Causal inference
Difference-in-differences, Synthetic controls

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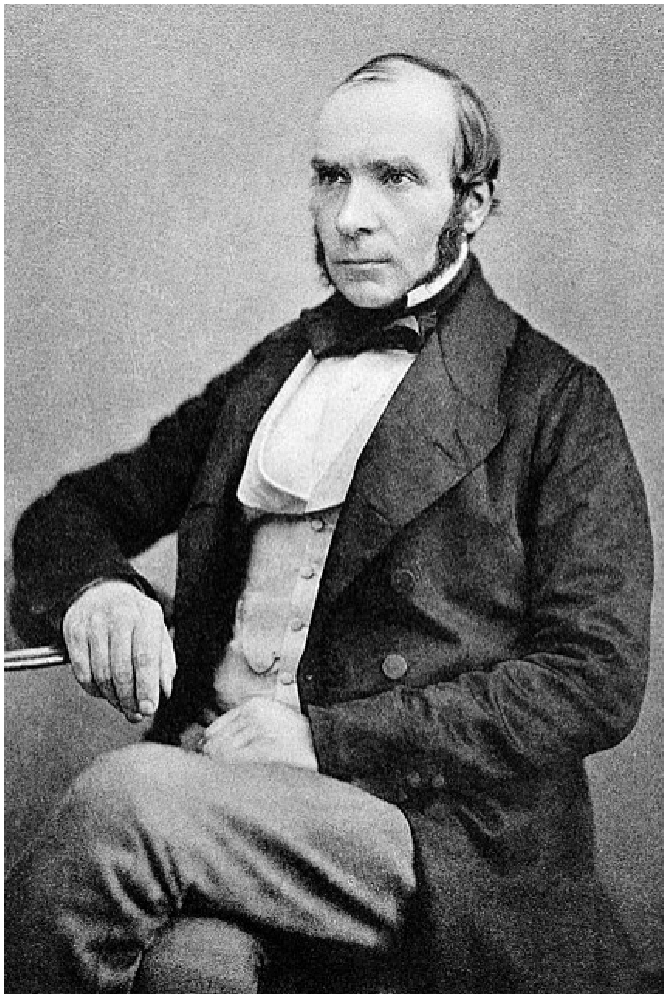
Difference-in-differences I

The story of J. Snow



Cholera in South London

- Change in water supply intake of one water company in 1852 reduced waste water contamination

	1849	1854
S & V Water	London	London
Lambeth Waterworks	London	Seething Wells

- All serve comparable households

Cholera case counts from Snow (1855)

	Cholera cases	
	1849	1854
S & V Water	135	147
Lambeth Waterworks	85	19

Note: cases per 1E4 households supplied by Lambeth vs. Southwark and Vauxhall

The basic idea of DiD

- Cholera deaths, Y
- Treatment (clean[er] water sypply), D
- Denote by ξ_L and ξ_{SV} constant aggregate-level differences between households served by either water company

Some possible comparisons

- Comparing *between* water suppliers, we have

	Outcome
Lambeth Waterworks	$Y = D + \xi_L$
S & V Water	$Y = \xi_{SV}$

- A naive causal estimate from this comparison is $D + (\xi_L - \xi_{SV})$

- Comparing *within* water suppliers, we have

	Time	Outcome
Lambeth Waterworks	Pre	$Y = \xi_L$
Lambeth Waterworks	Post	$Y = \xi_L + (\delta_t + D)$

- Here the causal estimate is contaminated by natural changes in Cholera cases over time, δ_t
- The (obvious) solution...

	Time	Outcome	Diff.	Diff.
Lambeth	Pre	$Y = \xi_L$	$\delta_t + D$	D
Lambeth	Post	$Y = \xi_L + (\delta_t + D)$		
S & V	Pre	$Y = \xi_{SV}$	δ_t	
S & V	Post	$Y = \xi_{SV} + \delta_t$		

The crucial assumption

- The calculation on the previous slide assumes that there is no company-specific unobservable that changes over time (e.g., no changes in households served by Lambeth that increase or decrease cholera)
- In other words $\delta_{t,L} = \delta_{t,SV} = \delta_t$
- This is referred to as the *parallel trends assumption*
- Expressing the DiD strategy in terms of potential outcomes clarifies this issue
- The sample calculation of a simple 2x2 (treated [T] vs control [C], pre vs post) difference in difference is

$$\hat{\delta} = \left(E[Y^T|post] - E[Y^T|pre]\right) - \left(E[Y^C|post] - E[Y^C|pre]\right)$$

- Expressed in terms of potential outcomes (and some rearranging) gets us the decomposition

$$\hat{\delta} = (E[Y_1^T|post] - E[Y_0^T|post]) \\ + \left(E[Y_0^T|post] - E[Y_0^T|pre]\right) - \left(E[Y_0^C|post] - E[Y_0^C|pre]\right)$$

  where

  - $(E[Y_1^T|post] - E[Y_0^T|post])$ represents the ATT
  - the rest represents bias due to non-parallel trends
    - It being zero requires the equality of $E[Y_0|post] - E[Y_0|pre]$ for the treated and untreated groups
    - Note that $(Y_0|post)$ is a *counterfactual* term!
  - Thus the parallel trend assumption is untestable

