1 Infiniteness of delay-1, arity-2 unary deterministic oritatami system

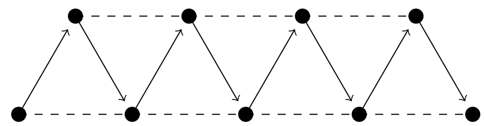


Fig. 1. zig-zag conformation

1.1 Introduction

In this section, we prove that unary oritatami system can form infinitely at delay 1 and arity 2 deterministically and moreover that the only infinite conformations which its oritatami system can yield is only the zig-zag conformation shown in Fig.1.

Let Ξ be a deterministic oritatami system of delay 1 and arity 2. Assume its seed σ consists of n beads. For $i \geq 0$ let C_i be the unique elongation of σ by $w[1..i]$ that is foldable by Ξ. Hence $C_0 = \sigma$.

Let us consider the stabilization of the i-th bead a_i upon C_{i-1}. The bead cannot collaborate with any succeeding bead $w[i+1], w[i+2], \cdots$ at delay 1. There are just two ways to get stabilized at delay 1. One way is to be bound to another bead. The other way is through a *tunnel section*. A tunnel section consists of four beads that occupy four neighbors of a point (Fig.2). Assume that four of the six neighbors of a point p are occupied by beads $a_{j_1}, a_{j_2}, a_{j_3}, a_{j_4}$ while the other two are free. We call such beads as p *inside of a tunnel* and such beads as p' *entrance of a tunnel* without a case that p' is inside of a tunnel. If the beads $w[i-2]$ and $w[i-1]$ are stabilized respectively at one of the two free neighbors and at p one after another, then the next bead $w[i]$ cannot help but be stabilized at the other free neighbor. In this way, $w[i]$ can get stabilized without being bound.

If a bead is stabilized through a tunnel section, then it can provide two binging capabilities and create tunnel sections.

Theorem 1 (Tunnel Troll Theorem). *Let Ξ be an unary oritatami system of delay 1 and arity 2. If a bead is stabilized through a tunnel section, then it consume some binging capabilities.*

1.2 Proof of Tunnel Troll Theorem

Assume Ξ is deterministic. Let us represent its transcript w as $w = w_1 w_2 w_3 \cdots$ for beads $w_1, w_2, w_3, \cdots \in \Sigma = \{e\}$. Each of these beads is stabilized either by

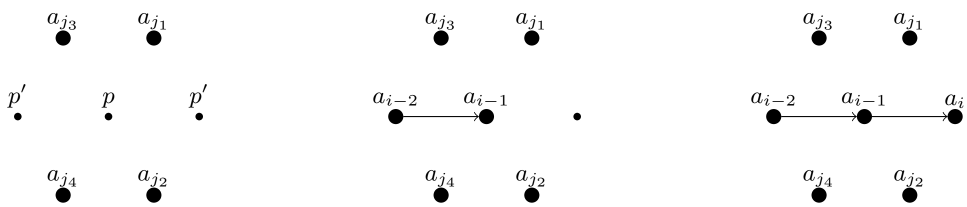


Fig. 2. Through a tunnel section

being bound or through a tunnel section (or by both). How they are stabilized can be described by a binary sequence S of b's (bound) and t's (tunnel section); priority is given to t, that is, $S[i] = t$ if the i-th bead w_i is stabilized not only by being bound but also through a tunnel section. On the other hand, each of beads are bound either inside of tunnel or outside (Fig. 5). If a bead is stabilized at inside of tunnel, then its successor is already decided the position either inside of a tunnel or outside. Moreover, If a bead is stabilized at outside of a tunnel, then its position is an entrance of a tunnel or otherwise.

Tunnel sections have three possible shape with considering symmetry such as acute turn, straight and obtuse turn (Figure 3). Let us focus entrances of a tunnel such as p', then Entrances have two possible shape (Figure 4).

Let us consider tunnel sections only tunnel A and B. See Fig. 6. From appendix (Entrance of Tunnel A, B and Exit of Tunnel), we add maximum number which each transitions provide binding capabilities for each edge where a is number of consuming binding capabilities when the bead is stabilized at position of *successer in outside*.

Next, we consider on tunnel C section. If $w[i]$ is stabilized by tunnel C and $S[i+1]$ is t, then $w[i+1]$ is stabilized by tunnel A or B because if $w[i+1]$ is stabilized by tunnel C, then C_{i+1} is a terminal. Hence, tunnel C section is devided cases sush as Figure 7. Cases of $S[i...] = bt^l (l \geq 2)$ are already considered (Upper). According to appendix (Tunnel C), cases of $S[i..i+2] = btb$ also consume some binding capabilities (Lower).

