

UNIT-IV

①

SEQUENTIAL LOGIC

Sequential circuits, latches, flip-flops Analysis of clocked sequential circuits, state Reduction and assignment, Design procedure, Registers, shift Registers, Ripple counters, synchronous counters, other counters.

* Introduction:

- Digital Electronics is classified into combinational logic and sequential logic.
- Combinational logic output depends on the present input levels, whereas sequential logic output depends on stored levels and also the input levels (i.e. present input and past outputs).

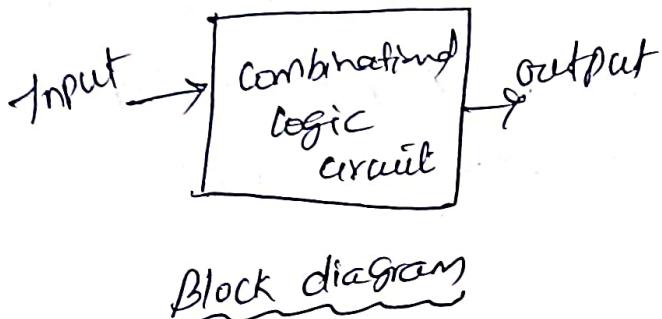
* Basic Architectural difference b/w combinational and logic circuits

- The memory elements are devices capable of storing binary information. The binary information stored in the memory elements at any given time defines the state of the sequential circuit.
- The input and the present state of the memory element determines the output. Memory elements next state is also a function of external inputs and present state.
- A sequential circuit is specified by a time sequence of inputs, outputs, and internal states.

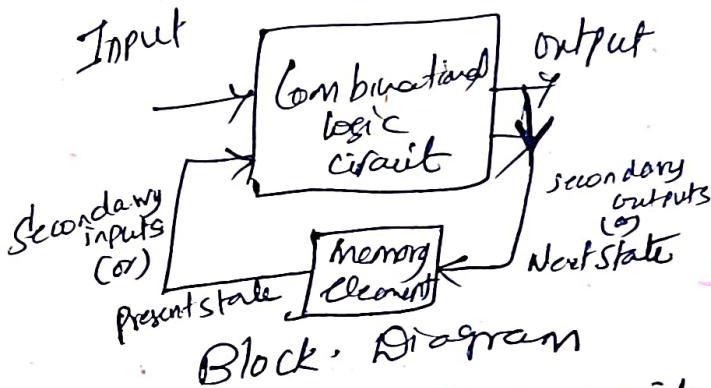
Differences Between Circuits

Combinational and sequential logic

- ① Combinational logic circuit



sequential logic circuit



- ② In combinational circuits, the output variables are at all times dependent on the combination of input variables

- ③ memory unit is not required in combinational circuits

- ④ Combinational circuits are faster than in speed because the delay between input and output is due to propagation delay of gates.

- ⑤ Easy to design

- ⑥ There is no feedback clock signal is not required

- ② In sequential circuits, the output variables depend ~~not~~ only on the present input variables but they also depend upon the past history of these input variables

- ③ memory unit is required to store the past

- ④ Sequential circuits are ~~faster~~ slower than the combinational circuits.

- ⑤ Harder to design.

- ⑥ There is at least one memory element in the feedback path.

- ⑦ clock signal is required

(8) Time is not an important parameter

(9) Time is an important parameter. For timing and synchronizing of different circuit elements, a clock signal is necessary.

(10) They are easier to implement with the help of basic logic gates but costly; their implementation requires more hardware.

(11) Used for Arithmetic as well as Boolean operations

(12) Elementary building blocks: logic gates.

(13) Circuits don't have CLOCK, may don't require triggering.

(14) Examples of combinational units are half adder, full adder, magnitude comparator, multiplexer, demultiplexers etc.

(15) faster in speed

(16) parallel adder, one more example

(17) They are difficult to implement but less costly than sequential circuits.

(18) Mainly used for storing data.

(19) Elementary building blocks: flip flops.

(20) Circuits are clock dependent; they need triggering.

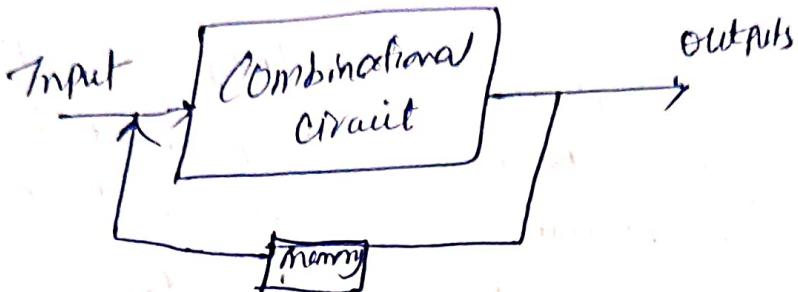
(21) Examples are flip flops, registers, counters, etc.

(22) slower compared to combinational circuit

(23) serial adder one more example

* Sequential Circuits

- Sequential CKT is a combinational CKT with memory
- The output of Sequential CKT depends upon present inputs and past outputs (Present state)



- The information stored in Sequential CKT represents present state.
- The present state & present inputs are define output next state
- There are classified two types

Synchronous

clock pulse present

Eg: flip flops

Eg. SR-Flip flop
JK-Flip flop
T-Flip flop
D-Flip flop

Asynchronous

clock pulse not required

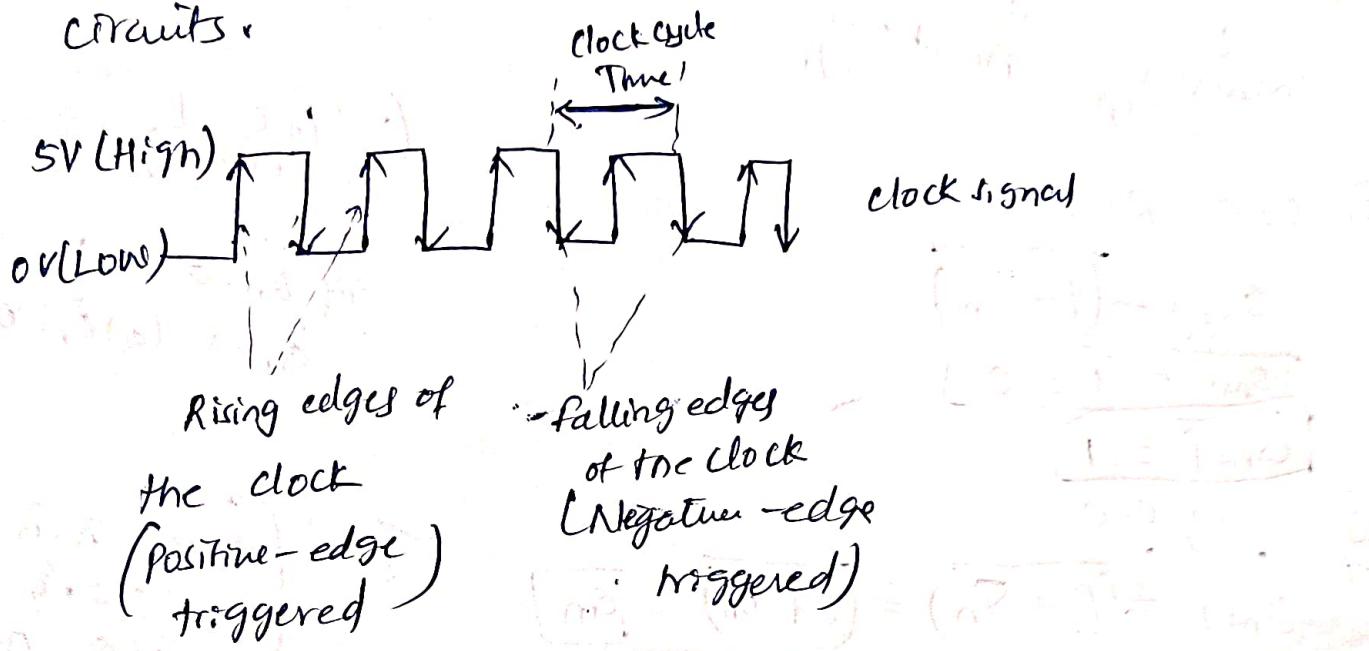
Eg. Latches, Eg SP-latch
JK-latch
T-latch
D-latch.

* Latch * Latch is a digital electronics device that will stores single bit information either '0' and '1'.

* flip flop * A flip flop is an electronic circuit with two stable states that can be used to store binary data. The stored data can be changed by applying varying inputs.

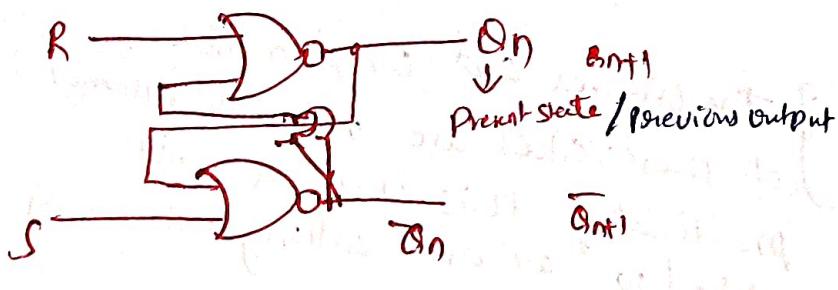
→ flip flops and latches are fundamental building blocks of digital electronics systems used in computers, communication, and many other types of systems.

* Clock Pulse * A clock pulse is a signal that oscillates between high and low states, is used to synchronize the operations of flip flops & other components in digital circuits.

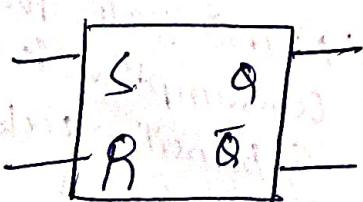


* Hatches *

* SR Latch (NOR) *



logic symbol



| | | Present State | | |
|---|---|---------------|-------------|-----------|
| S | R | Q_n | \bar{Q}_n | |
| 0 | 0 | 0 | 1 | No change |
| 0 | 0 | 1 | 0 | |
| 0 | 1 | 0 | 1 | Reset |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 1 | Set |
| 1 | 0 | 1 | 0 | |
| 1 | 1 | 0 | x | undefined |
| 1 | 1 | x | x | |

(i) $S=0; R=0$

$$Q_{n+1} = \overline{0 + \bar{Q}_n} = \bar{\bar{Q}}_n = Q_n \Rightarrow Q_{n+1} = Q_n$$

$$\overline{Q_{n+1}} = \bar{Q}_n$$

\downarrow

Next state O/P

present state O/P.

$$(\because 1+0=1 \\ 1+1=1)$$

(ii) $S=0; R=1$

$$Q_{n+1} = (1 + \bar{Q}_n)$$

$$\overline{Q_{n+1}} = \bar{1} = 0$$

$$Q_{n+1} = 1$$

$$\text{If } Q_n = 0 \text{ (or)} \\ \bar{Q}_n = 1 \quad \text{(or)} \quad \bar{\bar{Q}}_n = 0$$

(iii) $S=1; R=0$

$$Q_{n+1} = (R + \bar{Q}_n) = (\bar{0} + \bar{Q}_n) = \bar{Q}_n$$

$$\overline{Q_{n+1}} = \bar{\bar{Q}}_n$$

$$Q_{n+1} = 1$$

$$\begin{aligned} \overline{A+B} &= \bar{A} \cdot \bar{B} \\ &= 0 + \bar{Q}_n \\ &= Q_n \end{aligned}$$

$$\overline{Q_{n+1}} = (S + \bar{Q}_n)$$

$$= (1 + \bar{Q}_n)$$

$$\rightarrow = \bar{T} = 0$$

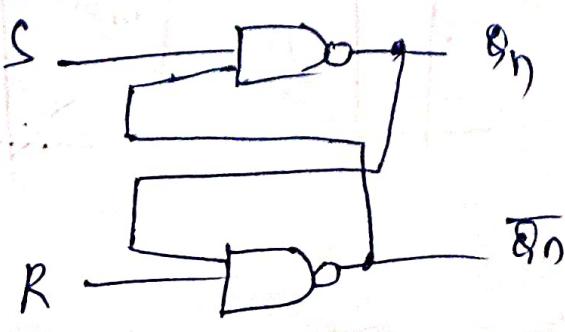
(iv) $S=1, R=1$

$$Q_{n+1} = \overline{(1 + \bar{Q}_n)} = \bar{T} = 0$$

$$\overline{Q_{n+1}} = (\overline{1 + \bar{Q}_n}) = \bar{T} = 0$$

} the original and complements
of these values are not valid (same)
particularly this state is not
existed so we are calling this one as
"undetermined state"

* SR Latch using NAND *



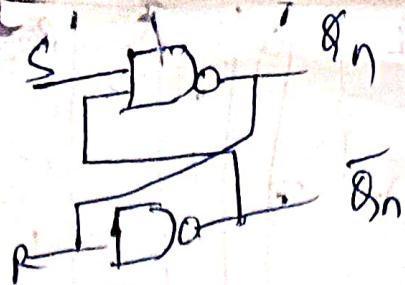
(\because NAND gate power
consumption is less
& transition rate is
fast)

$$\textcircled{1} \quad f = 0 \text{ and } Q_n = 0$$

$$Q_n = 1; \bar{Q}_n = 0$$

$$S = 1; R = 1 \text{ memory state}$$

$$Q_n = 1; \bar{Q}_n = 0$$



| NAND gate | | y |
|-----------|---|-----|
| A | B | |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$$\textcircled{2} \quad S = 1; R = 0; Q_n = 0; \bar{Q}_n = 1$$

$$S = 1; R = 1; Q_n = 0; \bar{Q}_n = 1$$

memory state

$$\textcircled{3} \quad [S = 0; R = 0; Q_n = 1, \bar{Q}_n = 1] \rightarrow \text{doesn't exist}$$

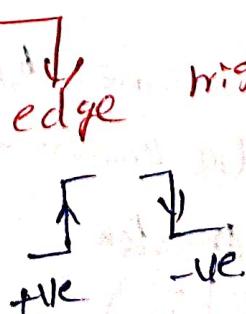
| S | R | Q | \bar{Q} |
|---|---|--------------|-----------|
| 0 | 0 | invalid. | |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | memory state | no change |

Grated SR Latch
 \rightarrow They are synchronizing with enable input
 \rightarrow They are level triggered.

