Syllabus

Unit-I :- Mathematical Logic

Introduction, statements and Notation, connectives, Normal forms, Theory of Inference for statement calculus, The Predicate calculus, Inference Theory of the Predicate calculus.

Unit-II :- Set Theory:

Introduction, Basic concepts of set Theory, Representation of Discrete structures, Relations and ordering, Functions.

Unit-III :- Algebraic structures

Introduction, Algebraic systems, semi groups and Monoids, lattices as partially ordered sets, Boolean Algebra.

Unit-IV : Elementary combinatorics

Basics of counting, combinations and Permutations; Enumeration of combinations and Permutations, Enumerating combinations and Permutations with Repititions, Enumerating permutation with constrained Repititions, Binomial coefficient, The Binomial and Multinomial Theorems, The principle of Exclusion.

Unit-V Graph Theory

Basic concepts, Isomorphism and Subgraphs, Trees and their properties, spanning Trees, Directed trees, Binary Trees, Planar Graphs, Euler's formula, Multi-graphs and Euler circuits, Hamiltonian Graphs, chromatic Numbers, The Four-color problem.

DM is the branch of mathematics deals with discrete objects.

Discrete objects means that they are

→ countable → separable → distinct [different]

[not connected] → not continuous

Ex :- whole numbers, Integers, No. of houses, Buses, people etc.

Advantages

→ It develops your mathematical thinking

→ helps in coding, programming languages, software development etc.

Statement [(or) proposition]

A sentence which is either true or false, but not both

Note: Questions, Exclamations and commands are not statements.

Ex of propositions	Ex of not propositions
1) $3+2=5$ (T)	1) Do u speak english
2) $3+2=6$ (F)	2) open the door
3) The sun rises in the east.	3) what a good day!
4) $3+5=7$ is a statement & it is false	4) $x+y>2$ let $x=1, y=2 \rightarrow 3>2$ (If $x=0, y=1 \rightarrow 1>2$ (F

Note :- we use letters "p, q, r" to represent propositions

Ex:- p : It is raining

q : she is good.

Atomic statement

A declarative sentences which cannot be further split into simpler sentences are called Atomic / primitive statements

Ex: 1) Today is holiday

2) The sun is shining

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Molecular statement (or) compound statement

A statement which contains 2 (or) more atomic statements along with connectives are called molecular statement.

Ex:- Ram is a boy and → connective Sita is a girl.

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logical connectives

1) Negation, 2) conjunction 3) Disjunction
4) conditional 5) Biconditional
6) Tautology 7) contradiction

Symbol	Name	compound form
1) $\sim$, $\neg$	Negation (not)	$\neg p$ (or) $\sim p$
2) $\wedge$	conjunction (And)	$p \wedge q$
3) $\vee$	Disjunction (or)	$p \vee q$
4) $\rightarrow$ (or) $\Rightarrow$	Implication (or) conditional [If...then]	$p \rightarrow q$ $p \Rightarrow q$
5) $\leftrightarrow$ $\Leftrightarrow$	Biconditional. If and only if	$p \leftrightarrow q$ $p \Leftrightarrow q$

Write the following statements to symbolic form

1) p : He is tall

q : He is handsome.

2) He is tall and handsome

$p \wedge q$

2) He is tall but not handsome

$p \wedge \sim q$

2) It is false that he is not tall or handsome

$\sim(\sim p \vee q)$

2) He is tall or he is not tall and handsome.

$p \vee (\sim p \wedge q)$

2) He is neither tall nor handsome.

[not tall and not hands]

$\sim p \wedge \sim q$

a) It is not true that he is not tall or not handsome.

$\sim(\sim p \vee \sim q)$.

---

2) p : The moon is out

q : It is snowing

r : Ram goes out for a walk.

a) If the moon is out and It is not snowing, then Ram goes out for a walk.

$(p \wedge \sim q) \rightarrow r$.

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a) If the moon is out, then if it is not snowing then Ram goes out for a walk.

$p \rightarrow (\sim q \rightarrow r)$

Q) Arun can access the internet from campus only if she is computer science student or she is not a fresher.

p : Arun can access the internet from campus.

q : she is cse student

r : she is fresher

Ans :- $p \rightarrow (q \vee \sim r)$

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p : It is raining

q : There are clouds in sky

r : The sun is shining

Q) If it is raining then there are clouds in the sky.

$p \rightarrow q$

Q) If it is not raining then the sun is not shining and there are clouds in the sky

$\sim p \rightarrow (\sim r \wedge q)$

a) Sun is shining if and only if It is not raining

$r \Leftrightarrow \sim p$.

If either Ram takes C++ or Kumar takes pascal then latha will take java.

Sol:- p : Ram takes C++

q : Kumar takes pascal

r : latha will take java.

$(p \vee q) \rightarrow r$.