Thus, if a bead is stabilized through a tunnel section, then it consume some binging capabilities.

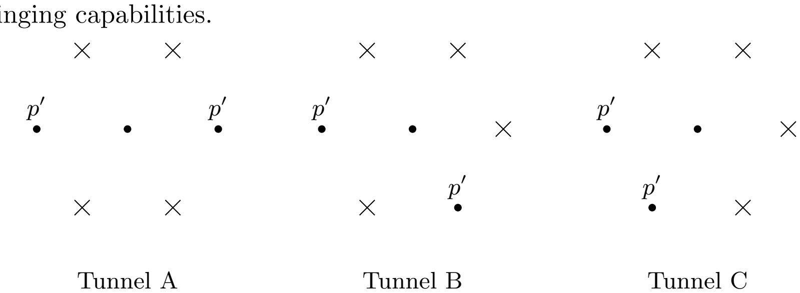


Fig. 3. All possible tunnel sections

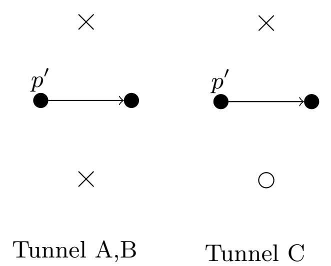


Fig. 4. Entrance of a tunnel

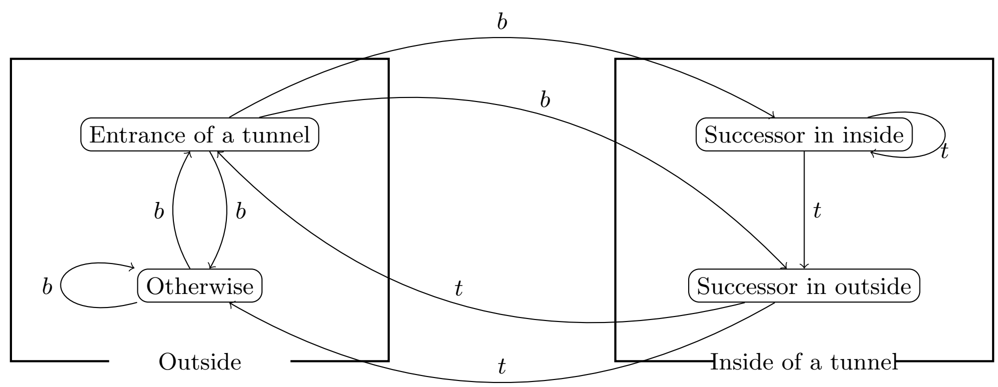


Fig. 5. Cases on position of a bead

1.3 Appendix of Tunnel Troll Theorem

Entrance of TunnelA,B Fig.8 exhibits all the three kinds of entrance of tunnel A, B. Any cases in $\delta = 1, \alpha = 2$ consume some binding capabilities into the follows.

- Case of t_0
  Let us consider points of c, d either occupied or not. A point c or d is free because if both of them are occupied, p' is inside of tunnel. If c is free, then p' has to be bound to a bead except A due to deterministically stabilize. In this situation, at least three neighbors of A are free that is at most three neighbors of A are occupied. A leave at least one binging capability because beads of beighbors are predecessor and successor in addition A is able to consume itself binging capabilities only one-time. Hence, p' must be bound to A. Thus, a case of t_0 consumes some binding capabilities.

