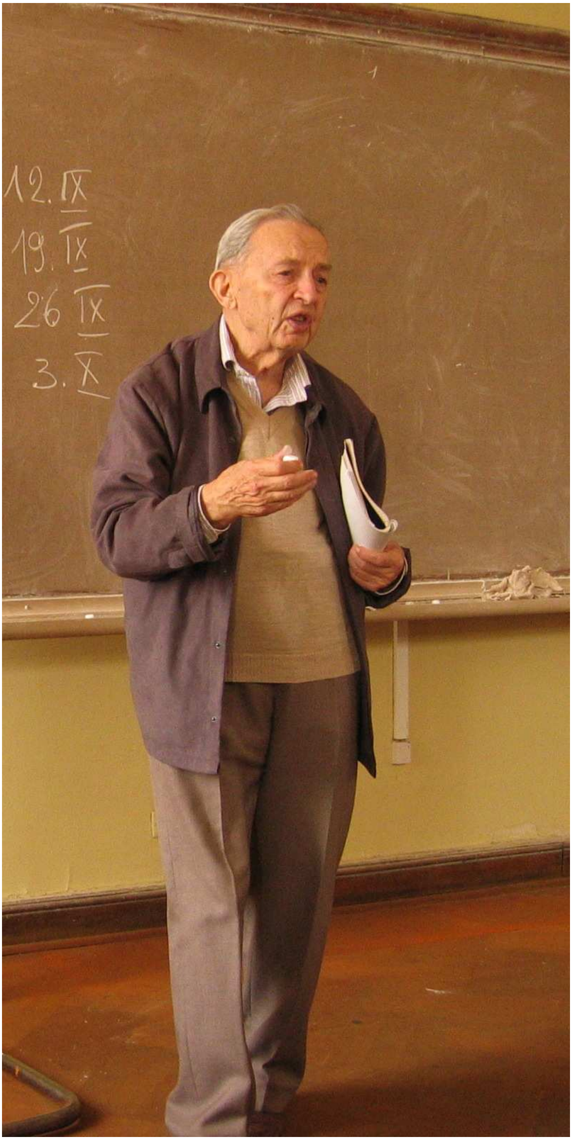
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Institute for Information Transmission Problems RAS
(Kharkevich Institute)

Lomonosov Moscow State University

DIFFERENTIAL EQUATIONS AND APPLICATIONS

International Conference
in Honour of Mark Vishik
On the occasion of his 90th birthday

Moscow, June 4-7, 2012

Abstracts of talks

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Mark Iosifovich Vishik

Mark Iosifovich Vishik was born on October 19, 1921 in Lwów. He lost his father when he was 8. In 1939 he graduated from the Lwów Classical Gymnasium and was accepted to the Department of Physics and Mathematics at Lwów University. His professors there were Stephan Banach, Juliusz Schauder, Stanisław Mazur, Bronislaw Knaster, and Edward Szpilrajn. In 1941 he went to Krasnodar and then to Makhachkala. In 1942 he moved to Tbilisi and in 1943 graduated with honours from Tbilisi University (its Dean at the time was Nikoloz Muskhelishvili). In 1943-1945 he took graduate studies at Tbilisi Mathematical Institute under the guidance of Ilia Vekua.

In 1945 Mark Vishik transferred to continue his graduate studies in Moscow Mathematical Institute where took active part in the I.G. Petrovsky Seminar. In 1947 he completed his dissertation under the guidance of L.A. Lusternik. In 1951 he completed his habilitation. In 1953-1965 Mark Vishik taught at the Mathematics Department of Moscow Institute of Energy. From 1965 until 1993 he is a professor at the Chair of Differential Equations at Department of Mechanics and Mathematics of Moscow State University. Since 1993 he is the Leading Scientist at the Institute for Information Transmission Problems of Russian Academy of Sciences. In 1966-1991 he worked part-time at the Institute for Problems in Mechanics AS USSR, and from 1993 until now – at the Chair of General Problems of Control Theory at the Department of Mechanics and Mathematics of Moscow State University.

Mark Vishik is the world-renowned scientist working in the field of Partial Differential Equations and Functional Analysis, who significantly enriched the theory of elliptic and parabolic boundary value problems, the boundary layer problems, nonlinear partial differential equations, pseudo-differential operators, equations with infinite number of variables, statistical hydrodynamics, attractors of dissipative equations, and others. He authored more than 250 research papers and 8 monographs. He created an excellent school which combines a large number of leading scientists, working in different branches of mathematics in many countries. He personally guided 57 PhD. students (with 30 of them having later finished habilitation), while influencing an incomparably larger number of mathematicians.

Remarks on strongly elliptic systems in Lipschitz domains

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We discuss some fundamental facts of the theory of strongly elliptic second-order systems in bounded Lipschitz domains. We propose a simplified choice of the right-hand side of the system and the conormal derivative in the Green formula. Using "Weyl's decomposition" of the space of solutions, we obtain two-sided estimates for solutions of the Dirichlet and Neumann problems. We remove the algebraic restriction in the generalized Savaré theorem on the regularity of solutions of these problems for systems with Hermitian principal part. The corollaries for potential type operators and Poincaré–Steklov operators on the boundary are strengthened. We consider the transmission problems for two systems in domains with common Lipschitz boundary without assumption of the absence of jumps in the coefficients on this boundary. We construct examples of strongly elliptic second-order systems, for which the Neumann problem does not have the Fredholm property.

The parabolic Harnack inequality for integro-differential operators

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We generalize the elliptic Harnack inequality proved in [1]. Additionally we relax the assumptions of [1]: the integral term measure in it is absolutely continuous with respect to the Lebesgue measure. The proof exploits the basic construction of Krylov-Safonov for elliptic operators and its modernization for the purpose of adding integral terms by R. Bass.

Let $B(x,r)$ denote $\{y \in \mathbb{R}^d : |y-x| < r\}$, Q be the cylinder $\{t \in [0,2), |x| \in B(0,1)\}$, and W be the space of Borel measurable bounded non-negative functions on $\mathbb{R}^{d+1}$ with restriction on Q belonging to the Sobolev space $W^{1,2}(Q)$. And let $\mathcal{L}$ be an operator with Borel measurable coefficients, acting on $u \in W$ according to the formula

$$\begin{aligned} \mathcal{L}u(t,x) &= \frac{1}{2}\sum_{i,j=1}^{d} a_{ij}(t,x)\frac{\partial^2 u(t,x)}{\partial x_i \partial x_j} + \sum_{i=1}^{d} b_i(t,x)\frac{\partial u(t,x)}{\partial x_i} \\ &+ \int_{\mathbb{R}^d\setminus\{0\}} [u(t,x+h) - u(t,x) - 1_{(|h|\le 1)} h\cdot\nabla u(t,x)]\mu(t,x;dh). \end{aligned}$$

ASSUMPTIONS There exist positive constants λ, K, k and β such that for all t, x:

1) $\lambda|y|^2 \le y^T a(t,x)y,\ y \in \mathbb{R}^d$;

2) $\|a(t,x)\| + |b(t,x)| + \int_{\mathbb{R}^d}(|h|^2 \wedge 1)\mu(t,x;dh) \le K$;

3) for any $r \in (0,1]$, $y_1,\ y_2 \in B(x,r/2) \cap B(0,1)$ and borel A with $\operatorname{dist}(x,A) \ge r$ holds $\mu(t,y_1;\{h : y_1+h \in A\}) \le kr^{-\beta}\mu(t,y_2;\{h : y_2+h \in A\})$.

Theorem 1 *There exists a positive constant $C(d,\ \lambda,\ K,\ k,\ \beta)$ s.t. for every $u \in W$ satisfying*

$$\frac{\partial}{\partial t} + \mathcal{L}u = 0 \ \textit{a.e. in}\ (t,x)$$

holds: for all $|x| \ \le\ \frac{1}{2}$

$$u_{0,x} \ \ge\ Cu_{1,0}.$$

The crucial point of the proof is the extension of Prop. 3.9 [1] to the cylinder.

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[1] The author is supported by RFBR grants 10-01-00767 and 10-08-01068

Relativistic point dynamics and Einstein's formula as a property of localized solutions of a nonlinear Klein-Gordon equation

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Relativistic mechanics includes the relativistic dynamics of a mass point and the relativistic field theory. In a relativistic field theory the relativistic field dynamics is derived from a relativistic covariant Lagrangian, such a theory allows to define the total energy and momentum, forces and their densities but does not provide a canonical way to define the mass, position or velocity for the system. For a closed system *without external forces* the total momentum has a simple form $\mathbf{P} = M\mathbf{v}$ where $\mathbf{v}$ is a *constant velocity*, allowing to define naturally the total mass M and to derive from the Lorentz invariance Einstein's energy-mass relation $E = Mc^2$ with $M = m_0\gamma$, with γ the Lorentz factor and m_0 the rest mass; according to Einstein's formula the rest mass is determined by the internal energy of the system.

The relativistic dynamics of a mass point is described by a relativistic version of Newton's equation where the rest mass m_0 of a point is prescribed; in Newtonian mechanics the mass M reveals itself in *accelerated motion* as a measure of inertia which relates the point acceleration to the external force. The question which we address is the following: Is it possible to construct a mathematical model where the internal energy of a system affects its acceleration in an external force field as the inertial mass does in Newtonian mechanics?

We construct a model which allows to consider in the same framework the uniform motion in the absence of external forces (a closed system) and the accelerated motion caused by external fields; the internal energy is present both in uniform and accelerating regimes. The model is based on the nonlinear Klein-Gordon (KG) equation which is a part of our theory of distributed charges interacting with electromagnetic (EM) fields, [1]-[4]. We prove that if a sequence of solutions of a KG equation concentrates at a trajectory $\mathbf{r}(t)$ and their local energies converge to $E(t)$ then the trajectory satisfies the relativistic version of Newton's equation where the mass is determined in terms of the energy by Einstein's formula, and the EM forces are determined by the coefficients of the KG equation. We prove that the concentration assumptions hold for the case of a general rectilinear accelerated motion.

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On the propagation of Monokinetic Measures with Rough Momentum Profile

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This is a report on a work in progress jointly with Francois Golse, Peter Markowich and Thierry Paul.

The analysis of the global flow defined by the Hamiltonian system

$$\begin{aligned} \dot{X}_t &= \nabla_\xi H(X_t, \Xi_t) \quad X_0(x, \Xi) = x \\ \dot{\Xi}_t &= \nabla_x H(X_t, \Xi_t) \quad \Xi_0(x, \xi) = \nabla_x U(x) \end{aligned}$$

is a standard tool in the WKB asymptotics.

In the present contribution it will be interpreted as the propagation of a monokinetic measure: The push forward by the Hamiltonian flow of a measure of the form:

$$\mu(x, \xi) = \rho^{\mathrm{in}}(x)\delta_{U(x)}(\xi)$$

evolving according to the Liouville equation:

$$\partial_t \mu + \{H, \mu\} = 0\,.$$

This approach leads us to an estimate of the number of folds of the Lagrangian Manifold even for rough initial data. We also provide informations on the structure of the push-forward $\rho(t, x)$ measure under the canonical projection of the space $\mathbb{R}_x \times \mathbb{R}_\xi$ on $\mathbb{R}_x$.

