Table 1 Regularization in early vision

Problem	Regularization principle
Edge detection	$\int [(Sf-i)^2+\lambda(f_{xx})^2]\,dx$
Optical flow (area based)	$\int [i_x u + i_y v + i_t)^2 + \lambda(u_x^2+u_y^2+v_x^2+v_y^2)]\,dx\,dy$
Optical flow (contour based)	$\int [(V\cdot N - V^N)^2 + \lambda((\partial/\partial_s)V)^2]\,ds$
Surface reconstruction	$\int [S\cdot f - d)^2 + \lambda(f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2)^2]\,dx\,dy$
Spatiotemporal approximation	$\int [(S\cdot f - i)^2 + \lambda(\nabla f\cdot V + ft)^2]\,dx\,dy\,dt$
Colour	$\lVert I^{\nu} - Az\rVert^2 + \lambda\lVert Pz\rVert^2$
Shape from shading	$\int [(E - R(f,g))^2 + \lambda(f_x^2 + f_y^2 + g_x^2 + g_y^2)]\,dx\,dy$
Stereo	$\int \{[\nabla^2 G * (L(x,y) - R(x+d(x,y),y))]^2 + \lambda(\nabla d)^2\}\,dx\,dy$

Some of the early vision problems that have been solved in terms of variational principles. The first five are standard quadratic regularization principles. In edge detection[26,27] the data on image intensity ($i = i(x)$) (for simplicity in one dimension) are given on a discrete lattice: the operator S is the sampling operator on the continuous distribution f to be recovered. A similar functional may be used to approximate time-varying imagery. The spatio-temporal intensity to be recovered from the data $i(x, y, t)$ is $f(x, y, t)$; the stabilizer imposes the constraint of constant velocity V in the image plane (ref. 61). In area-based optical flow[18], i is the image intensity, u and v are the two components of the velocity field. In surface reconstruction[21,22] the surface $f(x, y)$ is computed from sparse depth data $d(x, y)$. In the case of colour[32] the brightness is measured on each of three appropriate colour coordinates $I^{\nu}(\nu = 1, 2, 3)$. The solution vector z contains the illumination and the albedo components separately; it is mapped by A into the ideal data. Minimization of an appropriate stabilizer enforces the constraint of spatially smooth illumination and either constant or sharply varying albedo. For shape from shading[19] and stereo (T.P. and A. Yuille, unpublished), we show two non-quadratic regularization functionals. R is the reflectance map, f and g are related to the components of the surface gradient, E is the brightness distribution[19]. The regularization of the disparity field d involves convolution with the laplacian of a gaussian of the left (L) and the right (R) images and a Tikhonov stabilizer corresponding to the disparity gradient.