1 Infiniteness of delay-1 unary deterministic oritatami system

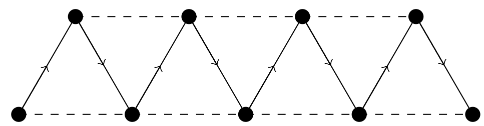


Fig. 1. zig-zag conformation

1.1 Introduction

In this section, we consider on finiteness of structures produced deterministically at delay 1. In result, cases of arity 1 and 3 can only yield finite structures which is size of $\mathcal{O}(n)$, and cases of arity 4 and more can only yield finite structures which is size of $mathcalO(n^2)$, and a case of arity 2 can yield infinite structures but they are only the zig-zag conformation shown in Fig.1.

Let Ξ be a deterministic oritatami system of delay 1 and arity 2. Assume its seed σ consists of n beads. For $i \geq 0$ let C_i be the unique elongation of σ by $w[1..i]$, that is, foldable by Ξ. Hence $C_0 = \sigma$.

Let us consider the stabilization of the i-th bead a_i upon C_{i-1}. The bead cannot collaborate with any succeeding bead $w[i+1], w[i+2], \cdots$ at delay 1. There are just two ways to get stabilized at delay 1. One way is to be bound to another bead. The other way is through a *tunnel section*. A tunnel section consists of four beads that occupy four neighbors of a point (Fig.2). Accordingly, how they are stabilized can be described by a binary sequence S of b's (bound) and t's (tunnel section); priority is given to t, that is, $S[i] = t$ if the i-th bead w_i is stabilized not only by being bound but also through a tunnel section.

Assume that four of the six neighbors of a point p are occupied by beads $a_{j_1}, a_{j_2}, a_{j_3}, a_{j_4}$ while the other two are free. We call such a bead p as $inside\ of\ a\ tunnel$ and such beads p' as *entrance of a tunnel* without a case that p' is inside of a tunnel. If the beads $w[i-2]$ and $w[i-1]$ are stabilized respectively at one of the two free neighbors and at p one after another, then the next bead $w[i]$ cannot help but be stabilized at the other free neighbor. In this way, $w[i]$ can get stabilized without being bound.

We say that point p is reachable from a conformation C if there exists a directed path P' from the last point of C that does not cross the path of C. We define *binding capability* with reachable.

Definition 1 (binding capability). *Let B_i be $(\{(h,i)|^{\forall}h < i\} \cup \{(i,j)|^{\forall}j > i\}) \cap H$. Moreover, let R_i be a set of neighbors of $w[i]$ that are free and reachable from C_j where C_j is a conformation which stabilized until $w[j]$. We represent* *the number of binding capabilities of a conformation C_j as $\#bc(C_j)$. $\#bc(C_j)$ is defined by $\sum_{k=-n+1}^{j} \min\{|B_k|, |R_k|\}$.*

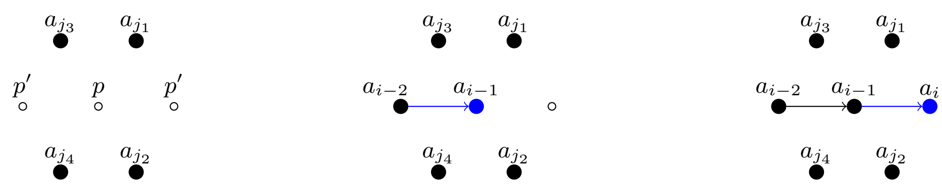


Fig. 2. Through a tunnel section

Theorem 1 (Tunnel Troll Theorem). *Let Ξ be a unary oritatami system of $\delta = 1, \alpha = 2$. If there are indices i and j such that $S[i..j+1] = bbt^{(j-i-1)}b$, then $\#bc(C_{i-1}) > \#bc(C_j)$ and if $S[i..j+1] = bt^l bt^m b \quad (l + m = j - i - 1)$, then $\#bc(C_{i-1}) > \#bc(C_j)$. On the other hand, at $\delta = 1$ and $\alpha \geq 3$, if $S[k] = t$, then $\#bc(C_{k-1}) > \#bc(C_k)$.*

Proof. Assume Ξ is deterministic. Each bead in the transcript is bound either inside a tunnel or outside. If a bead is stabilized inside a tunnel, then its successor is already decided the position either inside of a tunnel or outside. Moreover, If a bead is stabilized outside a tunnel, then its position is an entrance of a tunnel or otherwise.

