

Operators with symbolic hierarchies on stratified spaces

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Manifolds M with higher corners or edges of order $k \in \mathbb{N}$ are (in our notation) special stratified spaces, where $k = 0$ corresponds to smoothness, $k = 1$ to conical or edge singularities, especially smooth boundaries. Manifolds with singularities of order k form a category \mathcal{M}_k . The stratification $s(M) = (s_0(M), s_1(M), \dots, s_k(M))$ induces a principal symbolic hierarchy

$$\sigma(A) = (\sigma_0(A), \sigma_1(A), \dots, \sigma_k(A))$$

of operators A over $s_0(M)$, degenerate in a typical way in the representation over the stretched version \mathbb{M} of M . The component $\sigma_0(A)$ is the standard homogeneous principal symbol on the main stratum $s_0(M)$; the component $\sigma_j(A)$, $j > 0$, lives on $s_k(M)$ and is operator-valued. The symbolic hierarchy admits notions of ellipticity and the construction of parametrices within suitable algebras of degenerate pseudo-differential operators. We present some new developments in this field which has a long history through achievements of numerous Russian authors and other schools worldwide. Further progress is stimulated by the desire to reach new models of applications, see, for instance, [2]. Moreover, the tower of operator algebras with increasing k still contains many new challenges. The methods of the author have been stimulated very much by the works [4], [5], [1], since the case of manifolds with edge contains boundary value problems with and without the transmission property at the boundary.

References

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