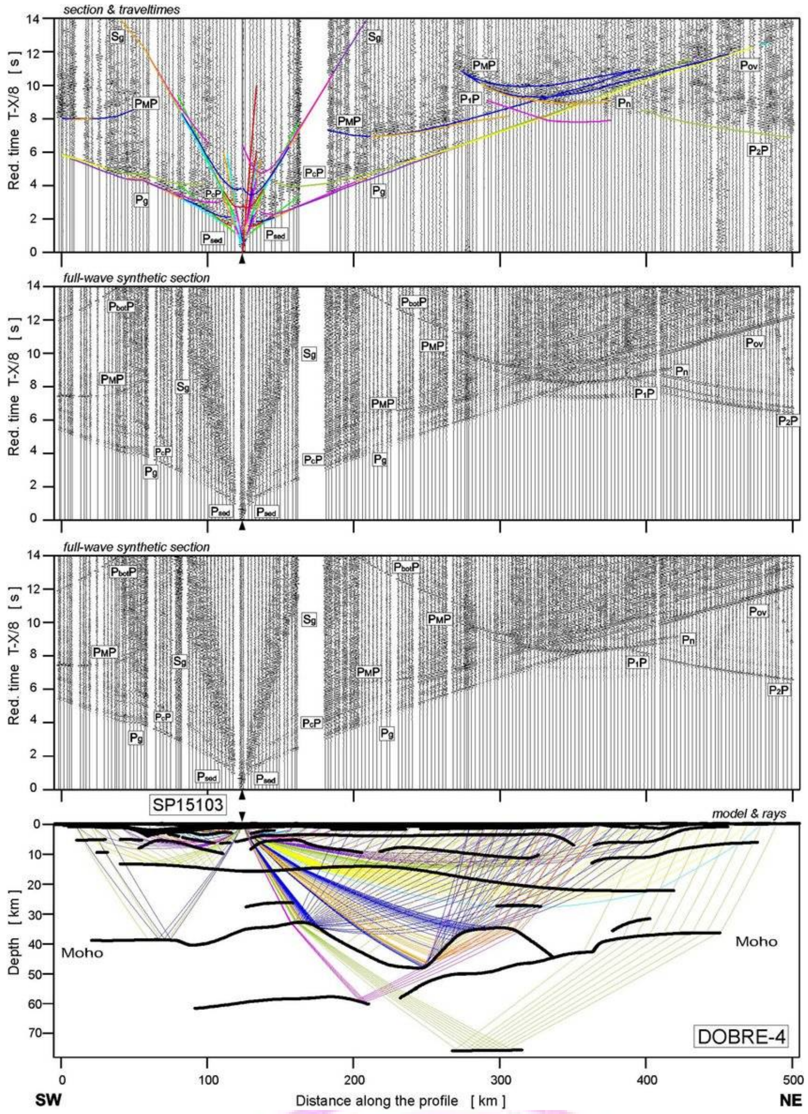
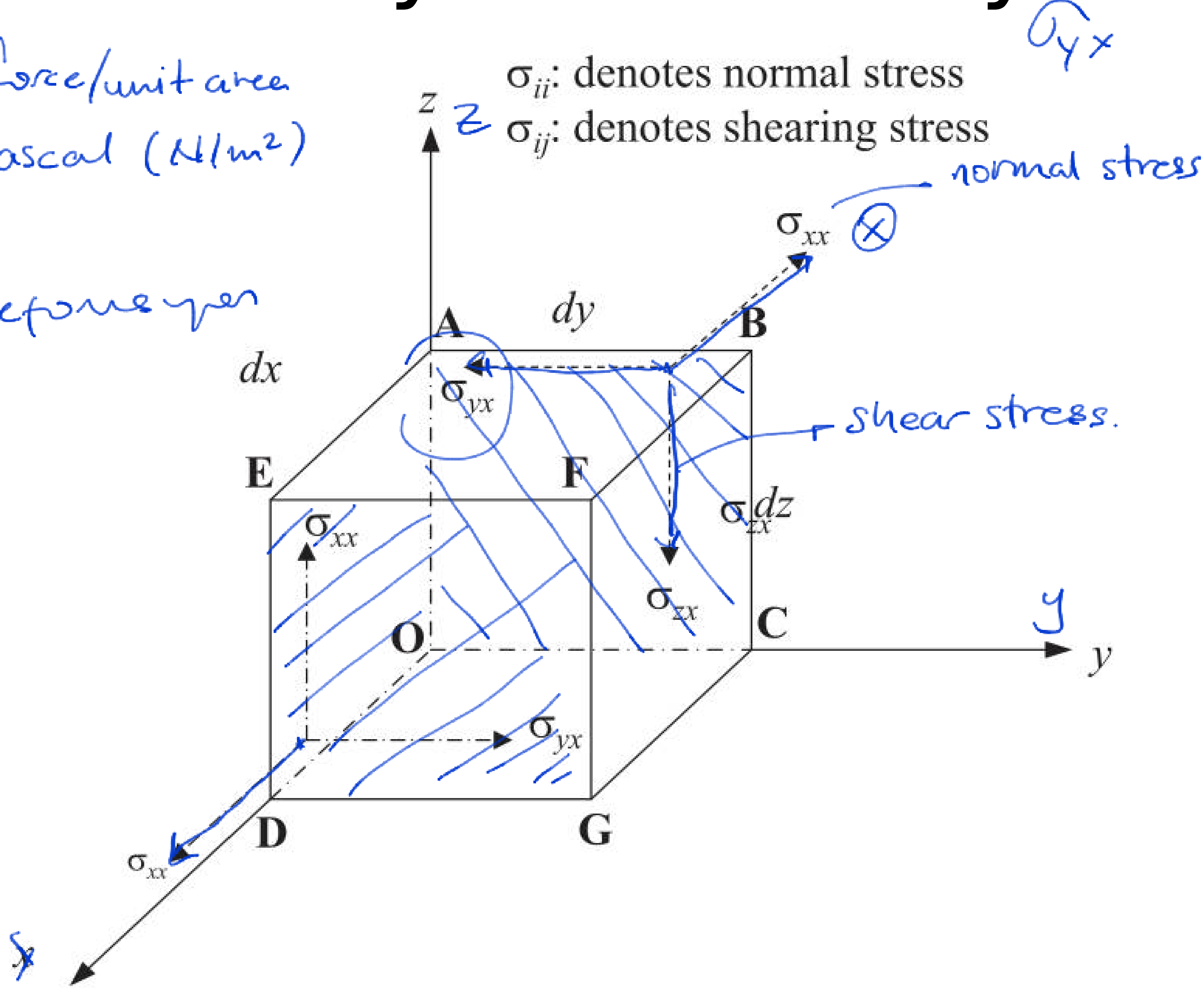
Seismic Theory



Theory of Elasticity

$\sigma \rightarrow$ force/unit area
Pascal (N/m^2)

strain : deformation
ε



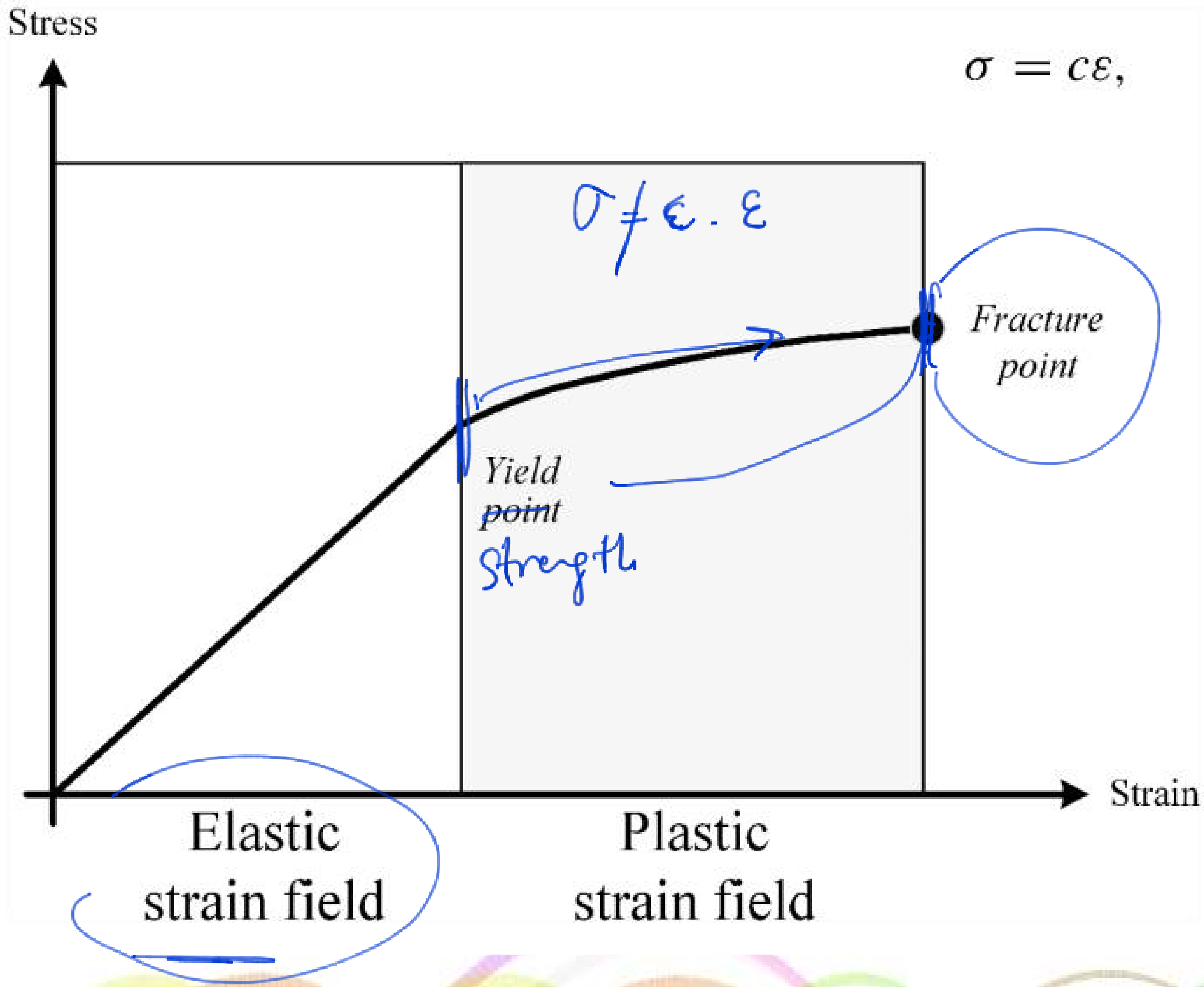
Theory of Elasticity

$\sigma = C \, \varepsilon$

Stiffness tensor

λ lame

μ

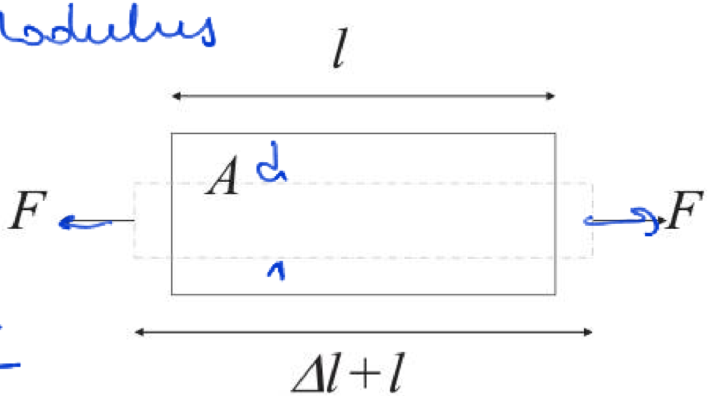


Theory of Elasticity

Young Modulus

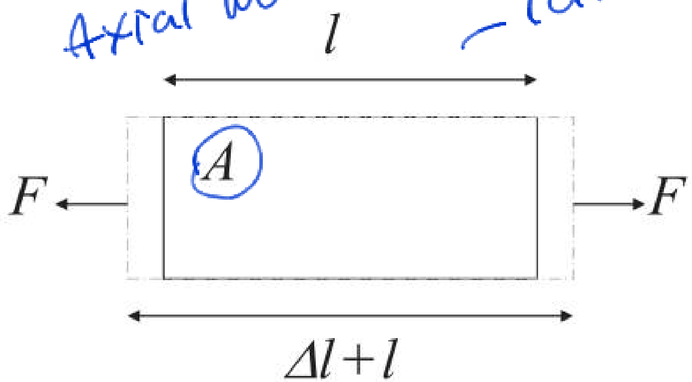
Axial modulus.

