MATRICES AND CALCULUS

UNIT – I MATRICES

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Define rank of a matrix and give one example	L1	CO1	PO1
2	Define Hermitian and skew - Hermitian matrices.	L1	CO1	PO1
3	Find the value of k such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.	L2	CO1	PO2
4	State the different conditions in non - homogeneous system of equations.	L2	CO1	PO1
5	Find the rank of the matrix A=$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to echelon form.	L2	CO1	PO2
6	Define symmetric matrix and give a suitable example.	L1	CO1	PO1
7	Define an orthogonal matrix and give one example.	L1	CO1	PO1
8	Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is aunitary matrix.	L2	CO1	PO2
9	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$	L2	CO1	PO2
10	Prove that the transpose of a unitary matrix is unitary.	L2	CO1	PO1

S.NO	**Part –B (Long answer questions)**	BT	CO	PO
1(a)	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 5 & 6 \end{bmatrix}$, by reducing it to the normal form.	L2	CO1	PO2

1(b)	Find the Inverse of a matrix $A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by using Gauss-Jordan method.	L3	CO1	PO2
2 (a)	Reduce the Matrix $A = \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ into Echelon form. Hence find its Rank.	L2	CO1	PO2
2(b)	Examine for what values of p and q , so that the equations 2x+3y+5z = 9, 7x+3y+2z=8, 2x+3y+pz=q $have$ (i) No solution (ii) Unique solution (iii) Infinitely many solutions.	L4	CO1	PO2
3(a)	Solve system of equations x+y+w= 0, y+z= 0 , x+y+z+w = 0 , x+y+2z= 0 .	L3	CO1	PO1
3(b)	Solve the equations 3x+y+2z=3, 2x-3y-z=-3, x+2y+z=4 using gauss elimination method.	L3	CO1	PO1
4	Solve the system of equations by gauss seidel method 20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25 .	L4	CO1	PO3
5(a)	Find the rank of the value of k , if the rank of the matrix A is 2 , where A= $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix}$	L2	CO1	PO1
5(b)	Show that the equations x+2y-z=3, 3x-y+2z =1, 2x-2y+3z = 2, x-y+z = -1 are consistent and solve them.	L2	CO1	PO1
6	Solve $2x - 7y + 4z = 9,\ x + 9y - 6z = 1,$ $-3x + 8y + 5z = 6$ by LU-decomposition method.	L3	CO1	PO3

UNIT-2 : Eigen values-Eigen vectors and Quadratic forms

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Define model and spectral matrices.	L1	CO2	PO1
2	Find the sum and product of the Eigen values of	L2	CO2	PO1

	$A = \begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$			
3	Using Cayley Hamilton theorem find A^8, if $A=\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.	L2	CO2	PO2
4	Find the Eigen values of A^{-1} where $A=\begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$	L2	CO2	PO1
5	Find the symmetric matrix corresponding to the quadratic form $x^2 + 6xy + 5y^2$.	L1	CO2	PO2
6	Find the characteristic roots of the matrix $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$	L2	CO2	PO1
7	Compute the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$	L2	CO2	PO1
8	Prove that zero is eigen value of a matrix iff it is singular.	L2	CO2	PO1
9	Find the Eigen values of the matrix $\begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$	L2	CO2	PO2
10	State Cayley – Hamilton theorem.	L1	CO2	PO1

S.NO	**Part-B (Long answer questions)**	**BT**	**CO**	**PO**
1	Find the Eigen values and Eigen vectors of a Matrix $$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$	L3	CO2	PO2
2(a)	Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation hence find A^{-1}.	L2	CO2	PO2
2(b)	Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$	L3	CO2	PO1
3(a)	Find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$ for the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$	L2	CO2	PO1
3(b)	Verify Cayley Hamilton theorem for the matrix	L3	CO2	PO1

	$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence find A^4 .			
4	Diagonalize the matrix$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$	L3	CO2	PO3
5	Reduce the Quadratic form $3{x_1}^2 + 3{x_2}^2 + 3{x_3}^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into Canonical form and hence state nature, rank, index and signature of the Quadratic form.	L4	CO2	PO3
6	Diagonalize the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by Orthogonal Reduction.	L4	CO2	PO3