Triggering



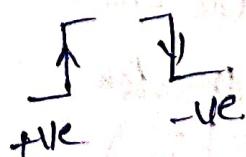
[Occurs in latches]



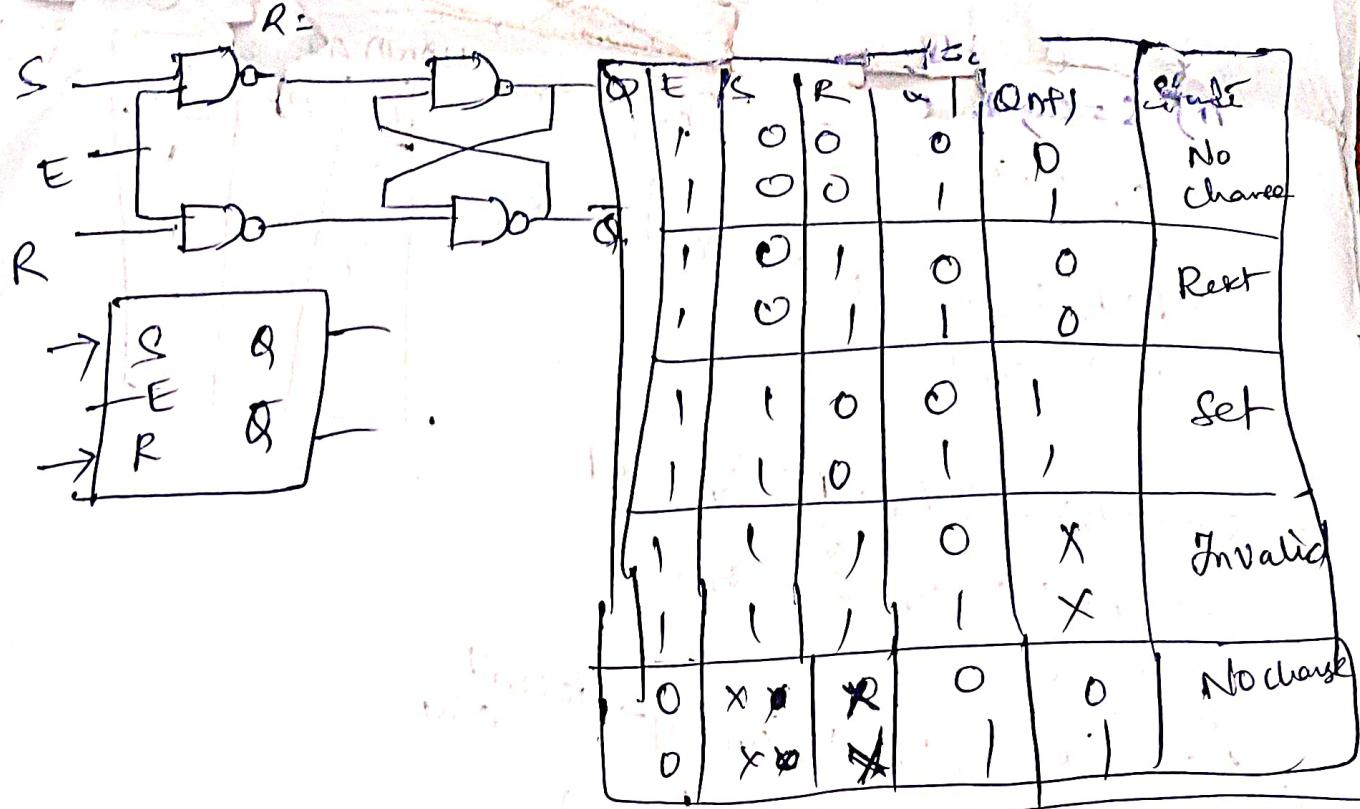
low to high \rightarrow +ve edge
 high to low \rightarrow -ve edge

(Triggering is the process of applying an external signal to induce transition from one state to another state)

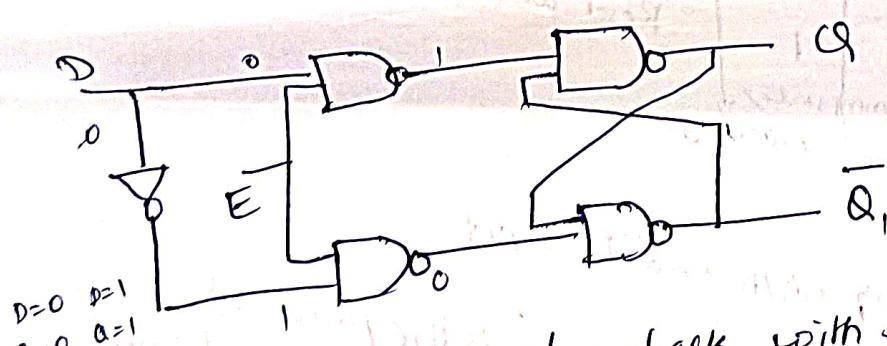
Triggering: The signal used for triggering is either a pulse of short duration or a step voltage.



low to high \rightarrow +ve edge
 high to low \rightarrow -ve edge



D-Latch



$D=0 \quad D=1$
 $Q=0 \quad Q=1$
 $\bar{Q}=1 \quad \bar{Q}=0$

→ There is a one drawback with SR latch that when the inputs $S \& R = 1$, the next state values cannot be predicted

→ This will be overcome by using D-Latch
 → Data latch is also known as D-latch

Truth table D-latch

| E | D | Q | \bar{Q} |
|---|---|---|-----------|
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

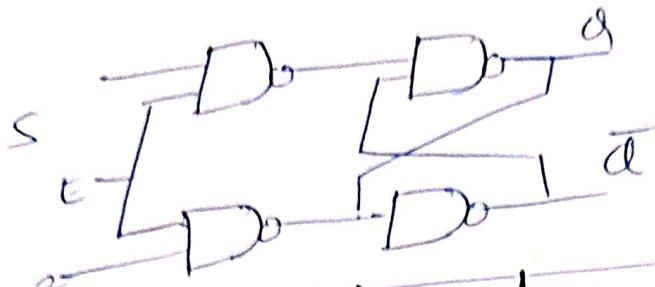
$SR \rightarrow$ Set & Reset
 $D \rightarrow$ Data transparent
 $JK \rightarrow$ Jack Kilby
 Invented by Jack Kilby in 1950

T → Toggle

* Gate SR latch *

→ It is also known as Synchronous. They are synchronized with enable input. They are level triggered.

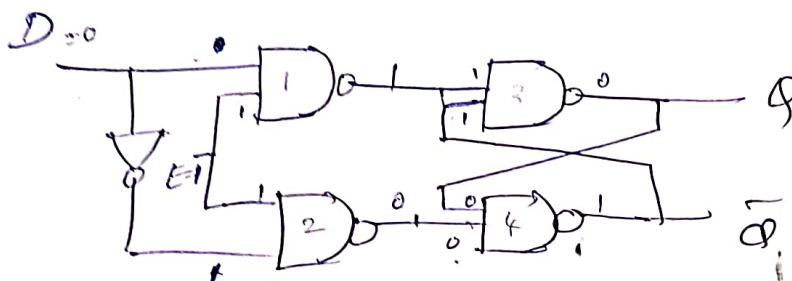
③



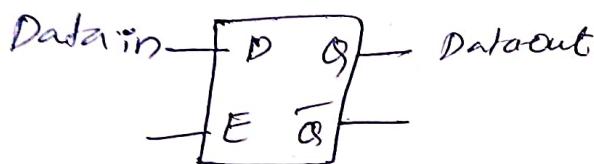
| E | S | R | Q_n | Q_{n+1} | State |
|---|---|---|-------|-----------|-----------|
| 1 | 0 | 0 | 0 | 0 | No change |
| 1 | 0 | 0 | 1 | 1 | |
| 1 | 0 | 1 | 0 | 0 | Reset |
| 1 | 0 | 1 | 1 | 0 | |
| 1 | 1 | 0 | 0 | 1 | Set |
| 1 | 1 | 0 | 1 | 1 | |
| 1 | 1 | 1 | 0 | X | Invalid |
| 1 | 1 | 1 | 1 | X | |
| 0 | X | X | 0 | 0 | No change |
| 0 | X | X | 1 | N.S. | |

* D-Latch *

→ The D-latch is a logic ckt most frequently used for storing data in digital systems. It is based on the SR-latch, but doesn't have an "undefined (or) invalid state problem."



| NAND gate | | |
|-----------|---|------------|
| A | B | \bar{AB} |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |



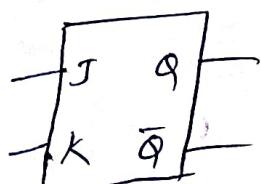
D-latch symbol

- If $D=0$; & $E=1$ next, 1st NAND gate o/p is 1;
 & not gate 1, $E=1$ " 2nd " " " is 0;
 Next 4th NAND gate o/p we don't know, then we are assuming
 that $Q=0$ (or) 1, but in this case i am considering $Q=0$;
 next 4th gate i/p 0 & $Q=0$, then o/p of 4th NAND gate is 1.
 → Next 3rd gate i/p's are 1 & $\bar{Q}=1$, then o/p of 3rd NAND gate is 0

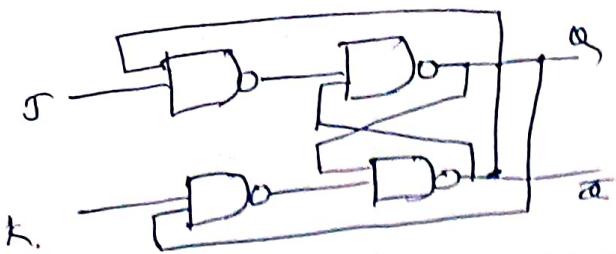
→ So if we apply $D=0$; $Q=0$; $\bar{Q}=1$
 If $D=1$; $Q=1$; $\bar{Q}=0$

- In D-latch the same input is received at the output side. so it is known as "transparent latch".
 & It is taking some time to give response at the o/p side, so it is known "Delay latch"

* JK latch *



JK Latch symbol



NAND

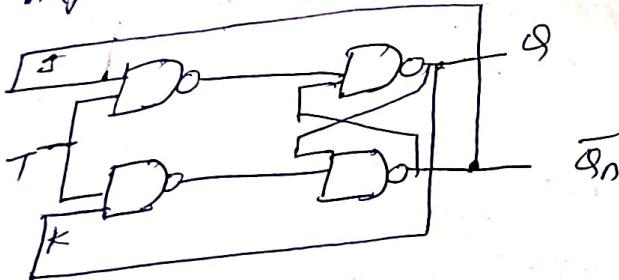
| A | B | $\bar{A} \cdot \bar{B}$ |
|---|---|-------------------------|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

⑥

| J | K | Q_n | Q_{n+1} | state |
|---|---|-------|-----------|-----------|
| 0 | 0 | 0 | 1 | No change |
| 0 | 0 | 1 | 1 | |
| 0 | 1 | 0 | 0 | Reset |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 1 | Set |
| 1 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 1 | Toggle |
| 1 | 1 | 1 | 0 | |

* T-Latch *

→ When the JK inputs of JK Latch are shorted we get the 'T-latch'. In T-latch QP's are toggled when the inputs are high.



| T | Q_n | Q_{n+1} |
|---|-------|-----------|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

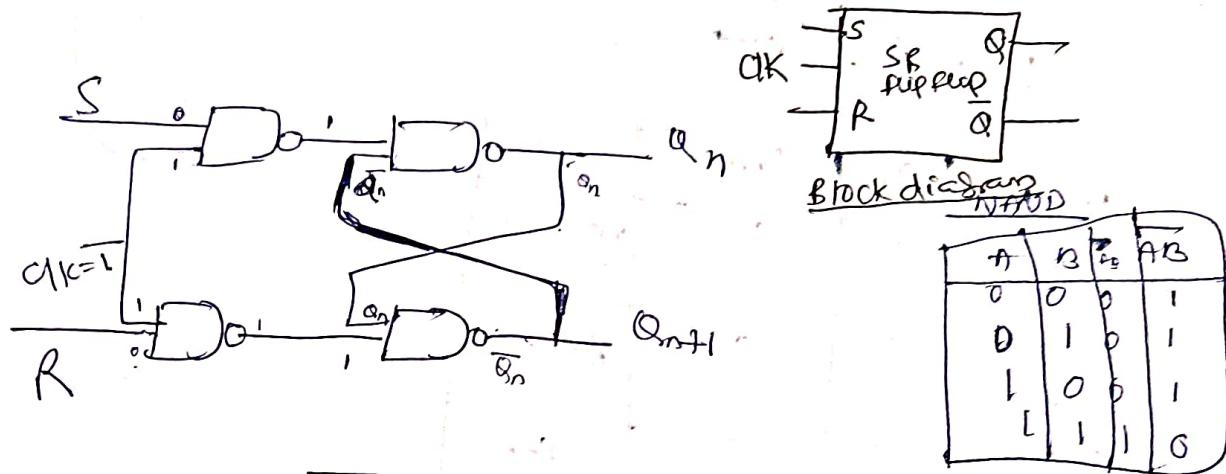
} Toggle

* flip flops *

→ It is a sequential ckt which has two states and can be used to store state inform. each flip flop can store 1 bit information. We will use the clock pulses in flip flops. → flip flops are fundamental building blocks of digital electronics systems used in computers, communications, and many other types of systems.

Types:- ① SR flip flop ② JK flip flop
③ D-Flip flop ④ T-Flip flop

① SR flip-flop: It is SR latch using NMOS gates & an addition enable input (i.e. clock).



$$\Rightarrow \text{when } S=0, R=0; \quad \overline{1 \times \bar{Q}_n} = Q_n \\ \overline{1 \times Q_n} = \bar{Q}_n \quad \left. \begin{array}{l} \text{No change} \\ \end{array} \right\}$$

$$\Rightarrow \text{when } S=0, R=1; \quad \overline{1 \times \bar{Q}_n} = Q_n = 0 \quad (\text{Q}_{n+1}) \\ \overline{0 \times Q_n} = \bar{Q} = 1 \quad (\text{Q}_{n+1})$$

$$S=0, R=1 \quad \overline{1 \times \bar{Q}} = \bar{Q}_n = 0 \\ S=0, R=1 \quad \overline{0 \times Q_n} = 0 \quad \overline{0 \times \bar{Q}n} = 0$$

$$\Rightarrow \text{when } S=1, R=0; \quad \overline{1 \times \bar{Q}_n} = \bar{Q} = 1 \quad (\text{Q}_{n+1}) \\ \overline{1 \times Q_n} = \bar{Q} = 0$$

$$S=1, R=0 \Rightarrow \overline{0 \times \bar{Q}_n} = 0 \\ \therefore \quad \overline{0 \times Q_n} = 0$$

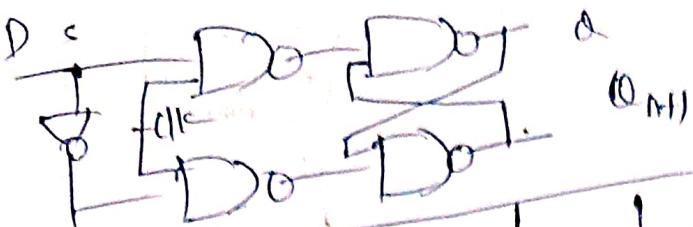
$$\Rightarrow \text{when } S=1, R=1 \Rightarrow Q_{n+1} = \overline{0 \times \bar{Q}_n} \div \bar{Q} = 1 \\ \overline{Q_{n+1}} = \overline{0 \times Q_n} = \bar{Q} = 1$$

$$S=1, R=1 \quad \overline{0 \times \bar{Q}_n} = \bar{Q} = 1$$

| Truth Table | | | P-S | Next State | |
|-------------|---|---|----------------|----------------|------------------|
| CK | S | R | Q _n | Q _n | Q _{n+1} |
| 1 | 0 | 0 | X | 0 1 | 0 1 |
| 1 | 0 | 1 | X | 0 1 | 0 0 |
| 1 | 1 | 0 | X | 0 1 | 1 1 |
| 1 | 1 | 1 | X | 0 1 | X |

Not possible.