— lateral strain



(a)

$Y = \frac{F/A}{\Delta l/l}$

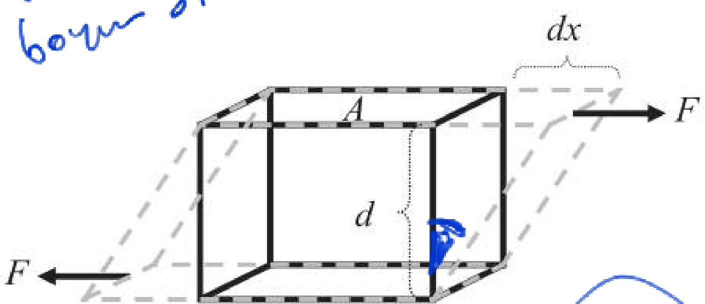


(b)

$k =$ Bulk modulus $= -\frac{P}{\Delta V/V}$

↑ ↓ volume

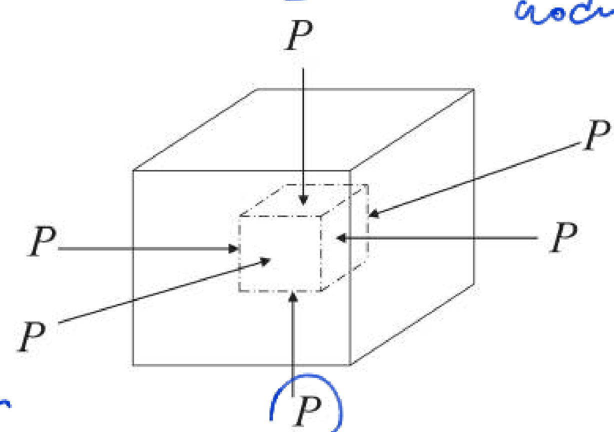
Poisson ratio = lateral strain / longitudinal strain



(c)

rigidity → Young modulus shear modulus.

$M = \frac{F/A}{dx/d}$ shear stress / shear strain



(d)

$P = \frac{F}{A}$

Theory of Elasticity

Exp: $F = 100kg.$

$A = 0.1 \times 10^{-4} m^2$

$1m$ $\quad 0.2 \times 10^{-2} m$

$\sigma = ?$ $\quad \frac{F}{A} = \frac{M \cdot g}{A} = \frac{100 \times 9.8}{0.1 \times 10^{-4}} = \underline{\underline{9.8 \times 10^{7} N/m^2}}\ \sigma$

$\varepsilon = ?$ $\quad \frac{\Delta l}{l} = \frac{0.2 \times 10^{-2}}{1}$ tensile strain.

$Y = ?$ $\quad \frac{\sigma}{\varepsilon} = 490 \times 10^{9} N/m^2$

$g = 9.8\, m/s^2$

$K = -\frac{P}{\Delta\xi/\xi} = -\frac{P\xi}{K} = -\frac{(2 \times 10^{7})(0.8 m^3)}{7.7 \times 10^{9}} = V$

Wave Equation & d'Alembert Solution

$$\nabla^2 u(x,y,z,t) = \frac{1}{v^2}\frac{\partial^2 u(x,y,z,t)}{\partial t^2}$$

Hooke's law $\sigma = c \cdot \varepsilon$

Newton law $F = m \cdot a$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$

1-D wave form

$$u(x,t) = A \cdot \exp\left[j\, 2\pi f\left(t - \frac{x}{v}\right)\right]$$

$$\frac{\partial u(x,t)}{\partial x} = -\frac{j 2\pi f}{v} A \exp\left[j\, 2\pi f\left(t - \frac{x}{v}\right)\right]$$

$$\frac{\partial^2 (x,t)}{\partial x^2} = \left(-\frac{4\pi^2 f^2}{v^2}\right) A \exp[\;\;]$$

$= j 2\pi f \; A \exp[-u \;\;]$

$\frac{d}{dx} \quad \partial x^2 = -\frac{4\pi^2 f^2}{v^2}$

Wave Equation & d’Alembert Solution

$$\nabla^2 u(x,y,z,t) = \frac{1}{v^2}\frac{\partial^2 u(x,y,z,t)}{\partial t^2}$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$

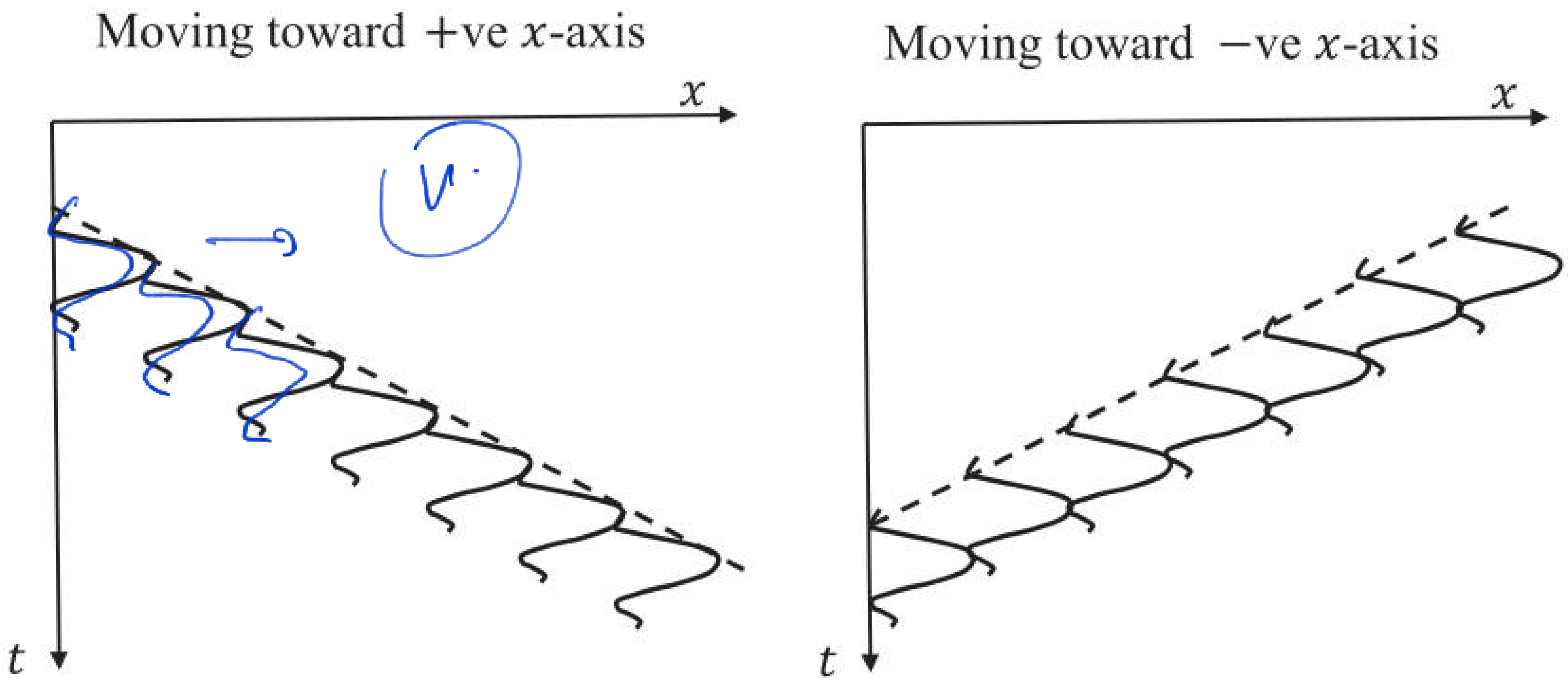
$$u(x,t) = B(x+vt) + F(x-vt).$$

backward

forward

$$u(x,t) = F(x-\cancel{vt}) = F(x)$$

Wave Equation & d’Alembert Solution



$F(x - vt)$
Forward propagation

$B(x - vt)$
Backward propagation

$u(x,t) = A\,s$

Wave Equation &
d'Alembert Solution

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$

$$\begin{cases} u(x,t) = A\sin[k(x - vt)], \\ u(x,t) = A\cos[k(x - vt)], \end{cases}$$