Tunnel sections have three possible shapes up to symmetry : straight(A), obtuse(B) and acute(C) turn (Fig. 3), and we will consider each of those.

Lemma 1. *For unary transcripts at $\delta = 1$, If a bead has no free hand, then at least $\alpha + 2$ of its neighbors have to be occpied.*

Lemma 2. *Let Ξ be an oritatami system at $\delta = 1, \alpha = 2$ Assume Ξ stabilizes transcript until $w[i-1]$. If $w[i]$ is stabilized at an entrance point of tunnel A or B, then $\#bc(C_{i-1}) > \#bc(C_i)$.*

Lemma 3. *Let $w[i]$ be a bead which is stabilized in exit of a tunnel. At $\delta = 1, \alpha = 2$, If we assume $S[h..i+1] = bt^{(i-h)}b \quad (h < i)$, then $\#bc(C_{h-1}) \geq \#bc(C_i)$ and $\#bc(C_{i-2}) \geq \#bc(C_i)$. On the other hand, if we assume $S[k] = t(k \leq i)$ at $\delta = 1, \alpha \geq 3$, then $\#bc(C_{k-1}) > \#bc(C_k)$.*

Lemma 4. *Let Ξ be a unary oritatami system of $\delta = 1, \alpha = 2$. We assume $S[h..i+1] = bt^{(i-h)}b \quad (1 < h < i)$. If at least one of $w[h+1..i]$ is stabilized by tunnel C, then $\#bc(C_{h-3}) > \#bc(C_i)$.*

Let us first consider cases of $\delta \geq 3, \alpha = 1$. These cases are clearly true because we use lemma3.

Next, we consider the case of $\delta = 2, \alpha = 1$. We assume there is index h such that $S[h-1..h+1] = bbt$ or $S[h-1..h+1] = tbt$. According to lemma2, if $w[h+1]$ is stabilized by tunnel A or B, then $\#bc(C_{h-1}) > \#bc(C_h)$. Also, According to lemma3, if $w[h+1]$ is stabilized by tunnel C, then $\#bc(C_{h-1}) > \#bc(C_h)$, too. On the other hand, if $S[k..l] = bt^{l-k}b$, then $\#bc(C_{k-1}) \geq \#bc(C_l)$. Therefore, if there are indices i and j such that $S[i..j+1] = bbt^{(j-i-1)}b$ or $S[i..j+1] = bt^m bt^n b \quad (m + n = j - i - 1)$, then $\#bc(C_{i-1}) > \#bc(C_j)$.

□

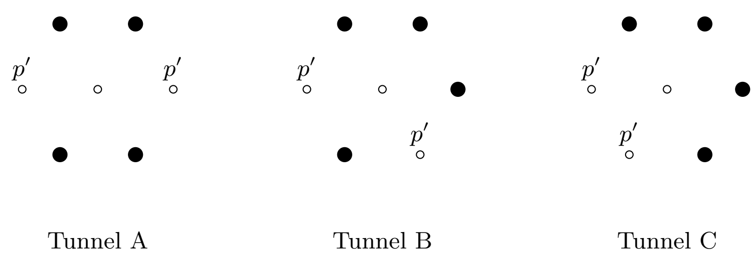


Fig. 3. All possible tunnel sections: straight, obtuse turn, and acute turn

Proof (lemma 1). A transcript bead in has predecessor and successor except the first and last beads. If the bead does not have any free hand, then it uses hands with α neighbors. Thus, lemma 1 is clearly true.

Proof (lemma 2). Fig.4 exhibits all the three kinds of entrance of tunnel A, B. Let $w[i]$ be stabilized at an entrance point of Tunnel A or B. All cases are $\#bc(C_{i-1}) > \#bc(C_i)$ as follows.

- Case of t_0
  Let us consider points n_3, n_4. At least one of the points n_3 or n_4 is free because if both of them are occupied, p' is inside of tunnel. If n_3 is free, then p' has to be bound to a bead other than n_1 due to deterministically stabilize. In this situation, at least three neighbors of n_1 are free, that is, n_1 has at least one free hand from lemma 1. Hence, p' must be bound to n_1. Thus, a case of t_0 consumes two hands and it does not supply any binding capabilities.
- Case of $t_{\pm 60}$
  In this case, a point n_4 or n_5 is free, too. If n_5 is free, p' has to be bound to n_1 or n_2. If n_5 is occupied, then n_4 is free. This time, n_2 has some free hands so p' has to be bound to n_2.
  In this situation, p' is able to supply a binding capabilities which could p' bind a bead into n_4 or n_5. However, n_2 and n_3 are part of a contiguous conformation. According to Jordan curve theorem, any successors of p' cannot reach a point n_4 or n_5 so this capability is inactive. Thus, a case of $t_{\pm 60}$ is$\#bc(C_{i-1}) > \#bc(C_i)$.