D-Flip Flop



(D)

| clk | D | Q _n | Q _{M11} | state |
|-----|---|----------------|------------------|-----------|
| 0 | 0 | 1 | 0 | Rest |
| 1 | 1 | 0 | 1 | Set |
| 1 | 1 | 1 | 1 | Set |
| 0 | X | 0 | 0 | No change |
| 0 | X | 1 | 1 | No change |

SR flip flop

Truth table

| S | R | PS Q _n | NS Q _{M11} |
|---|---|----------------------|------------------------|
| 0 | 0 | X | Q _n |
| 0 | 1 | X | 0 |
| 1 | 0 | X | 1 |
| 1 | 1 | X | Indefinied |

Excitation table

| PS Q _n | NS Q _{M11} | S | R |
|----------------------|------------------------|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | X | 0 |

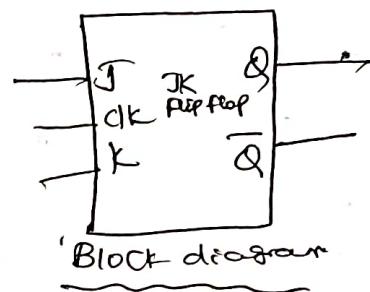
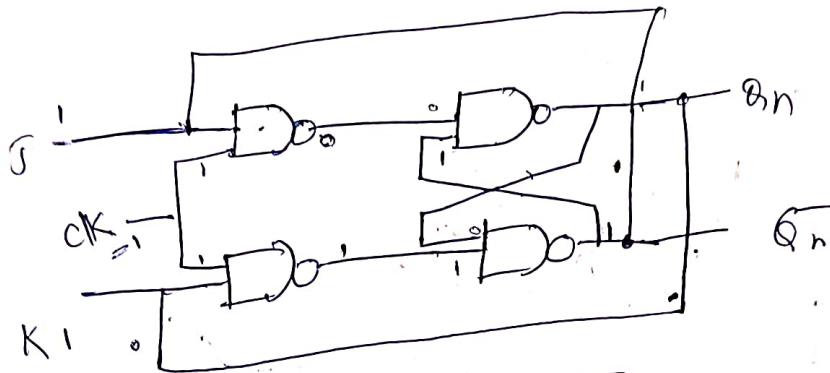
① S R
0 0
0 1
0 X

② only ①

S R
0 0
0 1

X (0) (1) (0)

② JK flip flop :-



| J | K | Q_n | Q_{n+1} |
|---|---|-------|----------------------|
| 0 | 0 | X 0 | 0 \bar{Q}_n (M.S) |
| 0 | 1 | X 0 | 0 (Reset) |
| 1 | 0 | X 1 | 1 (Set) |
| 1 | 1 | X 1 | 1 \bar{Q}_n Toggle |

NAND gate

| A | B | out B |
|---|---|-------|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

① $J=0; K=0 (G)$ $\Rightarrow Q_{n+1} = 1 \cdot \bar{Q}_n = Q_n$
 $\Rightarrow \bar{Q}_{n+1} = \overline{1 \cdot \bar{Q}_n} = \bar{Q}_n$

② $J=0; K=1 (P)$

$$Q_{n+1} = 1 \cdot \bar{Q}_n = Q_n = 0$$

$$\bar{Q}_{n+1} = \overline{Q_n \cdot \bar{Q}_n} = \bar{0} = 1$$

③ $J=1; K=0 (S)$

$$Q_{n+1} = \overline{Q_n \cdot \bar{Q}_n} = \bar{0} = 1$$

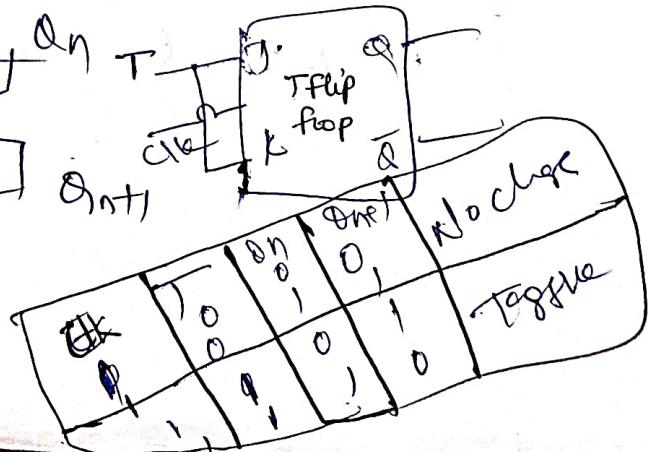
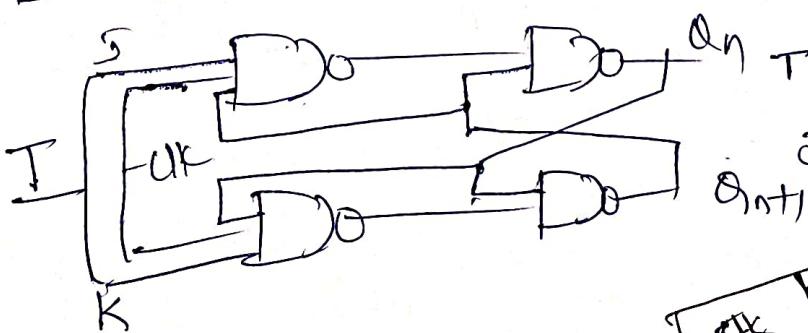
④ $J=1; K=1 (R)$

Assume $Q_n = 0$ & $\bar{Q}_n = 1$

$$Q_{n+1} = 0 \cdot \bar{Q}_n = \bar{0} = 1$$

$$\bar{Q}_{n+1} = \overline{1 \cdot \bar{Q}_n} = \bar{Q}_n = 0$$

T flip flop



Characteristic eqn

| S | R | PS Q_n | QNS Q_{n+1} |
|---|---|-------------|------------------|
| 0 | 0 | X | Q_n |
| 0 | 1 | X | 0 |
| 1 | 0 | X | 1 |
| 1 | 1 | X | Q_n |

| SR | Q_n | Q_{n+1} |
|-----|-------|-----------|
| 0 0 | 0 | 0 |
| 0 0 | 1 | 1 |
| 0 1 | 0 | 0 |
| 0 1 | 1 | 0 |
| 1 0 | 0 | 1 |
| 1 0 | 1 | 1 |
| 1 1 | 0 | X |
| 1 1 | 1 | X |

K-map

| | 00 | 01 | 11 | 10 |
|----|----|----|----|----|
| 00 | | | X | 1 |
| 01 | | X | | |
| 11 | X | | | |

$Q_{n+1} = Q \bar{R} + S$

Truth table

| J | K | PS Q_n | QNS Q_{n+1} |
|---|---|-------------|------------------|
| 0 | 0 | X | Q_n |
| 0 | 1 | X | 0 |
| 1 | 0 | X | 1 |
| 1 | 1 | X | Q_n |

| J | K | Q_n | Q_{n+1} |
|---|---|-------|-----------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | X |
| 1 | 1 | 1 | X |

| J | K | Q_n | Q_{n+1} |
|---|---|-------|-----------|
| 0 | 0 | X | |
| 0 | 1 | | X |

0, 0
1, 0

Characteristics eqn JK.

| J | K | Q_n | Q_{n+1} |
|---|---|-------|-----------|
| 0 | 0 | X | Q_n |
| 0 | 1 | X | 0 |
| 1 | 0 | X | 1 |
| 1 | 1 | X | Q_n |

| J | K | Q_n | Q_{n+1} |
|---|---|-------|-----------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

①

reset

set

tossle

| Q_n | J | K | Q_{n+1} |
|-------|-----|-----|-----------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 |

$$Q_{n+1} = \overline{Q_n J} + Q_n \overline{K}$$

Excitation table
*** D flip flop ***

| D | Q_n | Q_{n+1} |
|---|-------|-----------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

Excitation table

| PS | NS | D |
|-------|-----------|---|
| Q_n | Q_{n+1} | |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

*** T flip flop ***

T truth table

| T | PS | NS | Q_{n+1} |
|---|----|----|------------------|
| 0 | x | 0 | Q_n |
| 1 | x | 1 | $\overline{Q_n}$ |

Excitation table

| Q_n | Q_{n+1} | T |
|-------|-----------|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

* Flip flop conversions *

④

- ① Identify available and required flip flop
- ② make characteristic table for required flip flop
- ③ make excitation table for available flip flops
- ④ write boolean expression for available flip flop
- ⑤ draw the chart

* Convert JK to D - flip flop *

$$\text{Available FF} = JK$$

$$\text{Required FF} = D$$

for required characteristic
table for D

| S_n | Q_{n+1} | D | J | K |
|-------|-----------|---|---|---|
| 0 | 0 | 1 | 0 | X |
| 0 | 1 | 1 | 1 | X |
| 1 | 0 | 0 | X | 1 |
| 1 | 1 | 1 | X | 0 |

Excitation table for JK

| S_n | Q_{n+1} | J | K |
|-------|-----------|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | X |
| 1 | 0 | X | 1 |
| 1 | 1 | X | 0 |

$$J = D \quad K = D$$

for J

| S_n | D | 0 | 0 | 1 |
|-------|---|---|---|---|
| 0 | 0 | X | X | |
| 1 | X | X | | |

$$J = D$$

For K

| S_n | D | 0 | 0 |
|-------|---|---|---|
| 0 | 0 | X | X |
| 1 | X | | |

$$JK = D$$

* D flipflop to T flipflop conversions

Available \Rightarrow D-FF

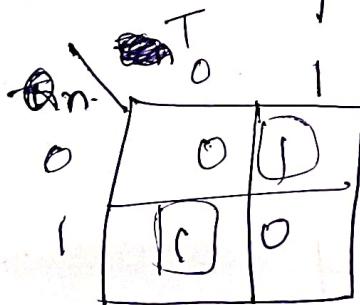
Required \Rightarrow T-FF

Required characteristic table
for T-FF

Excitation table for DFF

| Q_n | Q_{n+1} | T | D |
|-------|-----------|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

| Q_n | Q_{n+1} | D |
|-------|-----------|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |



$$D = \overline{T}Q_n + Q_n T$$

$$D = T \oplus Q_n$$

* Conversions of flip flops *

(16)

$$JK \text{ to } D\text{-FF} \Rightarrow [J=D \text{ and } K=\bar{D}]$$

$$JK \text{ to } T\text{-FF} \Rightarrow [J=K=T]$$

$$JK \text{ to } SR\text{-FF} \Rightarrow [J=S \text{ and } K=R]$$

$$SR\text{-FF to } D\text{-FF} \Rightarrow [S=D \text{ and } R=\bar{D}]$$

$$SR\text{-FF to } T\text{-FF} \Rightarrow [S=T\bar{Q} \text{ and } R=\bar{T}Q]$$

$$SR\text{-FF to } JK\text{-FF} \Rightarrow [S=J\bar{Q} \text{ and } R=KQ]$$

$$D\text{-FF to } SR\text{-FF} \Rightarrow [D=S+\bar{R}Q]$$

$$D\text{-FF to } T\text{-FF} \Rightarrow [D=T\oplus Q]$$

$$D\text{-FF to } JK\text{-FF} \Rightarrow D=[J\bar{Q} + \bar{K}Q]$$

$$T\text{-FF to } D\text{-FF} \Rightarrow [T=D\oplus S]$$

$$T\text{-FF to } JK\text{-FF} \Rightarrow [T=J\bar{Q} + KQ]$$

$$T\text{-FF to } SR\text{-FF} \Rightarrow [T=S\bar{Q} + \bar{R}Q]$$

* Race around condition (or) racing in JK flip flop *

⇒ Race around condition is arise in JK flip flop.

⇒ Race around condition is work

⇒ When $J=K=1$ race around condition is work

⇒ Race around condition is avoided by using Master slave flip flop

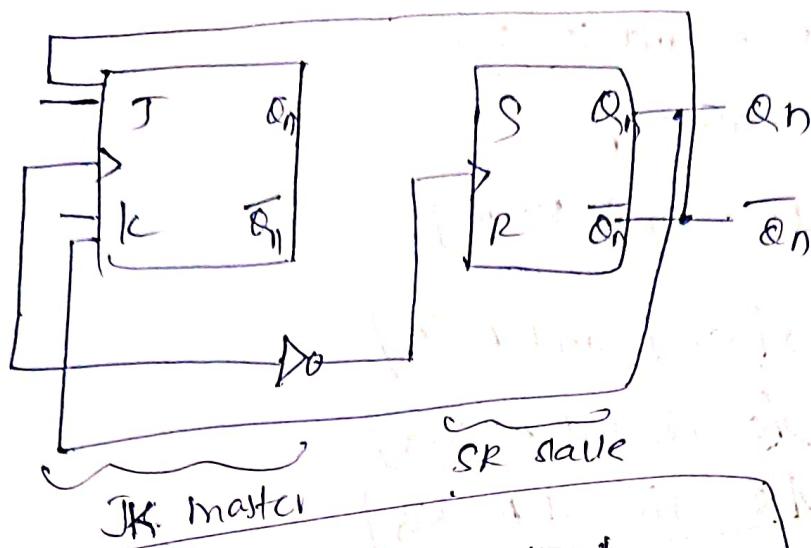
⇒ Master work on one clock cycle

slave work on other clock cycle.

⇒ for JK flip flop if $J=K=1$ & if clock is too long.