An asymptotic of a certain Riemann–Hilbert problem under singular deformation of a domain

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The Riemann–Hilbert problem is considered in a decagonal domain G on complex plane, which is an exteriour of a system Γ of cuts Γ_j with excluded infinity. The sought analytic function $\mathcal{F}$ satisfies to the boundary condition $\mathrm{Re}(h\,\mathcal{F}) = c$ on Γ, where h and c are prescribed piece–wise constant functions; $\mathcal{F}$ is continuous in $\overline{G} \setminus \{\infty\}$ and satisfies to a certain growth condition at infinity. The solution $\mathcal{F}$ has been constructed in analytic form. Asymptotics for function $\mathcal{F}$ have been found for two limit cases of geometry of Γ; first case corresponds to $|\Gamma_j| \to \infty$, and second case to $|\Gamma_j| \to 0$ for some numbers j. The Riemann–Hilbert problem under consideration originates from magnetic hydrodinamics, in model [1]–[4] of the effect of magnetic field reconnection in Solar flares. The model includes a current layer and shock–waves attached to its end–points. The constructed solution $\mathcal{F}$ and its asymptotics possess clear physical meaning. For construction the asymptotics we used an approach [2], [5] and asymptotics for Schwarz — Christoffel integral parameters, that have been found in [3].

This work is supported by the RFBR proj. No. 10-01-00837, Program No. 3 of the Division of Mathematical Sciences of the RAS and the "Contemporary Problems of Theoretical Mathematics" Program of the RAS.

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Inertial manifolds for strongly damped wave equations

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Consider the boundary value problem for a semilinear strongly damped wave equation in a bounded domain Ω:

$$u_{tt} - 2\gamma\Delta u_t = \Delta u + f(u), \quad u\big|_{\partial\Omega} = 0, \tag{1}$$

$$u\big|_{t=0} = u_0(x) \in H_0^1(\Omega), \quad u_t\big|_{t=0} = p_0(x) \in L_2(\Omega). \tag{2}$$

Here $\gamma > 0$ is a coefficient of strong dissipation, and the nonlinearity $f(u)$ satisfies the global Lipschitz condition:

$$|f(v_1) - f(v_2)| \leqslant L\,|v_1 - v_2| \qquad \forall v_1, v_2 \in \mathbb{R}.$$

The following theorem gives sufficient condition for the existence of an inertial manifold for equation (1).

Theorem 2 *Let λ_k, $0 < \lambda_1 < \lambda_2 \leqslant \cdots \to +\infty$, be eigenvalues of the operator $-\Delta$ in the domain Ω under the Dirichlet boundary conditions. Suppose that there is an N such that the following inequalities hold*

$$\lambda_N < \lambda_{N+1} < 1/(2\gamma^2), \tag{3}$$

$$2L < \sup_{\gamma\lambda_N \leqslant \Phi < \gamma\lambda_{N+1}} \{(\gamma\lambda_{N+1} - \Phi)\min\{\varkappa_1(\Phi), \varkappa_N(\Phi), \varkappa_{N+1}(\gamma\lambda_{N+1})\}\}, \tag{4}$$

where we have used the notation

$$\varkappa_k(\Phi) = \Phi - \gamma\lambda_k + \sqrt{\Phi^2 - 2\gamma\lambda_k\Phi + \lambda_k}.$$

Then, in the phase space $H_0^1(\Omega) \times L_2(\Omega)$, there exists a $2N$-dimensional inertial manifold that exponentially attracts (as $t \to +\infty$) all the solutions of problem (1), (2).

The proof is based on the construction of a new inner product in the phase space in which gap property holds and thus an inertial manifold exists (the corresponding general theorem for an abstract differential equation in a Hilbert space one can find, e.g., in [1]).

Remark. If γ, λ_N and λ_{N+1} are fixed and they satisfy (3), then we state the existence of an inetial manifold for sufficiently small L.

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Trajectory attractors for equations of mathematical physics

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The report is based on joint works with M.I.Vishik.

We describe the method of trajectory dynamical systems and trajectory attractors and we apply this approach to the study of the limiting asymptotic behaviour of solutions of non-linear evolution equations. This method is especially useful in the study of dissipative equations of mathematical physics for which the corresponding Cauchy initial-value problem has a global (weak) solution with respect to the time but the uniqueness of this solution either has not been established or does not hold. An important example of such an equation is the 3D Navier–Stokes system in a bounded domain (see [1]). In such a situation one cannot use directly the classical scheme of construction of a dynamical system in the phase space of initial conditions of the Cauchy problem of a given equation and find a global attractor of this dynamical system. Nevertheless, for such equations it is possible to construct a trajectory dynamical system and investigate a trajectory attractor of the corresponding translation semigroup.

This universal method is applied for various types of equations arising in mathematical physics: for general dissipative reaction-diffusion systems, for the 3D Navier–Stokes system, for dissipative wave equations, for non-linear elliptic equations in cylindrical domains, and for other equations and systems. Special attention is given to using the method of trajectory attractors in approximation and perturbation problems arising in complicated models of mathematical physics.

The work partially supported by the Russian Foundation of Basic Researches (Projects no. 11-01-00339 and 10-01-00293).

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Quantum dissipative Zakharov model in a bounded domain

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We consider an initial boundary value problem for a quantum version (introduced in [1]) of the Zakharov system arising in plasma physics:

$$\begin{cases} n_{tt} - \Delta \left(n + |E|^2\right) + h^2\Delta^2 n + \alpha n_t = f(x), \quad x \in \Omega,\ t > 0, \\ iE_t + \Delta E - h^2\Delta^2 E + i\gamma E - nE = g(x), \quad x \in \Omega,\ t > 0. \end{cases}$$

Here $\Omega \subset \mathbb{R}^d$ is a bounded domain, $d \leq 3$, $E(x,t)$ is a complex function and $n(x,t)$ is a real one, $h > 0$, $\alpha \geq 0$ and $\gamma \geq 0$ are parameters and $f(x)$, $g(x)$ are given (real and complex) functions. We also impose some boundary and initial conditions on E and n.

We prove the global well-posedness of this problem in some Sobolev type classes and study properties of solutions. This result confirms the conclusion recently made in physical literature concerning the absence of collapse in the quantum Langmuir waves. (see a discussion in [2]). In the dissipative case the existence of a finite dimensional global attractor is established and regularity properties of this attractor are studied. For this we use the recently developed method of quasi-stability estimates (see [3, 4]). In the case when external loads are C^∞ functions we show that every trajectory from the attractor is C^∞ both in time and spatial variables. This can be interpret as the absence of sharp coherent structures in the limiting dynamics. For some details we refer to [5].

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Vishik–Lyusternik's method and the inverse problem for plasma equilibrium in a tokamak

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Control over thermonuclear fusion reactions (including suppression of instabilities of the plasma discharge) depends essentially on how well the information about the current density through plasma is taken into account. In the case cylindrical approximation (when the tokamak (toroidal magnetic) chamber and the resulting plasma discharge are modeled in the form of infinite cylinders $\mathcal{S}\times\mathbb{R}$ and $\omega\times\mathbb{R}$ with simply connected cross-sections $\mathcal{S}\Subset\mathbb{R}^2$ and $\omega\Subset\mathcal{S}$), the required current distribution is given by the mapping $f_u:\omega\ni(x,y)\mapsto f\big(u(x,y)\big)\geq 0$, where the *required* functions $u\in C^2(\omega)$ and f are as follows:

$$\frac{\partial^2 u(x,y)}{\partial x^2}+\frac{\partial^2 u(x,y)}{\partial y^2}=f\big(u(x,y)\big)\quad\text{in}\quad\omega\,,\qquad\text{and}\qquad u=0\quad\text{on}\quad\gamma=\partial\omega,$$

$$\sup_{P\in\gamma}\Big|\frac{\partial u}{\partial\nu}(P)-\ \Phi(P)\Big|\leq\lambda\sup_{P\in\gamma}\Big|\Phi(P)\Big|\,,\qquad\int_\gamma\frac{\partial u}{\partial\nu}\,d\gamma=1=\text{the total current}\,.$$

Here, $\gamma=\overline{\omega}\setminus\omega$ is the boundary of the domain ω, $\lambda\geq 0$ is small parameter, ν is the outward unit normal to the curve $\gamma=\partial\omega$ (with respect to the domain ω). Both the function Φ and the curve $\gamma=\partial\omega$ (and hence the domain ω) may be regarded as known: they are determinable from measurements of the magnetic field at the tokamak chamber $\partial\mathcal{S}$.

Within the class of affine functions $f:u\mapsto f(u)=au+b$, Vishik–Lyusternik's method is capable of showing that

$$\left|\frac{\partial u}{\partial\nu}\bigg|_{s\in\gamma}-\left(\frac{1}{|\gamma|}-\frac{k(s)-|\gamma|^{-1}\int_\gamma k(s)\,ds}{2|\gamma|\sqrt{a}}\right)\right|\leq\frac{C_\gamma(a)}{\sqrt{a}}\,,\ C_\gamma(a)\to 0\ \text{ as }\ a\to\infty\,,$$

$$\left|\frac{d}{da}\frac{\partial u}{\partial\nu}\bigg|_{s\in\gamma}-\frac{k(s)-|\gamma|^{-1}\int_\gamma k(s)\,ds}{4|\gamma|\,a^{3/2}}\right|\leq\frac{C_\gamma(a)}{a^{3/2}}\,,\quad C_\gamma(a)\to 0\quad\text{as}\quad a\to\infty\,,$$

where $k(s)$ is the curvature of γ at $s\in\gamma$. Using these asymptotic relations, it follows that there is only one affine distribution f_u (for a large class of domains ω) if $\lambda=0$. However, for any arbitrarily small $\lambda>0$, there is an infinite number $\{f_u^j\}_{j\in\mathbb{N}}$ of distributions, for which $\|f_u^{j_1}\|\ll\|f_u^{j_2}\|$, $j_1\neq j_2$, where $\|f_u^j\|=\max\limits_{(x,y)\in\omega}\big|f_u^j(x,y)\big|$. It is shown that all these different distributions are necessarily members of a sequence converging to the δ-function supported on γ (the so-called skinned current). Hence these distributions are not essentially different from the physical standpoint.

Two truly physically essentially different current distributions f_u^1 and f_u^2 are found in the class of polynomials $f:u\mapsto f(u)=\sum_{m=0}^3 a_m u^m$ of third degree (see Russian J. Math. Physics, **17** (1), 56–65 (2010) and Asymptotic Analysis, **74** (1), 95–121 (2011)).

Pseudodifferential operator, adiabatic approximation and averaging of linear operators

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One use the averaging methods for partial differential equations in the case when their coefficients are rapidly oscillating functions. There exists a great number of publications devoted to averaging, we mention well known monographes by V.Zhikov, S.Kozlov and O.Oleinik, N.Bakhvalov and G.Panasenko, E.Khruslov and V.Marchenko. As a rule averaging methods are used for construction such asymptotic solutions that their leading term is quite smooth function. From the other hand there exist interesting. The several scale are in this situation and it reasonable to use the variant of adiabatic approximation based on the V.Maslov operator methods and pseudodifferential operators. We illustrate this approach using the Srödiger and Klien-Gordon type equations with rapidly oscillating velocity and potential.

This work was supported by RFBR grant 11-01-00973 and DFG-RAS project 436 RUS 113/990/0-1.

