

# **Introduction to Causal Inference, week 2**

**Daniel Stegmueller**

Duke University

## The problem of *causal* inference

## Causal inference questions and statistics

Typical questions of interest:

- ▶ Could government action have prevented the crisis?
- ▶ How effective is this treatment in preventing this disease?
- ▶ Is there evidence for discrimination in these policing records?

Do changes in one variable cause changes in another. And, if so, how much?

(Until recently) relative silence in statistical education

- ▶ Focus on data summary / parameter “inference”.
- ▶ Often silent on if the estimated parameters reflect the wanted quantity (large fraction of statistics books do not even have the term causal in their index)
- ▶ Rather, well-worn, but not very helpful advice that “correlation does not imply causation” etc.

## Why study causal effects?

*“WHAT IS?”*

is central to many scientific questions, to guide action, policies

- ▶ Effect of smoking on lung cancer
- ▶ Effect of institutional features on behavior
- ▶ Effect of education on future salaries
- ▶ ...

*“WHY and HOW is it?”*

- ▶ Understanding how and why causes influence their effects (think ‘mal-air’ vs. mosquitos and what remedial action they imply...)

## Causal inference and statistics

But how do considerations of causality differ from “statistics”?

Consider this famous puzzle; Simpson’s paradox (after E. Simpson, 1951)

- ▶ Effect of drug on disease outcome
- ▶ A group of sick patients is offered a novel drug
- ▶ Among those who took the drug, outcomes are *worse!* (a lower percentage recovers)
- ▶ When partitioning by gender, one finds:
  - ▶ Men who took drug have higher recovery rate than those who did not
  - ▶ Women who took drug have higher recovery rate than those who did not
- ▶ Thus, drug helps men and women; hurts the general population

## Simpson's paradox

- 700 patients, 350 take drug, 350 do not

	Drug		No Drug	
Men	81 of 87 recovered	93%	234 of 270 recovered	87%
Women	192 of 263 recovered	73%	55 of 80 recovered	69%
Combined	273 of 350 recovered	78%	289 of 350 recovered	83%

- Is the drug safe? Can we only prescribe it knowing gender (nonsensical)? What is the relevant causal effect?

## Simpson's paradox

- 700 patients, 350 take drug, 350 do not

	Drug		No Drug	
Men	81 of 87 recovered	93%	234 of 270 recovered	87%
Women	192 of 263 recovered	73%	55 of 80 recovered	69%
Combined	273 of 350 recovered	78%	289 of 350 recovered	83%

- Is the drug safe? Can we only prescribe it knowing gender (nonsensical)? What is the relevant causal effect?
- Consider two details of the *data generating process* ([common cause](#)):
  - Estrogen has a negative effect on recovery (women are less likely to recover regardless of the drug)
  - More women take the drug compared to men
  - A randomly selected drug user is more likely to be a women and less likely to recover
  - The disaggregated data shows the true causal effect

## Simpson's paradox

However, disaggregation does not always work to reveal the causal effect

- ▶ Extend drug experiment by recording blood pressure at end of experiment
- ▶ Drug effect recovery by lowering blood pressure (but also has a toxic side effect)

	No Drug		Drug	
Low BP	81 of 87 recovered	93%	234 of 270 recovered	87%
High BP	192 of 263 recovered	73%	55 of 80 recovered	69%
Combined	273 of 350 recovered	78%	289 of 350 recovered	83%

- ▶ Again, understanding of this pattern stems from understanding the data generating process
  - ▶ In general population, drug improves recovery by lowering BP
  - ▶ Among those with low or high (*post-treatment*) blood pressure, we only see the drug's toxic effect
  - ▶ Thus, correct answer lies in using the aggregate data

## Causal inference and statistics

- ▶ Note that both tables contain the same numbers
- ▶ No statistical model can tell us if one or the other strategy should be used
- ▶ Extraneous information was the determinant (timing of the measurements, treatment affects BP, etc..)

*Encoding such information is the role of a causal language – whether in potential outcomes or graphical form. Next week(s).*

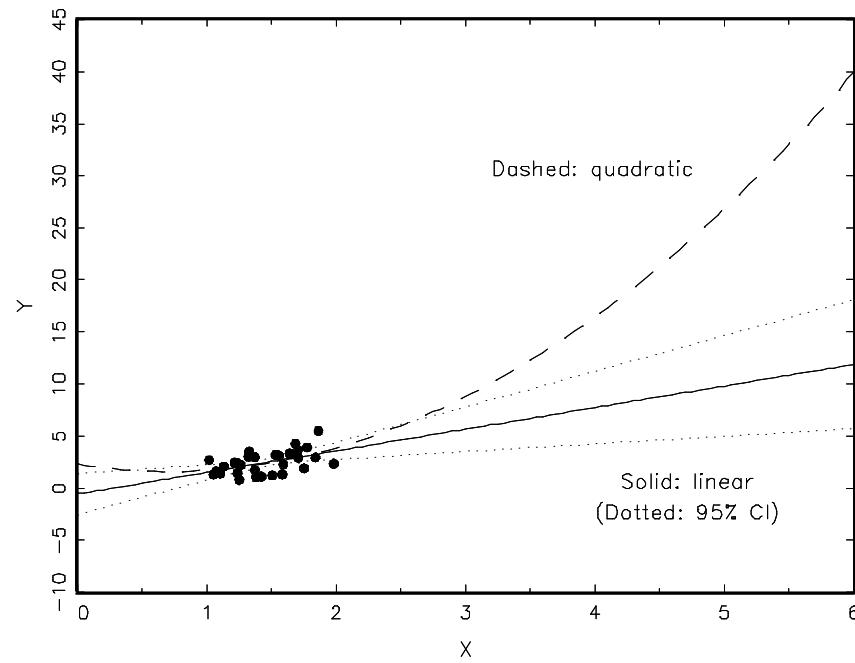
## The problem of counterfactuals and model dependence

## Counterfactuals

- ▶ Types of counterfactuals
  - ▶ Forecasts (will candidate win election?)
  - ▶ What-if (what would have happened if US had not signed free-trade agreement)
  - ▶ Effects of cause (what is the causal effect of protest on court decisions?)
- ▶ Central to scientific inquiry
- ▶ Usually based on models
- ▶ How to identify *sensible* counterfactuals?