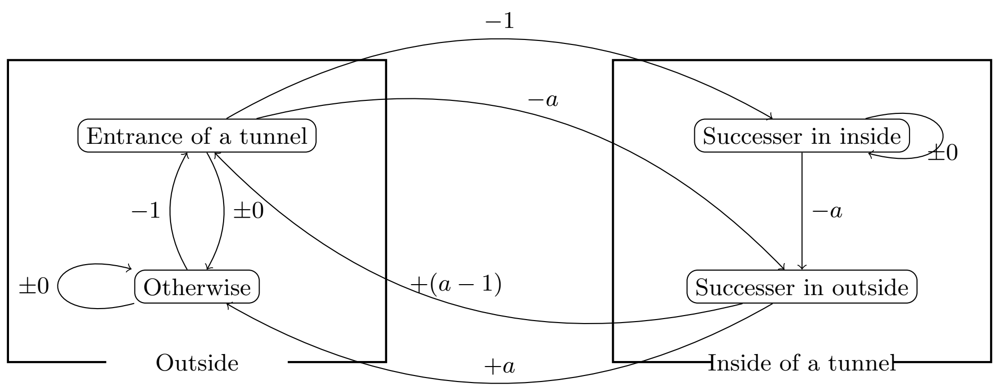


Fig. 6. Increment on Tunnel A,B

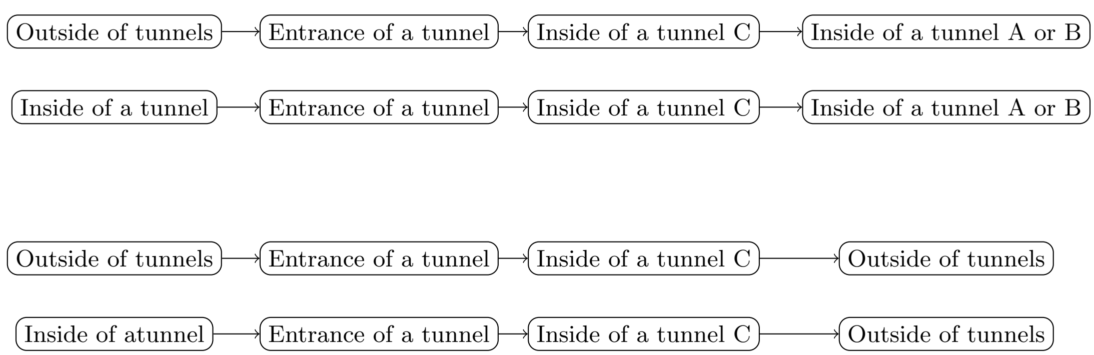


Fig. 7. Case of Tunnel C

– Case of $t_{\pm 60}$
  In this case, a point c or d is free, too. If c is free, p' has to be bound to A or B. If c is occupied, then d is free. This time, B has some binding capabilitie so p' has to be bound to B.
  In this situation, p' is able to supply a binding capability. if this capability is active, p' bind a bead into c or d. However, B and P are exist in back bone. According to Jordan curve theorem, any successors of p' cannot reach a point c or d so this capability is inactive. Thus, a case of $t_{\pm 60}$ consumes some binding capabilities.
– Case of $t_{\pm 120}$
  Binding capabilities that p' supply are inactive according to Jordan curve theorem on A and P. Moreover, p' has to be bound to one of c, d,e in order to deterministically stabilize. Thus, a case of $t_{\pm 120}$ consumes some binging capabilities.

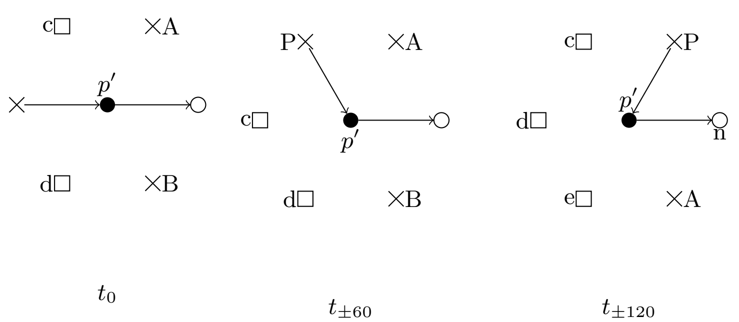


Fig. 8. Direction into a entrance

Exit of Tunnel Fig.9 exhibits all the two kings of exit of tunnel. At least one of point c or d is free because if both of them are occupied, p' is inside of tunnel.

$\delta = 1, \alpha = 2$ Any cases of $\delta = 1, \alpha = 2$ supply at most a binding capabilities into follows where a is number of predecessor of p' consumes binging capabilities.

– Case of c and d are free
  This case can be regarded same situation as entrance. See Fig.9 (Left). Predecessor P has to be bound A and B because each of A and B leave binding capabilities. Hence, at least $a = 2$. This time, $\alpha = 2$ that is this case supply at most only a binding capabilities.
– Case of c is occupied
  See Fig.9 (Right). If c is pccupied, then d is free so that P has to be bound B. Hence, at least $a = 1$. This case can supply two binding capabilities but p' can bind to only one of e or d because e or d will be occupied a successor of p'. Therefore, this case supply at most only $a = 1$ active binding capabiliry.