UNIT-III

CALCULUS

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Verify Rolle's theorem for f(x)= $2x^3+x^2-4x-2$ in $[-\sqrt{3},\sqrt{3}]$.	L2	CO4	PO1
2	Verify Lagrange's mean value theorem for f(x) =$\log_e x$ in [1,e] .	L2	CO4	PO2
3	Define beta and gamma functions.	L1	CO4	PO1
4	Find the value of $\Gamma(\frac{1}{2})$	L2	CO4	PO1
5	Evaluate $\int_0^1 x^5 (1-x)^3\, dx$	L1	CO4	PO1
6	Find c of Cauchy's mean value theorem for f(x)=$\sqrt{x}$ and g(x)=$\frac{1}{\sqrt{x}}$ in [a,b] where 0<a<b.	L2	CO4	PO2
7	Using Rolles theorem show that g(x)= $8x^3-6x^2-2x+1$ has a zero between 0 and 1 .	L1	CO4	PO2
8	Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}}$ dx = $\frac{1}{5}B(\frac{2}{5},\frac{1}{2})$	L1	CO4	PO2
9	Find the value of $\Gamma(\frac{5}{2})$	L1	CO4	PO2
10	Compute $\int_0^\infty e^{-x}\ x^3 \quad dx$	L1	CO4	PO2

S.NO	Part-B(Long answer questions)	BT	CO	PO
1(a)	Verify Rolle's theorem for $f(x) = (x-a)^m(x-b)^n$ where m, n are positive integers in [a , b].	L3	CO4	PO2
1(b)	Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\left(\frac{3}{5}\right) > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem.	L3	CO4	PO2
2(a)	Verify generalized mean value theorem for f(x) = e^x , g(x)= e^{-x} in [3,7] and find the value of c.	L3	CO4	PO2
2(b)	Prove that β (m,n) =$\beta(m+1,n) + \beta(m,n+1)$.	L3	CO4	PO3
3(a)	Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^5}}\, dx$ in terms of Beta function.	L3	CO4	PO2
3(b)	Evaluate $\int_0^1 x^7 \ (1-x)^5\, dx\ by$ using β-Γ functions.	L2	CO4	PO2
4(a)	Evaluate $\int_0^{\frac{\pi}{2}} Sin^6\,\theta\ Cos^7\theta\ d\theta$ using β-Γ functions .	L2	CO4	PO2
4(b)	Show that $\Gamma(n) = \int_0^1 (log\,\frac{1}{x})^{n-1}\ dx,\ n > 0$.	L2	CO4	PO2
5	Establish the relation between Beta and Gamma functions .	L3	CO4	PO2
6(a)	Show that $4\int_0^{\infty} \frac{x^2}{1+x^4}$ dx = $\sqrt{2}\,\pi$.	L4	CO4	PO2
6(b)	Evaluate $\int_0^1 x^3\sqrt{(1-x)}$ dx using β-Γ functions .	L4	CO4	PO2

UNIT-IV

Multi variable Calculus (Partial Differentiation and Applications)

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	State Euler's theorem for homogeneous function in x and y.	L1	CO5	PO1
2	Determine whether the functions u =e^x siny , v = e^x cosy are functionally dependent or not.	L2	CO5	PO2
3	If x=u(1+v) , y=v(1+u) then prove that $\frac{\partial(x,y)}{\partial(u,v)}$ = 1+u+v .	L2	CO5	PO2

4	Write the working rule to find the maximum and minimum values of f(x,y) .	L2	CO5	PO1
5	Verify $\frac{\partial^2 u}{\partial x \partial y}$ = $\frac{\partial^2 u}{\partial y \partial x}$ for the function u = $\tan^{-1}\frac{x}{y}$.	L2	CO5	PO2
6	Find the first and second order partial derivatives of x^3+y^3-3axy and verify $\frac{\partial^2 f}{\partial x \partial y}$ = $\frac{\partial^2 f}{\partial y \partial x}$	L2	CO5	PO2
7	Verify Euler's theorem for the function xy+yz+zx .	L1	CO5	PO2
8	If $u=x^2-2y$, v= x+y+z , w=x-2y+3z find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	L1	CO5	PO2
9	Verify if u= 2x-y+3z , v= 2x-y-z ,w=2x-y+z are functionally dependent and if ,so find the relation between them.	L2	CO5	PO2
10	Find the maximum an minimum values of $f(x,y)=x^3+3xy^2-3x^2-3y^2+4$	L2	CO5	PO2