Statistical Hydrodynamics and Reynolds averaging

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We shall revisit the Reynolds method of averaging to obtain equations for turbulent flow in the spirit of M.I. Vishik and A.V. Fursikov's approach to statistical hydrodynamics. In particular, for statistically homogeneous flows we shall examine how space averages over domains (as in the original Reynolds ideas) are close to statistical averages on a phase space of vector fields.

We shall also examine Reynolds averages obtained from microscopic statistical mechanics at a hydrodynamic limit via measure disintegration.

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To be announced

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Pseudovariational operators and Yang-Mills Millennium problem

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The second quantization of a complexified Gelfand triple produces the Kree nuclear triple of sesqui-holomorphic functionals on it. Any continuous operator in the Kree triple is a pseudovariational operator, a strong limit of second quantized pseudodifferential operators on $\mathbb{R}^n$, $n \to \infty$.

A thorough analysis of the Noether Yang-Mills energy functional of Cauchy data shows that it is the anti-normal symbol of a selfadjoint elliptic operator in variational derivatives. Such quantum Yang-Mills energy operator has a mass gap at the bottom of its spectrum. This is a solution of the Yang-Mills Millennium problem.

Key words: Second quantizations; infinite-dimensional pseudodifferential operators, symbolic calculus; infinite-dimensional ellipticity; essentially hyperbolic nonlinear partial differential equations; Yang-Mills Millennium problem.

Acoustic and optical black holes

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Acoustic and optical black holes appear in the study of wave equations describing the wave propagation in the moving medium. They include the black holes of the general relativity when the corresponding Lorentz metric is the solution of Einstein equation.

We investigate the existence and the stability of the black and white holes in the case of two space dimensions and in the axisymmetric case. The case of nonstationary, i.e. time-dependent metrics, also will be considered.

Perturbation theory for systems with multiple stationary regimes

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I will consider deterministic and stochastic perturbations of dynamical systems and stochastic processes with multiple invariant measures. Long-time evolution of the perturbed system will be described as a motion on the cone of the invariant measures of the non-perturbed system.

Quasilinear parabolic equations with a small parameter in the higher derivatives [3], perturbations of non-linear oscillators [6], [1], and of the Landau–Lifshitz equation for magnetization [2], linear elliptic PDE's with a small parameter [4], [5], [6] will be considered as examples.

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Generic properties of eigenvalues of a family of operators

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Let $\Omega(t)$, $0 \leq t \leq 1$, be a smooth family of bounded Euclidean domains, and let $\Delta(t)$ be the Dirichlet Laplacian in $\Omega(t)$. We call a family spectrally simple if the spectrum of $\Delta(t)$ is simple for all t. We prove that spectrally simple families form a residual set in the space of all families. A similar result holds in other situations, e.g. Laplace–Beltrami operators that correspond to a family of Riemannian metrics on a manifold, a family of Schrödinger operators (the potential depends on t).

Normal parabolic equation corresponding to 3D Navier–Stokes system

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Energy estimate is very important tool to study 3D Navier–Stokes system. Absents of such bound in phase space H^1 is very serious obstacle to prove nonlocal existence of smooth solutions.

Semilinear parabolic equation is called equation of normal type if its nonlinear term B satisfies the condition: vector $B(v)$ is collinear to vector v for each v. Since the property $B(v) \perp v$ implies energy estimate, equation of normal type does not satisfies energy estimate "‘in the most degree"’. That is why we hope that investigation of normal parabolic equations should make more clear a number problems connected with existence of nonlocal smooth solutions to 3D Navier–Stokes equations.

In the talk we will start from Helmholtz equations that is analog of 3D Navier–Stokes system in which the curl of fluid velocity is unknown function. We will derive normal parabolic equations (NPE) corresponding to Helmholtz equations and will prove that there exists explicit formula for solution to NPE with periodic boundary conditions. This helped us to investigate more less completely the structure of dynamical flow corresponding to NPE. Its phase space V can be decomposed on the set of stability $M_-(\alpha)$, $\alpha > 0$ (solutions with initial condition $\omega_0 \in M_-(\alpha)$ tends to zero with prescribed rate $e^{-\alpha t}$ as time $t \to \infty$), set of explosions M_+ (solutions with initial condition $\omega_0 \in M_+$ blow up during finite time), and intermediate set $M_I(\alpha) = V \setminus (M_-(\alpha) \cup M_+)$. The exact description of all these sets will be given.

Relative version of the Titchmarsh convolution theorem

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We consider the algebra $C_u = C_u(\mathbb{R})$ of uniformly continuous bounded complex functions on the real line $\mathbb{R}$ with pointwise operations and sup-norm. Let I be a closed ideal in C_u invariant with respect to translations, and let $\mathrm{ah}_I(f)$ denote the minimal real number (if it exists) satisfying the following condition. If $\lambda > \mathrm{ah}_I(f)$, then $(\hat{f} - \hat{g})|_V = 0$ for some g_I , where V is a neighborhood of the point λ. The classical Titchmarsh convolution theorem is equivalent to the equality $\mathrm{ah}_I(f_1 \cdot f_2) = \mathrm{ah}_I(f_1) + \mathrm{ah}_I(f_2)$, where $I = \{0\}$. We show that, for ideals I of general form, this equality does not generally hold, but $\mathrm{ah}_I(f^n) = n \cdot \mathrm{ah}_I(f)$ holds for any I. We present many nontrivial ideals for which the general form of the Titchmarsh theorem is true.

Negative eigenvalues of two-dimensional Schrödinger operators

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Given a non-negative L^1_{loc} function $V(x)$ on $\mathbb{R}^n$, consider the Schrödinger operator $H_V = -\Delta - V$ where $\Delta = \sum_{k=1}^n \frac{\partial^2}{\partial x_k^2}$ is the Laplace operator. More precisely, H_V is defined as a form sum of $-\Delta$ and $-V$, so that, under certain assumptions about V, the operator H_V is self-adjoint in $L^2(\mathbb{R}^n)$.

Denote by $\mathrm{Neg}(V)$ the number of non-positive eigenvalues of H_V (counted with multiplicity), assuming that its spectrum in $(-\infty, 0]$ is discrete. For example, the latter is the case when $V(x) \to 0$ as $x \to \infty$. We are are interested in obtaining estimates of $\mathrm{Neg}(V)$ in terms of the potential V in the case $n = 2$.

For the operator H_V in $\mathbb{R}^n$ with $n \geq 3$ a celebrated inequality of Cwikel-Lieb-Rozenblum says that

$$\mathrm{Neg}(V) \leq C_n \int_{\mathbb{R}^n} V(x)^{n/2}\, dx. \tag{3}$$

For $n = 2$ this inequality is not valid. Moreover, no weighted L^1-norm of V can provide an upper bound for $\mathrm{Neg}(V)$. In fact, in the case $n = 2$ instead of the upper bounds, the lower bound in (3) is true.

The main result is the estimate (4) below that was obtained jointly with N.Nadirashvili. For any $n \in \mathbb{Z}$, set

$$U_n = \begin{cases} \{e^{2^{n-1}} < |x| < e^{2^n}\}, & n > 0, \\ \{e^{-1} < |x| < e\}, & n = 0, \\ \{e^{-2^{|n|}} < |x| < e^{-2^{|n|-1}}\}, & n < 0. \end{cases}$$

Define for any $n \in \mathbb{Z}$ the following quantities:

$$A_n = \int_{U_n} V(x)\left(1 + |\ln|x||\right) dx\ , \qquad B_n = \Big(\int_{\{e^n < |x| < e^{n+1}\}} V^p(x)\, |x|^{2(p-1)}\, dx \Big)^{1/p},$$

where $p > 1$ is fixed. Then the following estimate holds

$$\mathrm{Neg}(V) \leq 1 + C \sum_{\{n \in \mathbb{Z}: A_n > c\}} \sqrt{A_n} + C \sum_{\{n \in \mathbb{Z}: B_n > c\}} B_n, \tag{4}$$

where C, c are positive constants depending only on p.

For example, (4) implies the finiteness of $\mathrm{Neg}(V)$ provided V is locally bounded and $V(x) = o\Big(\frac{1}{|x|^2 \ln^2 |x|}\Big)$ as $x \to \infty$, which cannot be seen by any previously known method.

Incompressible limit of the linearized Navier–Stokes equations

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We consider initial–boundary value problem for linearized equations of viscous barotropic fluid motion in a bounded domain. We briefly discuss results on existence, uniqueness and estimates of weak solutions to this problem (see [1, 2, 3]). Then we focus on the asymptotic behaviour of the solutions as the compressibility tends to zero, i.e. on the passage to so-called *incompressible limit* (see [4, 5]). Briefly, we show that

- in general case the velocity field converges *weakly* in $L^2(0,T;H_0^1)$;
- if the initial condition for the velocity is divergence-free then the velocity converges *strongly* and the pressure converges $*$-*weakly* in $L^\infty(0,T;L^2)$;
- if, in addition, the *initial condition* for the pressure is compatible with the *initial value* of the pressure in the incompressible problem then the convergence of the pressure is *strong*. (A similar compatibility condition was obtained in [6] as a *necessary condition* of strong convergence of the solutions.)

We also demonstrate the necessity of these sufficient conditions using explicit solutions which are available for simplified data.

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On a compactness problem

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Let V, H be two real Hilbert spaces, $V \subset H$ with compact and dense imbedding and let $A : V \to V'$ be bounded, self ajoint and coercive. Under reasonable assumptions on g , a compactness property of the range is established for energy-bounded solutions of an abstract equation

$$u'' + Au + g(u') = h(t), \quad t \geq 0$$

when h is S^1-uniformly continuous with values in H. This property allows to deduce :

1) asymptotic almost-periodicity of all solutions of wave or plate equations in presence of an almost-periodic source term when the damping term is strong enough.

2) convergence to equilibrium of all solutions of some equations of the form

$$u'' + Au + f(u) + g(u') = h(t), \quad t \geq 0$$

when f is the gradient of a potential satisfying the Łojasiewicz gradient inequality, g is sufficiently coercive globally and $h(t) \to 0$ sufficiently fast for t tending to infinity.

These results generalize, mainly to the case of non-local damping terms, some previous works by the author and his colleagues and rely on methods developped during more than 30 years, cf. e.g. [1, 2, 3] for applications of type 1) and [4, 5] for applications of type 2). There are still challenging open problems in this direction which will be mentionned during the lecture.

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Bony and thick attractors

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Understanding of the structure of attractors of generic dynamical systems is one of the major goals of the theory. A vast general program suggested by Palis presents numerous conjectures about this structure. Various particular cases of these conjectures are proved in numerous papers that we do not quote here. Main part of these investigations is related to diffeomorphisms of closed manifolds. Our investigation is parallel to this direction of research. In the first part of the talk, attractors of *manifolds with boundary onto themselves* are studied. At present, locally generic properties of attractors of such maps are established, that are not yet observed (and plausibly do not hold) for the case of closed manifolds. For instance, an open set of diffeomorphisms of manifolds with boundary onto themselves may have attractors with intermingled basins. The strongest result of this kind is in [3].