amplitude

wave number

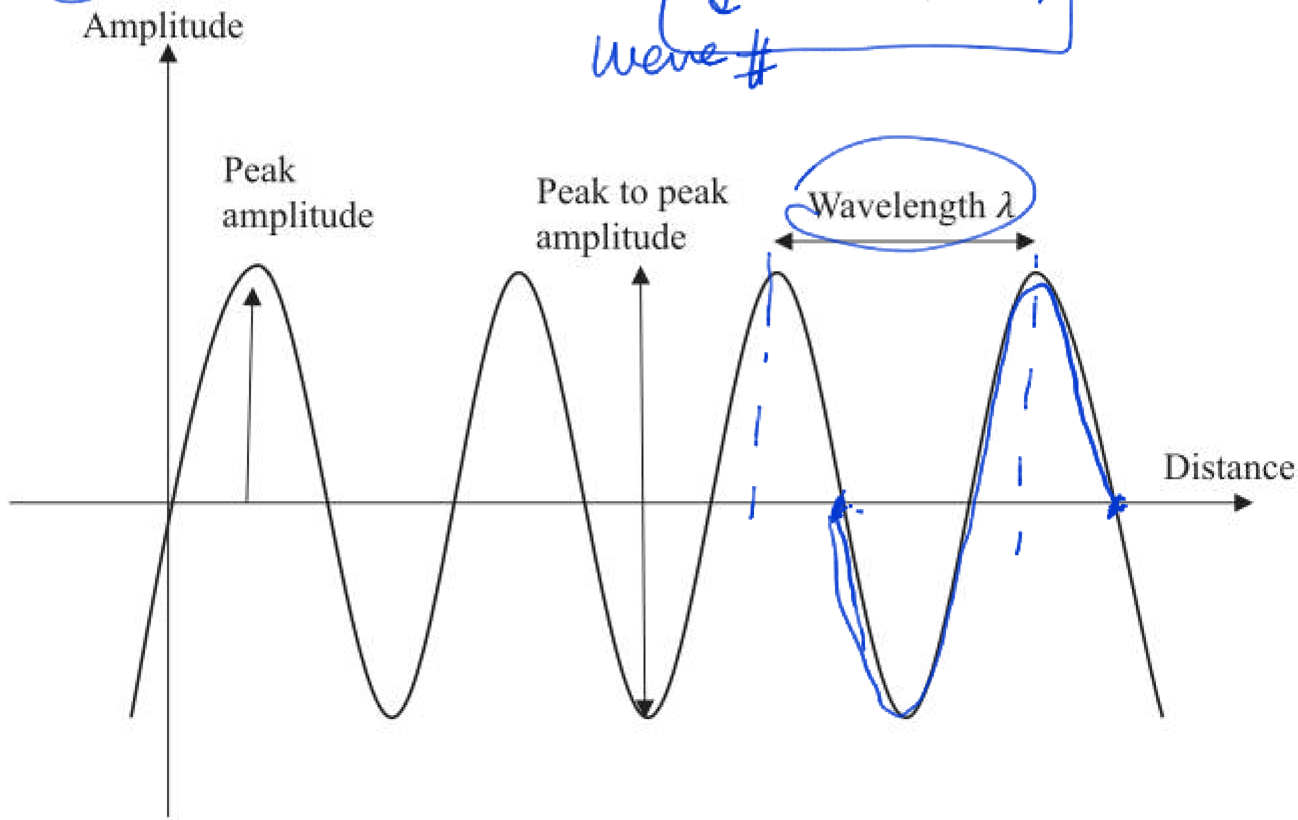
cycle/unit dist = rad/m

Wave Equation & d’Alembert Solution

$$u(x,t) = A\sin[k(x-vt)] \rightarrow \boxed{k = \frac{2\pi}{\lambda}}$$

$k \downarrow$ wave #

$\lambda \rightarrow$ wavel.



Wave Equation & d’Alembert Solution

$\boxed{k \cdot v \cdot T = 2\pi}$

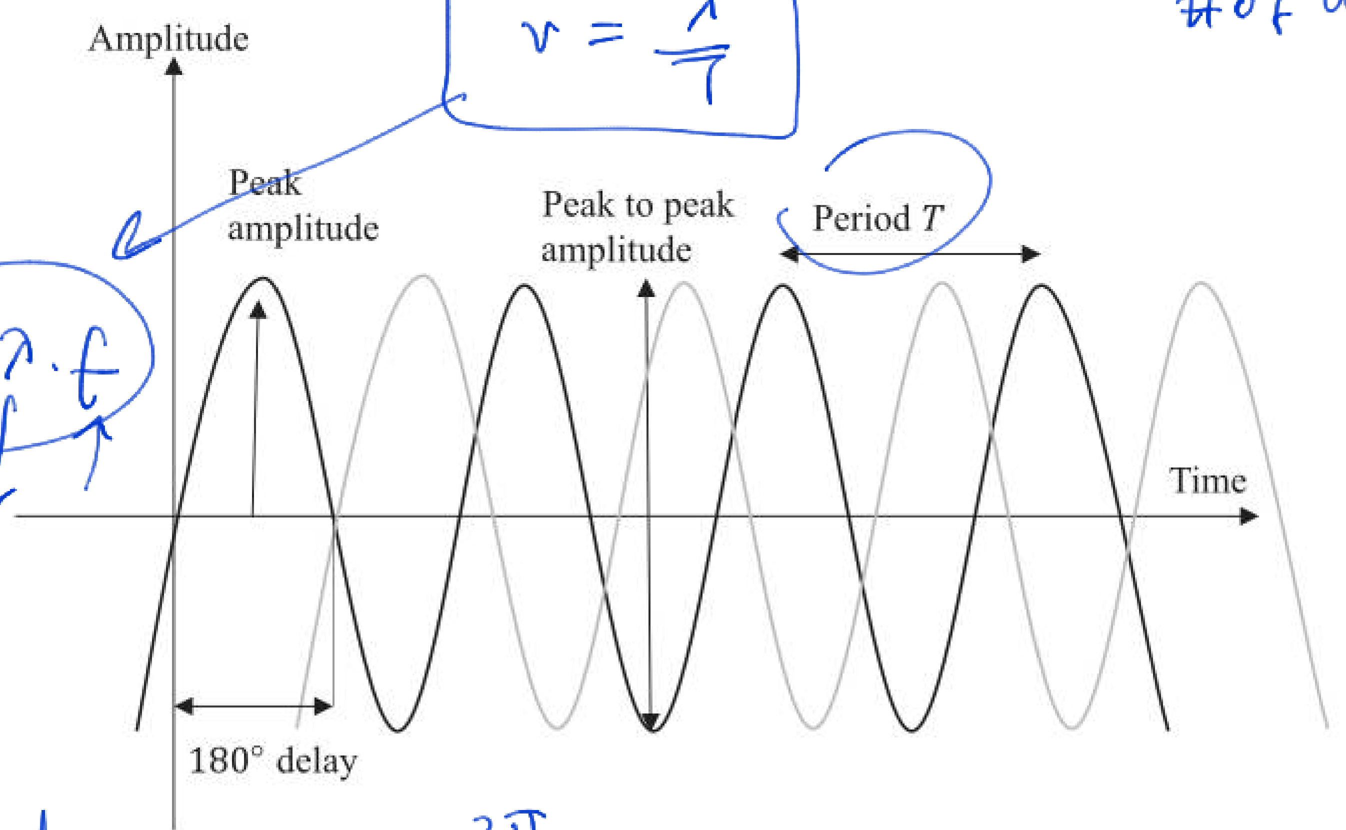
$u(x,t) = \sin[k(x - vt)] = \sin[k(x - vt) \mp 2\pi]$

