The regularization of the ill-posed problem of finding z from the 'data' y

$$Az = y \tag{1}$$

requires the choice of norms $\|\cdot\|$ and of a stabilizing functional $\|Pz\|$. In standard regularization theory, A is a linear operator, the norms are quadratic and P is linear. Two methods that can be applied are[8,13]: (1) among z that satisfy $\|Az-y\| \leqslant \varepsilon$ find z that minimizes (ε depends on the estimated measurement errors and is zero if the data are noiseless)

$$\|Pz\|^2 \tag{2}$$

(2) find z that minimizes

$$\|Az-y\|^2 + \lambda \|Pz\|^2 \tag{3}$$

where λ is a so-called regularization parameter.

The first method computes the function z that is sufficiently close to the data and is most 'regular', that is minimizes the 'criterion' $\|Pz\|^2$. In the second method, λ controls the compromise between the degree of regularization of the solution and its closeness to the data. Standard regularization theory provides techniques for determining the best λ[12,15]. Thus, standard regularization methods impose the constraints on the problem by a variational principle, such as the cost functional of equation (3). The cost that is minimized reflects physical constraints about what represents a good solution: it has to be both close to the data and regular by making the quantity $\|Pz\|^2$ small. P embodies the physical constraints of the problem. It can be shown for quadratic variational principles that under mild conditions the solution space is convex and a unique solution exists. It must be pointed out that standard regularization methods have to be applied after a careful analysis of the ill-posed nature of the problem. The choice of the norm $\|\cdot\|$, of the stabilizing functional $\|Pz\|$ and of the functional spaces involved is dictated both by mathematical properties and by physical plausibility. They determine whether the precise conditions for a correct regularization hold for any specific case.

Variational principles are used widely in physics, economics and engineering. In physics, for instance, most of the basic laws have a compact formulation in terms of variational principles that require minimization of a suitable functional, such as the energy or the lagrangian.