Vishik's approach to general boundary value problems for elliptic operators. Recent development

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In his pioneering paper [1] M.I. Vishik proposed a new approach to the extension theory of symmetric operators as well as dual pairs of operators in a Hilbert space. In the framework of this approach the proper extensions are parameterized in terms of (abstract) boundary conditions. Moreover, he applied general constructions to investigate the properties of solvability and complete solvability of boundary value problems for (not necessarily symmetric) elliptic operators on bounded domains.

During three last decades this approach has been formalized in the concept of boundary triplets for dual pairs of operators and elaborated in great detail. The revival of interest to this approach has been motivated by numerous applications to boundary value problems for differential and difference operators (see for instance publications [2], [3], [4], [5] and references therin).

I plan to recall the main results and basic constructions of the Vishik's paper [1] as well as to discuss its influence on development of the extensions theory.

Next I plan to discuss applications of to elliptic boundary value problems in domain with compact boundary. Some spectral properties of different realizations of elliptic differential expressions will be discussed too.

References

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