– Case of $t_{\pm 120}$
  Binding capabilities that p' supplies are inactive according to Jordan curve theorem on n_1 and n_2. Moreover, p' has to be bound to one of n_3, n_4, n_5 in order to deterministically stabilize. Thus, a case of $t_{\pm 120}$ is $\#bc(C_{i-1}) > \#bc(C_i)$.

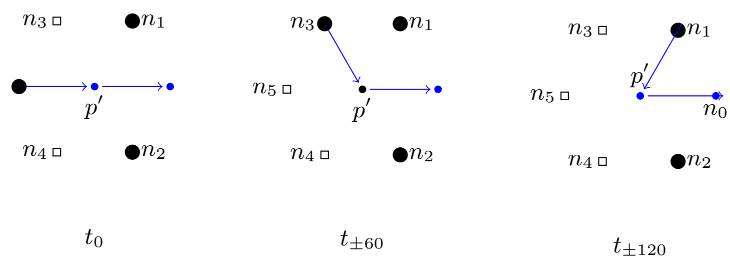


Fig. 4. Direction into a entrance

Proof (lemma 3). Fig.5 exhibits all the two kinds of exit of tunnel. At least one of points n_1 or n_2 is free because if both of them are occupied, p' is inside of tunnel.

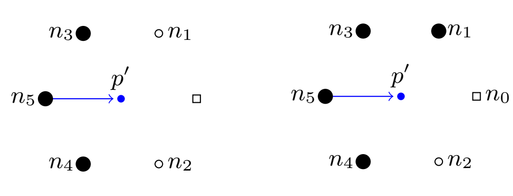
$\delta = 1, \alpha = 2$
Let Ξ be a unary oritatami system at $\delta = 1, \alpha = 2$. We assume $S[h..i+1] = bt^{(i-h)}b \quad (h < i)$. If let a be $\#bc(C_{i-2}) - \#bc(C_{i-1}) = a$, then $\#bc(C_i) - \#bc(C_{i-1}) \leq a$ as follows. Also, if $h - i > 1$ and let j be $h < j < i$, then $\#bc(C_{j-1}) \geq \#bc(C_j)$ because any neighbors of $w[j]$ are occupied by beads of tunnel so that any $w[i+1..]$ cannot reach neighbors of $w[j]$. Thus, $\#bc(C_{h-1}) \geq \#bc(C_i)$ and $\#bc(C_{i-2}) \geq \#bc(C_i)$.

– Case of n_1 and n_2 being free
  This case can be regarded the same as entrance. See Fig.5 (Left). Predecessor n_5 has to be bound to n_4 and n_5 because both of n_3 and n_4 have binding capabilities. Hence, $a \geq 2$. This time, $\alpha = 2$, that is, this case $\#bc(C_i) - \#bc(C_{i-1}) \leq a$.
– Case of n_1 is occupied
  See Fig.5 (Right). If n_1 is occupied, then n_2 is free so that n_5 has to be bound n_4. Hence, $a \geq 1$. This case can supply two binding capabilities but p' can bind to only one of n_0 or n_2 because n_0 or n_2 will be occupied by the successor of p'. Therefore, this case $\#bc(C_i) - \#bc(C_{i-1}) \leq a$.

$\delta = 1, \alpha \geq 3$
Let Ξ be a unary oritatami system at $\delta = 1, \alpha \geq 3$. We assume $w[i]$ is stabilized at exit of tunnel. Any cases $\#bc(C_{i-1}) > \#bc(C_i)$. Moreover, if $S[k] = t(k \leq i)$,

then $\#bc(C_{k-1}) > \#bc(C_k)$ because both of sides of the path p in Fig.6 (n_1, n_2) have two free points and one of n_1, n_2 is not the predecessor so that it has hand and moreover that $w[k]$ supplies any binding capabilities because its neighbors are occupied by beads of tunnel.