# of units/cycle

$\boxed{v = \dfrac{\lambda}{T}}$

$f = \text{Hz}$

$f = \dfrac{1}{T}$ $\quad v = \lambda \cdot f$



$\omega = \dfrac{\text{radial}}{\text{sn.}} = 2\pi f = \dfrac{2\pi}{T}$

Wave Equation & d'Alembert Solution

λ, k, T, f $\quad u(x,t) = A \sin[k(x - vt)]$

$u(x,t) = 10 \sin(0.1\pi x - 20\pi t)$

- Amplitude?
- k?
- $w = ?$
- $f = ?$
- $v = ?$
- $T = ?$
- $\lambda = ?$

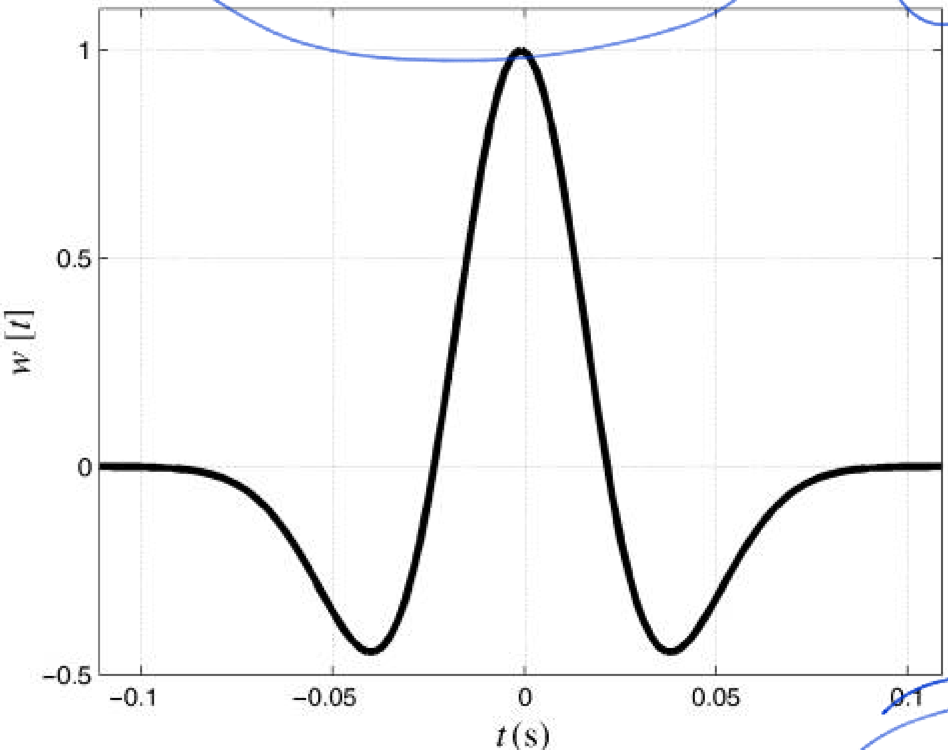
Wave Equation & d'Alembert Solution

$$f_{dom} = \frac{f_1 + f_2}{2}$$

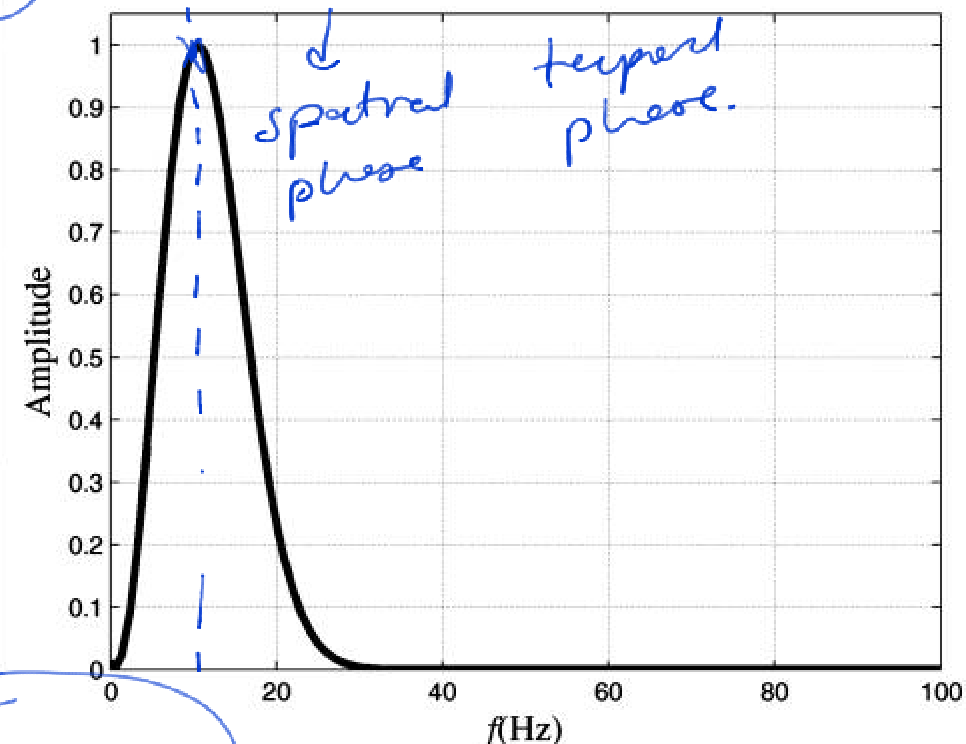
$$u(x,t) = A\sin[kx - \omega t]$$

$$\phi(x,t) = \underbrace{kx}_{\text{spatial phase}} - \omega t$$

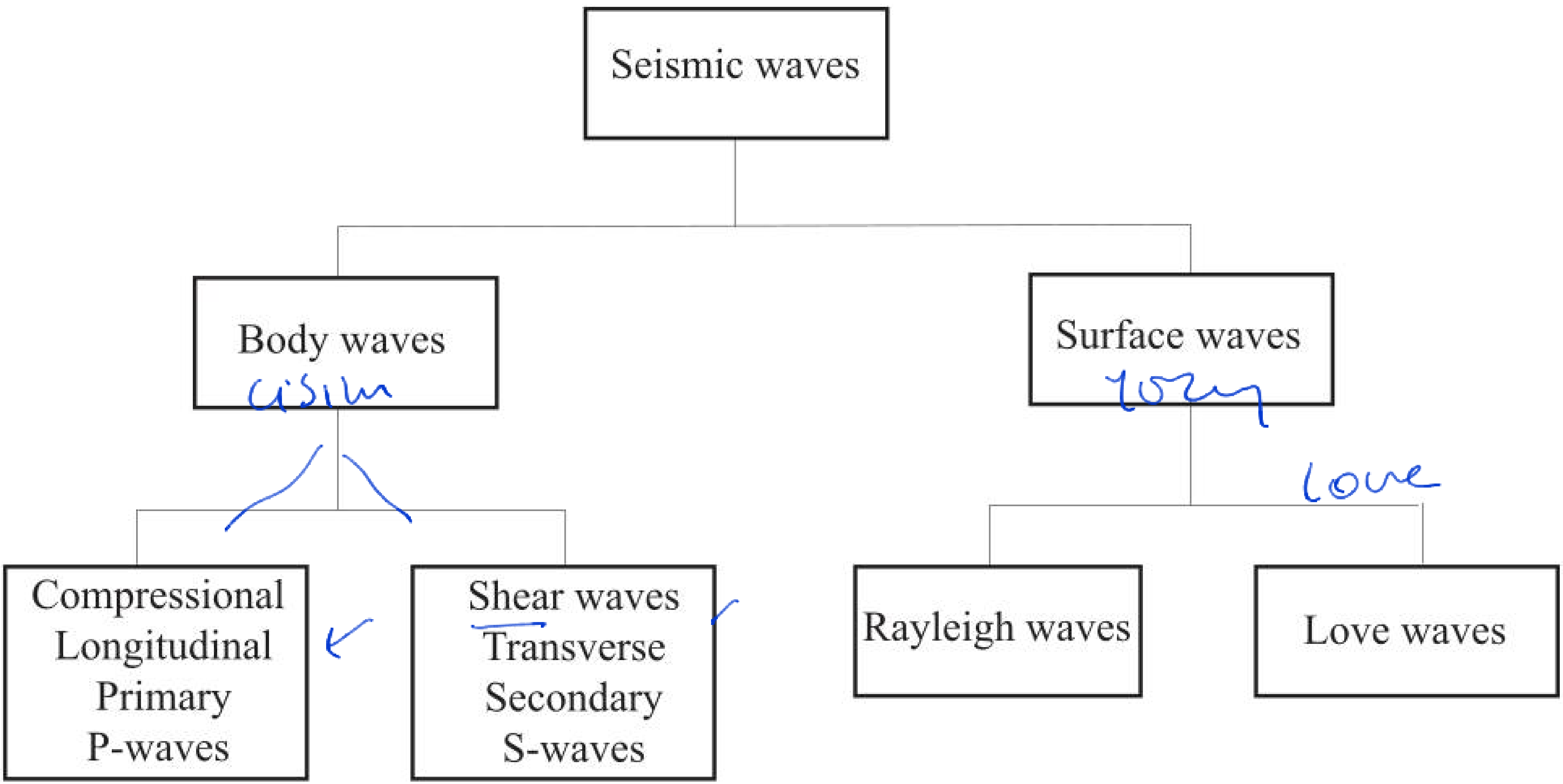
temporal phase



FFT



Seismic Waves



Body Waves

Material Type	v_p (m/s)	v_s (m/s)	ρ (kg/m^3)
Air	331	0	3.8
Limestone	3,500–6,100	2,000–3,300	2,400–2,700
Dolomite	3,500–6,500	1,900–3,600	2,500–2,900
Sandstone	2,000–4,300	700–2,800	2,100–2,400
Soil	300–900	120–360	1,700–2,400
Stiff mud	1,600	0	1,500
Water	1,500	0	1,000