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s : Anita goes out for a walk

t : The moon is out.

u : It is snowing.

Translate symbolic form to statemen

1) $(t \wedge \sim u) \rightarrow s$.

If The moon is out and It is not snowing then Anita goes out for a walk.

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2) $t \to (\sim u \to s)$

If The moon is out then, If it is not snowing then Anita goes out for a walk.

---

8)

H.W. p: There is life on Mars.

q: There is life on Europe.

Q) There is life on both Mars and Europe

$p \wedge q$.

Q) There is life on neither Mars nor Europe.

$\sim p \wedge \sim q$

Q) There is life on Mars, but not on Europe.

$p \wedge \sim q$

Q) There is either life on Mars or no life on Europe

$p \vee \sim q$.

Truth Tables

conjunction :- $p \wedge q$ is true only when both are true.

Disjunction :- $p \vee q$ is False only when both are false.

conditional :- $p \rightarrow q$ true to false is only false.

Biconditional :- $p \leftrightarrow q$ is true when both p and q are true or both p & q are false.

Negation :- If p is true then $\sim p$ is false & vice verse.

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conj → T, T → T

Dis : → F, F → F

cond → T, F → F

Bicond → $\left.\begin{matrix} T, T \\ F, F \end{matrix}\right\} \rightarrow T$

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construct the truth table for

1) $p \vee \sim q$

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

2) $(p \vee q) \vee \sim p$

p	q	$\sim p$	$p \vee q$	$(p \vee q) \vee \sim p$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

3) $p \wedge (\sim q \vee q)$

p	q	$\sim q$	$(\sim q \vee q)$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

$p \wedge (p \to q)$

p	q	$p \to q$	$p \wedge (p \to q)$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

$(p \wedge q) \leftrightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \leftrightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T

$\sim(p \vee q) \vee (\sim p \wedge \sim q)$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	Q
T	T	T	F	F	F	F	F
T	F	T	F	F	T	F	F
F	T	T	F	T	F	F	F
F	F	F	T	T	T	T	T

Logical Equivalence → next lower

Two statements S_1, S_2 are said to be logically equivalent if both statements have the same truth values. It is denoted by $S_1 \equiv S_2$

Use truth table to show that

$p \to q \equiv \sim p \vee q$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

→ Equal

The truth values of $p \to q$ & $\sim p \vee q$ are same.

H.W. S.T $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

first
Tautology: A compound statement is called a tautology if it is true in the last column of compound proposition

contradiction: A compound statement is called a contradiction if it is false in the last column of compound proposition.

contigency: A proposition that is neither tautology nor contradiction is called a contigency.

Ex: $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Equal.

8. T.P $p \vee \sim(q \wedge r)$ and

$(p \vee \sim q) \vee \sim r$ are equivalent

Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is tautology.

p	q	r	$(q \to r)$	$p \to (q \to r)$	$p \to q$	$p \to r$	$(p \to q) \to (p \to r)$	Question
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

tautology

S.T $(p \to q) \wedge (q \to r) \wedge (\sim p \wedge \sim r)$
is a ~~contradiction.~~ $\to$ contigency

$\to$ S.T $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
is tautology.

$\to$ $\sim(p \vee (q \wedge r)) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
contradiction $\to$

$\to$ $p \wedge \sim p$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$	$\sim(p \vee (q \wedge r))$	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$	Question
T	T	T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	T	T	F
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	T	T	F
F	T	T	T	T	F	T	T	T	F
F	T	F	F	F	T	T	F	F	F
F	F	T	F	F	T	F	T	F	F
F	F	F	F	F	T	F	F	F	F

contradiction

Equivalent formulas

(or) laws of logic

$(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ } Associative law

$p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$ } commutative law.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ } Distributive law.

$\neg (p \vee q) \equiv \neg p \wedge \neg q$
$\neg (p \wedge q) \equiv \neg p \vee \neg q$ } De-Morgan's law.

$p \vee p \equiv p$
$p \wedge p \equiv p$ } Idempotent law

$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$ } Absorption law.

$p \vee T \equiv T$
$p \wedge F \equiv F$ } Domination law.

$\sim(\sim p) \equiv p$ } Double Negation law.

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$p \vee F \equiv p$
$p \wedge T \equiv p$ } Identity law.

$p \vee \sim p \equiv T$
$p \wedge \sim p \equiv F$ } Inverse law.

State and prove demorgan's laws.

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Prove Distributive law.

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$p \vee \sim p \equiv T$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Without truth table sons

1) S.T $(p \wedge q) \rightarrow (p \vee q)$ is tautology.

$\therefore [p \rightarrow q \equiv \sim p \vee q]$

Sol:

$\sim(p \wedge q) \vee (p \vee q)$

$(\sim p \vee \sim q) \vee (p \vee q)$ [De-morgan's]

$(\sim p \vee p) \vee (\sim q \vee q)$

$T \vee T$ [Inverse law]

$T \rightarrow$ Tautology.//

---

2) S.T $(p \wedge q) \rightarrow p$ is tautology.