Another property of this kind is *having thick attractors*. It is a general belief that attractors of typical smooth dynamical systems (diffeomorphisms and flows) on closed manifolds either coincide with the whole phase space or have Lebesgue measure zero. In this talk we show that this is not the fact for diffeomorphisms of manifolds with boundary onto themselves. Namely, in the space of diffeomorphisms of $T^2 \times [0,1]$, there exists an open set such that any map from a complement of this set to a countable number of hypersurfaces, has a thick attractor: a transitive attractor that has positive Lebesgue measure together with its complement [1]. The problem whether thick attractors exist for locally generic diffeomorphisms of a closed manifold remains widely open.

In the second part we study so called bony attractors. These are attractors of skew products over a Bernoulli shift with the following unexpected property: the map has an invariant manifold, and the intersection of the attractor with that manifold, called *a bone*, is much larger than the attractor of the restriction of the map to the invariant manifold. Bony attractors with one-dimensional bones were discovered in [4]. We construct bones of arbitrary dimension [2]. It is expected that bony attractors are in a sense locally generic.

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Sharp two-term Sobolev inequality and applications to the Lieb–Thirring estimates

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For a function $\varphi \in H^1(\mathbb{R})$ the following inequality is well known

$$\|\varphi\|_\infty^2 \le \|\varphi\|\|\varphi'\|,$$

where the norms on the right-hand side are the L_2-norms. The constant 1 in the inequality is sharp and the unique extremal unction is $\varphi^*(x) = e^{-|x|}$. The same inequality clearly holds on a finite interval, that is, for $\varphi \in H_0^1(0,L)$. However, since $\varphi^*(x) > 0$, no extremal functions exist. The following result provides a sharp correction term for this inequality (the correction term in the periodic case was found in [1]).

Theorem 1. *Let $f \in H_0^1(0,L)$. Then* $\|\varphi\|_\infty^2 \le \|\varphi\|\|\varphi'\|\bigl(1 - 2e^{-\frac{L\|\varphi'\|}{\|\varphi\|}}\bigr)$.
The coefficients of the two terms on the right-hand side are sharp and no extremal functions exist.

As in the periodic case considered in [2], this theorem makes it possible to obtain a simultaneous bound for the negative trace and the number of negative eigenvalues for the 1D Schrödinger eigenvalue problem on $(0,L)$

$$-y_j'' - Vy_j = \nu_j y_j \tag{5}$$

with Dirichlet boundary conditions $y(0) = y(L) = 0$ and potential $V(x) \ge 0$.

Theorem 2. *Suppose that there exist N negative eigenvalues $\nu_j \le 0$, $j = 1, \dots, N$ of the operator* (5). *Then both the negative trace and the number N of negative eigenvalues satisfy for any $\varepsilon \ge 0$*

$$\sum_{j=1}^N |\nu_j| + N \cdot \frac{\pi^2}{L^2}\left(\frac{c(\varepsilon)}{1+\varepsilon}\right)^2 \le \frac{2}{3\sqrt{3}} \cdot (1+\varepsilon)\int_0^L V(x)^{3/2}dx,$$

where $c(\varepsilon) = \min_{x\ge1}\bigl(\varepsilon x + 2xe^{-\pi x}\bigr)$.

Remark. We have $c(\varepsilon) \ge \varepsilon$, and the optimal ε for the bound involving only the number of negative eigenvalues N is $\varepsilon = 2$.

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Symplectic projection methods of deriving long-time asymptotics for nonlinear PDEs

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We consider some systems which describe a field-particle interaction, namely a charged particle coupled to the scalar wave field, to the Klein-Gordon field, and to the Maxwell field. Since the systems are Hamiltonian, methods of symplectic projection onto invariant finite-dimensional manifolds of soliton-type solutions turn to be helpful in deriving long-time asymptotics of solutions, [1, 2, 3, 4].

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On the uniform attractors of finite-difference schemes

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The theory of global uniform attractors of non-autonomous differential systems has been constructed in [1]. It is important in applications how close the attractors of discrete approximations to mathematical models are to their true attractors. For autonomous equations, this problem was studied in [2], where a theorem on the semicontinuous dependence of attractors of a family of semidynamical systems on the parameter was proved. A similar result was obtained in [3] for uniform attractors of families of semiprocesses corresponding to non-autonomous evolution equations. It was assumed in [2, 3] that the considered families have a common time semigroup; therefore, when studying finite-difference, the grid increment was represented in the form $\tau = \tau_n = T_0/n$ where T_0 is some positive number and $n \in \mathbb{N}$. In this paper we prove a theorem on the upper semi-continuous dependence on the parameter of the uniform attractors of families of semiprocesses [4] which allows us to investigate the convergence of the attractors of the numerical schemes in which the discretization parameter is not subjected to any law and can tend to zero in an arbitrary manner. This result is applied to the study of the uniform attractor of the explicit finite-difference scheme for the Lorenz system with time-dependent coefficients [5].

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Structure and regularity of the global attractor of reaction-diffusion equation with non-smooth nonlinear term

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In a bounded domain $\Omega \subset \mathbb{R}^3$ with sufficiently smooth boundary $\partial\Omega$ we consider the problem

$$\begin{cases} u_t - \Delta u + f(u) = h, & x \in \Omega,\ t > 0, \\ u|_{\partial\Omega} = 0, \end{cases} \tag{6}$$

where $f \in C(\mathbb{R})$ satisfies suitable growth and dissipative conditions, but there is no condition ensuring uniqueness of the Cauchy problem. When the nonlinear term f is smooth and f' satisfies additional assumptions, it is well known [1], that the problem (6) generates semigroup, which has global attractor and it coincides with the unstable set, emanating from the set of stationary points and with stable one as well. In general case (2), when the uniqueness of Cauchy problem is not guaranteed, we have existence of trajectory attractor [2], and existence of global attractor of multivalued semiflow [3]. Our aim is to study the structure of the global attractor in multi-valued case. We prove that the attractor of the multi-valued semiflow generated by all weak solutions of (6) in the phase space $L^2(\Omega)$ is the closure of the union of all stable manifolds of the set of stationary points. Also, for multi-valued semiflow, generated by regular solutions, we prove the existence of global attractor, which is compact in $H_0^1(\Omega)$ and we establish that it is the union of all unstable manifolds of the set of stationary points and of the stable ones as well.

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On global attractors of nonlinear hyperbolic PDEs

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We consider Klein-Gordon and Dirac equations coupled to U(1)-invariant nonlinear oscillators. Solitary waves of the coupled nonlinear system form two-dimensional submanifold in the Hilbert phase space of finite energy solutions.

Main Theorem. Let all the oscillators be strictly nonlinear. Then any finite energy solution converges, in the long time limit, to the solitary manifold in the local energy seminorms.

The investigation is inspired by Bohr's postulates on transitions to quantum stationary states. The results are obtained for:

- 1D KGE coupled to one oscillator [1,2,3], and to finitely many oscillators [4];
- nD KGE and Dirac coupled to one oscillator via mean field interaction [5, 6].

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Weak attractor for the Klein-Gordon equation with a nonlinear oscillator in discrete space-time

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We consider the Klein-Gordon equation in the discrete space-time interacting with a nonlinear oscillator. In [1], we prove that the weak attractor of all finite energy solutions coincides with the set of all multifrequency solitary waves,

$$\sum_{j=1}^{N} \phi_j(x) e^{-i\omega_j t}, \qquad (x,\,t) \in \mathbb{Z}^n \times \mathbb{Z}, \quad \phi_j \in l^2(\mathbb{Z}^n), \quad \omega_j \in \mathbb{R} \mod 2\pi.$$

More precisely, we show that there are only one-, two-, and four-frequency solitary waves. In the continuous limit, only the one-frequency solitary waves survive. The convergence to the attractor takes place weakly (on finite subsets or in the weighted norms). The proof is based on a version of the Titchmarsh convolution theorem proved for distributions supported on a circle [3].

The result generalizes an earlier result for the Klein-Gordon equation in the continuous space-time [2].

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Dispersive estimates for magnetic Klein-Gordon equation

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We obtain a dispersive long-time decay in weighted energy norms for solutions of 3D Klein-Gordon equation with magnetic and scalar potentials. The decay extends the results of [3], [5] and [4] for the Schrödinger, wave and Klein-Gordon equations with scalar potentials. For the proof we develop the spectral theory of Agmon, Jensen and Kato and minimal escape velocities estimates of Hunziker, Sigal and Soffer.

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Disprove of the commonly recognized belief that the foreign exchange currency market is self-stabilizing

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In economics and finance, arbitrage is the practice of taking advantage of a price difference between two or more markets. The act of exploiting an arbitrage opportunity resulting from a pricing discrepancy among three different currencies in the foreign exchange market is called triangular arbitrage (also referred to as cross currency arbitrage or three-point arbitrage). The commonly recognized belief in economics and finance is that

> ... Arbitrage has the effect of causing prices in different markets to converge. As a result of arbitrage, the currency exchange rates, the price of commodities, and the price of securities in different markets tend to converge...

see, e.g. `http://en.wikipedia.org/wiki/Arbitrage`.

In the talk, the triangle arbitrage operations will be reformulated in terms of the so-called asynchronous systems. This will allow to disprove the above belief by a set of examples. It will be demonstrated that the foreign exchange currency market may exhibit periodical regimes and exponential growth of exchange rates but also unexpectedly strong instability: the so-called double-exponential growth of exchange rates.

The structure of the solution sets for generic operator equations

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Consider an abstract operator equation $x = F(x, \lambda)$ in a Banach space X (all constructions are interesting even in $\mathbb{R}^2$) with compact and continuous operator F depending on a parameter $\lambda \in \Lambda$. Here Λ is a compact set, say an interval or the circle S^1. The set of all solutions of the equation $x = F(x, \lambda)$ in the space $X \times \Lambda$ may have very complicated form (e.g. may have a fractal structure). However, these sets have some common properties, for example such sets are always compact. Under generic topological assumptions (partially, if F satisfies the Schauder principle conditions: it maps some ball in its interior part) such equations always have large connected components. The first results in this direction were obtained by Mark Krasnosel'skii in 50's, later his ideas were continued and developed by various authors.

In my talk I would like to present some new results on the structure of the solution sets for generic operator equations. Possible applications to boundary value problems are evident, as an example I present some statements on nontrivial asymptotic bifurcation points in the problems on periodic forced oscillations for higher order ODE. In these statements the set of all solutions in the space $X \times \Lambda$ is nonconnected and consists from the infinite sequence of bounded cyclic branches going to infinity.

Around the Cauchy-Kowalevski theorem

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I will present a general approach which allows to prove the propagation of analyticity for solutions of various classes of quasilinear and nonlinear PDEs. In particular, it implies that under the assumptions of the Cauchy-Kowalevski or Ovsiannikov-Nirenberg theorems classical solutions stay analytic till they +exist. This is a joint work with N. Nadirashvili.

Critical manifold in the space of contours in Stokes-Leibenson problem for Hele-Shaw flow

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We here deal with the Stokes-Leibenson problem for a punctual Hele-Shaw flow. By using a geometrical transformation inspired by Helmholtz-Kirchhoff method, we introduce an integro-differential problem which leads to the construction of a discrete model. We first give a short recall about the source-case: global in time existence and uniqueness result for an initial contour close to a circular one, investigation of the evolutionary structure of the solution. Our main subject concerns the development of a numerical model in order to get some qualitative properties of the motion. This model provides numerical experiments which confirm the existence of a critical manifold of codimension 1 in some space of contours. This manifold contains one attractive point in the source-case corresponding to a circular contour centered at the source-point. In the sink-case, every point of this manifold seems to be attractive. In particular, we present some numerical experiments linked to fingering effects.