← isotropic

yön bağımsız

anisotropy →

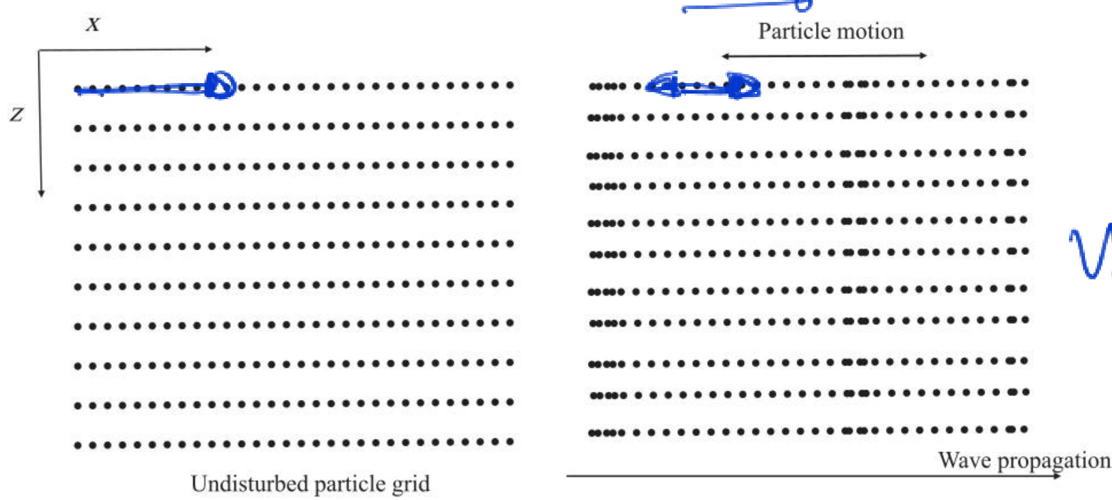
$$v = \frac{c}{\rho}$$

non-dispersive

Body Waves

P wave
Primary
Comp

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

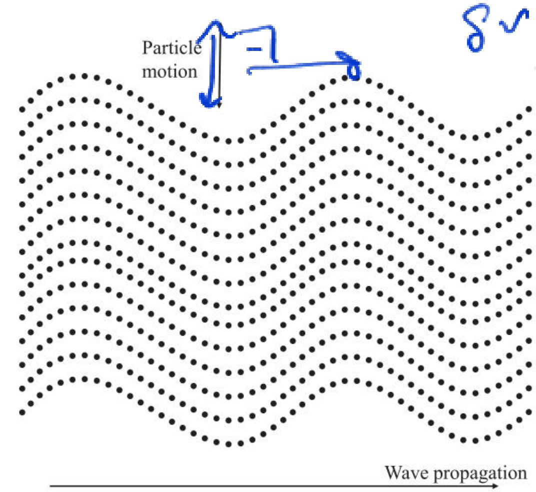


$$V_s = \sqrt{\frac{\mu}{\rho}}$$

SV/SH

$V_p > V_s$

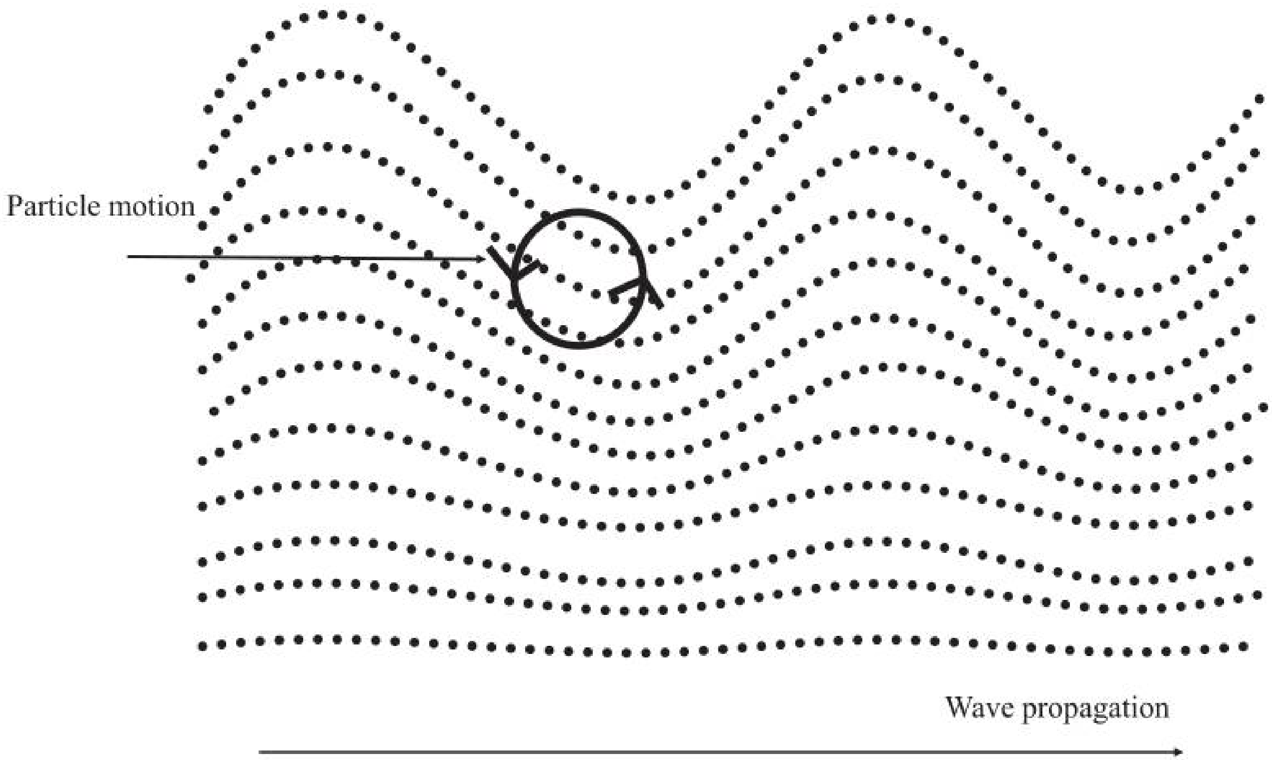
Shear wave
Secondary
S wave



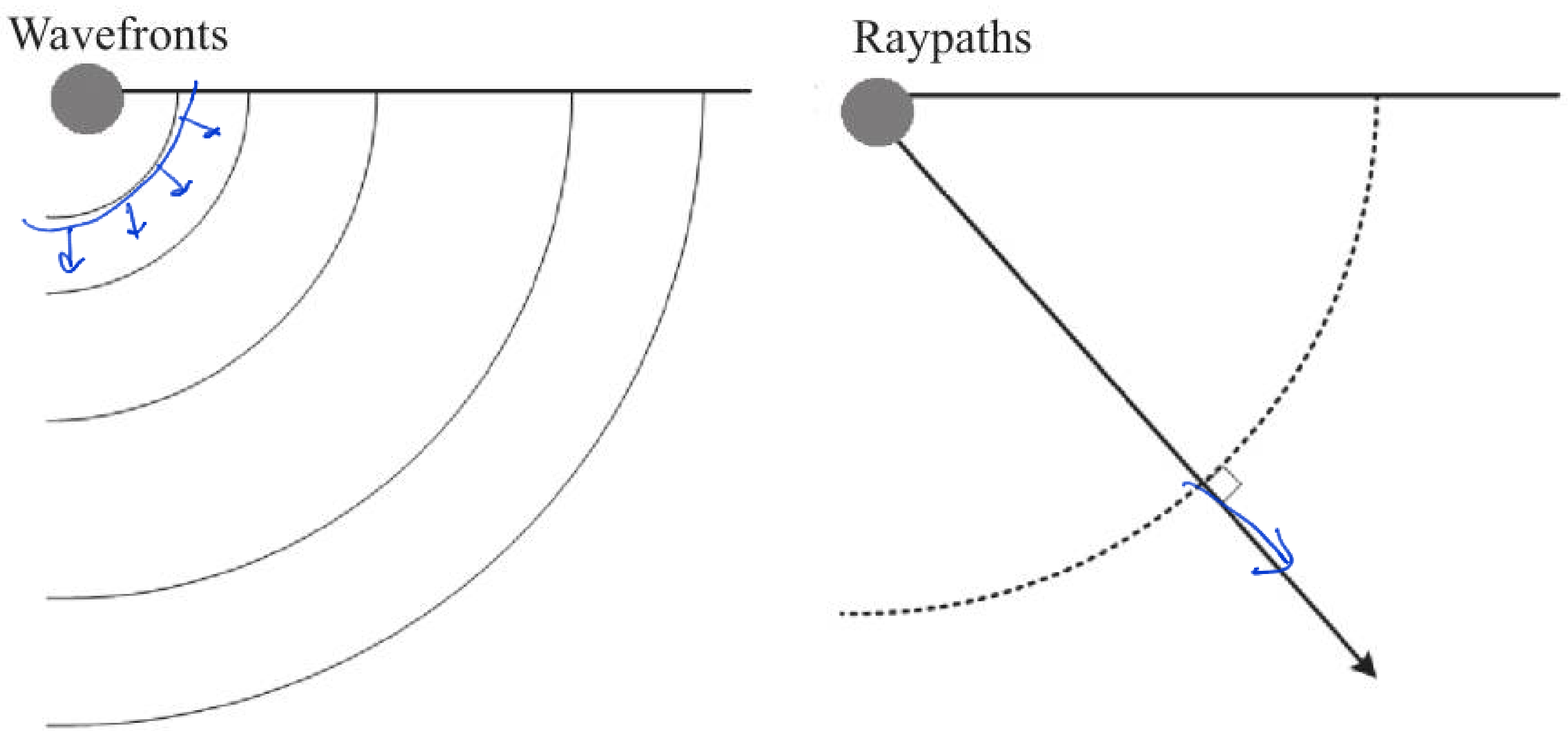
Surface Waves

Rayleigh / Love.

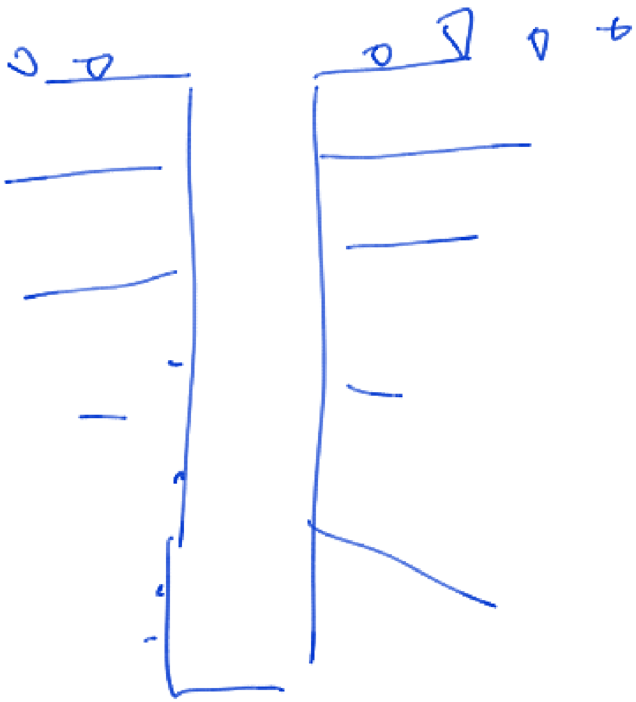