A well-known example: Minimum wage

- Running example: impact of minimum wage on employment. Comparison of NJ (increase in mw) and PA (no increase)
- 400 fast food stores before and after mw increase
- Let: Y^1_{ist} be the potential employment outcome in 'restaurant' i in state s at time t under an increased mw; Y^0_{ist} is potential outcome under low minimum wage Using superscripts to denote potential outcomes
- We only observe one state for each restaurant (not the potential outcomes)
- But can assume
$$E[Y^0_{ist}|s,t] = \gamma_s + \lambda_t$$
absent treatment, employment is a function of state-specific idiosyncratic effect and a time effect (common for all states)

Differences in Differences

- Let D_{st} indicate high mw states in a given time period
- Under conditional independence assumption average treatment effect is

$$\delta = E[Y_{ist}^1 - Y_{ist}^0 | s, t]$$

- Observed employment is then:

$$Y_{ist} = \gamma_s + \lambda_t + \delta D_{st} + \epsilon_{ist}$$

Differences in Differences: calculation

- Differences PA-NJ before mw increase - differences after mw increase

New Jersey

- Employment February:

$$E(Y_{ist}|s = NJ, t = Feb) = \gamma_{NJ} + \lambda_{Feb}$$

- Employment November:

$$E(Y_{ist}|s = NJ, t = Nov) = \gamma_{NJ} + \lambda_{Nov} + \delta$$

- Difference

$$\lambda_{Nov} - \lambda_{Feb} + \delta$$

Differences in Differences: calculation

Pennsylvania

- Employment February:

$$E(Y_{ist}|s = PA, t = Feb) = \gamma_{PA} + \lambda_{Feb}$$

- Employment November:

$$E(Y_{ist}|s = PA, t = Nov) = \gamma_{PA} + \lambda_{Nov}$$

- Difference

$$\lambda_{Nov} - \lambda_{Feb}$$

Differencing the two differences removes the time effects

Differences in Differences: calculation

- Population difference-in-difference estimated using sample analog of population means

$$\begin{aligned}\hat{\delta} = &\big(E(Y_{ist}|s = NJ, t = Nov) - E(Y_{ist}|s = NJ, t = Feb)\big)\\ &\quad - \big(E(Y_{ist}|s = PA, t = Nov) - E(Y_{ist}|s = PA, t = Feb)\big)\\ = &(\lambda_{Nov} - \lambda_{Feb} + \delta) - (\lambda_{Nov} - \lambda_{Feb})\\ = &\delta\end{aligned}$$

	Stores by state		
Variable	PA (i)	NJ (ii)	Difference, NJ – PA (iii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	−2.89 (1.44)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	−0.14 (1.07)
3. Change in mean FTE employment	−2.16 (1.25)	0.59 (0.54)	2.76 (1.36)

Differences in Differences: calculation

- Alternatively calculate in regression form
- In our mw example

$$Y_{ist} = \alpha + \gamma NJ_s + \lambda d_t + \delta(NJ \times d)_{st} + \epsilon_{ist}$$

where $NJ = 1$ if case is in NJ, d is equal to 1 if case is from November (post-period)

  - (a) PA before: α
  - (b) PA after: $\alpha + \lambda$
  - (c) NJ before: $\alpha + \gamma$
  - (d) NJ after: $\alpha + \gamma + \lambda + \delta$

(b-a)-(d-c) $= \delta$

Differences in Differences: identifying assumption

- δ above is the ATET (under the assumptions discussed)
- Parallel trends assumption (remember: involves counterfactual states!)
- Empirical strategy based on plausibility checks
  - look for parallel pre-treatment trends to gain some confidence about evolution (time shocks would have to be D-specific). Often conducted graphically or by including leads in the model
  - Placebo tests

Can go further with additional control group = DiDiD (cost: more parallel trend assumptions)

DiD with repeated observations

- Usual two-way FE model

$$Y_{ist} = \alpha + \delta D_{st} + \lambda_t + \xi_i + \epsilon_{ist}$$

- Units i belong to group s (states, countries, districts etc) at time t
- Treatment period begins at some t for some s
- λ_t and ξ_i are time and unit constants (“fixed effects”)
- A version with time-varying covariates is

$$Y_{ist} = \alpha + x_{ist}\beta + \delta D_{st} + \lambda_t + \xi_i \epsilon_{ist}$$

Exploring parallel pre-trends

- First order of business is always a plot!!
- But we can explore a hypothesis of parallel linear pre-trends
- Some housekeeping first
- Denote by c_i an indicator variable equal to 1 if a unit is ever treated
- Denote by d_{t0} the pre-treatment time periods and by d_{t1}
- Then, the model from the last slide is augmented to

$$Y_{ist} = \alpha + \delta D_{st} + \lambda_t + \xi_i + tc_i d_{t0}\theta_0 + tc_i d_{t1}\theta_1 + \epsilon_{ist}$$

- A test of $\theta_0 = 0$ tests the null hypothesis of parallel pre-trends

But do remember that parallel pre-trends are neither a sufficient (or necessary) condition for parallel counterfactual trends!

After the break...

- How do we deal with treatments lasting multiple periods?
- What if treatments are not “switched on” at the same time?
- What if treatments are applied multiple times?
- Can we relax parallel trends assumptions?