S.NO	Part-B(Long answer questions)	BT	CO	PO
1(a)	If z= $\log(e^x + e^y)$ $show\ that\ rt - s^2$=0 , where r= $\frac{\partial^2 z}{\partial x^2}$, t= $\frac{\partial^2 z}{\partial y^2}$,s= $\frac{\partial^2 z}{\partial x\, \partial y}$	L3	CO5	PO2
1(b)	If $\sin u$ = $\frac{x^2y^2}{x^2+y^2}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = Sin2u.$	L3	CO5	PO2
2(a)	If u=x+y+z , y+z= uv , z=uvw show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ = u^2 v.	L3	CO5	PO2
2(b)	If u = $x^2 - y^2$, v= 2xy where x=r cos θ ,y = r sinθ show that $\frac{\partial(u,v)}{\partial(r,\theta)}$ = $4r^3$.	L2	CO5	PO2
3(a)	If x= $\frac{u^2}{v}$, y = $\frac{v^2}{u}$ find $\frac{\partial(u,v)}{\partial(x,y)}$.	L2	CO5	PO2
3(b)	If x= u$\sqrt{(1-v^2)}$+v$\sqrt{(1-u^2)}$ and y= $\sin^{-1} u + \sin^{-1} v$ then show that x and y are functionally related , also find the relationship.	L3	CO5	PO3
4	Find the maximum and minimum values of the function $f(x,y) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$.	L4	CO5	PO1
5(a)	Find the maximum and minimum values of the function f(x, y)= $x^3y^2(1-x-y)$.	L4	CO5	PO2
5(b)	Find the maximum and minimum values of x+y+z subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ by Lagrange's method of undetermined multipliers.	L3	CO5	PO3

6	Find the dimensions of the rectangular parallelepiped box open at top of max capacity whose surface area is 256 sq. inches.	L4	CO5	PO3

UNIT-V

S.No		Questions	BT	CO	PO
Part – A (Short Answer Questions)					
1		Evaluate $\int_0^2 \int_0^3 xydxdy$	L1	CO3	PO1
2		Evaluate $\int_0^2 \int_0^x ydxdy$	L2	CO3	PO2
3		Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$	L1	CO3	PO1
4		Evaluate $\int_0^\pi \int_0^{asin\theta} rdrd\theta$	L1	CO3	PO1
5		Write the limits after changing the order of integration $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy^2dydx$	L3	CO3	PO2
6		Find $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dxdydz$	L3	CO3	PO2
7		Find $\int_0^1 \int_1^2 \int_2^3 xyzdxdydz$	L1	CO3	PO1
8		Find $\int_0^1 \int_1^2 \int_2^3 (x+y+z)dxdydz$	L3	CO3	PO2
9		Shade the region bounded by the $y = x^2$ and $x = y^2$.	L3	CO3	PO2
10		Evaluate $\int_0^1 \int_0^2 y^2dxdy$	L3	CO3	PO2
Part – B (Long Answer Questions)					
11	a)	Evaluate $\int_0^1 \int_x^{\sqrt{x}}(x^2+y^2)dxdy$	L4,L5	CO3	PO3
	b)	Evaluate $\oiint y^2dxdy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.	L3	CO3	PO2
12	a)	Evaluate $\iint(x^2+y^2)$dxdy in the positive quadrant for which x+y ≤1.	L3	CO3	PO2
	b)	Evaluate $\int_0^\pi \int_0^{asin\theta} rdrd\theta$.	L3	CO3	PO2

13	a)	Evaluate $\iint r^3 dr d\theta$ over the area included between the circles r=2sinθ and r=4sinθ.	L4,L5	CO3	PO3
	b)	Evaluate $\int_0^1 \int_{x^2}^{2-x} xydydx$ by change of order of integration.	L3	CO3	PO2
14	a)	Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dxdydz.$	L3	CO3	PO2
15		Evaluate $\iiint xyzdxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$.	L2,L3	CO3	PO2
16		Find the volume of the greatest rectangular parallelopiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$	L2,L3	CO3	PO2

*Blooms Taxonomy Level(BT) (L1-Remembering; L2- Understanding:L3-Applying;L4-Analyzing;L5-Evaluating;L6-Creating)

Course Outcomes(CO)

Program Outcomes(PO).