amplitd



Seismic Wavefronts and Raypaths



Seismic Wave Velocity and Rocks



$$v_s^2 = v_p^2 \frac{1-2\alpha}{2(1-\alpha)}$$

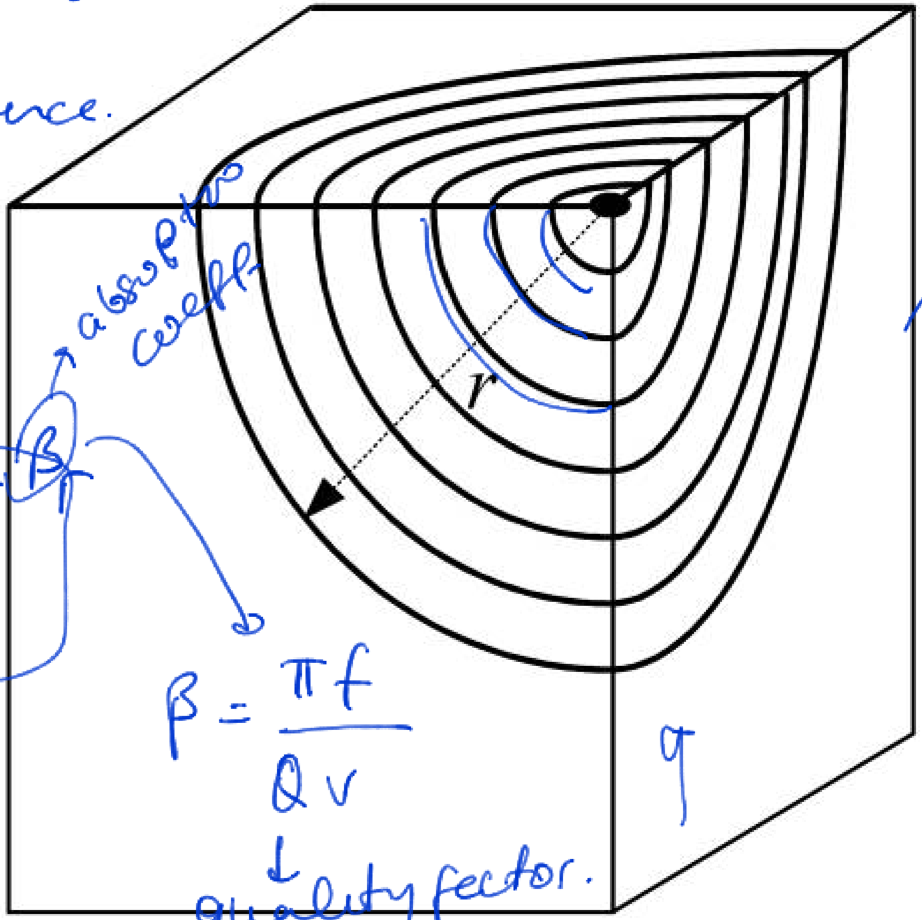
α poisson ratio.

Amplitude Attenuation

1- transmission losses

2- Geometric divergence.

3- Absorption.



absorption coeff.

$$A = A_0 \frac{1}{r} e^{-\beta r}$$

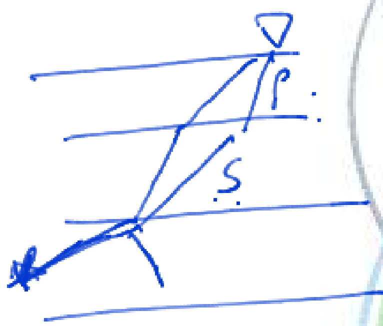
amplitude

keynote amp.

radius.

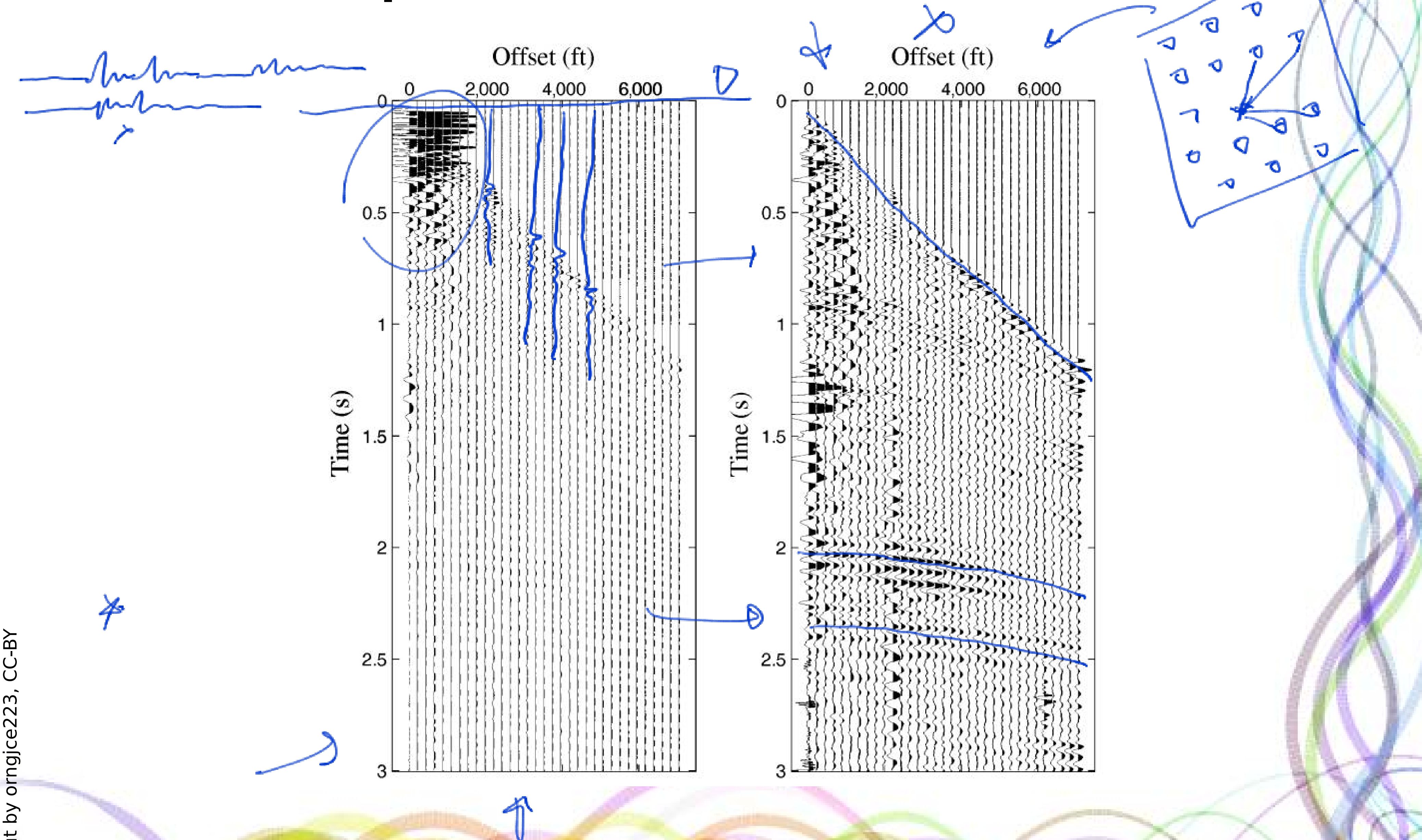
$$\beta = \frac{\pi f}{Q v}$$

quality factor.