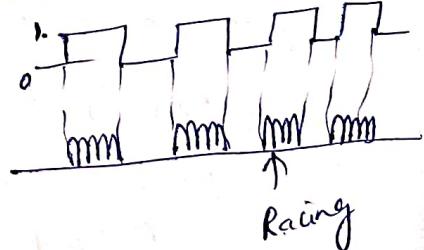
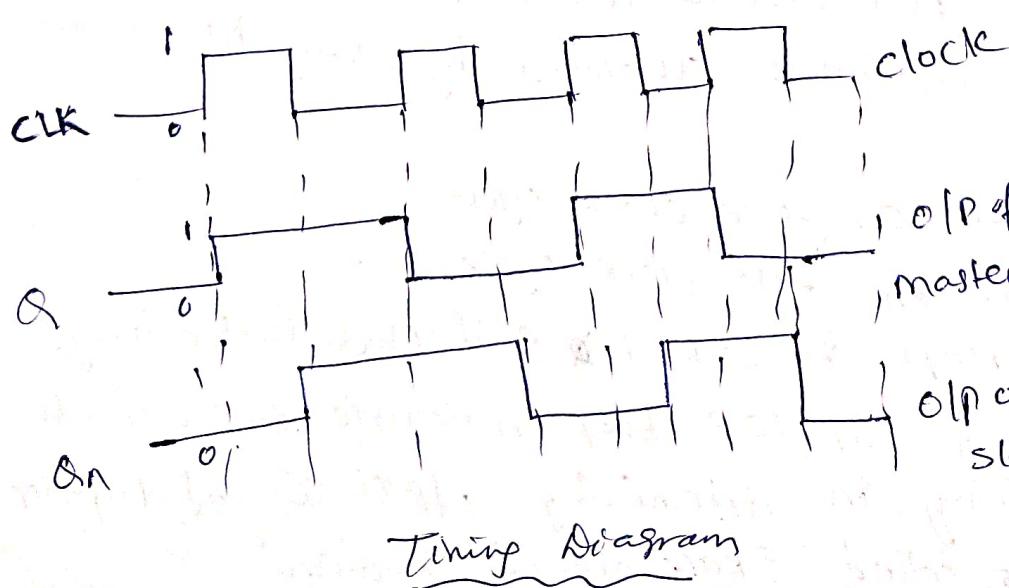
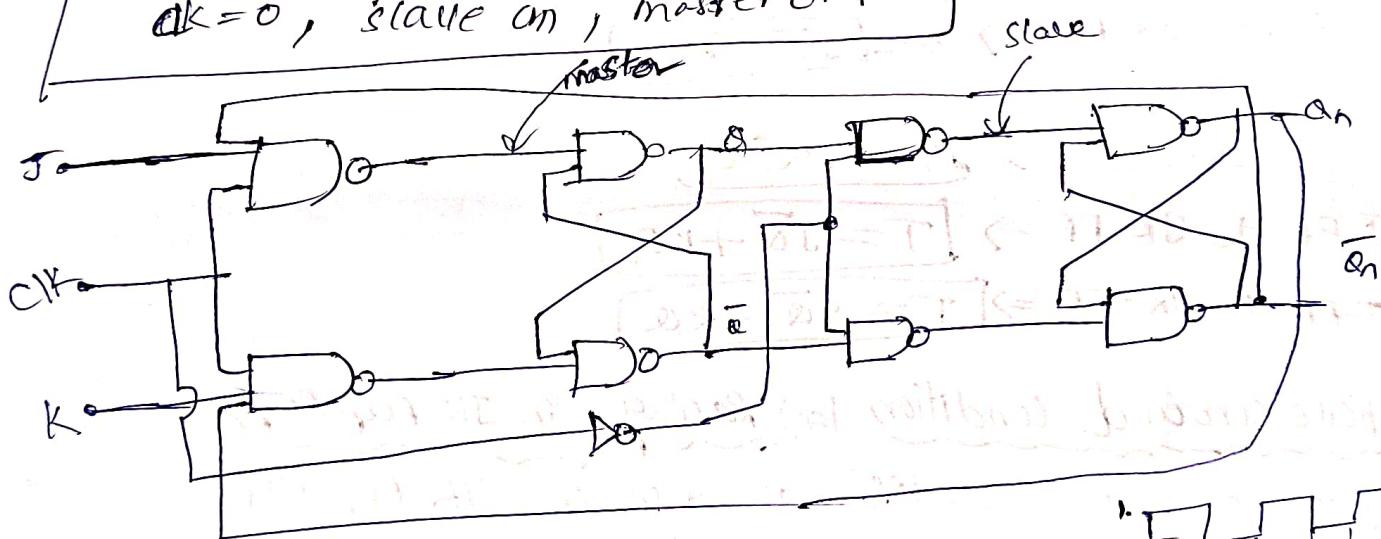
then state of flip flop keep on toggle which leads to uncertainty in determining off state of flip flop

This problem is called "Race around condition"



$$\text{Clock} = 1; J = 1; K = 1$$

$\text{clk} = 1$, master on, slave off
 $\text{clk} = 0$, slave on, master off



So, if we write the value of our QP when J=1 & our clock is high, then the QP Q_{n+1} will be \bar{Q}_n ie

| Clock | J | K | Q_{n+1} |
|-------|---|---|-------------|
| 1 | 1 | 1 | \bar{Q}_n |

(11)

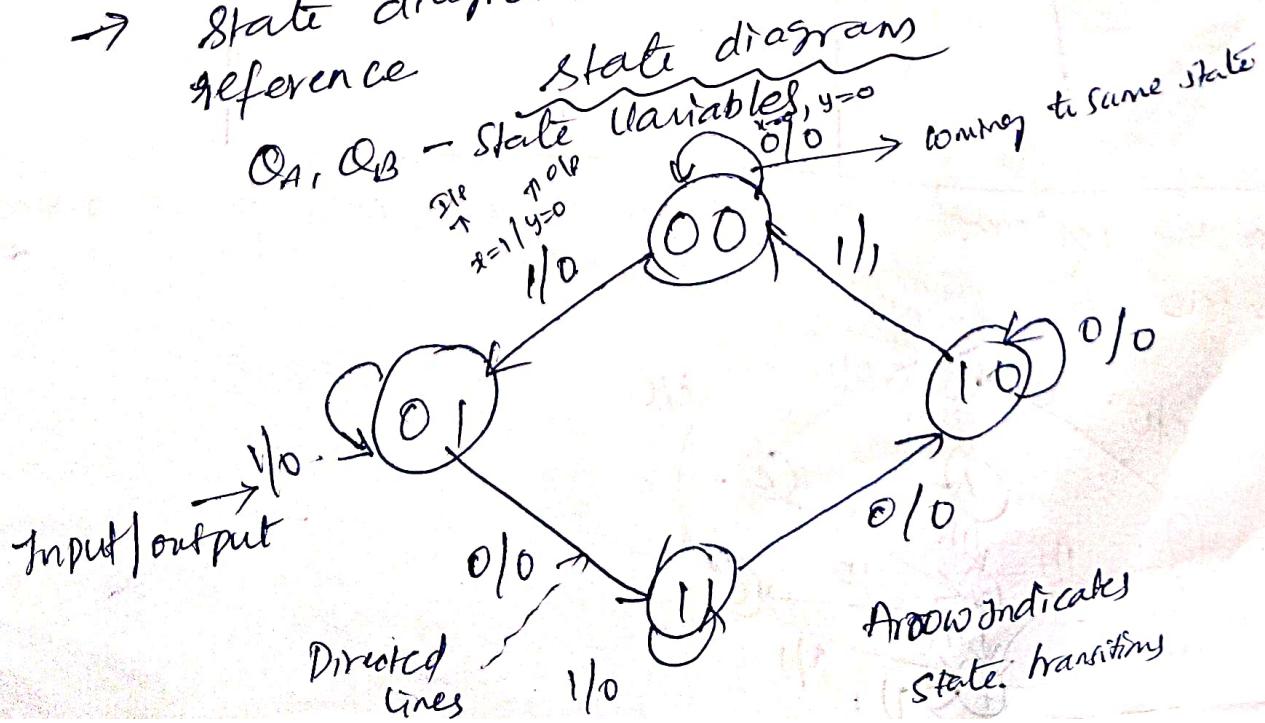
* State Reduction *

- State reduction in digital electronics is the process of reducing the number of states in a state table while maintaining the same external input-output requirements. This is done by identifying & merging states that have the same next state entries & outputs.
- State reduction is important because it can lead to simpler designs, which can improve efficiency & reduce resource consumption. [i.e Reducing the no. of flip flops in a sequential ckt is referred to as the State Reduction] *
- Sequential circuits that may simplify a design by reducing the number of gates & flip flops it uses, reducing the no. of flip flops reduces the cost of a circuit.

* State Assignment *

* State diagram : The pictorial representation of the relationships between the present state, the input & the next state and the output of a sequential ckt.

- Information available in state table is represented graphically using state diagram.
- State diagram is drawn by using state table as reference



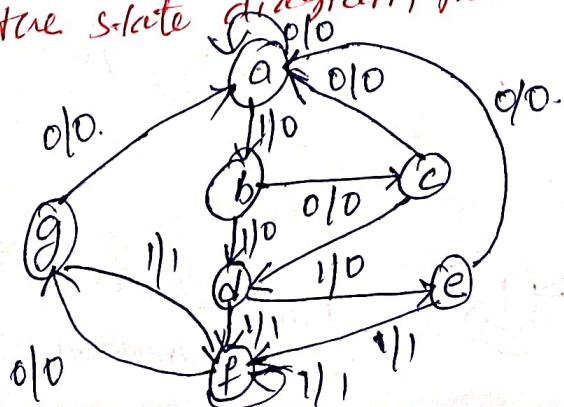
* State table *

- Relates the inputs & outputs
- Outputs Q_A, Q_B are state inputs
- These are called state variables
- x represents external input
- y represents the output
- y depends on the state variables and external input x .

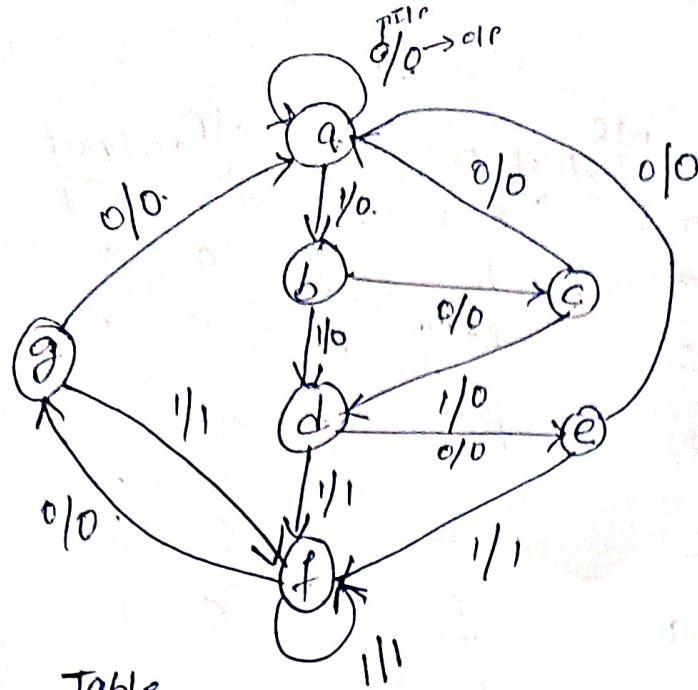
* General form of State table * → relates present state Next state & output

| Present state | Next state $x=0 \cdot x=1$ $Q_{A+1} \ Q_{B+1}$ | output y $x=0 \ x=1$ |
|---------------|--|---------------------------|
| $Q_A \ Q_B$ | | |

⑥ Draw the reduced state table or reduced state diagram for the state diagram given below.



(12)


 $x \rightarrow \text{input}$
 $y \rightarrow \text{output}$
State Table.

| Present state | Next state | | output | |
|---------------|------------|-------|--------|-------|
| | $x=0$ | $x=1$ | $x=0$ | $x=1$ |
| a | a | b | 0 | 0 |
| b | c | d | 0 | 0 |
| c | a | d | 0 | 1 |
| d | e | f | 0 | 1 |
| e | a | f | 0 | 1 |
| f | g, e | f | 0 | 1 |
| g | a | f | 0 | 1 |

$\boxed{c = g}$

Reducing table

Reducing table :

| P.S Present state | NS Next state | | O/P Output | |
|----------------------|------------------|-------|---------------|-------|
| | $x=0$ | $x=1$ | $x=0$ | $x=1$ |
| a | a | b | 0 | 0 |
| b | c | d | 0 | 0 |
| c | a | d | 0 | 0 |
| <u>d</u> | e | f | 0 | 1 |
| e | a | f | 0 | 1 |
| <u>f</u> | e | f | 0 | 1 |

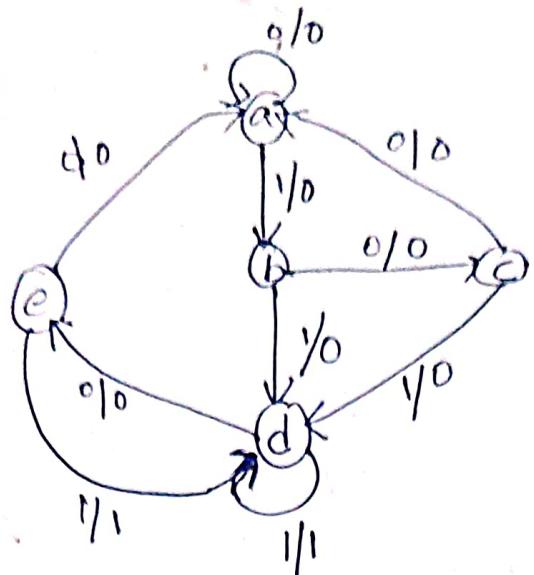
$$\therefore d=f$$

Reducing table

| Present state | Next state | | output | |
|---------------|------------|-------|--------|-------|
| | $x=0$ | $x=1$ | $x=0$ | $x=1$ |
| a | a | b | 0 | 0 |
| b | c | d | 0 | 0 |
| <u>c</u> | a | d | 0 | 0 |
| d | e | d | 0 | 1 |
| <u>e</u> | a | d | 0 | 1 |

[-: By comparing cie the next states are equal but outputs are not equal \therefore this is final reduced state table]

Now we have to draw state diagram for reduced state table



Reduced state diagram

State assignment:

$$\text{No. of states} = m$$

$$\text{No. of flip flops} = n$$

where

$$2^n \geq m$$

$$m = 5$$

$$2^3 \geq 5$$

$$8 \geq 5$$

condition is satisfied

| Present state | Assignment 1 Binary | Assignment 2 Gray code | Assignment 3 one hot |
|---------------|------------------------|---------------------------|-------------------------|
| a | 0 0 0 | 0 0 0 | 0 0 0 1 |
| b | 0 0 1 | 0 0 1 | 0 0 1 0 |
| c | 0 1 0 | 0 1 1 | 0 0 1 0 0 |
| d | 0 1 1 | 0 1 0 | 0 1 0 0 0 |
| e | 1 0 0 | 1 1 0 | 1 0 0 0 0 |

Reduced state table with binary assignment 1

| Present state | Next state $x=0$ $x=1$ | | output $x=0$ $x=1$ | |
|---------------|---------------------------|-------|-----------------------|-------|
| | 0 0 0 | 0 0 1 | 0 0 | 0 0 |
| 0 0 1 | 0 1 0 | 0 1 1 | 0 0 | 0 0 |
| 0 1 0 | 0 0 0 | 0 1 1 | 0 0 | 0 0 |
| 0 1 1 | 1 0 0 | 0 1 1 | 0 1 | 0 1 |
| 1 0 0 | 0 0 0 | 0 1 1 | 0 1 | 0 1 |

This Reduced state table with binary Assignment 1,
 → This table is called as transition table of sequential circuit. This table is used for implementing the CLTs.

Design procedure :-

* Definitions

* State assignment :- To represent states in the state diagram using binary values instead of ~~alphabets~~ alphabets.

* State diagram :- Information available in state table is represented graphically using state diagram.

→ State diagram is drawn by using state table as reference. Arrows indicate state transitions.

* Design procedure for clocked sequential circuits

① State diagram

② State table

③ State Reduction

④ State assignment

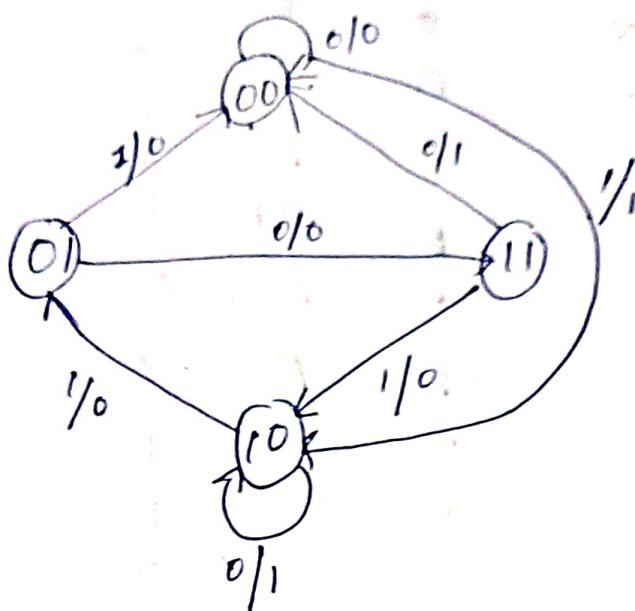
Determine number of flip flops required

⑤ Derive circuit excitation table from state table.