- Case of n_1 and n_2 are free
  In $\alpha \geq 3$, if three neighbors of a bead leave, then it can supply two binding capabilities. Therefore Predecessor n_5 has to be bound n_3 and n_4, and p', too. In this case, at least consumes four bindings and supplies at most two bindings. Thus, it consumes some binding capabilities, totally.
- Case of c is occupied
  In this case, n_4 leave at least two bindings and n_3, n_1 also leave at least one binding. Therefore n_5 has to be bound n_3 and n_4, and p' also has to be bound n_1 and n_4. In this case, at least consumes four bindings and supplies at most one binding. Thus, it consumes some binding capabilities, totally.



Point n_1 and n_2 are free　　Point n_1 is occupied

Fig. 5. Exit of Tunnel

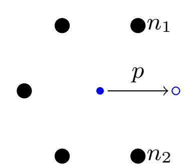


Fig. 6. Inside tunnel

Proof (lemma 4). Let Ξ be a unary oritatami system at $\delta = 1, \alpha = 2$. Assume $S[h..i+1] = bt^{i-h}b(h < i)$. If at least one of $w[h+1..i]$ are stabilized by tunnel C, then only $w[h+1]$ can use tunnel C because if $w[g]$ which is one of $w[h+2..i]$ is stabilized by tunnel C, C_g is a terminal.

Let us consider kinds of stabilization $S[h-1..h+1] = tbt$ or $S[h-1..h+1] = bbt$ into follows. In result, $\#bc(C_{h-3}) > \#bc(C_{h+1})$. In addition according to lemma3 $\#bc(C_{h+1}) \geq \#bc(C_i)$. Thus, $\#bc(C_{h-3}) > \#bc(C_i)$.

Case of $S[h-1..h+1] = tbt$ Fig.7 exhibits all the two kinds of stabilization depending on structures of tunnel C.

- Left of Fig.7
  In this figure, Bead n_4 has at least one binding so that $w[h-1]$ has to bound n_4. Moreover, $w[h]$ has to bound one of n_1, n_2, n_3 in order to stabilize deterministically. On the other hand, $w[h+1]$ can supply two bindings but

free neighbors of $w[h+1]$ are two points. One of them is occupied by a successor. Therefore $w[h+1]$ can only bind one of n_5, n_6, that is, $w[h+1]$ supplies at most one binding. Thus, this case $\#bc(C_{h-1}) > \#bc(C_{h+1})$.

– Right of Fig.7
  This cases are divided on number of capabilities that $w[h-1]$ consumes.
  - $w[i]$ does not consume any bindings
    According to lemma3, $\#bc(C_{h-3}) \geq \#bc(C_{h-1})$ because of $S[h-1] = t$. $w[h]$ has to bound one of n_1, n_2, n_3 in order to stabilize deterministically so that $\#bc(C_{h-1}) > \#bc(C_h)$. $w[h+1]$ has to be bound to $w[h-1]$ because $w[h-1]$ has bindings, that is, $w[h+1]$ consumes at least one hand and supplies at most one hand so that $\#bc(C_h) \geq \#bc(C_{h+1})$. Thus, in this cases $\#bc(C_{h-3}) > \#bc(C_{h+1})$.
  - w_{h+1} consumes one binding
    In this case, w_h has to be bound one of n_1, n_2, n_3. In addition, $w[h-1]$ and $w[h+1]$ are not supply any bindings. Thus, in this cases consume some binding capabilities.
  - w_i consumes two bindings
    In this case, $w[h-1]$ already consumes two binding. $w[h]$ has to be bound. $w[h+1]$ supplies two bindings. Thus, in this cases $\#bc(C_{h-1}) > \#bc(C_{h+1})$.

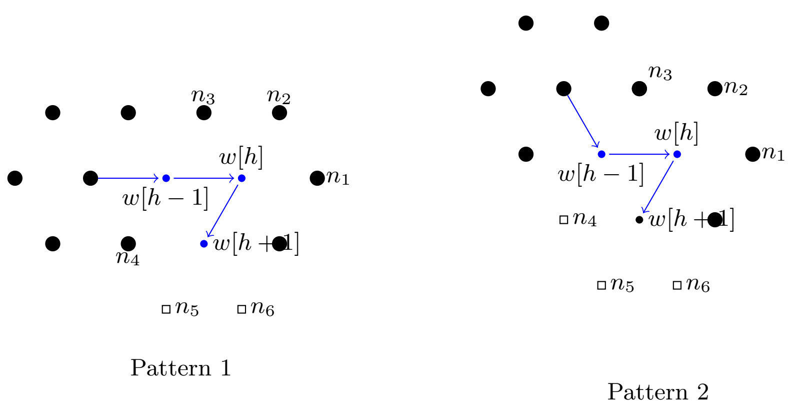


Fig. 7. Case of $S[h-1..h+1] = tbt$

Case of $\boldsymbol{S[h-1..h+1] = bbt}$ Let us consider number of consumed by $w[h-1]$ (Fig.8).