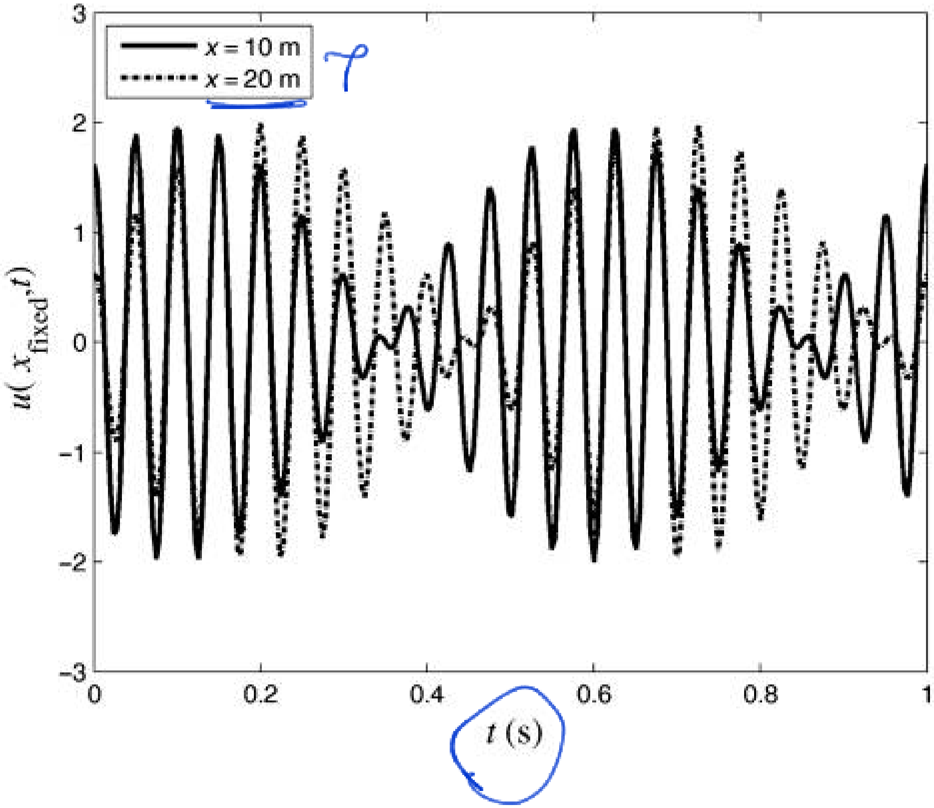


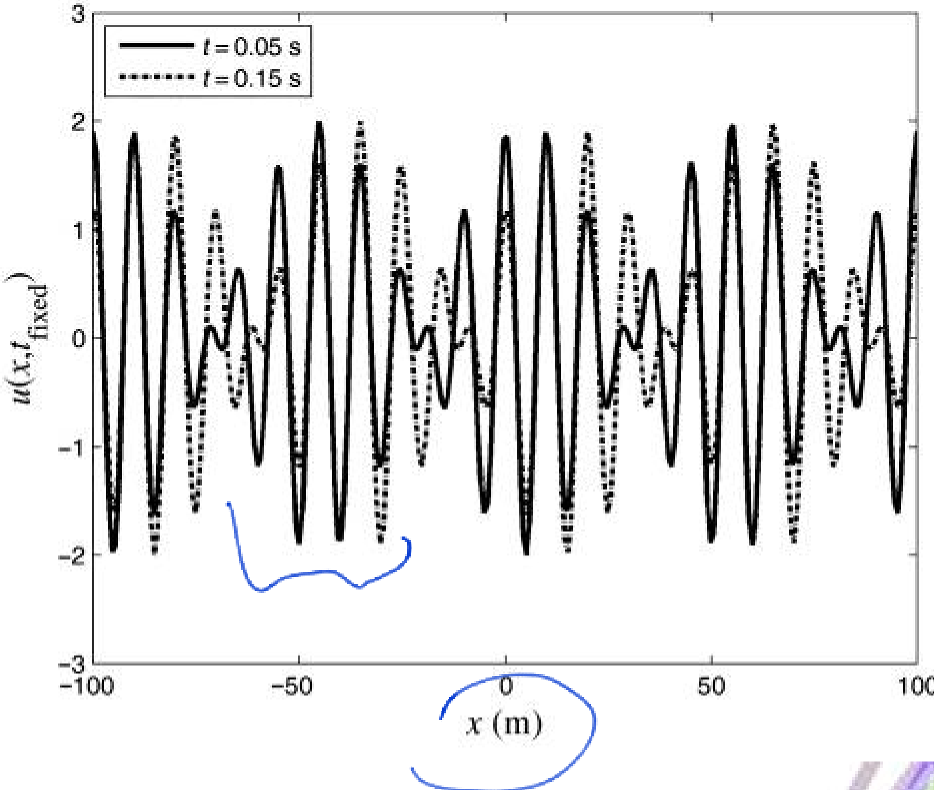
$$\frac{E}{4\pi r^2}$$

Amplitude Attenuation

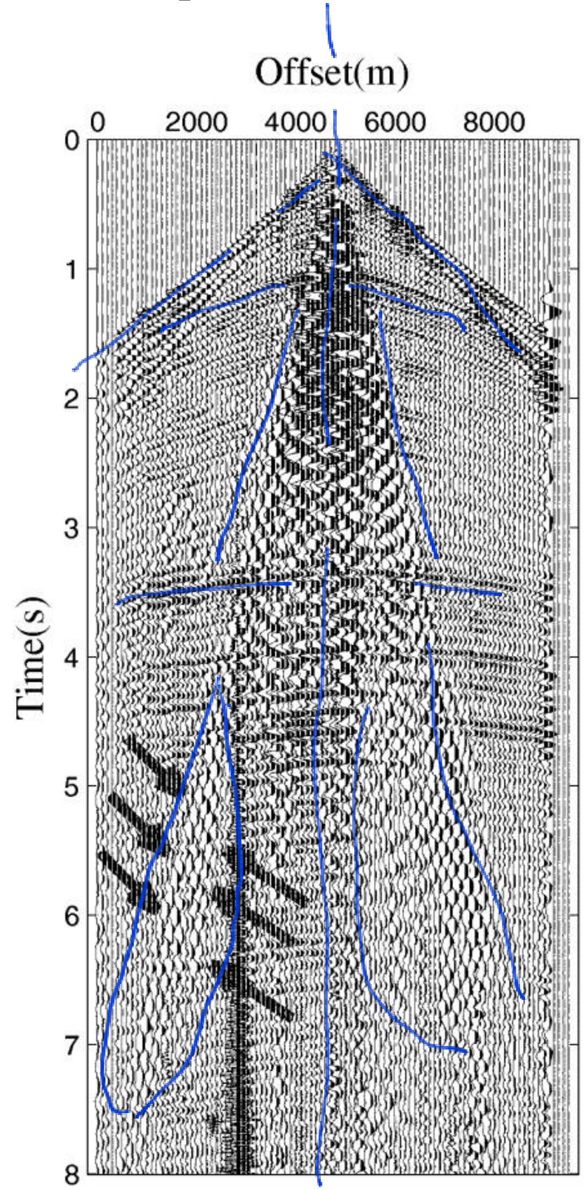


Dispersion

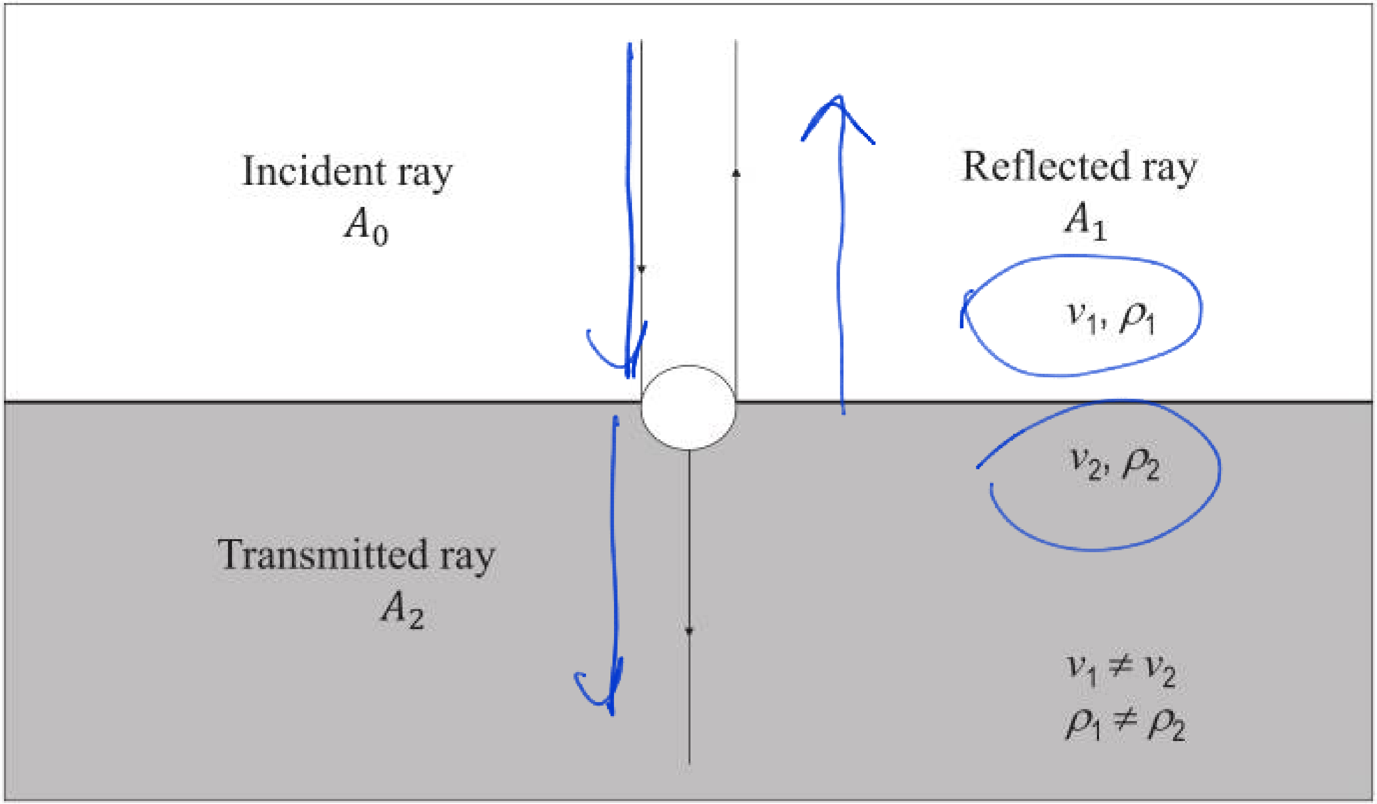




Dispersion



Raypaths in Layered Media



Reflection & Refraction

Snell's law

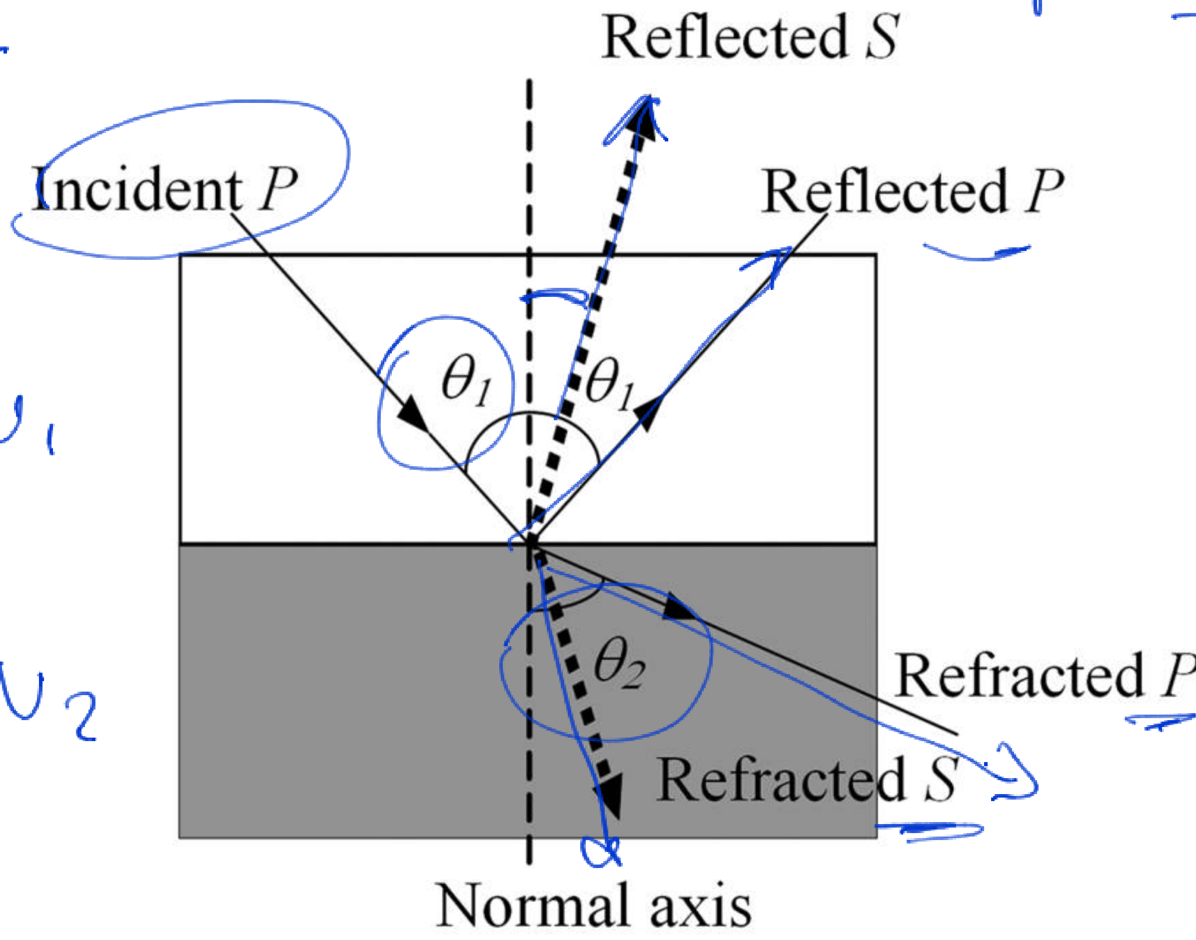
$$\frac{\sin Q_1}{V_1} = \frac{\sin Q_2}{V_2}$$

$P = \frac{\sin Q_1}{V}$

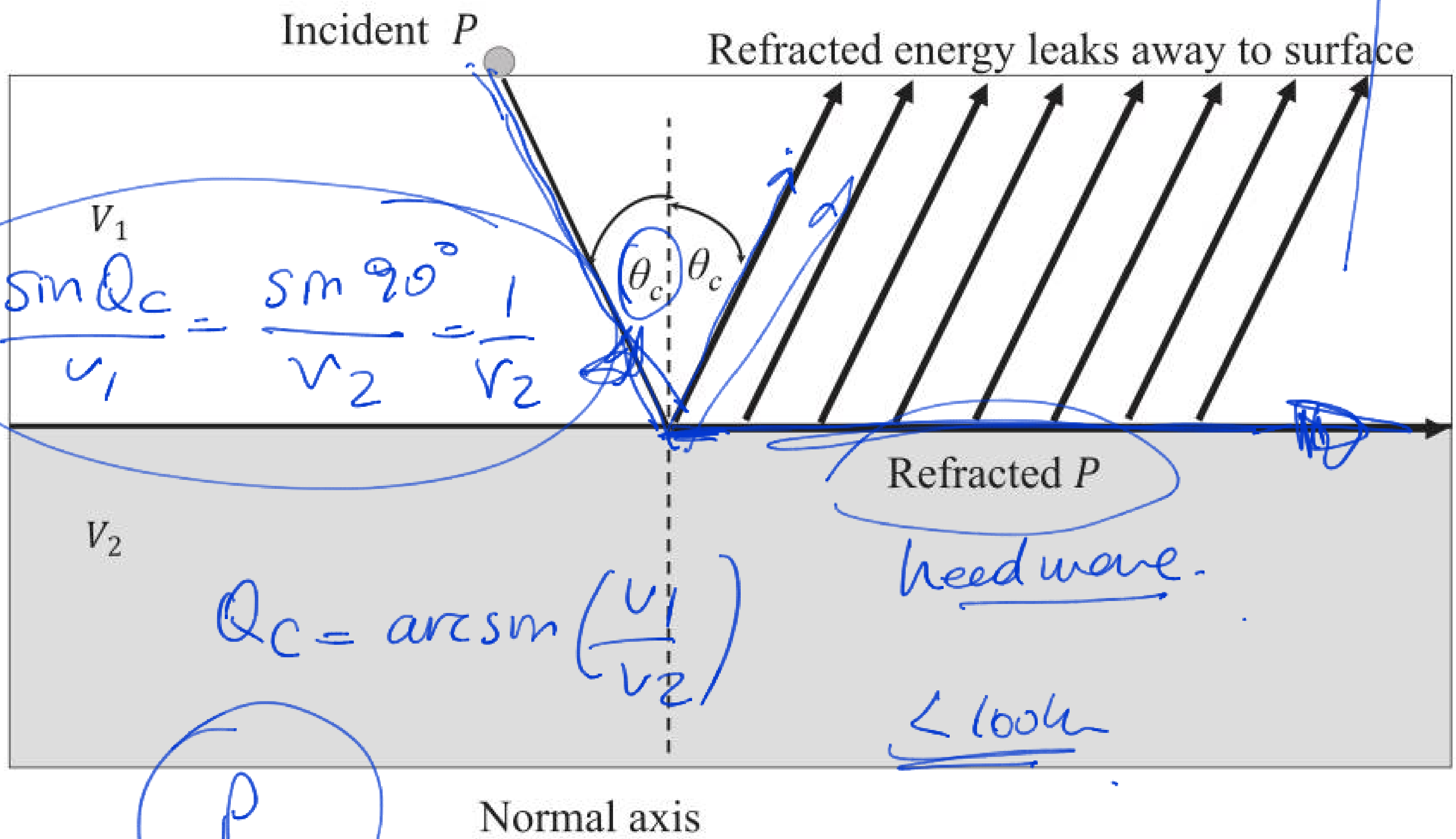
V_1

V_2

$V_2 > V_1$



Critical Refraction



$$\frac{\sin Q_c}{v_1} = \frac{\sin 90^\circ}{v_2} = \frac{1}{v_2}$$

$$Q_c = \arcsin\left(\frac{v_1}{v_2}\right)$$

head wave.