⑥ Implement circuit

⑦

⑥ Design a sequential circuit that has 2 inputs and 1 output state diagram is shown.



| Present state | | Next state | | | Output | |
|---------------|---|------------|---|-------|--------|-------|
| A | B | $x=0$ | | $x=1$ | $x=0$ | $x=1$ |
| | | A | B | AB | y | y |
| 0 | 0 | 0 | 0 | 10 | 0 | 1 |
| 0 | 1 | 1 | 1 | 00 | 0 | 0 |
| 1 | 0 | 1 | 0 | 01 | 1 | 0 |
| 1 | 1 | 0 | 0 | 10 | 1 | 0 |

→ State table shows that circuit goes through four states therefore we require 2 flip flops.

$m = \text{no. of states}$
 $n = \text{no. of flip flops}$
 using T-flip flop:
 Excitation table

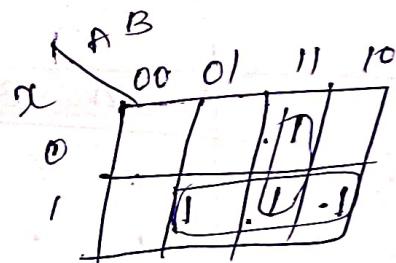
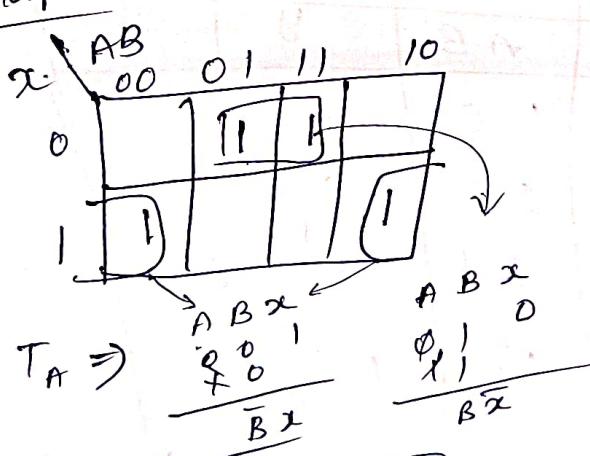
$$T = Q_n \oplus Q_{n+1}$$

| Q_n | Q_{n+1} | T |
|-------|-----------|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

* circuit excitation table *

| present state A B | Input X | Next stat. A B | | flip flop | | output Y |
|----------------------|------------|-------------------|---|-----------|----|-------------|
| | | A | B | TA | TB | |
| 0 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 1 | 1 | 1 | 0 | 0 | 1 | 0 |

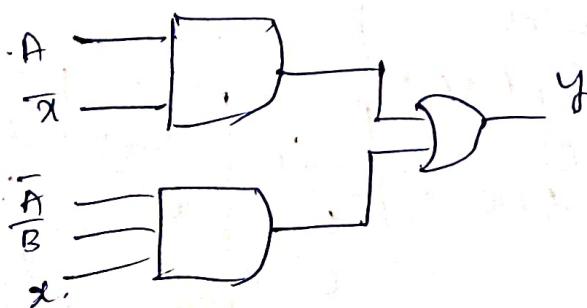
K-map:

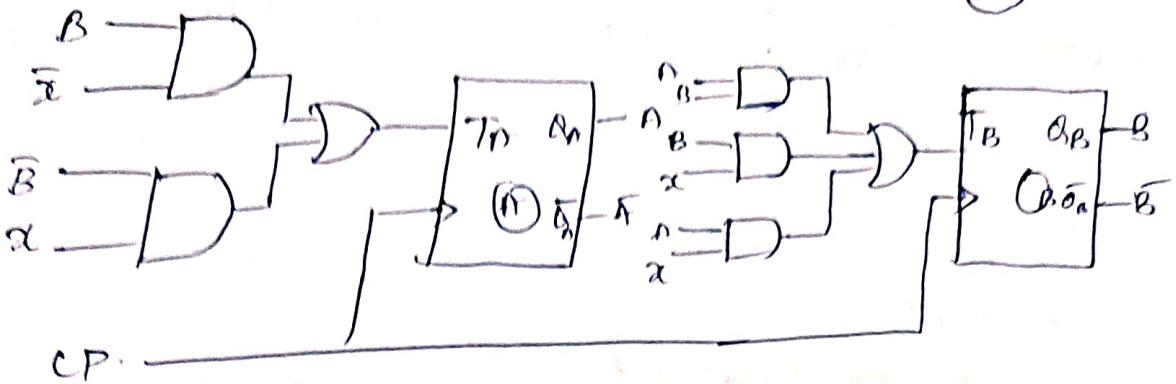


$$T_B = AB + BX + A\overline{x}$$

$$Y = A\overline{x} + \overline{A}\overline{B}x$$

$$T_A = B\overline{x} + \overline{B}x$$





Circuit diagram

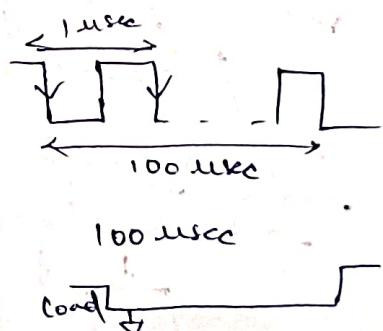
Registers

- flip flop is a 1-bit memory cell.
- To increase the storage capacity we have to use group of flip flops. This group of flip flops is known as "Register".
- The n-bit register consist of 'n' number of flip flops and is capable of storing "n-bit" word.

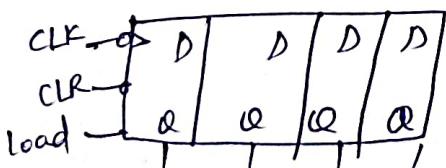
| |
|-----------------------|
| Nibble = 4 bits |
| Byte = 8 bits |
| word = 16 bits |
| double word = 32 bits |
| quad word = 64 bits |

If $f_{clk} = 1 \text{ MHz} = 10^6 \text{ Hz}$

$$\text{Time period} = \frac{1}{f} \text{ sec.}$$



$$T = 1 \mu\text{sec.}$$



$$\frac{1}{10^6} = 10^{-6}$$

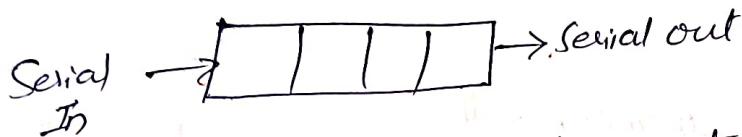
1 usec

In synchronous circuits = clock ↑ and load ↑ (i.e. clock increases & load is increased).
 Asynchronous circuits = Only load ↑ ses.

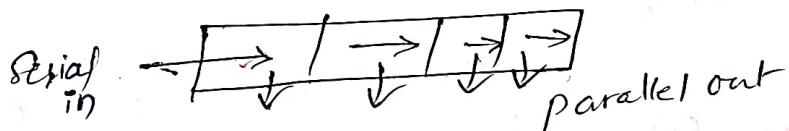
To store the 4 bits, we need 4 flipflops.
 → ~~Shift registers~~
 → Shift register are used to transfer the data.

- ① SISO
- ② CIPO
- ③ PISO
- ④ PIPO

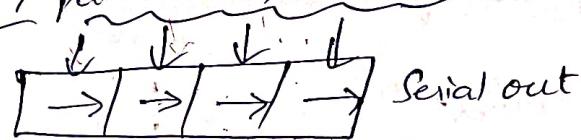
① SISO → serial in serial out
 → Data in right (or) left direction



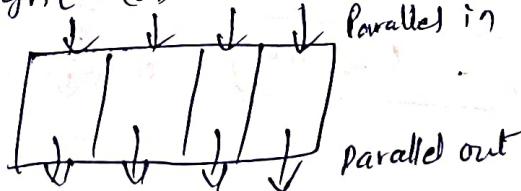
② SIPO → serial in parallel out
 → Data in right (or) left direction



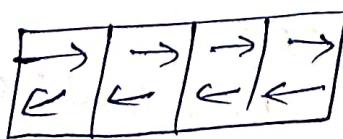
③ PISO → parallel in serial out



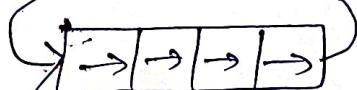
④ PIPO → parallel in parallel out
 → Right (or) left Direction.



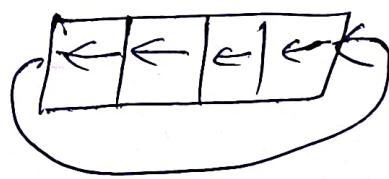
⑤ Bidirectional



⑥ Rotate Right → Anti clock wise to Right side



⑦ Rotate Left → Anti clock wise left side



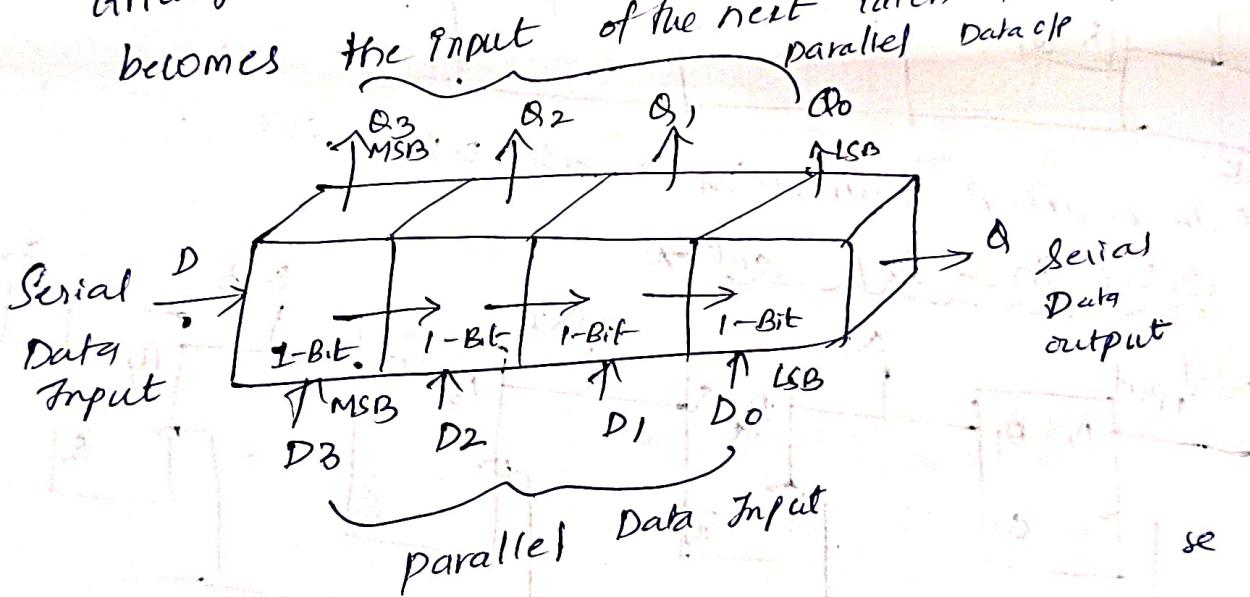
* Applications *

(16)

- Data storage device → In calculators
- Data transmission device → In computers
- Serial to parallel | convert do one form to another form
- Parallel to serial

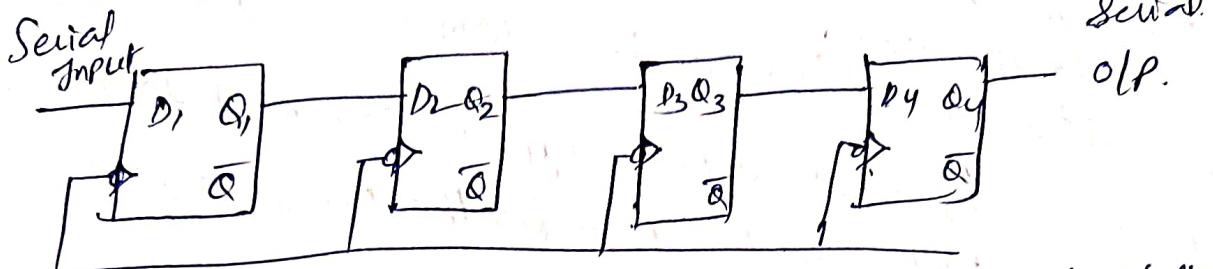
* The shift register is another type of sequential logic circuit that can be used for the storage (or) the transfer of binary data.

The shift register basically consists of several single bit "D-type data latches", one for each data bit, either a logic '0' (or) a '1', connected together in a serial arrangement so that the output from one data latch becomes the input of the next latch & so on.