– $w[h-1]$ consumes one binding
  In this situation, $w[h-1]$ supplies one active binding whereas $w[h+1]$ consumes this binding. In addition, $w[h]$ has to bound to one of n_1, n_2, n_3. Thus, in this cases consume some binding capabilities.

– $w[h-1]$ consumes two bindings
  In this case, $w[h-1]$ already consumes two binding. $w[h]$ has to be bound. $w[h+1]$ supplies at most two bindings. Thus, in this cases consume some binding capabilities.

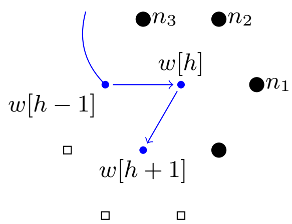


Fig. 8. Case of $S[h-1..h+1] = bbt$

1.2 On structures provided by a unary and $\delta = 1$ oritatami system

Theorem 2 **($\delta = 1, \alpha = 2$).** *Let Ξ be a unary oritatami system of $\delta = 1, \alpha = 2$. It can yield infinite structures but they are only zig-zag conformation.*

Proof. By Tunnel Troll Theorem, any tunnel sections which represented in bbt^+ or bt^+bt^+ consume binding capabilities. If the sequence S is free from any subsequence of the form bt^+bt^+, then it can factorize as $S = u_1u_2u_3\cdots$ for some $u_1, u_2, u_3, \cdots \in \{b\} \cup bbt^+$. Assume the length of σ is n, seed supplies at most $2n$ binding capabilities. Therefore formula 1 hold.

$$\exists i \in \mathbb{N} \quad s.t. \quad u_{i-1}, u_i, u_{i+1}, u_{i+2}, \cdots \in \{b\} \tag{1}$$

Let us represent S as $S[i.i+1...] = v_iv_{i+1}v_{i+2}\cdots$ for some $v_i, v_{i+1}, v_{i+2}, \cdots \in \{a, o\}$ where if v_k is a, then v_{k+1} is bound to v_{k-1}, if v_k is o, then v_{k+1} is NOT bound to v_{k-1}.

Let us consider the case of v_k is o. See Fig.9. $w[i-1]$ consumes some binding capabilities because v_{i-1} is b. If the number of $w[i-1]$'s bindings is one binding, then $w[i+1]$ has to be bound except n_1 or n_2 so that $w[i+1]$ must consumes two bindings except the case of n_1 and n_2 are occupied and $w[i]$ consumes at least one binding. If n_1 and n_2 are occupied, then $w[i-1]$'s bindings are inactive, that is, $w[i-1]$ consumes two binding capabilities. Therefore, this case consumes binding capabilities. If $w[i-1]$ dose Not have any bindings, then $w[i-1]$ already consumes two bindings. In addition, $w[i]$ and $w[i+1]$ consume at least one binding. Therefore this case consumes binding capabilities. Thus, the formula 2 hold and according to the formula 1 and the formula 2, the formula 3 is hold. Thus, in this case, oritatami system can yield infinite structures but they are only zig-zag conformation.

$$\exists j \in \mathbb{N} \quad s.t. \quad u_j, u_{j+1}, u_{j+2}, \cdots \in \{a\} \tag{2}$$

$$|S| > \forall m \in \mathbb{N} \quad \rightarrow \quad \exists n \in \mathbb{N} \quad s.t. \quad S[n], S[n+1], \cdots \in \{a\} \tag{3}$$

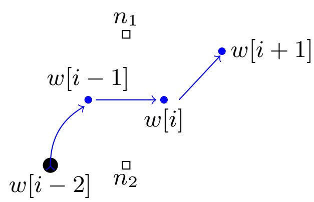


Fig. 9. Case of $S[i]$

Theorem 3 **($\delta = 1, \alpha = 3$).** *Let Ξ be a unary oritatami system of $\delta = 1, \alpha = 3$. It can yield only finite structures whose size is $\mathcal{O}(n)$.*

