consumes two bindings
    In this case, w_{i-2} already consumes two binding. w_{i-1} has to be bound. w_i supplies two bindings. Thus, in this cases consume some binding capabilities.

Case of $S[i-2..i] = bbt$ Let us consider number of consumed by w_{i-2} (Fig.11).

– w_{i-2} consumes one binding
  In this situation, w_{i-2} supplies one active binding whereas w_i consumes this binding. In addtion, w_{i-1} has to bound to one of A, B, C. Thus, in this cases consume some binding capabilities.
– w_{i-2} consumes two bindings
  In this case, w_{i-2} already consumes two binding. w_{i-1} has to be bound. w_i supplies at most two bindings. Thus, in this cases consume some binding capabilities.

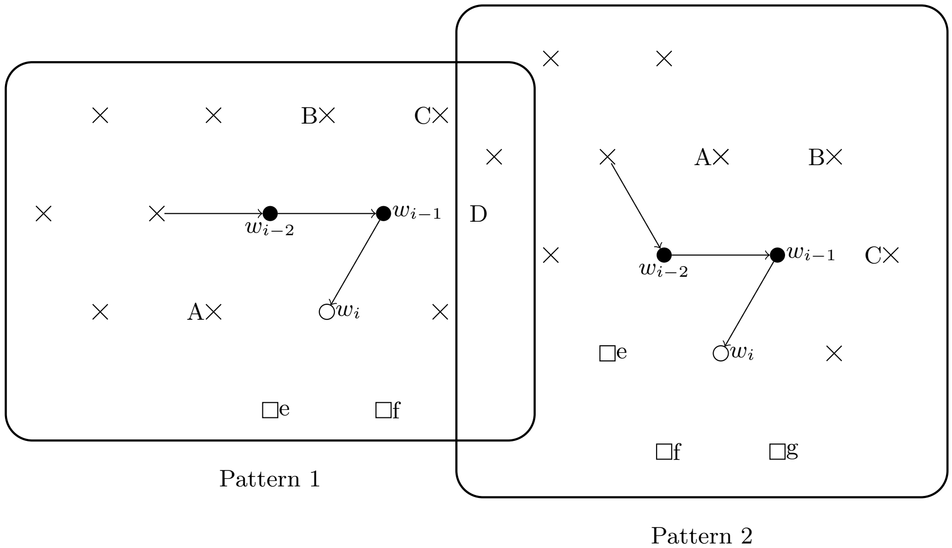


Fig. 10. Case of $S[i-2..i] = tbt$

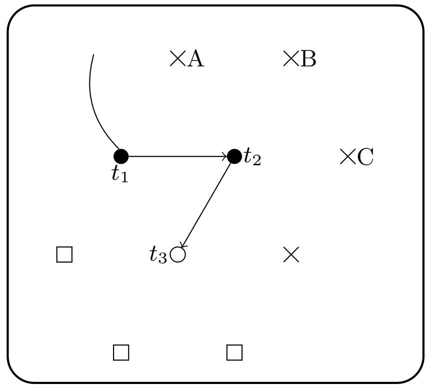
By Tunnel Troll Theorem, any tunnel sections which represented in bbt^+ or bt^+bt^+ consume binding capabilities. If the sequence S is free from any subsequence of the form bt^+bt^+, then it can factorize as $S = u_1u_2u_3\cdots$ for some $u_1, u_2, u_3, \cdots \in \{b\} \cup bbt^+$. Assume the length of σ is n, seed supplies at most $2n$ binding capabilities. Therefore formula **??** hold.

$$\exists i \in \mathbb{N} \quad s.t. \quad u_{i-1}, u_i, u_{i+1}, u_{i+2}, \cdots \in \{b\} \tag{1}$$

Let us represent S as $S[i.i+1...] = v_iv_{i+1}v_{i+2}\cdots$ for some $v_i, v_{i+1}, v_{i+2}, \cdots \in \{a, o\}$ where if v_k is a, then v_{k+1} is bound to v_{k-1}, if v_k is o, then v_{k+1} is NOT bound to v_{k-1}.

Let us consider the case of v_k is o. See Fig.12. v_{i-1} consumes some binding capabilities because $S[i-1]$ is b. If the number of v_{i-1}'s bindings is one binding, then v_{i+1} has to be bound except A or B so that v_{i+1} must consumes two bindings except the case of A and B are occupied and v_i consumes at least one binding. If A and B are occupied, then v_{i-1}'s binginds are inactive that is v_{i-1} consumes two binding capabilities. Therefore, this case consumes binding capabilities. If v_{i-1} dose Not have any bindings, then v_{i-1} already consumes two bindings. In addition, v_i and v_{i+1} consume at least one binding. Therefore this case consumes binding capabilities. Thus, the formula 2 hold and according to the formula 1 and the formula 2, the formula 3 is hold.

$$\exists j \in \mathbb{N} \quad s.t. \quad u_j, u_{j+1}, u_{j+2}, \cdots \in \{a\} \tag{2}$$



Pattern 1

Fig. 11. Case of $S[i-2..i] = bbt$

$$|S| > \forall m \in \mathbb{N} \quad \rightarrow \quad \exists n \in \mathbb{N} \quad s.t. \quad S[n], S[n+1], \cdots \in \{a\} \tag{3}$$

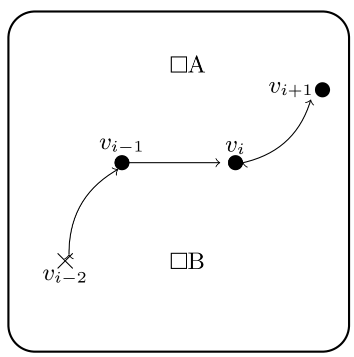


Fig. 12. Case of $S[i$