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The trajectory attractor of the nonlinear hyperbolic equation, contain a small parameter by the second derivative with respect to time

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In many articles dealt with the convergence of the attractor of the nonlinear autonomous hyperbolic equation, contain a small parameter by the second derivative with respect to time, to the attractor of the limit ($\varepsilon = 0$) parabolic equation (for example: [1], [2], [3]). It was assumed that the Cauchy problem for the limit of the parabolic equation has a unique solution. In the present report focuses on the case when there is no uniqueness of solutions of the Cauchy problem for these equations. It is shown that the trajectory attractor of a hyperbolic equation converges to the trajectory attractor of the limit parabolic equation in an appropriate topology.

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Vishik's approach to general boundary value problems for elliptic operators. Recent development

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In his pioneering paper [1] M.I. Vishik proposed a new approach to the extension theory of symmetric operators as well as dual pairs of operators in a Hilbert space. In the framework of this approach the proper extensions are parameterized in terms of (abstract) boundary conditions. Moreover, he applied general constructions to investigate the properties of solvability and complete solvability of boundary value problems for (not necessarily symmetric) elliptic operators on bounded domains.

During three last decades this approach has been formalized in the concept of boundary triplets for dual pairs of operators and elaborated in great detail. The revival of interest to this approach has been motivated by numerous applications to boundary value problems for differential and difference operators (see for instance publications [2], [3], [4], [5] and references therin).

I plan to recall the main results and basic constructions of the Vishik's paper [1] as well as to discuss its influence on development of the extensions theory.

Next I plan to discuss applications of to elliptic boundary value problems in domain with compact boundary. Some spectral properties of different realizations of elliptic differential expressions will be discussed too.

[1] M.I. Vishik. *On general boundary problems for elliptic differential equations.* Am. Math. Soc., Transl., II. Ser.. **24**(1952), 107–172.

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[4] A. S. Kostenko and M. M. Malamud. *1-D Schrödinger operators with local point interactions on a discrete set.* J. Differ. Equations. **249(2)**(2010), 253–304.

[5] M. M. Malamud. *Spectral theory of elliptic operators in exterior domains.* Russ. J. Math. Phys., **17(1)**(2010), 96–125.

New phenomena in large systems of ODE and classical models of DC

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We consider the system

$$M\frac{d^2x_i}{dt^2} = -\frac{\partial U}{\partial x_i} + F(x_i) - A\frac{dx_i}{dt}, i = 1, ..., N$$

of N ordinary differential equations describing Newtonian dynamics of N particles (electrons), initially at the points

$$x_1(0) < x_2(0) < ... < x_N(0),$$

on the interval $[0, L) \in R$ with periodic boundary conditions, that is on the circle of length L. Here $M > 0, A \geq 0$ are the parameters, $F(x)$ is the external force, and

$$U(x_1, ..., x_N) = \sum_{i=1}^{N} \frac{\alpha}{|x_{i+1} - x_1|}.\alpha > 0,$$

(where of course $x_{N+1} \equiv x_1$) is the Coulomb repulsive interaction between nearest neighbors.

We review new results concerning this system: fixed points, quasi-homogeneous regime (Ohm's law) and very fast propagation of the "effective" external field, which is initially zero on the most part of the circle.

All these phenomena are closely related to many problems with DC (direct electric current), that the statistical physics was unable to understand. The following is a picturesque description of one of DC enigmas in the famous Feynman lectures, v. 6, pp. 33-34: "The force pushes the electrons along the wire. But why does this move the galvanometer, whis is so far from the force? Because when the electrons which feel the magnetic force try to move, they push - by electric repulsion - the electrons a little farther down the wire; they, in turn, repel the electrons a little farther on, and so on for a long distance. An amazing thing. It was so amazing to Gauss and Weber - who first built a galvanometer - that they tried to see how far the forces in the wire would go. They strung the wire all the way across the city."

Asymptotic solutions of the Navier-Stokes equations and scenario of turbulence development

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We discuss asymptotic solutions of the Navier-Stokes equations, describing periodic collections of vortices in 3D space. These solutions are connected with topological invariants of divergence-free vector fields. Equations, describing evolution of vortices, are defined on a graph – Reeb graph of the stream function or Fomenko molecule of the Liouville foliation. Homogenization with respect to the periodic structure leads to equations coinciding with Reynolds equations. It is well known that existence of the Reynolds stresses leads to the growth of the energy and entropy of the fluid. As the entropy reaches certain critical value, the molecules of the fluid have to form "clusters" which leads to the occurrence of turbulence.

A Cahn–Hilliard model with dynamic boundary conditions

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Our aim in this talk is to discuss the dynamical system associated with the Cahn-Hilliard equation with dynamic boundary conditions. Such boundary conditions take into account the interactions with the walls for confined systems. We are in particular interested in a model which accounts for the conservation of mass, both in the bulk and on the walls.

The structure of the population inside the propagating front (the qualitative analysis of FKPP equation)

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The talk will contain several results describing the local structure of the particles field near the front of the reaction in the classical FKPP model. The central fact is the fragmentation or intermittency of the field in the agreement with the experimental data.

Geometry of stream lines of ideal fluid

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On singular solutions of fully nonlinear elliptic equations

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Extensions are made of the strong maximum principle for solutions with singularities, including viscosity solutions. A number of other results will be presented : on removable singularities, and for parabolc operators. The talk will be expository.

Structure of the minimum-time damping of a physical pendulum

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We study the minimum-time damping of a physical pendulum by means of a bounded control. In the similar problem for a linear oscillator each optimal trajectory possesses a finite number of control switchings from the maximal to the minimal value. If one considers simultaneously all optimal trajectories with any initial state, the number of switchings can be arbitrary large. We show that for the nonlinear pendulum there is a uniform bound for the switching number for all optimal trajectories. We find asymptotics for this bound as the control amplitude goes to zero.

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[5] F.L. Chernousko, L.D. Akulenko, and B.N. Sokolov, *Upravlenie kolebaniyami.* Nauka, Moscow, 1980.

A uniform reconstruction formula in integral geometry

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We address the problem of reconstruction of a function on a manifold from data of its integrals over a family of hypersurfaces. The archetypes are reconstruction of a function on a sphere from data of big circle integrals (Minkowski-Funk) and of a function in plane from data of line integrals (Radon). A general integral formula will be presented that covers all known cases where such an explicit reconstruction is known and also several unknown cases. Possibility of the uniform reconstruction method depends on vanishing of some singular integrals over a sphere.

On the general theory of multi-dimensional linear functional operators with applications in Analysis

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The talk is devoted to the linear multi-dimensional functional operator

$$(\mathcal{P}F)(x) := \sum_{j=1}^{N} c_j(x)(F \circ a_j)(x), \ \ x \in D \subset \mathbb{R}^n.$$

Here $F \in C(I)$ with $I = (-1, 1)$ and $|F|$ norm in C, *coefficients* c_j and *arguments* a_j of $\mathcal{P}$ are continuous functions $D \to \mathbb{R}$ and $D \to I$, respectively; D is a domain with compact closure. These operators are of interest both in Analysis and in applying fields. If time allows I'll mention some problems in Integral geometry and PDE closely connected with them. As to the intrinsic problems relating to the operator $\mathcal{P}$ we'll discuss *the asymptotic behavior* of solutions to the equation $\mathcal{P}u = h_\varepsilon$ depending on a small parameter $\varepsilon \to 0$ under condition $h_\varepsilon = O(\varepsilon)$. Recent speaker's results make significally more precise analogous information based on the solution to well known Ulam problem. It turned out that this problem (as it is formulated in his book "A Collection of Mathematical Problems") is not well posed: the input information ($|\mathcal{P}F(x)| < \varepsilon$ for *all* $x \in D$) is redundant. As a matter of fact, to describe the asymptotic behavior of the function F the latter relation should be valid only at points x of some one-dimensional submanifold $\Gamma \subset D$ (subject to determining), but not everywhere in D. This result will be discussed together with a new Inverse problem for the equation $\mathcal{P}F = H_\varepsilon$ (reconstruction of the operator $\mathcal{P}$ using the given asymptotic behavior of the solution F).

A uniform Gronwall-type lemma with parameter and applications to nonlinear wave equations

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We discuss a uniform Gronwall-type lemma depending on a small parameter $\varepsilon > 0$, based on an integral inequality that predicts blow up in finite time of the involved unknown function for any fixed ε. The result permits to establish uniform estimates even if the function itself depends on ε.
As an application, we consider the asymptotics of the strongly damped nonlinear wave equation

$$u_{tt} - \Delta u_t - \Delta u + f(u_t) + g(u) = h$$

with Dirichlet boundary conditions, which serves as a model in the description of thermal evolution within the theory of type III heat conduction. In particular, the nonlinearity f acting on u_t is allowed to be nonmonotone and to exhibit a critical growth of polynomial order 5.

[1] V. Pata, *Uniform estimates of Gronwall type*, Journal of Mathematical Analysis and Applications **373** (2011), 264–270.

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Ground state asymptotics for a singularly perturbed second order elliptic operator with oscillating coefficients

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The talk will focus on the Dirichlet spectral problem in a smooth bounded domain for a singularly perturbed second order elliptic operator with locally periodic rapidly oscillating coefficients. We will study the limit behaviour of the first eigenvalue, the logarithmic asymptotics of the first eigenfunction and, for convection-diffusdion operators, the second term of the asymptotics.

Critical nonlinearities in Partial Differential Equations

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Longitudinal correlation functions and the intermittency

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A time dependence for second-order and third-order longitudinal correlation functions are considered in the intermittency model [1], i.e. we use the following model: flow is considered as a mixture of turbulent and viscous regimes. Both regimes have Loitsyansky invariants and Kolmogorov's (for turbulent regime) and Millionshchikov's (for viscous regime) self-similarities.

Gradient hypothesis of Lytkin and Chernykh [2] is used to make Karman-Howarth equation closed by the expression of the two-point third-order correlation moment through the two-point second-order correlation moment in the regime of Kolmogorov turbulence in the inertial range.

A model dependence obtained for the longitudinal correlation coefficient has asymptotically exponential form of decay and is in good agreement with the experimental data of Batchelor-Townsend-Stewart [3].

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[2] Yu.M. Lytkin, G.G. Chernykh *One method of closing the Karman-Howarth equation* [in Russian] Dynamics of Continuous Media **27** (1976), 124–130 .

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On global solutions to the Cauchy problem for discrete kinetic equations

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The kinetic theory considers the gas as a collection of a huge number of randomly moving particles, in some way interacting with each other. As a result of these interactions the particles exchange momenta and energies. Interaction can be through direct collisions or by certain forces. To elucidate the mathematical scheme describing such phenomena, we consider [1] the so-called discrete kinetic Boltsmann equations and give a phenomenological derivation of the Boltsmann equation for the model of gas with finitely many particle velocities and finitely many different interactions (Broadwell-type model [2]):

$$\partial_t n_j + (\omega_{ix}\partial_x + \omega_{iy}\partial_y + \omega_{iz}\partial_z)n_j = \sum_{k,l,j;k\neq i,l\neq i,j\neq i} \sigma_{kl}^{ij}(n_k n_l - n_i n_j), \quad 1 \leq i \leq N.$$

For the discrete kinetic equations [3] (in dimensions $d = 1, 2, 3$) we prove the existence of a global solution, its decomposition with respect to smoothness, and consider the influence of oscillations born by the interaction operator.