Lemma 5. *Let p be a point whose neighbors is occupied at least two point. If $w[i]$ is not stabilized and $w[i-1]$ includes neighbors of p, then $w[i]$ is stabilized at p with at least one bond, $w[i]$ is stabilized at another point of p otherwise with at least two bond except any neighbors of p is occupied.*

Proof (proof of lemma). Assume the transcript is stabilized until $w[i-1]$. One of neighbors of p is not $w[i-1]$ where this bead regards n_1. If $w[i-1]$ include neighbors of p and $w[i]$ is stabilized at another point of p with one bond. Then, any neighbors do not have bond without $w[i-1]$. Neighbors of n_1 have to be occupied at least five according to lemma 1 and two of them include neighbors of p where each of them regards n_2, n_3. In the same way, five neighbors of n_2 and n_3 are occupied and each of one of them includes neighbors of p where they regard n_4, n_5. one of n_5's neighbors includes neighbors of p where it regards n_6. Then, any neighbors of p are occupied. That is, if some neighbors of p are free, then there exists a bead which has bonds in neighbors.

Proof. Let us show that $\#bc(C_{i-1}) > \#bc(C_i)$, that is, when $w[i]$ is stabilized, $w[i]$ uses at least two hands. Let us assume $w[i]$ is able to be stabilized with using one hand. Fig.10 exhibits all the three kinds of possibility of stabilized $w[i]$. Then, $w[i]$ can be also stabilized at n_3.

Case of straight

- Case of n_3 is free
  According to assumption, $w[i]$ uses only one hand. Therefore, any neighbors of n_3 are occupied according to the lemma5. n_3 and the point which is stabilized $w[i]$ are free so that n_1 has some bond by lemma1. Accordingly, this situation is non-deterministic. Thus, n_3 and n_4 have to be occupied because of symmetry.
- Otherwise
  Because of $S[i] = b$, at least one of n_1 and n_2 have to be free. Let us regard that n_1 is free. Neighbors of n_1 have to be occupied and at least two neighbors of n_{-1} have to be free for n_1 and $w[i]$. According to lemma1, n_{-1} have some hand. Therefore $w[i]$ can be also stabilized n_1, that is, this situation is non-deterministic. Thus, one of n_3 and n_4 has to be free.

Therefore, this case is false.

Case of obtuse

- Case of n_3 is free
  Any neighbors of n_3 have to be occupied but the point which is stabilized $w[i]$ is free. Thus n_3 has to be occupied.
- Case of n_4 is free
  According to lemma5, n_2 has to be occupied because n_4 is free. Also n_0 has to be occupied from lemma1. Thus, only one of n_0, n_3 leave some hands or both of them do not leave any hands because $w[i]$ use only one bond.
  If n_0 has some hands, then n_3 does not have any hands so that n_{-3} is occupied. Also n_{-3} must not have any hands so that n_{-2} is occupied and also n_{-1} is occupied. Therefore any neighbors of $w[i]$ are occupied so that $w[i+1]$ cannot provide.
  If n_3 has some hands, then n_0 does not have any hands so that n_{-1} is occupied. In the same previous way, any n_{-2}, n_{-3} are occupied. Therefore any neighbors of $w[i]$ are occupied.
  If both of n_0, n_3 do not have any hands, then both of n_{-1}, n_{-3} are occupied. If one of n_{-1}, n_{-3} has some hands, the other does not have any hands so that n_{-2} is occupied. If both of n_{-1}, n_{-3} do not have any hands, n_{-2} has to be occupied and n_{-2} has some hands.Therefore any neighbors of $w[i]$ are occupied so that $w[i+1]$ cannot provide.
  Thus n_3 has to be occupied in order to yield infinite structures.
- Case of n_2 is free
  Any neighbors of n_2 have to be occupied so that n_0 is occupied. Any neighbors of n_0 except n_2 have to be also occupied but the point which is stabilized $w[i]$ is free. Thus n_2 has to be occupied.
- Case of n_0 is free
  Any neighbors of n_0 have to be occupied so that n_{-1} is occupied. Any neighbors of n_{-1} except n_0 have to be also occupied but the point which is stabilized $w[i]$ is free. Thus n_0 has to be occupied.

Therefore, any situations contradict $S[i] = b$.