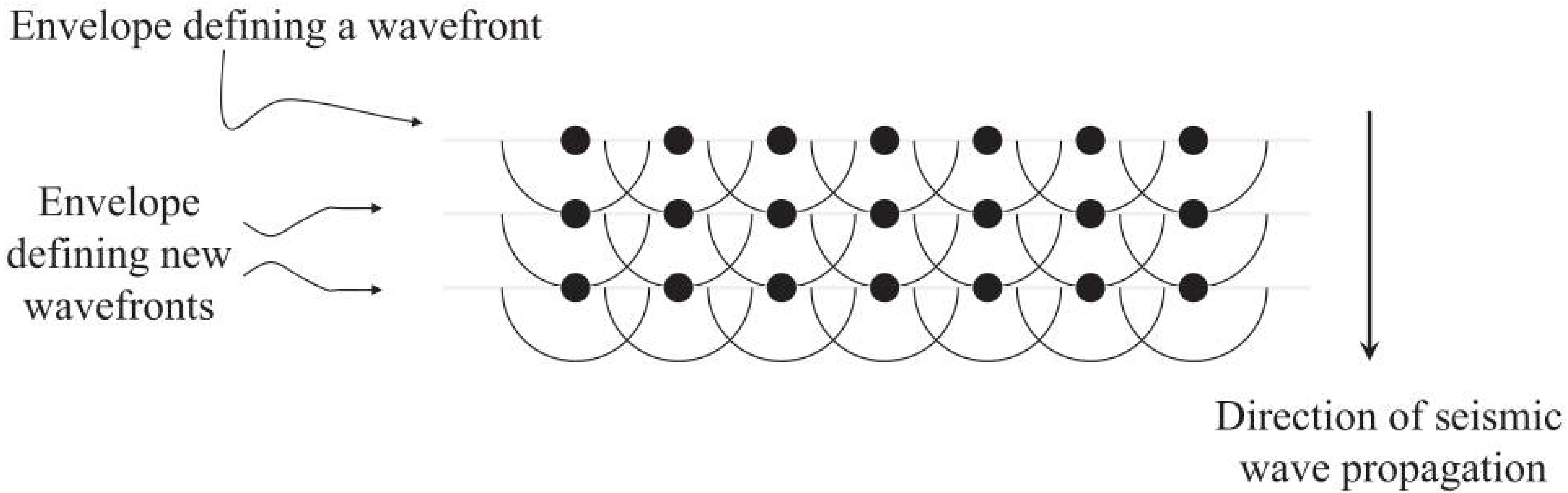
< 100k

~100k

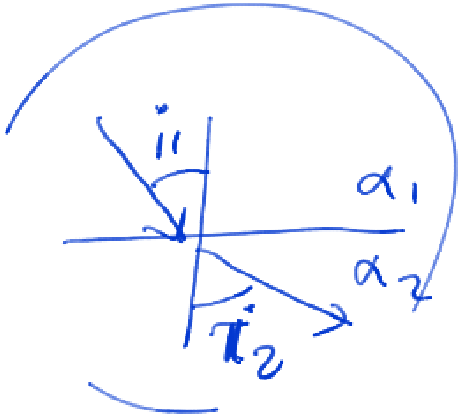
P_n

Diffraction

Huygens Principle

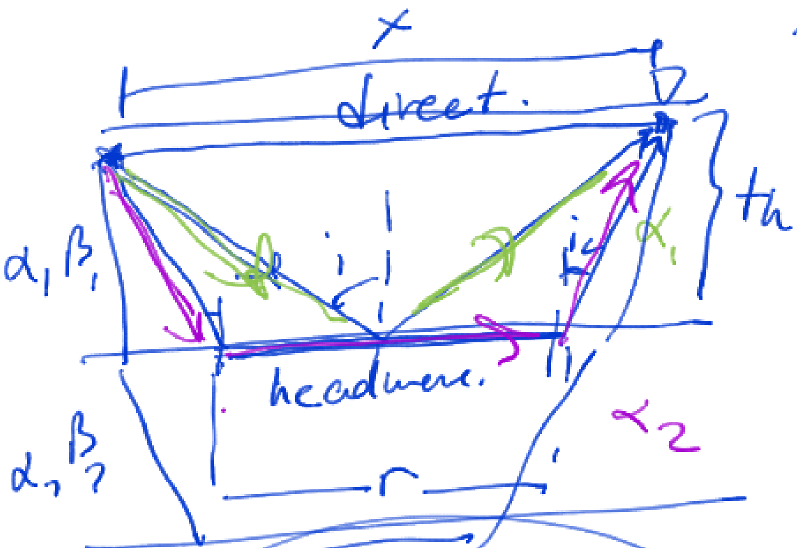


Seismic Event Geometry in Layered Media



$$\frac{\sin i_1}{\alpha_1} = \frac{\sin i_2}{\alpha_2}$$

$$T_{direct} = \frac{x}{\alpha_1}$$

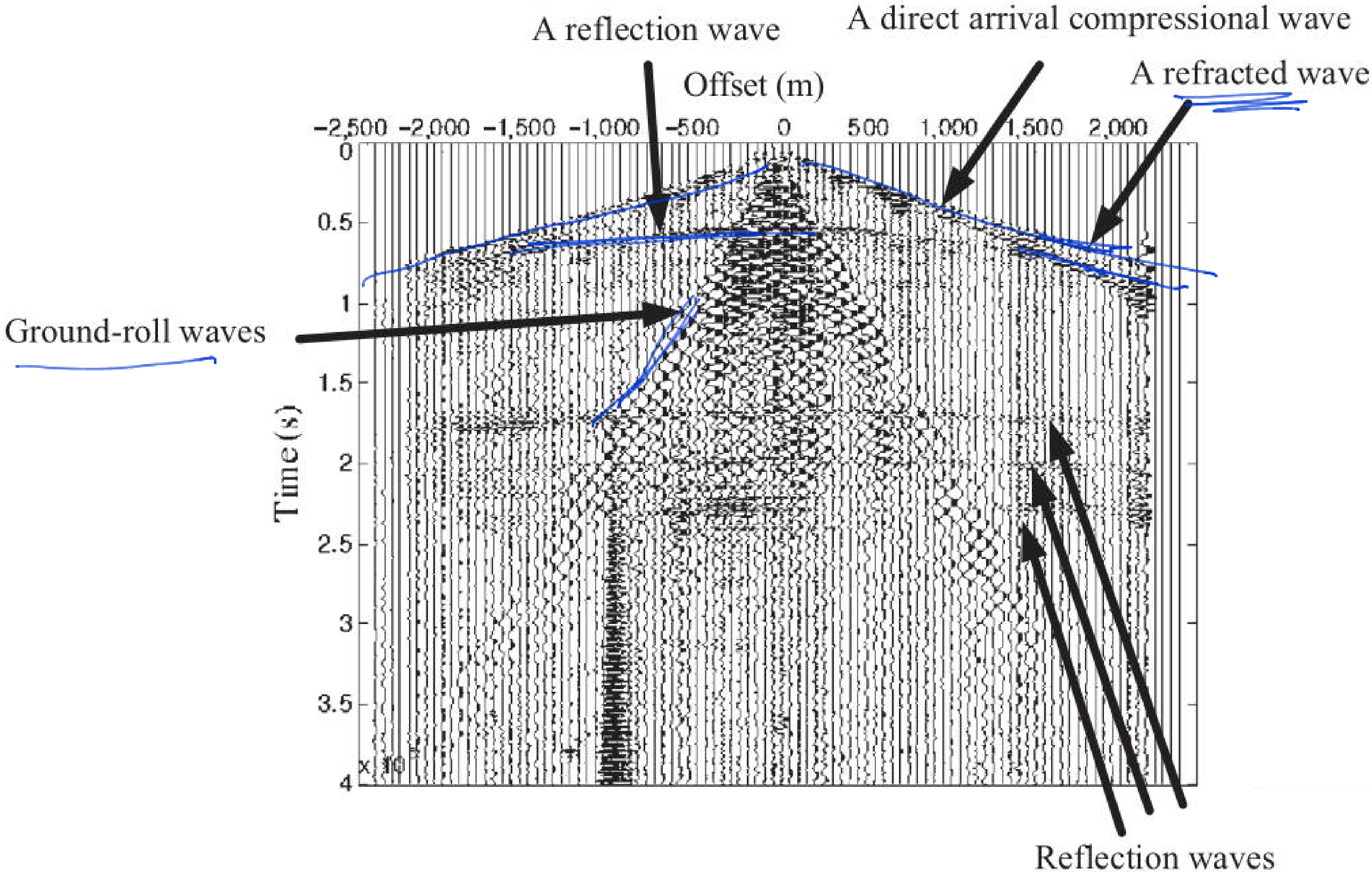


$$T = \frac{2th}{\cos i} \cdot \frac{1}{\alpha_1}$$

$$T = \frac{r}{\alpha_2} + \frac{2th}{\cos i_c} \cdot \frac{1}{\alpha_1}$$

Seismic Event Geometry in Layered Media

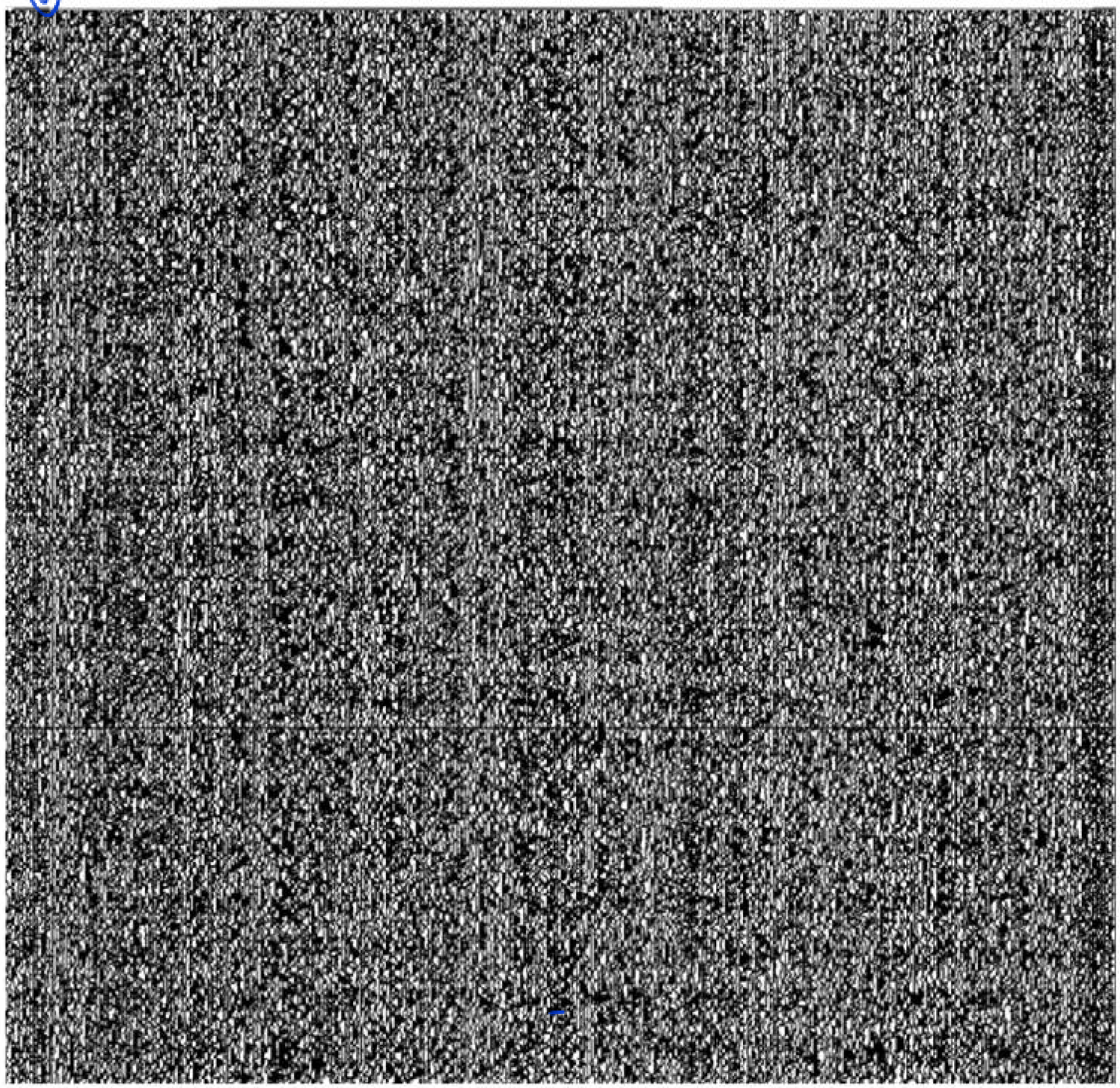
Seismic Event Geometry in Layered Media



NOISE

SNR

$\frac{S}{N} < 1$



Critical Refraction

Critical Refraction