$\delta = 1, \alpha \geq 3$ Any cases of $\delta = 1, \alpha \geq 3$ consume some binding capabilities into follows.

– Case of c and d are free
  In $\alpha \geq 3$, if three neighbors of a bead leave, then it can supply two binding capabilities. Therefore Predecessor P has to be bound A and B, and p', too. In this case, at least consumes four bindings and supplies at most two bindings. Thus, it consumes some binding capabilities, totally.
– Case of c is occupied
  In this case, B leave at least two bindings and A, c also leave at least one binding. Therefore P has to be bound A and B, and p' also has to be bound B and c. In this case, at least consumes four bindings and supplies at most one binding. Thus, it consumes some binding capabiliries, totally.

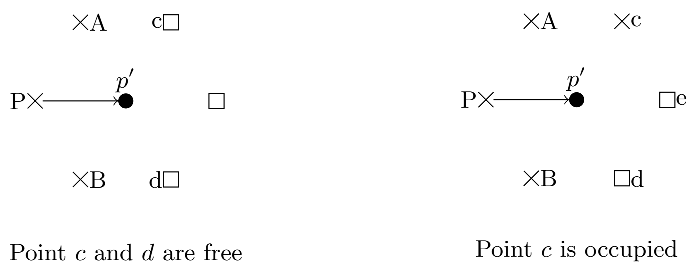


Fig. 9. Exit of Tunnel

Tunnel C Assume $w[i]$ is a bead which stabilized by tunnel C. Let us consider kinds of stabilization $S[i-2..i] = tbt$ or $S[i-2..i] = bbt$ except cases of $w[i]$ is inside of tunnel A, B.

Case of $S[i-2..i] = tbt$ Fig.10 exhibits all the two kinds of stabilization depending on structures of tunnel C.

– Left of Fig.10
  In this figure, Bead A has at least one binding so that w_{i-2} has to bound A. Moreover, w_{i-1} has to bound one of B, C, D in order to stabilize deterministically. On the other hand, w_i can supply two bindings but free neighbors of w_i are two points. One of them is occupied a successor. Therefore w_i can only bind one of e, f that is w_i supplies at most one binding. Thus, this case consumes some binding capabilities.
– Right of Fig.10
  This cases are divided on number of capabilities that w_{i-2} consumes.
  - w_i does not consume any bindings
    w_{i-1} has to bound one of B, C, D in order to stabilize deterministically. w_i has to be bound to w_{i-2} because w_{i-2} has bindings. This time, let us

consider either e is occupied or not. If e is occupied, then w_{i-2} has no active bindings that is this situation consumes some binding capabilities. If e is free and w_{i+1} is stabilized in e, then w_{i-2} has to bind w_{i+1}. Therefore, In this case, stabilization of $w[i-2..i+1]$ consumes some bindings. If e is free and w_{i+1} is stabilized except e, then this oritatami system has to use two binding capabilities in order to bind w_{i+1}. Therefore, in this case consumes some bindings. Thus in this cases consume some binding capabilities.

- w_i consumes one binding
  In this case, w_{i-1} has to be bound one of B, C, D. In addition, w_{i-2} and w_i are not supply any bindings. Thus, in this cases consume some binding capabilities.
- w_i consumes two bindings
  In this case, w_{i-2} already consumes two binding. w_{i-1} has to be bound. w_i supplies two bindings. Thus, in this cases consume some binding capabilities.

