

On a class of degenerate pseudodifferential operators and applications to mixed-type PDEs

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In a series of papers in 1969/70, Vishik and Grushin [2, 3] introduced a class of degenerate pseudodifferential operators roughly being of the form

$$|x|^{b-(l_*+1)m}P(x, y, |x|D_x, |x|^{l_*+1}D_y),$$

where m is the order of the operator and $b \in \mathbb{R}$ is a parameter ($b = ml_*$ in the work of Vishik and Grushin). Here, $l_* \in \mathbb{Q}$, $l_* > 0$, $x \in \mathbb{R}^q$, $y \in \mathbb{R}^d$, and $P(x, y, \xi, \eta)$ is a pseudodifferential symbol. A basic example is the Tricomi operator $\partial_x^2 + x\Delta_y$, where $l_* = 1/2$, $q = 1$, $m = 2$, and $b = 1$.

In this talk, we develop a pseudodifferential calculus for such operators including a full symbol calculus, discuss elliptic boundary problems [4] and the hyperbolic Cauchy problem [1] in this class, and eventually present applications to 2D mixed elliptic-hyperbolic equations (where $q = d = 1$) with variable coefficients. In the latter case, the problem is reduced to a cone-degenerate elliptic problem in the interface (which is an interval), where the equations under study change type.

References

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