## Dangerous counterfactuals....

- ▶ Which model to choose? Both fit the data rather well



- ▶ Prediction at  $x = 1.5$  vs. at  $x = 3$  or  $x = 6$

## Dangerous counterfactuals....

- ▶ Some answers do not exist in the data
- ▶ Results / estimates of quantities of interest can be highly model dependent

### *Model dependence*

- ▶ A model free estimate of  $E(Y|X = x)$ : average ‘many’ observed Y with value x
- ▶ Model dependence at  $x$  is a function of the distance from the counterfactual to the data

## Model dependence illustration

A hypothetical experiment on education and earnings

- ▶ Take a random sample of children
- ▶ Assign them at random to obtain different years of schooling [0, 6, 10, 12, 16 years]
- ▶ (assume away many complications, such as non-compliance, measurement error, etc.)
- ▶ Outcome of interest: yearly earnings in year 20
- ▶ Regression of earnings on education yields estimate of 2,000 USD

## Model dependence

- ▶ Suppose you have data on  $Y$ : earnings and  $X$ : education with 10 categories
- ▶ Estimation of  $E(Y|X)$  uses 10 parameters:  $E(Y|X = x_j)$ ,  $j = 1, \dots, 10$
- ▶ Model-free estimate: average observations of  $Y$  for each  $x_j$
- ▶ Model-based estimate: (typically) linear regression of  $Y$  on  $X$
- ▶ Difference in parameters between the two approaches (10 vs 2) is simple due to an *assumption* (linear functional form)

[What if  $X$  were continuous instead of discrete?]

## Model dependence, now with two explanatory variables

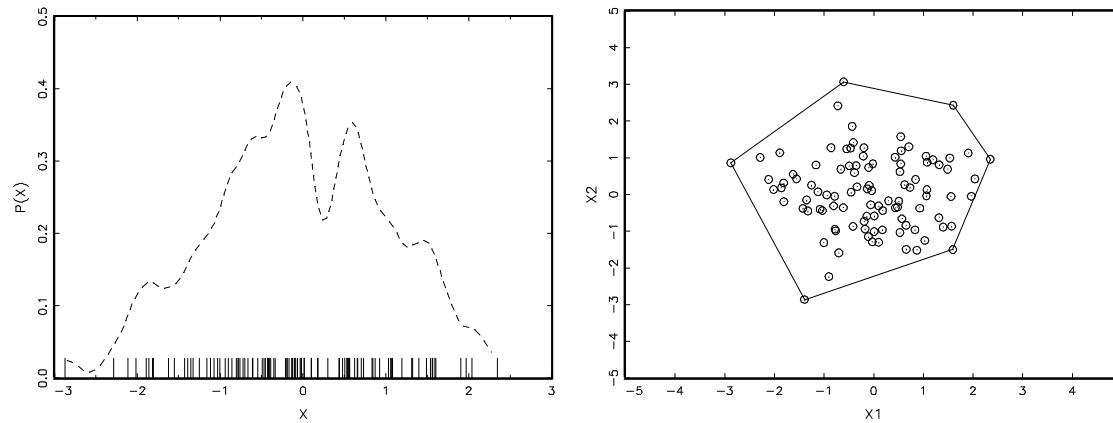
- ▶ Add new variable,  $Z$ : parental social class (with 7 categories)
- ▶ How many parameters do we need? (17? 70?)
- ▶ With more explanatory variables, model complexity (number of parameters) increases *geometrically*; a.k.a. the curse of dimensionality
- ▶ A linear regression as typically specified reduces this to 3 parameters
- ▶ Again, this reduction of the parameter space is (usually) arrived at via assumptions, not theory or prior evidence

## Model dependence, realistically

- ▶ Common model specifications easily include 10 or more variables (“controls”)
- ▶ 10 variables each with 5 categories
- ▶ Number of parameters:  $5^{10} = 9,765,625$
- ▶ Simple linear regression? 11 parameters
- ▶ Number of observations?

## Defensible simplicity?

- ▶ Is the counterfactual quantity produced by your statistical model close (enough) to the data to provide empirical answers?
- ▶ If no, answers likely depend on hard-to-defend model assumptions



- ▶ *Interpolation*: “inside” the data (convex hull)
- ▶ *Extrapolation*: “outside” the data (convex hull)

[WhatIf R package provides approx. convex hull checking]

## Impact on naive linear regression analyses

- Decomposition of bias in a typical regression

$$\tau = \text{mean}(Y|D=1) - \text{mean}(Y|D=0)$$

$$\text{bias} = \Delta_o + \Delta_p + \Delta_i + \Delta_e$$

- $\Delta_o$ : omitted variable bias
- $\Delta_p$ : post-treatment bias
- $\Delta_i$ : interpolation bias
- $\Delta_e$ : extrapolation bias

## Summa summarum

### *Model dependence*

- ▶ Both *interpolation* and *extrapolation* is possible and unproblematic if the statistical model is correct (or “true”)
- ▶ That is unlikely to be the case. Less ambitiously, the model might be locally correct
- ▶ Thus likely to get very different counterfactuals the further we move away from the data.