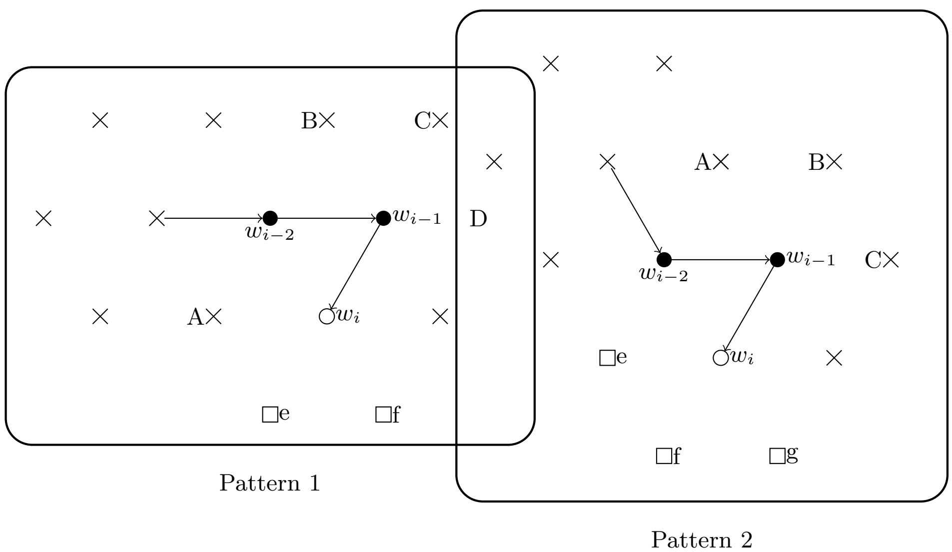


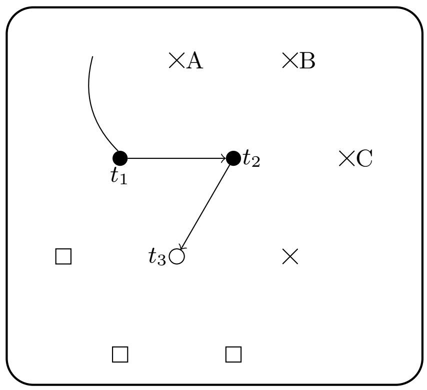
Fig. 10. Case of $S[i-2..i] = tbt$

Case of $S[i-2..i] = bbt$ Let us consider number of consumed by w_{i-2} (Fig.11).

– w_{i-2} consumes one binding
  In this situation, w_{i-2} supplies one active binding whereas w_i consumes this

binding. In addtion, w_{i-1} has to bound to one of A, B, C. Thus, in this cases consume some binding capabilities.

– w_{i-2} consumes two bindings
  In this case, w_{i-2} already consumes two binding. w_{i-1} has to be bound. w_i supplies at most two bindings. Thus, in this cases consume some binding capabilities.



Pattern 1

Fig. 11. Case of $S[i-2..i] = bbt$

By Tunnel Troll Theorem, any tunnel sections which represented in bbt^+ or bt^+bt^+ consume binding capabilities. If the sequence S is free from any subsequence of the form bt^+bt^+, then it can factorize as $S = u_1u_2u_3\cdots$ for some $u_1, u_2, u_3, \cdots \in \{b\} \cup bbt^+$. Assume the length of σ is n, seed supplies at most $2n$ binding capabilities. Therefore formula **??** hold.

$$\exists i \in \mathbb{N} \quad s.t. \quad u_{i-1}, u_i, u_{i+1}, u_{i+2}, \cdots \in \{b\} \tag{1}$$

Let us represent S as $S[i.i+1...] = v_iv_{i+1}v_{i+2}\cdots$ for some $v_i, v_{i+1}, v_{i+2}, \cdots \in \{a, o\}$ where if v_k is a, then v_{k+1} is bound to v_{k-1}, if v_k is o, then v_{k+1} is NOT bound to v_{k-1}.

Let us consider the case of v_k is o. See Fig.12. v_{i-1} consumes some binding capabilities because $S[i-1]$ is b. If the number of v_{i-1}'s bindings is one binding, then v_{i+1} has to be bound except A or B so that v_{i+1} must consumes two bindings except the case of A and B are occupied and v_i consumes at least one binding. If A and B are occupied, then v_{i-1}'s binginds are inactive that is v_{i-1} consumes two binding capabilities. Therefore, this case consumes binding capabilities. If v_{i-1} dose Not have any bindings, then v_{i-1} already consumes two bindings. In addition, v_i and v_{i+1} consume at least one binding. Therefore this case consumes binding capabilities. Thus, the formula 2 hold and according to the formula 1 and the formula 2, the formula 3 is hold.

$$\exists j \in \mathbb{N} \quad s.t. \quad u_j, u_{j+1}, u_{j+2}, \cdots \in \{a\} \tag{2}$$

$$|S| > \forall m \in \mathbb{N} \quad \rightarrow \quad \exists n \in \mathbb{N} \quad s.t. \quad S[n], S[n+1], \cdots \in \{a\} \tag{3}$$

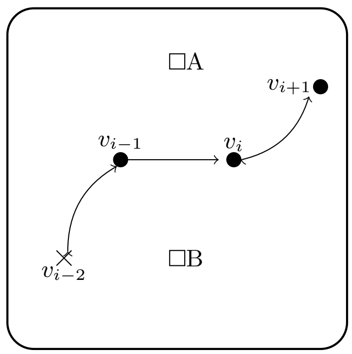


Fig. 12. Case of $S[i$