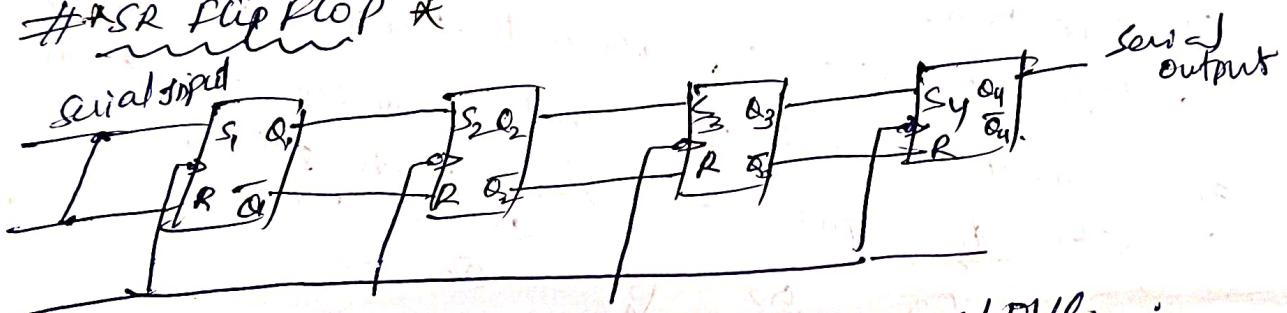
* Design 4 Bit Serial In & serial Out Shift Register

* D-flip flop *



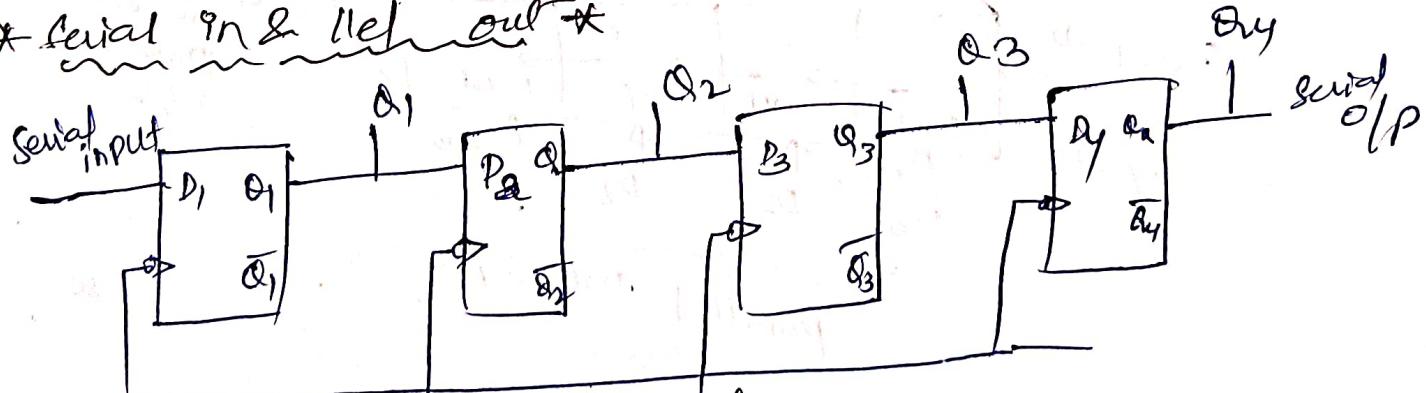
* CLK
* The shift can be changed from right-to-left (or) left-to-right by interchanging the D & Q states

* S-R flip flop *

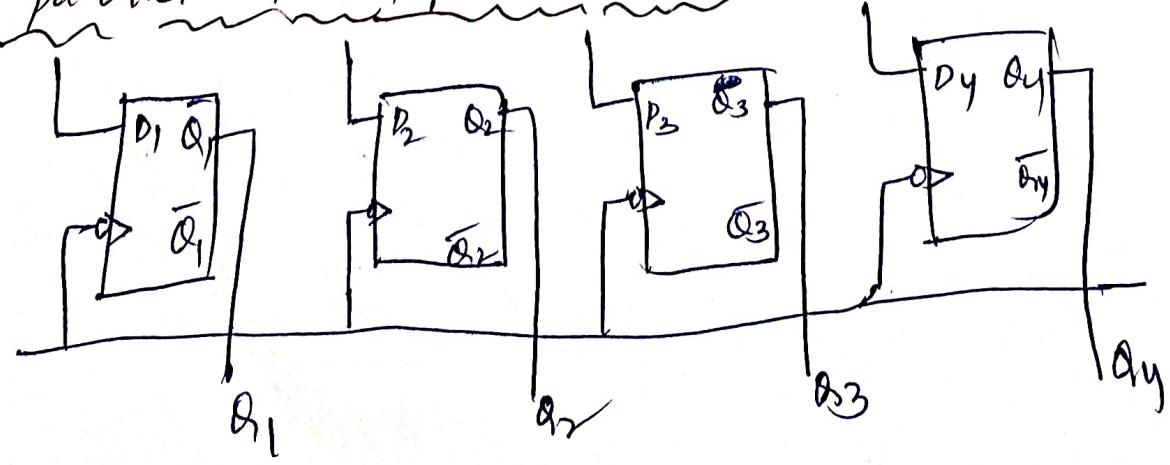


* CLK
* JK flip flop construction is also as above

* Serial In & Serial Out *

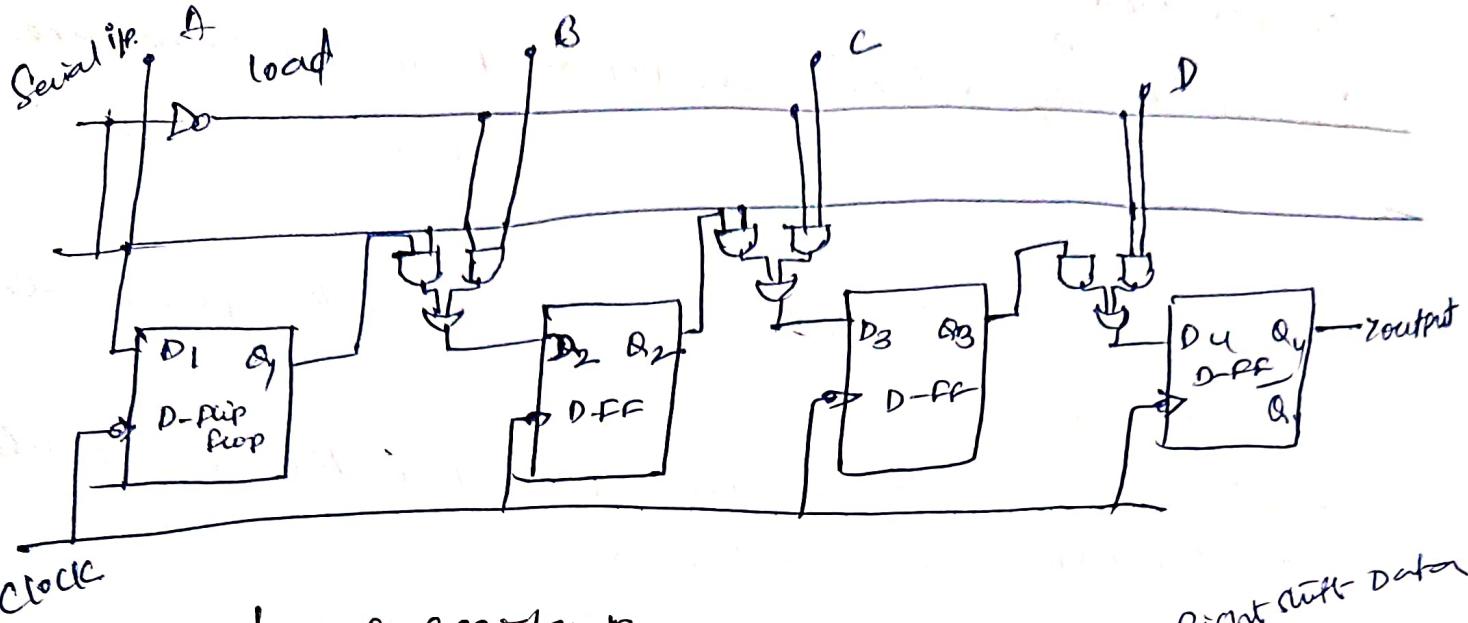


* Parallel In and Parallel Out

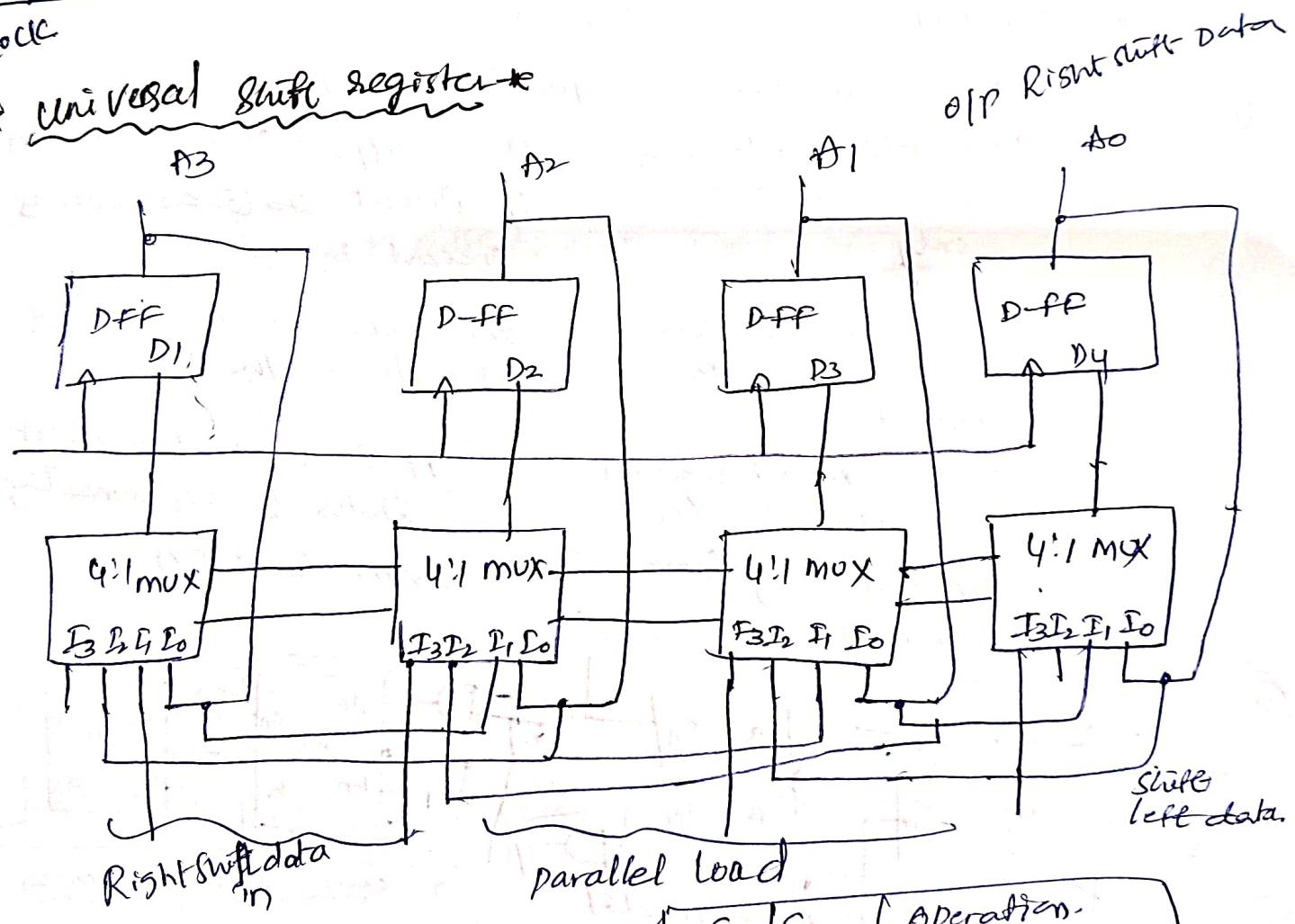


* Parallel in serial out

(17)



* Universal shift register



| S_1 | S_0 | Operation |
|-------|-------|------------------|
| 0 | 0 | NC |
| 0 | 1 | SR (Shift Right) |
| 1 | 0 | SL (Shift Left) |
| 1 | 1 | Parallel Load |

Q) List out the two types of clocked sequential circuits & compare them.

(or)

Compare mealy and moore model of finite state machine

→ The synchronous (or) clocked sequential circuits are represented by two models

Moore model:- The o/p depends only on the present state of flip flops.

Mealy model:- The o/p depends on both the present state of flip flop & on the inputs

①

Moore model
It's o/p is a function of present state only

mealy model

It's output is a function of present state as well as present input

②

Input changes does not effect on the output

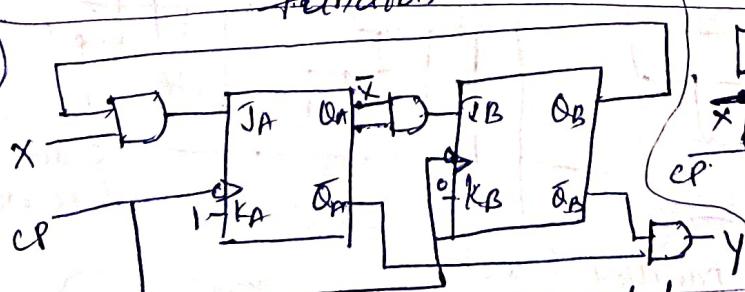
Input changes may affect the o/p of the ckt

③

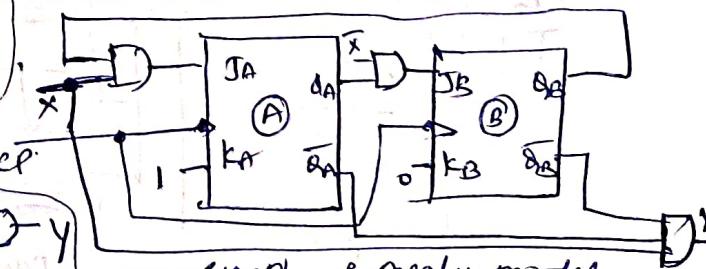
Moore model requires more no. of states for implementing same function

It requires less no. of states for implementing same function

④

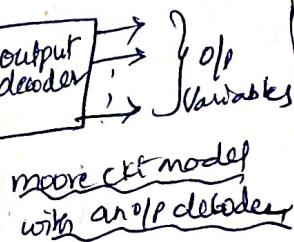
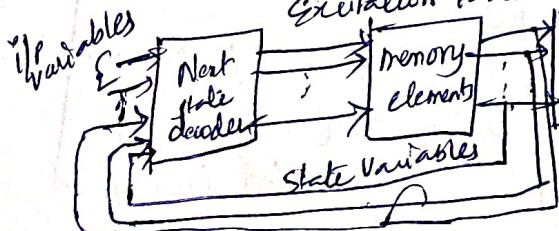