[1] S.K. Godunov and U.M. Sultangazin, *Discrete models of the Boltzmann kinetic equation*, Uspehi Mat. Nauk, **26** (1971), 3–51.

[2] J.E. Broadwell, *Study of rarefied shear flow by the discrete velocity method*, J. Fluid Mech. **19** (1964), 401–414.

[3] E. Radkevich, *On existence of global solutions to the Cauchy problem for discrete kinetic equations*, Journal of Mathematical Sciences **181** (2012), 232–280.

Branching random motions, nonlinear hyperbolic systems and traveling waves

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It is known that under certain assumptions for nonlinearities the following coupled nonlinear hyperbolic equations

$$\begin{cases} \dfrac{\partial u_+}{\partial t} - c\dfrac{\partial u_+}{\partial x} = \mu_+(u_- - u_+) - \lambda_+ u_+ + F_+(u_+, u_-), \\ \dfrac{\partial u_-}{\partial t} + c\dfrac{\partial u_-}{\partial x} = \mu_-(u_+ - u_-) - \lambda_- u_- + \lambda_- F_-(u_+, u_-), \end{cases}$$

have traveling-wave solutions.

We realize the McKean's program [1] for the Kolmogorov-Petrovskii-Piskunov equation in this hyperbolic case.

A branching random motion on a line, with abrupt changes of direction, is studied. The branching mechanism, being independent of random motion, and intensities of reverses are defined by a particle's current direction. A solution of a certain hyperbolic system of coupled non-linear equations (Kolmogorov type backward equation) have a so-called McKean representation via such processes. Commonly this system possesses travelling-wave solutions. The convergence of solutions with Heaviside terminal data to the travelling waves is discussed.

The Feynman-Kac formula plays a key role, [2].

[1] H.P. McKean, *Application of Brownian motion to the equation of Kolmogorov-Petrovskii-Piskunov*, Comm. Pure Appl. Math. XXVIII (1975), 323-331.

[2] N. Ratanov, *Branching random motion, nonlinear hyperbolic systems and travelling waves*, ESAIM: Probability and Statistics, **10** (2006), 236-257

Periodic solutions of some quasilinear evolutionary equations

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[1] I.A. Rudakov, *Periodic solutions of a quasilinear wave equation with variable coefficients*, Mat. Sb. **198** (2007), 83–100.

[2] *Periodic solutions of a quasilinear beam equation with homogeneous boundary conditions*, Differential equations **48** (2012).

Neurogeometry of vision and sub-Riemannian geometry

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The talk will be devoted to the following questions:

- Image inpainting
- The pinwheel model of the primary visual cortex V1 of a human brain,
- Sub-Riemannian problem on the group of rototranslations of a plane and its solution,
- Image inpainting via sub-Riemannian length minimizers,
- Curve cuspless reconstruction,
- Image inpainting via hypoelliptic diffusion.

[P1] J.Petitot, *The neurogeometry of pinwheels as a sub-Riemannian contact structure*, J. Physiology - Paris, **97** (2003), 265–309.

[P2] J.Petitot, *Neurogeometrie de la vision — Modeles mathematiques et physiques des architectures fonctionnelles*, (2008), Editions de l'Ecole Polytechnique.

[S1] Yuri L. Sachkov and Igor Moiseev, *Maxwell strata in sub-Riemannian problem on the group of motions of a plane*, ESAIM: COCV, **16** (2010), 380–399.

[S2] Yuri L. Sachkov, *Conjugate and cut time in the sub-Riemannian problem on the group of motions of a plane*, ESAIM: COCV, **16** (2010), 1018–1039.

[S3] Yuri L. Sachkov, *Cut locus and optimal synthesis in the sub-Riemannian problem on the group of motions of a plane*, ESAIM: COCV, **17** (2011), 293–321.

On the blow up phenomena in differential equations and dynamical systems

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The comparison is given of the phenomena of the Ω-blow up in dynamical systems and the phenomena of the blow up in the evolution differential equations. The comparison is based on the methods of multivalued analysis.
The examples are considered. In particular, the new example of C^0- Ω-blow up in C^1-smooth simplest skew products in the plane is described (see [1]).
The procedure is defined of the extension of the dynamical transformation of the space of the initial conditions of Cauchy problem is the case of the destruction of a solution or in the case of the appearance of the singularities in a finite time (see [2]).

[1] E.V. Blinova, L.S. Efremova, *On Ω-blow ups in simplest C^1-smooth skew products of maps of an interval*, J. Math. Sci. **157** (2009), 456–465.

[2] V.Zh. Sakbaev, *On the averaging of quantum dynamical semigroups*, TMPh. **164** (2010), 455–463.

Operators with symbolic hierarchies on stratified spaces

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Manifolds M with higher corners or edges of order $k \in \mathbb{N}$ are (in our notation) special stratified spaces, where $k = 0$ corresponds to smoothness, $k = 1$ to conical or edge singularities, especially smooth boundaries. Manifolds with singularities of order k form a category $\mathcal{M}_k$. The stratification $s(M) = (s_0(M), s_1(M), \ldots, s_k(M))$ induces a principal symbolic hierarchy

$$\sigma(A) = (\sigma_0(A), \sigma_1(A), \ldots, \sigma_k(A))$$

of operators A over $s_0(M)$, degenerate in a typical way in the representation over the stretched version $\mathbb{M}$ of M. The component $\sigma_0(A)$ is the standard homogeneous principal symbol on the main stratum $s_0(M)$; the component $\sigma_j(A)$, $j > 0$, lives on $s_k(M))$ and is operator-valued. The symbolic hierarchy admits notions of ellipticity and the construction of parametrices within suitable algebras of degenerate pseudo-differential operators. We present some new developments in this field which has a long history through achievements of numerous Russian authors and other schools worldwide. Further progress is stimulated by the desire to reach new models of applications, see, for instance, [2]. Moreover, the tower of operator algebras with increasing k still contains many new challenges. The methods of the author have been stimulated very much by the works [4], [5], [1], since the case of manifolds with edge contains boundary value problems with and without the transmission property at the boundary.

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[2] H.-J. Flad, G. Harutyunyan, R. Schneider, and B.-W. Schulze, *Explicit Green operators for quantum mechanical Hamiltonians.I. The hydrogen atom*, arXiv:1003.3150v1 [math.AP], 2010. manuscripta math. **135**(2011), 497-519.

[3] B.-W. Schulze *The iterative structure of the corner calculus*, Oper. Theory: Adv. Appl. **213**, Pseudo-Differential Operators: Analysis, Application and Computations (L. Rodino et al. eds.), Birkhäuser Verlag, Basel, 2011, pp. 79-103.

[4] M.I. Vishik and G.I. Eskin, *Convolution equations in a bounded region*, Uspekhi Mat. Nauk **20**, 3 (1965), 89-152.

[5] M.I. Vishik and G.I. Eskin, *Convolution equations in bounded domains in spaces with weighted norms*, Mat. Sb. **69**, 1 (1966),65-110.

On numerical methods and the study of the dynamics inside the attractor

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Control and mixing for 2D Navier–Stokes equations with space-time localised force

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We consider 2D Navie–Stokes equations in a bounded domain with smooth boundary and discuss the interconnection between controllability for the deterministic problem and mixing properties of the associated random dynamics. Namely, we first consider the problem of stabilisation of a given non-stationary solution, assuming that the control is localised in space and time and is finite-dimensional as a function of both variables. We next replace the control by a random force and prove that the resulting random dynamical system is exponentially mixing in the Kantorovich–Wasserstein distance. Some of the results of this talk are obtained in collaboration with V. Barbu and S. Rodrigues.

On the 2-point problem for the Lagrange-Euler equation

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Consider the motion of ideal incompressible fluid in a bounded domain (or on a compact Riemannian manifold). The configuration space of the fluid is the group of volume preserving diffeomorphisms of the flow domain, and the flows are geodesics on this infinite-dimensional group where the metric is defined by the kinetic energy. The geodesic equation is the Lagrange-Euler equation. The problem usually studied is the initial value problem, where we look for a geodesic with given initial fluid configuration and initial velocity field. In this talk we consider a different problem: find a geodesic connecting two given fluid configurations. The main result is the following

Theorem: *Suppose the flow domain is a 2-dimensional torus. Then for any two fluid configurations there exists a geodesic connecting them. This means that, given arbitrary fluid configuration (diffeomorphism), we can "push" the fluid along some initial velocity field, so that by time one the fluid, moving according to the Lagrange-Euler equation, assumes the given configuration.*

This theorem looks superficially like the Hopf-Rinow theorem for finite-dimensional Riemannian manifolds. In fact, these two theorems have almost nothing in common. In our case, unlike the Hopf-Rinow theorem, the geodesic is not, in general case, the shortest curve connecting the endpoints (fluid configurations). Moreover, the length minimizing curve can not exist at all, while the geodesic always exists.

The proof is based on some ideas of global analysis (Fredholm quasilinear maps) and microlocal analysis of the Lagrange-Euler equation (which may be called a "microglobal analysis").

Non-linear PDE of mKdV type with possibly unbounded coefficients at infinity

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We discuss solving mKdV-type equations in classes of temperate functions near infinity. Main ideas include generalized spectral decomposition of I.M. Gelfand, A.G. Kostyuchenko, and Yu.M. Berezanskii, as well a special wave front micro-localization technique.

Bifurcations of solutions to the Navier–Stokes system

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Eigenfunction of the Laplace operator in a tetrahedron

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Let T be an open and regular triangular pyramid (tetrahedron) in the space $\mathbb{R}^3$ with a boundary $\partial\Gamma$. Let α, β, γ, σ are barycentric coordinates of a point $(x, y, z) \in \mathbb{R}^3$ with respect to tetrahedron T which can be expressed in the variables x, y, z.

Theorem. The function $w = \sin(\alpha\pi/2)\sin(\beta\pi/2)\sin(\gamma\pi/2)\sin(\sigma\pi/2)$ is the eigenfunction of the Laplace operator $\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in T. The function w satisfies conditions: $w > 0$ in T and $w = 0$ on ∂T.

Let Π be unlimited cylinder in the space $\mathbb{R}^4$ which a cross-section with hyperplane is a quadrangular pyramid with edges of unit length (one-half of the octahedron). Let L be a second order linear differential operator in divergence form which uniformly elliptic with bounded measurable coefficients and η is its ellipticity constant. Let u be a solution of he mixed boundary value problem in Π for the equation $Lu = 0$ ($u > 0$) with homogeneous Dirichlet and Neumann data on the boundary of the cylinder. Our theorem allows us to continue this solution from the cylinder Π to the whole space $\mathbb{R}^4$ with the same ellipticity constant η.

This continuation allows us to prove a number of theorems about growth of the solution u in the cylinder Π.

The idea of using barycentric coordinates is taken from paper of A.P. Brodnikov, where it is used for the finding of eigenfunction of the Laplace operator in the triangle. Eigenfunction of the Laplace operator in hypertetrahedron from $\mathbb{R}^4$ and in $n+1$-dimensional simplex from $\mathbb{R}^n$ ($n \geq 2$) were constructed by the author.