Case of acute

- Case of n_4 is free
  n_4 and a point which is stabilized $w[i]$ are free so that $w[i-2]$ has some hands according to lemma1. However, $w[i]$ can be also stabilized n_4 in this case. Thus, n_4 has to be occupied.
- Case of n_2 is free
  According to lemma5, n_0 has to be occupied. n_1 has to be also occupied because of lemma1. We consider this case just like case of obtuse and that n_4 is free. Then if $w[i-2]$ binds $w[i]$, any n_{-1}, n_{-2}, n_{-3} are occupied. If n_1 binds $w[i]$, this case is same. Also if n_1 and $w[i-2]$ do not have any hand, any n_{-1}, n_{-2}, n_{-3} are occupied. Therefore, $w[i+1]$ cannot be provided.
- Case of n_0 is free
  Any neighbors of n_0 have to be occupied so that n_1 is occupied. Any neighbors of n_1 except n_0 have to be also occupied but the point which is stabilized $w[i]$ is free. Thus n_0 has to be occupied.
- Case of n_1 is free
  Any neighbors of n_1 have to be occupied so that n_{-1} is occupied. Any neighbors of n_{-1} except n_1 have to be also occupied but the point which is stabilized $w[i]$ is free. Thus n_1 has to be occupied.

Therefore, any situations contradict $S[i] = b$.

Hence, assumption that w[i] is able to be stabilized with using one hand is false. Therefore, when w[i] is stabilized, w[i] uses at least two hands.

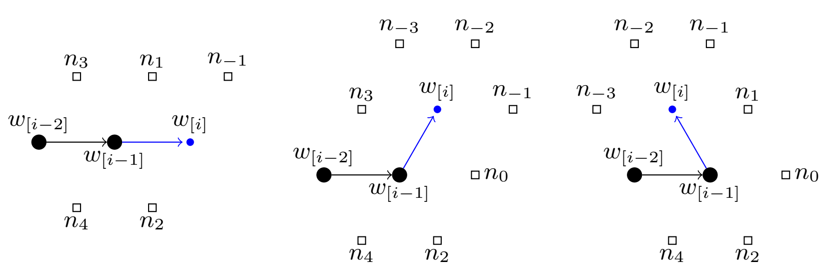


Fig. 10. All possible directions of $w[i]$: straight, obtuse, acute.

Theorem 4 **($\delta = 1, \alpha = 4$).** *Let Ξ be a unary oritatami system of $\delta = 1, \alpha = 4$. It can yield only finite structures whose size is $\mathcal{O}(n^2)$.*

Lemma 6. *Any beads which are already stabilized by some bonds use at least two bonds.*

Proof (proof of lemma). Let us consider when $w[i]$ is stabilized by only one bond. See Fig.11. According to lemma1, if n_3 is free, $w[i-2]$ has some hands. Thus, n_4 has to be occupied in order to stabilize deterministically. Moreover, also n_2 has to be occupied for deterministic and also n_0, n_1. n_1 has some hands because n_3 is free. Therefore, $w[i]$ is stabilized at n_3 and it has to use at least two hands. It contradict assumption.

Proof. According to lemma6, when $w[i]$ is stabilized, it has to use at least two bonds. Let us consider when a bead $w[i]$ which is the first bead out of $\hexagon^n_{w[-n+1]}$ is stabilized. See Fig.12. any n_0, n_1, n_3, n_5 is free because if some of them is occupied, $w[i]$ is not the first bead out of $\hexagon^n_{w[-n+1]}$. At least two neighbors of $w[i]$ except predecessor have to be occupied in order to bind. In this case, a point which is able to put a bead is only n_2. Therefore, any transcript cannot be stabilized in out of $\hexagon^n_{w[-n+1]}$. Hence oritatami system can yield only a finite structure whose size is $\mathcal{O}(n^2)$ in $\delta = 1, \alpha = 4$.

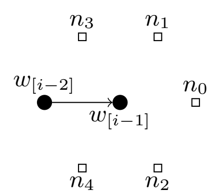


Fig. 11. $\alpha = 4$: when $w[i]$ is stabilized

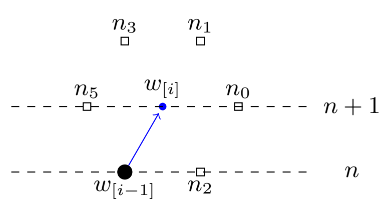


Fig. 12. the first bead out of $\hexagon^n_{w[-n+1]}$