Example of moore model



Example of mealy model

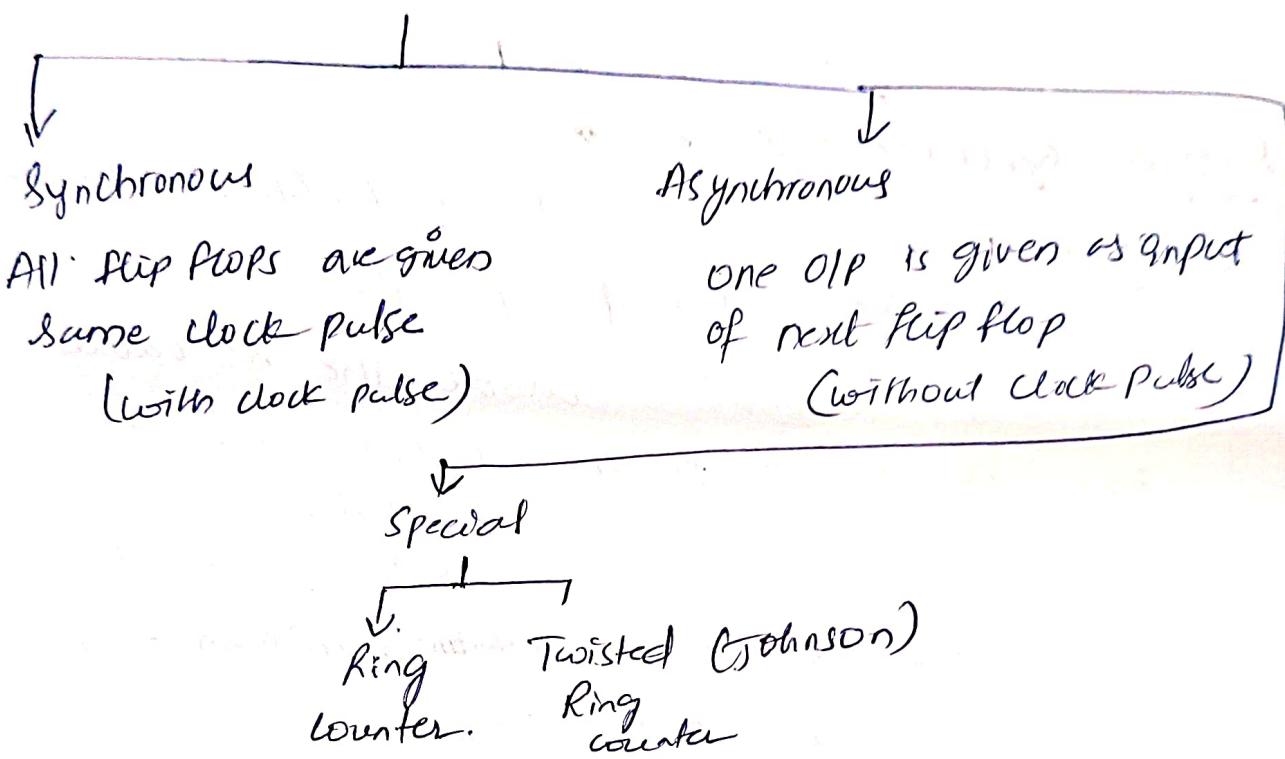
⑤



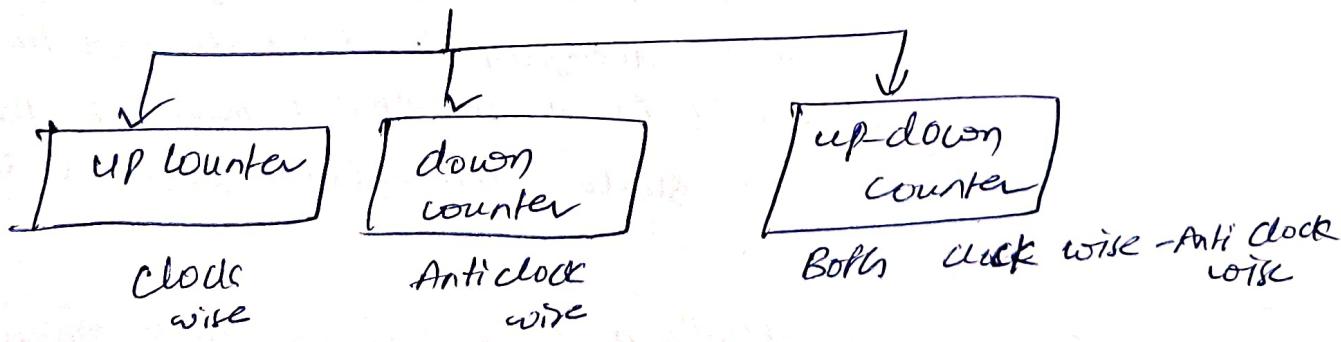
mealy CKT model

Counters

- Counter is a sequential circuit. A digital circuit which is used for counting pulses is known as counter.
- Counter is a device which stores (and sometimes displays) the no. of times a particular event (or) process has occurred often in relationship to a clock signal.
- Types of counters. (Depending on the clock pulses applied)
 - Counter
 - counters are two types),



① Synchronous counter



for example, in UP counter a counter increases count for every rising edge of clock. Not only counting, a counter can follow the certain sequence based on our design like any random sequence 0, 1, 3, 2... they can also be designed with the help of flip flops. They are used as frequency dividers where the freq. of given pulse waveforms is divided.

→ counters are sequential Ckt that count the no. of pulses can be either in binary code (or) BCD form. The main properties of counters are timing, sequencing, & counting. Counters works in two modes

- ① UP Counter
- ② Down Counter.

*Design Synchronous Counter *

Step 1 :- Based on the description of problem determined the required no. of flip flops

$$N \leq 2^n \quad \text{choose the } 'n' \text{ value}$$

has as small as possible. N is the mode of the counter
 \checkmark No. of states

$n \rightarrow$ is the required no. of flip flops

choose the ' n ' as ~~minimum~~ minimum as possible

$$\text{Ex: } 5 \leq 2^3 = 8$$

$$5 \leq 8$$

Step 2: State diagram & Draw the state with all possible state. The state diagram is also called as transition diagram. It is a graphical means of the sequence of state through the counter is progress.

Step 3: choose of flip flop & Excitation table given in the problem. otherwise choose default.

Step 4: Minimal expression for excitation obtains minimal Expression by K-map.

Step 5 : logic diagram

(17)

→ Draw the logic diagram according to the required expression.

a) Design mod-6, Gray code synchronous counter by using T-flip flop.

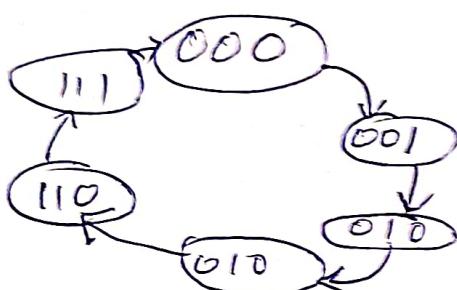
$$\textcircled{1} \quad N=6$$

$$6 \leq 2^3 \Rightarrow 6 \leq 8$$

\textcircled{2} The no. of flip flops is needed = 3

\textcircled{3} Here for designing we definitely chose up counter

\textcircled{4} 3-Bit Gray code is needed.



Excitation table

| 3 Bit Gray code | | |
|-----------------|-----|----|
| 0 | 000 | -1 |
| 1 | 001 | -2 |
| 2 | 010 | -3 |
| 3 | 011 | -4 |
| 4 | 110 | -5 |
| 5 | 111 | -6 |
| 6 | - | - |
| 7 | - | - |

| PS | NS | T ₃ | T ₂ | T ₁ |
|-----|-----|----------------|----------------|----------------|
| 000 | 001 | 0 | 0 | 1 |
| 001 | 011 | 0 | 1 | 0 |
| 011 | 010 | 0 | 0 | 1 |
| 010 | 110 | 1 | 0 | 0 |
| 110 | 111 | 0 | 0 | 1 |
| 111 | 100 | 1 | 1 | 1 |
| 100 | 000 | 1 | 1 | 1 |

T₃ (K-map)

| Q ₃ | Q ₂ | Q ₁ | T ₃ |
|----------------|----------------|----------------|----------------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |

T₂ (K-map)

| Q ₃ | Q ₂ | Q ₁ | T ₂ |
|----------------|----------------|----------------|----------------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |

T₁ (K-map)

| Q ₃ | Q ₂ | Q ₁ | T ₁ |
|----------------|----------------|----------------|----------------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |

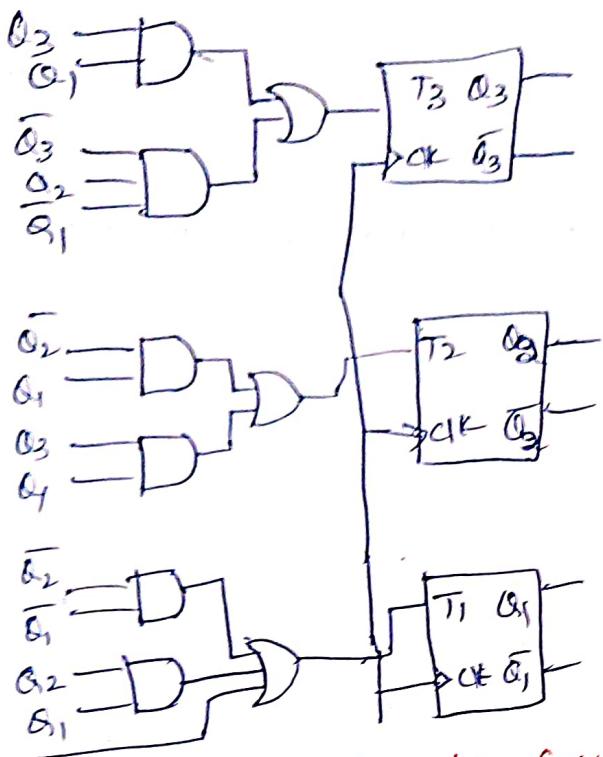
T₁ - K-map

| Q ₃ | Q ₂ | Q ₁ | T ₁ |
|----------------|----------------|----------------|----------------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |

$$T_3 = \bar{Q}_3 \cdot Q_2 \bar{Q}_1 + Q_3 Q_1$$

$$T_2 = \bar{Q}_2 Q_1 + Q_1 \bar{Q}_3$$

$$T_1 = \bar{Q}_2 \bar{Q}_1 + Q_2 Q_1$$



Q) Design decade counter using T-flip flop

$$N \leq 2^n$$

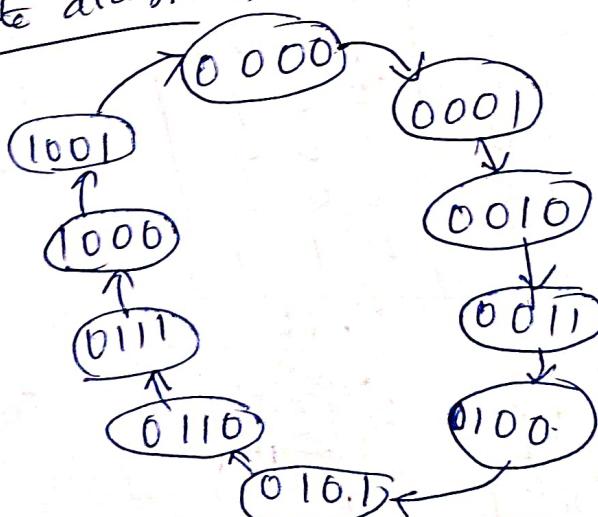
$$10 \leq 2^4$$

$$10 \leq 16$$

No. of states = 10
No. of flip flops = 4

| Q_n | 0 | 1 |
|-------|---|---|
| 0 | 0 | 0 |
| 1 | 1 | 1 |

state diagram



Truth table

Present state

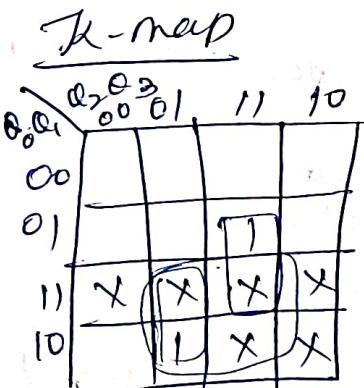
Next state

To T_1, T_2, T_3

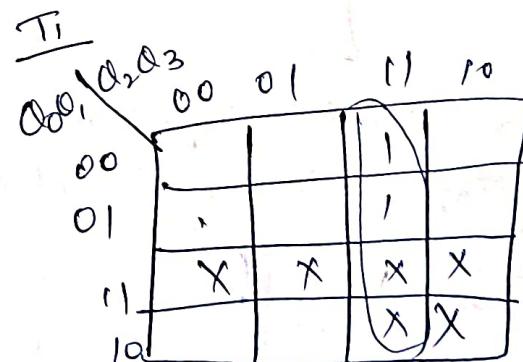
(20)

| $Q_0 Q_1 Q_2 Q_3$ | $Q_0 Q_1 Q_2 Q_3$ | $Q_0 Q_1 Q_2 Q_3$ |
|-------------------|-------------------|-------------------|
| 0 0 0 0 | 0 0 0 1 | 0 0 0 1 |
| 0 0 0 1 | 0 0 1 0 | 0 0 1 1 |
| 0 0 1 0 | 0 0 1 1 | 0 0 0 1 |
| 0 0 1 1 | 0 1 0 0 | 0 1 1 1 |
| 0 1 0 0 | 0 1 0 1 | 0 0 0 1 |
| 0 1 0 1 | 0 1 1 0 | 0 0 1 1 |
| 0 1 1 0 | 0 1 1 1 | 0 0 0 1 |
| 0 1 1 1 | 1 0 0 0 | x x x x |
| 1 0 0 0 | 1 0 0 1 | 1 0 0 1 |
| 1 0 0 1 | 0 0 0 0 | x x x x |
| 1 0 1 0 | x x x x | x x x x |
| 1 0 1 1 | x x x x | x x x x |
| 1 1 0 0 | x x x x | x x x x |
| 1 1 0 1 | x x x x | x x x x |
| 1 1 1 0 | x x x x | x x x x |
| 1 1 1 1 | x x x x | x x x x |

To



$$T_0 = Q_1 Q_2 Q_3 + Q_0 Q_2 Q_3$$



$$T_1 = Q_2 Q_3$$

T₂

| Q ₀ Q ₃ | | Q ₁ Q ₂ | | T ₂ |
|-------------------------------|----------------|-------------------------------|----------------|----------------|
| Q ₀ | Q ₃ | Q ₁ | Q ₂ | |
| 00 | 00 | 11 | 11 | |
| 01 | 01 | 11 | 11 | |
| 11 | X X | X X | X X | |
| 10 | | X X | X X | |

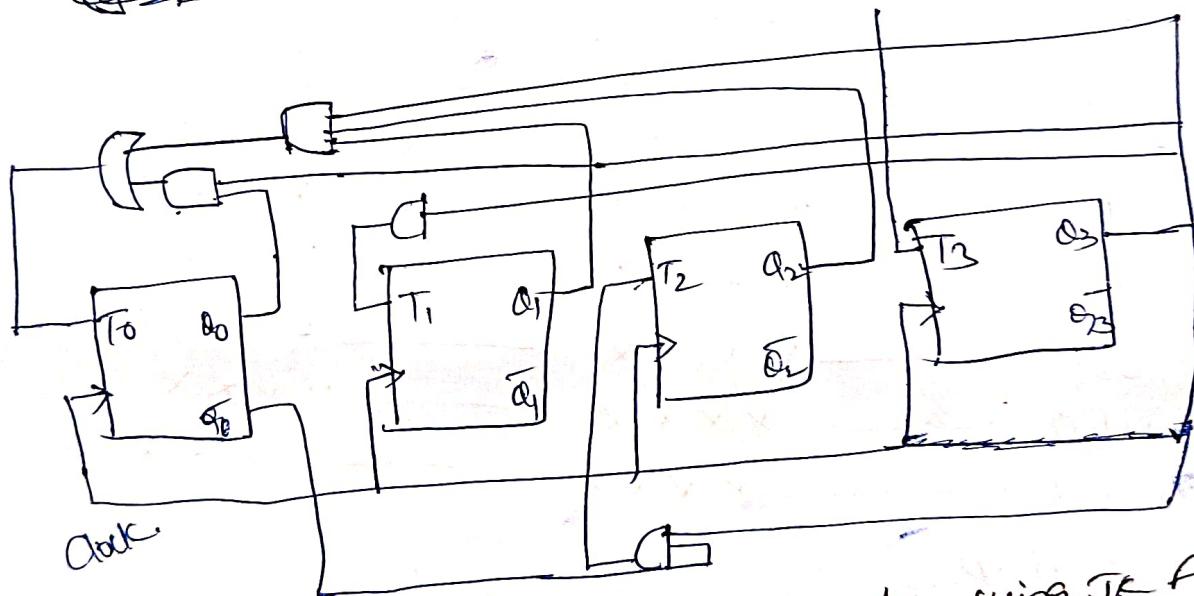
$T_2 = \bar{Q}_0 \cdot Q_3$

T₃

| Q ₀ Q ₃ | | Q ₁ Q ₂ | | T ₃ |
|-------------------------------|----------------|-------------------------------|----------------|----------------|
| Q ₀ | Q ₃ | Q ₁ | Q ₂ | |
| 00 | 00 | 0 | 1 | |
| 01 | 01 | 1 | 1 | 1 |
| 11 | X X | X X | X X | |
| 10 | | 11 | X X | |

$T_3 = 1$

logic diagram



⑩ Design a 2-Bit Synchronous Counter using JK flip flop.