Classical solutions of the Vlasov–Poisson equations in a half-space

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We consider the Vlasov system of equations describing the evolution of distribution functions of the density for the charged particles in a rarefied plasma. We study the Vlasov system in $\mathbb{R}^3_+ \times \mathbb{R}^3$ with initial conditions for distribution functions $f^\beta\big|_{t=0} = f_0^\beta(x,p)$, $\beta = \pm 1$, and the Dirichlet or Neumann boundary conditions for the potential of an electric field for $x_1 = 0$, where $f_0^\beta(x,p)$ is the initial distribution function (for positively charged ions if $\beta = +1$ and for electrons if $\beta = -1$) at the point x with impulse p, $\mathbb{R}^3_+ = \{x \in \mathbb{R}^3 \colon x_1 > 0\}$. Assume that initial distribution functions are sufficiently smooth and $\operatorname{supp} f_0^\beta \subset (\mathbb{R}^3_\delta \cap B_\lambda(0)) \times B_\rho(0)$, $\delta, \lambda, \rho > 0$, and the magnetic field $H(x)$ is also sufficiently smooth and has a special structure near the boundary $x_1 = 0$, where $\mathbb{R}^3_\delta = \{x \in \mathbb{R}^3 \colon x_1 > \delta\}$. Then we prove that for any $T > 0$ there is a unique classical solution of the Vlasov system in $\mathbb{R}^3_+ \times \mathbb{R}^3$ for $0 < t < T$ if $\|f_0^\beta\| < \varepsilon$, where $\varepsilon = \varepsilon(T, \delta, \rho, \|H\|)$ is sufficiently small.

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On free boundary problems of magnetohydrodynamics

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We prove local in time solvability of free boundary problem of magnetohydrodynamics of a viscous incompressible liquid assuming that the domain filled with the liquid can be multi-connected. Under some additional assumptions, the solution can be extended to the infinite time interval $t > 0$.

Homogenization of the elliptic Dirichlet problem: operator error estimates

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Let $\mathcal{O} \subset \mathbb{R}^d$ be a bounded domain of class $C^{1,1}$. In $L_2(\mathcal{O};\mathbb{C}^n)$, we consider a matrix elliptic differential operator $A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$ with the Dirichlet boundary condition. We assume that an $(m \times m)$-matrix-valued function $g(\mathbf{x})$ is bounded, uniformly positive definite and periodic with respect to some lattice Γ. The elementary cell of Γ is denoted by Ω. Next, $b(\mathbf{D}) = \sum_{j=1}^d b_j D_j$ is an $(m \times n)$-matrix first order differential operator (b_j are constant matrices). It is assumed that $m \geq n$ and the symbol $b(\boldsymbol{\xi}) = \sum_{j=1}^d b_j \xi_j$ has maximal rank, i. e., $\operatorname{rank} b(\boldsymbol{\xi}) = n$ for $0 \neq \boldsymbol{\xi} \in \mathbb{R}^d$. The simplest example is $A_\varepsilon = -\operatorname{div} g(\mathbf{x}/\varepsilon)\nabla$.

We study the behavior of the solution $\mathbf{u}_\varepsilon$ of the Dirichlet problem $A_\varepsilon \mathbf{u}_\varepsilon = \mathbf{F}$ in $\mathcal{O}$, $\mathbf{u}_\varepsilon|_{\partial\mathcal{O}} = 0$, where $\mathbf{F} \in L_2(\mathcal{O};\mathbb{C}^n)$. It turns out that $\mathbf{u}_\varepsilon$ converges in $L_2(\mathcal{O};\mathbb{C}^n)$ to $\mathbf{u}_0$, as $\varepsilon \to 0$. Here $\mathbf{u}_0$ is the solution of the "homogenized" Dirichlet problem $A^0\mathbf{u}_0 = \mathbf{F}$ in $\mathcal{O}$, $\mathbf{u}_0|_{\partial\mathcal{O}} = 0$. The *effective operator* A^0 is given by the expression $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ with the Dirichlet boundary condition. The effective matrix g^0 is a constant positive $(m \times m)$-matrix defined as follows. Denote by $\Lambda(\mathbf{x})$ the $(n \times m)$-matrix-valued periodic solution of the equation $b(\mathbf{D})^* g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m) = 0$ such that $\int_\Omega \Lambda(\mathbf{x})\,d\mathbf{x} = 0$. Then $g^0 = |\Omega|^{-1} \int_\Omega g(\mathbf{x})(b(\mathbf{D})\Lambda(\mathbf{x}) + \mathbf{1}_m)\,d\mathbf{x}$.

Theorem 1. (see [2]) *We have the following sharp order error estimate*:

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0\|_{L_2(\mathcal{O};\mathbb{C}^n)} \leq C\varepsilon \|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}.$$

Now we give approximation of $\mathbf{u}_\varepsilon$ in the Sobolev space $H^1(\mathcal{O};\mathbb{C}^n)$. For this, the first order corrector must be taken into account.

Theorem 2. (see [1]) 1) *Let $\Lambda \in L_\infty$, and denote $\Lambda^\varepsilon(\mathbf{x}) = \Lambda(\varepsilon^{-1}\mathbf{x})$. Then*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon\Lambda^\varepsilon b(\mathbf{D})\mathbf{u}_0\|_{H^1(\mathcal{O};\mathbb{C}^n)} \leq C\varepsilon^{1/2}\|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}.$$

2) *In the general case, we have*

$$\|\mathbf{u}_\varepsilon - \mathbf{u}_0 - \varepsilon\Lambda^\varepsilon b(\mathbf{D})(S_\varepsilon \widetilde{\mathbf{u}}_0)\|_{H^1(\mathcal{O};\mathbb{C}^n)} \leq C\varepsilon^{1/2}\|\mathbf{F}\|_{L_2(\mathcal{O};\mathbb{C}^n)}.$$

Here $\widetilde{\mathbf{u}}_0 = P_\mathcal{O}\mathbf{u}_0$ and $P_\mathcal{O} : H^2(\mathcal{O};\mathbb{C}^n) \to H^2(\mathbb{R}^d;\mathbb{C}^n)$ is a continuous extension operator, S_ε is the smoothing operator $(S_\varepsilon\mathbf{u})(\mathbf{x}) = |\Omega|^{-1}\int_\Omega \mathbf{u}(\mathbf{x} - \varepsilon\mathbf{z})\,d\mathbf{z}$.

[1] M. A. Pakhnin, T. A. Suslina, *Operator error estimates for homogenization of the Dirichlet problem in a bounded domain*, Preprint, 2012. Available at http://arxiv.org/abs/1201.2140.

[2] T. A. Suslina, *Homogenization of the elliptic Dirichlet problem: operator error estimates in L_2*, Preprint, 2012. Available at http://arxiv.org/abs/1201.2286.

Algebra of boundary value problems with small parameter

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In a singular perturbation problem one is concerned with a differential equation of the form $A(\varepsilon)u_\varepsilon = f_\varepsilon$ with initial or boundary conditions $B(\varepsilon)u_\varepsilon = g_\varepsilon$, where ε is a small parameter. The distinguishing feature of this problem is that the orders of $A(\varepsilon)$ and $B(\varepsilon)$ for $\varepsilon \neq 0$ are higher than the orders of $A(0)$ and $B(0)$, respectively. There is by now a vast amount of literature on singular perturbation problems for partial differential equations. A comprehensive theory of such problems was initiated by the remarkable paper of Vishik and Lyusternik [1]. In [2], Volevich completed the theory of differential boundary value problems with small parameter by formulating the Shapiro-Lopatinskii type ellipticity condition.

We contribute to the theory by constructing an algebra of pseudodifferential operators in which singularly perturbed boundary value problems can be treated. Given any $m, \mu \in \mathbb{R}$, denote by $\mathcal{S}^{m,\mu}$ the space of all smooth functions $a(x,\xi,\varepsilon)$ on $T^*\mathbb{R}^n \times \mathbb{R}_{\geq 0}$, such that $|D_x^\alpha D_\xi^\beta a| \leq C_{\alpha,\beta} < \xi >^{\mu-|\beta|} < \varepsilon\xi >^{m-\mu}$ for all multi-indices α and β, where $C_{\alpha,\beta}$ are constants independent of x, ξ and ε. For any fixed $\varepsilon > 0$, a function $a \in \mathcal{S}^{m,\mu}$ is a symbol of order m on $\mathbb{R}^n$ which obviously degenerates as $\varepsilon \to 0$. These symbols quantize to continuous operators $H^{r,s} \to H^{r-m,s-\mu}$ in a scale of Sobolev spaces on $\mathbb{R}^n$ whose norms depend on ε and are based on L^2 and weight functions $< \xi >^s < \varepsilon\xi >^{r-s}$. The family $\mathcal{S}^{m-j,\mu-j}$ with $j = 0, 1, \dots$ is used as usual to define asymptotic sums of homogeneous symbols. By the homogeneity of degree μ is meant the property $a(x, \lambda\xi, \lambda^{-1}\varepsilon) = \lambda^\mu a(x,\xi,\varepsilon)$ for all $\lambda > 0$. Let $\mathcal{S}^{m,\mu}_{\mathrm{phg}}$ stand for the subspace of $\mathcal{S}^{m,\mu}$ consisting of all polyhomogeneous symbols, i.e., those admitting asymptotic expansions in homogeneous symbols. For any $a \in \mathcal{S}^{m,\mu}_{\mathrm{phg}}$ there is well-defined principal homogeneous symbol $\sigma^\mu(a)$ of degree μ whose invertibility away from the zero section of $T^*\mathbb{R}^n$ is said to be the interior ellipticity with small parameter. Familiar techniques lead now to calculi of pseudodifferential operators with small parameter on diverse compactifications of smooth manifolds. Our results gain in interest if we realize that pseudodifferential operators with small parameter provide also adequate tools for studying Cauchy problems for elliptic equations.

This is a joint paper with my PhD student Evgeniya Dyachenko who studies singular perturbation problems.

[1] M. I. Vishik and L. A. Lyusternik, *Regular degeneration and boundary layer for linear differential equations with small parameter*, Uspekhi Mat. Nauk **12** (1957), Issue 5 (77), 3–122.

[2] L. R. Volevich, *The Vishik-Lyusternik method in elliptic problems with small parameter*, Trans. Moscow Math. Soc. **67** (2006), 87–125.

Pattern formation: The oscillon equation

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In this lecture, we will consider the (non autonomous) oscillon equation which is used in cosmology to model and represent some transient persistent structures. We will discuss questions of existence and uniqueness of solutions and of long time behavior of solutions.

Global well-posedness of an inviscid three-dimensional pseudo-Hasegawa-Mima model

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The three-dimensional inviscid Hasegawa-Mima model is one of the fundamental models that describe plasma turbulence. The model also appears as a simplified reduced Rayleigh-Bénard convection model. The mathematical analysis of the Hasegawa-Mima equation is challenging due to the absence of any smoothing viscous terms, as well as to the presence of an analogue of the vortex stretching terms. In this talk, we introduce and study a model which is inspired by the inviscid Hasegawa-Mima model, which we call a pseudo-Hasegawa-Mima model. The introduced model is easier to investigate analytically than the original inviscid Hasegawa-Mima model, as it has a nicer mathematical structure. The resemblance between this model and the Euler equations of inviscid incompressible fluids inspired us to adapt the techniques and ideas introduced for the two-dimensional and the three-dimensional Euler equations to prove the global existence and uniqueness of solutions for our model. This is in addition to proving and implementing a new technical logarithmic inequality, generalizing the Brezis-Gallouet and the Berzis-Wainger inequalities. Moreover, we prove the continuous dependence on initial data of solutions for the pseudo-Hasegawa-Mima model. These are the first results on existence and uniqueness of solutions for a model that is related to the three-dimensional inviscid Hasegawa-Mima equations.