Excitation table for JK flip flop:

| Q _n | Q _{n+1} | J | K |
|----------------|------------------|---|---|
| 0 | 0 | 0 | X |
| 0 | 1 | 1 | X |
| 1 | 0 | X | 1 |
| 1 | 1 | X | 0 |

equation

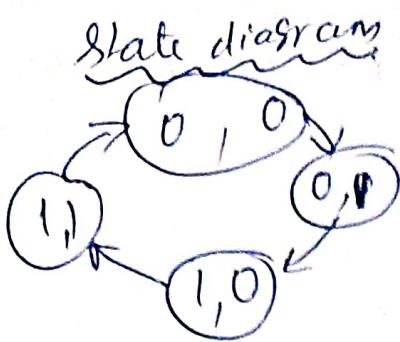
$$N \leq 2^n$$

$$N \leq 2^2 \quad n=2$$

$$n \leq 4 \quad u=4$$

No. of states = 4

No. of flip flops = 2



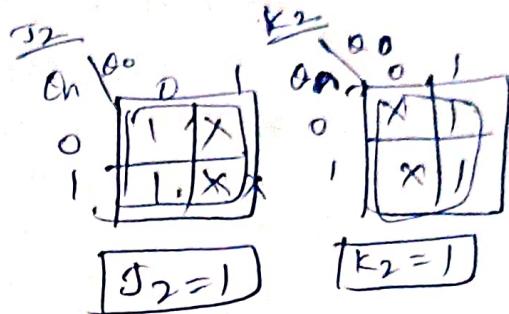
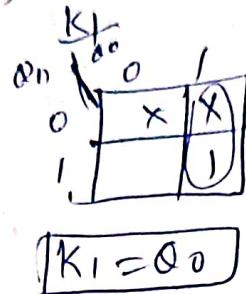
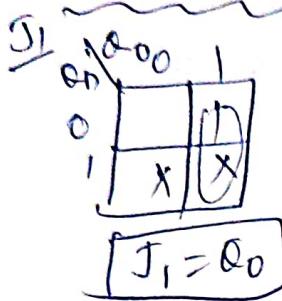
Truth Table

| R | S | N.S |
|---|---|-----|
| 0 | 0 | 01 |
| 0 | 1 | 10 |
| 1 | 0 | 11 |
| 1 | 1 | 00 |

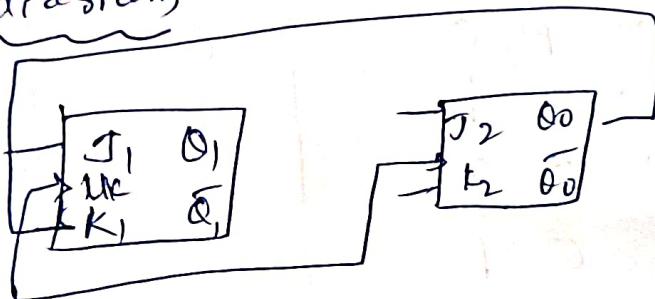
(21)

| J ₁ , K ₁ | J ₂ , K ₂ |
|---------------------------------|---------------------------------|
| 0X | 1X |
| 1X | X1 |
| X0 | 1X |
| X1 | X1 |

K-map for J₁



Logic diagram



* Asynchronous counters → In this type of counters there is a connection btw the o/p of 1st flip flop & clock input of next flip flop.

→ All flip flops are not clocked simultaneously.

→ clock is quite simple even for more no. of states

→ the performance of the device is low speed.

→ Example: Ripple counter.

→ Ripple counter is a special type of asynchronous counter in which the clock pulse ripples through the circuit. The no. of flip flops in the cascaded arrangement depends upon the no. of different logic states that it goes through before it repeats the sequence a parameter known as

modulus of the counter. A n-bit ripple counter can count up to 2ⁿ states. It is also known as MOD n counter.

~~RV-202~~ The other name for asynchronous counter is Ripple counter.

① Design Mod-6 Asynchronous Counter by using

T-flip flop

$$N \leq 2^n \quad N \leq 2^3$$

→ 3 T-flip flops are needed

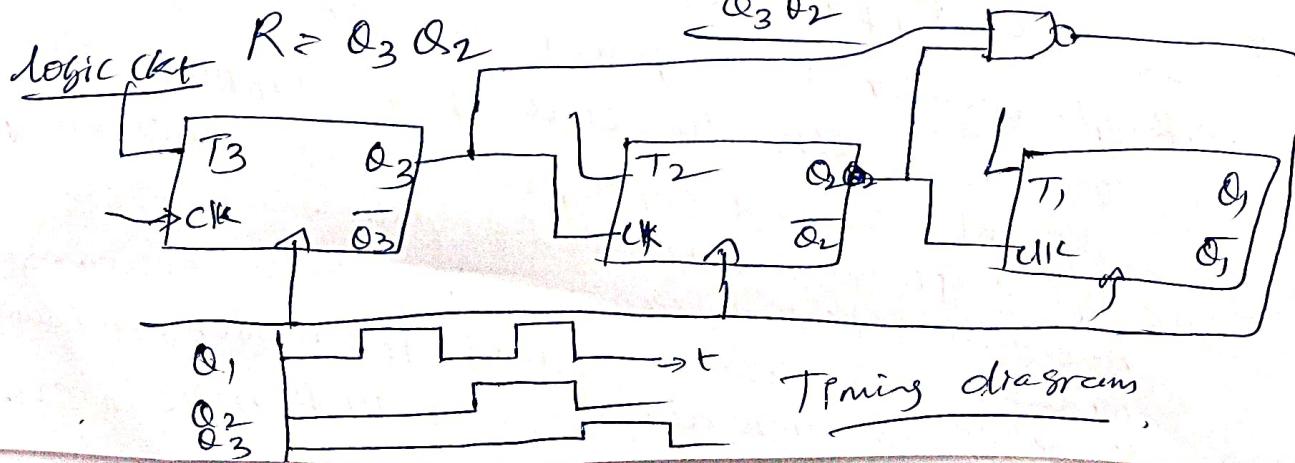
→ 0 to 7 numbers it means 8 states

→ We need 0 to 5 states it means 6 states.

| After Clock Pulse | | state $Q_3 \ Q_2 \ Q_1$ | R |
|-------------------|--|----------------------------|---|
| 0 | | 0 0 0 | 0 |
| 1 | | 0 0 1 | 0 |
| 2 | | 0 1 0 | 0 |
| 3 | | 0 1 1 | 0 |
| 4 | | 1 0 0 | 0 |
| 5 | | 1 0 1 | 0 |
| 6 | | 1 1 0 | 0 |
| 7 | | 1 1 1 | X |

| Q_3 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
|-------|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | X | 1 | 1 | 1 | 0 |

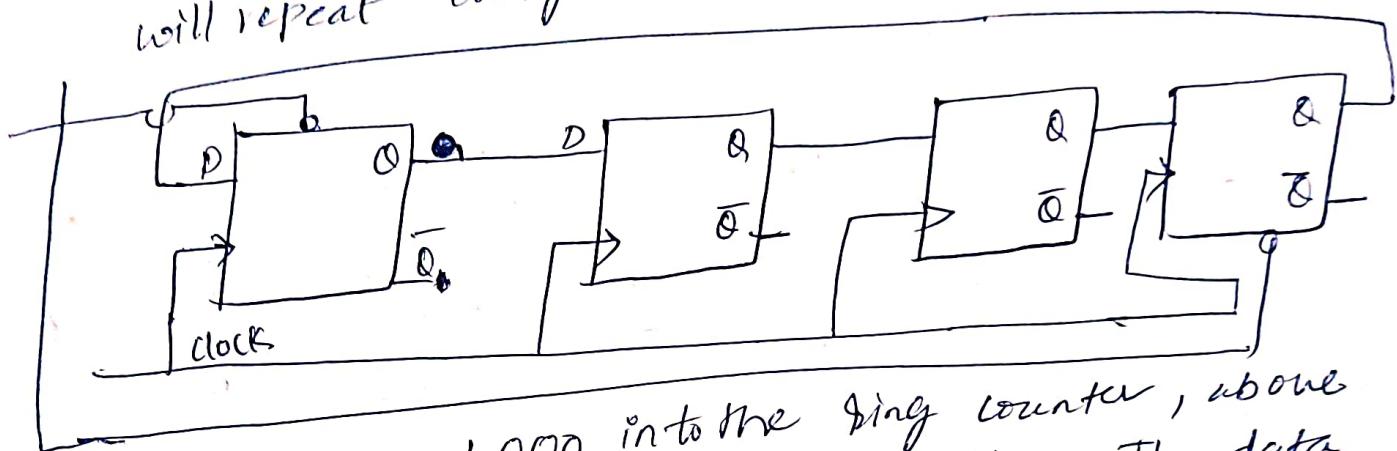
$$\begin{array}{l} Q_3 \ Q_2 \ Q_1 \\ 1 \ 1 \ X \\ \hline Q_3 \ Q_2 \end{array}$$



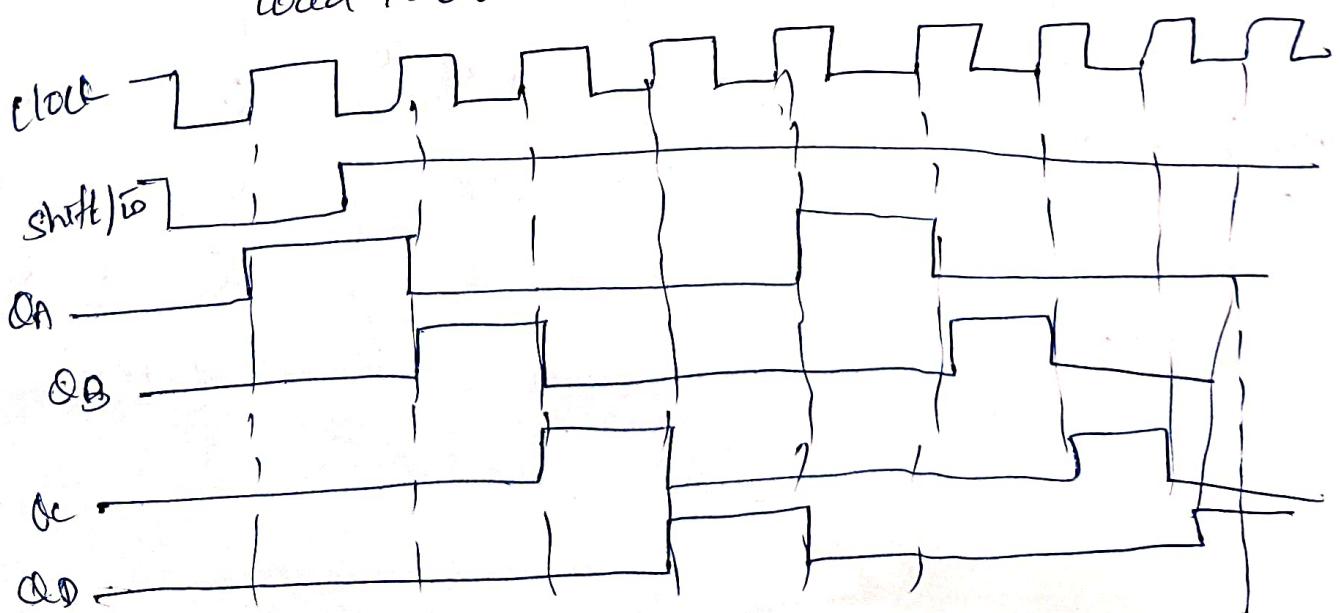
(22)

Ring counter

→ If the o/p of a shift register is fed back to the input, a ring counter results the data pattern contained within the shift register will recirculate as long as clock pulses are applied. For example, the data pattern will repeat every four clock pulses in the figure below.



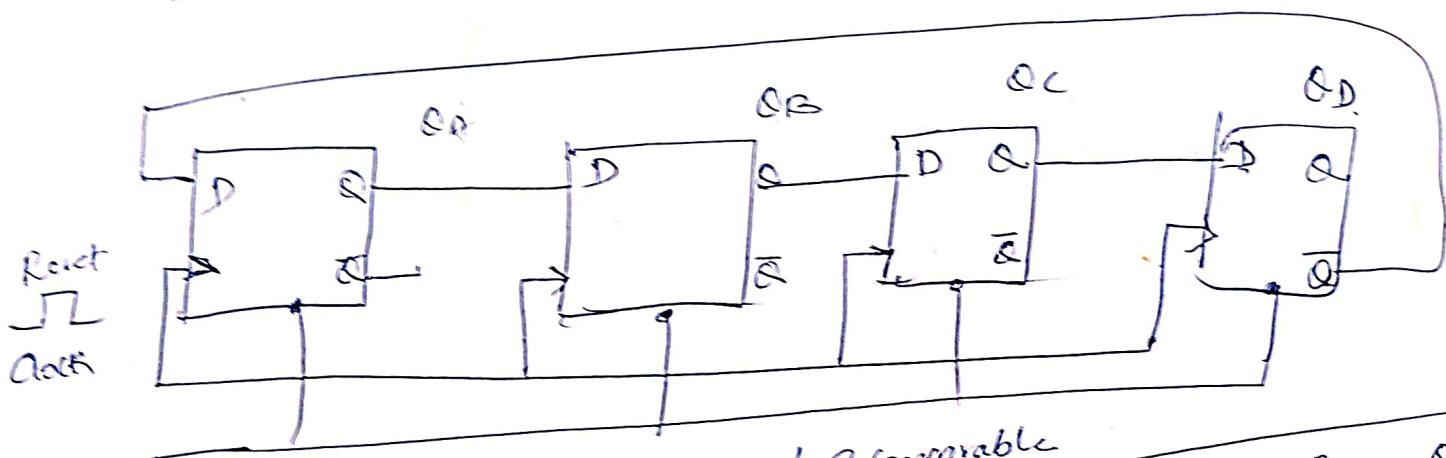
→ Loading binary 1000 into the ring counter, above prior to shifting yields a variable pattern. The data pattern for a single stage repeats every four clock pulses in our 4-stage examples. The waveform for all four stages look the same, except for the one clock time delay from one stage to the next see figure below.
Load 1000 into 4-stage ring counter & shift



Timing diagrams

Johnson Counter

The switch-tail ring counter, also known as the Johnson counter, overcomes some of the limitations of the ring counter. Like a ring counter a Johnson counter is a shift register fed back on itself.



- It requires half the stages of a comparable ring counter for a given division ratio. The complement D/P of a ring counter is fed back to the input instead of the true output, a Johnson counter results. The difference between a ring counter & a Johnson counter is, which D/P if the last stage is fed back (Q or Q'). Carefully compare the feedback connection below to the previous ring counter.
- | Q _A | Q _B | Q _C | Q _D |
|----------------|----------------|----------------|----------------|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 |
- Repeat