Weyl asymptotics for interior transmission eigenvalues

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Interior transmission eigenvalues are defined by the problem

$$-\Delta u - \lambda u = 0, \quad x \in \mathcal{O}, \quad u \in H^2(\mathcal{O}),$$

$$-\nabla A \nabla v - \lambda n(x) v = 0, \quad x \in \mathcal{O}, \quad v \in H^2(\mathcal{O}),$$

$$u - v = 0, \quad x \in \partial\mathcal{O},$$
$$\tfrac{\partial u}{\partial \nu} - \tfrac{\partial v}{\partial \nu_A} = 0, \quad x \in \partial\mathcal{O},$$

were $\mathcal{O} \subset R^d$ is a bounded domain with a smooth boundary, $H^2(\mathcal{O}),\ H^s(\partial\mathcal{O})$ are Sobolev spaces, $A(x),\ x \in \overline{\mathcal{O}}$, is a smooth symmetric elliptic $(A = A^t > 0)$ matrix with real valued entries, $n(x)$ is a smooth function, ν is the outward unit normal vector and the co-normal derivative is defined as follows

$$\frac{\partial}{\partial \nu_A} v = \nu \cdot A \nabla v.$$

The importance of these eigenvalues is based on their relation to the scattering of plane waves by inhomoginuety defined by A and n: a real $\lambda = k^2$ is an interior transmission eigenvalue if and only if the far-field operator has a non trivial kernel at the frequency k.

The problem above is not symmetric. However we will show that under some conditions it has infinitely many real eigenvalues. We will obtain the Weyl type bound from below for the counting function of these eigenvelues as well as some estimates on the first egenvalues.

These results are obtained together with E. Lakshtanov.

Example of equations with nonlinearity of type min[u,v]

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This paper considers a boundary-value problem for a nonlinear system of equations that are derived from a trading process model [1].

Let $u(x.t), v(x,t)$, $0 < x < 1$, $t > 0$, satisfy a system

$$\begin{aligned} \frac{\partial u(x,t)}{\partial t} &= -a_u \frac{\partial u(x,t)}{\partial x} \quad -b_u u(x,t) - c\min[u(x,t), v(x,t)], \\ \frac{\partial v(x,t)}{\partial t} &= \quad a_v \frac{\partial v(x,t)}{\partial x} \quad -b_v v(x,t) - c\min[u(x,t), v(x,t)], \end{aligned} \tag{7}$$

$$\frac{\partial u(0,t)}{\partial t} = d_u - a_u u(0,t) - b_u u(0,t) - c\min[u(0,t), v(0,t)], \tag{8}$$

$$\frac{\partial v(1,t)}{\partial t} = d_v - a_v v(1,t) - b_v v(1,t) - c\min[u(1,t), v(1,t)], \tag{9}$$

$$u(x,0) = u_0(x) \geq 0, \quad v(x,0) = v_0(x) \geq 0. \tag{10}$$

Here $a_{u/v}$, $b_{u/v}$, c, $d_{u/v}$ are positive.

Theorem 3 *For any initial data $u_0(x), v_0(x)$ for all $t > 0$ there exists a unique solution to (*7)-(10). As $t \to \infty$ a solution approaches a fixed point which is the unique solution to a system

$$\begin{aligned} -a_u \frac{\mathrm{d}u(x,t)}{\mathrm{d}x} \quad -b_u u(x,t) - c\min[u(x,t), v(x,t)] = 0, \\ a_v \frac{\mathrm{d}v(x,t)}{\mathrm{d}x} \quad -b_v v(x,t) - c\min[u(x,t), v(x,t)] = 0, \end{aligned} \tag{11}$$

$$d_u - a_u u(0,t) - b_u u(0,t) - c\min[u(0,t), v(0,t)] = 0, \tag{12}$$

$$d_v - a_v v(1,t) - b_v v(1,t) - c\min[u(1,t), v(1,t)] = 0. \tag{13}$$

Remark that the type of boundary conditions (8), (9) and (12), (13) depends on sign of $u - v$, that makes not evident the uniqueness of solution to (11)-(13).

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On the Gauss problem with Riesz potential

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This is a lecture on joint work with H. Harbrecht (U. Basel, Switzerland), G. Of (TU. Graz, Austria) and N. Zorii (Nat. Academy Sci. Kiev, Ukraine).

In $\mathbb{R}^n$, $n \geqslant 2$, we study the constructive and numerical solution of minimizing the energy relative to the Riesz kernel $|\mathbf{x}-\mathbf{y}|^{\alpha-n}$, where $1 < \alpha < n$, for the Gauss variational problem, considered for finitely many compact, mutually disjoint, boundaryless $(n-1)$–dimensional Lipschitz manifolds Γ_ℓ, $\ell \in L$, each Γ_ℓ being charged with Borel measures with the sign $\alpha_\ell = \pm 1$ prescribed. We show that the Gauss variational problem over an affine cone of Borel measures can alternatively be formulated as a minimum problem over an affine cone of surface distributions belonging to the Sobolev–Slobodetski space $H^{-\varepsilon/2}(\Gamma)$, where $\varepsilon := \alpha - 1$ and $\Gamma := \bigcup_{\ell \in L} \Gamma_\ell$. This allows the application of simple layer boundary integral operators on Γ and, hence, a penalty approximation. A corresponding numerical method is based on the Galerkin–Bubnov discretization with piecewise constant boundary elements. For $n = 3$ and $\alpha = 2$, multipole approximation and in the case $1 < \alpha < 3 = n$ wavelet matrix compression is applied to sparsify the system matrix. Numerical results are presented to illustrate the approach.

[1] G. Of, W.L. Wendland and N. Zorii: *On the numerical solution of minimal energy problems.* Complex Variables and Elliptic Equations **55** (2010) 991–1012.

[2] H. Harbrecht, W.L. Wendland and N. Zorii: *On Riesz minimal energy problems.* Preprint Series Stuttgart Research Centre for Simulation Technology (SRC Sim Tech) Issue No. 2010–80.

On a class of degenerate pseudodifferential operators and applications to mixed-type PDEs

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In a series of papers in 1969/70, Vishik and Grushin [2, 3] introduced a class of degenerate pseudodifferential operators roughly being of the form

$$|x|^{b-(l_*+1)m} P(x, y, |x|D_x, |x|^{l_*+1} D_y),$$

where m is the order of the operator and $b \in \mathbb{R}$ is a parameter ($b = ml_*$ in the work of Vishik and Grushin). Here, $l_* \in \mathbb{Q}$, $l_* > 0$, $x \in \mathbb{R}^q$, $y \in \mathbb{R}^d$, and $P(x, y, \xi, \eta)$ is a pseudodifferential symbol. A basic example is the Tricomi operator $\partial_x^2 + x\Delta_y$, where $l_* = 1/2$, $q = 1$, $m = 2$, and $b = 1$.

In this talk, we develop a pseudodifferential calculus for such operators including a full symbol calculus, discuss elliptic boundary problems [4] and the hyperbolic Cauchy problem [1] in this class, and eventually present applications to 2D mixed elliptic-hyperbolic equations (where $q = d = 1$) with variable coefficients. In the latter case, the problem is reduced to a cone-degenerate elliptic problem in the interface (which is an interval), where the equations under study change type.

[1] M. Dreher and I. Witt, *Edge Sobolev spaces and weakly hyperbolic equations*, Ann. Mat. Pura Appl. (4), **180** (2002), 451–482.

[2] M.I. Vishik and V.V. Grushin, *A certain class of degenerate elliptic equations of higher orders*, Mat. Sb. (N.S.) **79** (**121**) (1969), 3–36 (in Russian).

[3] M.I. Vishik and V.V. Grushin, *Degenerate elliptic differential and pseudo-differential operators*, Uspehi Mat. Nauk **25** (1970), 29–56 (in Russian).

[4] I. Witt, *A calculus for a class of finitely degenerate pseudodifferential operators.* In: Evolution equations, volume 60 of Banach Center Publ., Polish Acad. Sci., 2003, pp. 161-189.

Equation of coagulation process of falling drops

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We consider coagulation process of water drops which fall in the air. This process is described by an integro-differential equation for the density $\sigma(m,t,x)$ of the water liquid contained in drops of mass m. For the motion of drops we consider the their velocity $u(m)$ determined by the mass m and the velocity of the air; on the other hand for the coagulation process we consider a probability $\beta(m_1,m_2)$ of meeting between a drop of mass m_1 and one of mass m_2 (see for example [3]).

First we prove the existence of a stationay solution with a constant horizontal wind [2]. Secondly we prove the existence and uniqueness of the global solution in the absence of the wind [1]; the convergence of the global solution to the stationary solution is a corollary of this result.

[1] Belhireche, H., Aissaoui, M. Z., Fujita Yashima, H.: Solution globale de l'équation de coagulation des gouttelettes en chute. *Quaderno Dip. Mat. Univ. Torino, 2012.*

[2] Merad, M., Belhireche, H., Fujita Yashima, H.: Solution stationnaire de l'équation de coagulation de gouttelettes en chute avec le vent horizontal. To appear on *Rend. Sem. Mat. Univ. Padova.*

[3] V.M. Voloshtchuk, *Kinetic theory of coagulation.* Gidrometeoizdat, 1984.

Is free surface deep water hydrodynamics an integrable system?

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Infinite energy solutions for damped Navier–Stokes equations in $\mathbb{R}^2$

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The so-called damped Navier–Stokes equations in the whole 2D space:

$$\begin{cases} \partial_t u + (u, \nabla_x) u = \Delta_x u - \alpha u + \nabla_x p + g, \\ \operatorname{div} u = 0, \quad u\big|_{t=0} = u_0, \end{cases}$$

where α is a positive parameter, will be considered and the results on the global well-posedness, dissipativity and further regularity of weak solutions of this problem in the uniformly-local spaces $L^2_b(\mathbb{R}^2)$ will be presented. These results are obtained based on the further development of the weighted energy theory for the Navier–Stokes type problems. Note that any divergent free vector field $u_0 \in L^\infty(\mathbb{R}^2)$ is allowed and no assumptions on the spatial decay of solutions as $|x| \to \infty$ are posed.

In addition, the applications to the classical Navier–Stokes problem in $\mathbb{R}^2$ (which corresponds to $\alpha = 0$) will be also considered. In particular, the improved estimate on the possible growth rate of spatially non-decaying solutions as time goes to infinity:

$$\|u(t)\|_{L^2_b(\mathbb{R}^2)} \le C(t^5 + 1),$$

where C depends on u_0 and g, but is independent of t, will be presented. Note that the previous best known estimate was super-exponential in time:

$$\|u(t)\|_{L^\infty(\mathbb{R}^2)} \le C_1 e^{C_2 t^2},$$

see [1].

[1] O. Sawada and Y. Taniuchi, *A remark on L^∞-solutions to the 2D Navier–Stokes equations*, J. Math. Fluid Mech., **9** (2007), 533–542.

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