Sol:

$\sim(p \wedge q) \vee p$

$(\sim p \vee \sim q) \vee p$ [De-morgan's]

$(\sim p \vee p) \vee \sim q$

$T \vee \sim q \rightarrow T \vee F$

$T \rightarrow$ tautology.

3) S.T $(p\to r)\wedge(q\to r)\equiv(p\vee q)\to r$

Sol: L.H.S $(p\to r)\wedge(q\to r)$

$(\sim p\vee r)\wedge(\sim q\vee r)$

$[p\to q\equiv\sim p\vee q]$

$(\sim p\wedge\sim q)\vee r$

$\sim(p\vee q)\vee r$ (De-morgan's)

$(p\vee q)\to r$ [R.H.S]

Hence proved.

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4) S.T $p\to(q\to r)\equiv(p\wedge q)\to r$

Sol: L.H.S $p\to(q\to r)$

$\sim p\vee(q\to r)$ [$\because p\to q\equiv\sim p\vee q$]

$\sim p\vee(\sim q\vee r)$

$(\sim p\vee\sim q)\vee r$ [Associative law]

$\sim(p\wedge q)\vee r$ [De-morgan's]

$(p\wedge q)\to r\Rightarrow$ R.H.S

Hence proved.

The principle of Duality

Let 's' and 't' be 2 statements. s and t are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$ and

$\vee$ by $\wedge$,

T by F &

F by T

Write the duals of following

1) $(p \vee q) \wedge r$

Ans $(p \wedge q) \vee r$

2) $(p \wedge q) \vee T$

$(p \vee q) \wedge F$

3) $\sim(p \vee q) \wedge (p \vee \sim q) \wedge F$

$\sim(p \wedge q) \vee (p \wedge \sim q) \vee F$

4) $p \leftrightarrow q$

$(p \rightarrow q) \wedge (q \rightarrow p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$

$(\sim p \wedge q) \vee (\sim q \wedge p) \rightarrow$ Dual

Inverse, converse and contrapositive

Statement: $p \to q$

converse : $q \to p$

contrapositive : $\sim q \to \sim p$

Inverse : $\sim p \to \sim q$

p : It rains

q : The ground is wet.

$p \to q$: If it rains then the ground is wet.

converse $q \to p$

If the ground is wet, then it rains.

contrapositive: If the ground is not wet then it doesn't rain.

Inverse: If it doesnot rain then the ground is not wet.

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2) p : you work hard

q : you will be rewarded

3) p : It rains today

q : I buy an umbrella.

S.T $(p \vee q) \wedge \sim(\sim p \wedge q) \equiv p$.

Sol:- L.H.S $(p \vee q) \wedge \sim(\sim p \wedge q)$

$(p \vee q) \wedge \sim(\sim p) \vee \sim q$ [De-Morgan's]

$(p \vee q) \wedge (p \vee \sim q)$ [$\because \sim(\sim p) \equiv p$]

$p \vee (q \wedge \sim q)$ [Associative]

$p \vee F \equiv p$.

S.T $\sim(\sim(p \vee q) \wedge r) \vee \sim q \equiv q \wedge r$

Sol: L.H.S $\sim((\sim(p \vee q) \wedge r)) \wedge \sim(\sim q)$ [De-Morgans]

$((p \vee q) \wedge r) \wedge q$ [$\because \sim(\sim p) \equiv p$]

$((p \vee q) \wedge q) \wedge r \Rightarrow q \wedge r$ [Absorption law]

Normal forms

$\vee \to$ sum
$\wedge \to$ product.

DNF
CNF
PDNF
PCNF

1) Disjunctive Normal form

$\boxed{\wedge \vee \wedge}$

Sum of elementary products is called DNF.

Elementary product

It is a product of variables and their negation

Ex:- $p \wedge \sim q$, $p \wedge q$ [no negation no problem]

$p \wedge q \wedge \sim p \wedge q \wedge \sim p$ [can be repeated]

So DNF $\Rightarrow$

$\underbrace{(p \wedge \sim q)}^{\to pro} \underset{\text{sum}}{\vee} \overbrace{(p \wedge q)}^{\to pro} \underset{\text{sum}}{\vee} \overbrace{(\sim p \wedge \sim q)}^{\to pro}$

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1) Find DNF for $p \wedge (p \to q)$.

Sol:- $\underset{A}{p} \wedge (\underset{B}{\sim p} \vee \underset{C}{q})$ $\{\because p \to q \equiv \sim p \vee q\}$

$\{A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)\}$

$(p \overset{\to pro.}{\wedge} \sim p) \underset{som}{\vee} (p \overset{\to pro}{\wedge} q)$

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2) Find DNF for $p \wedge \sim (q \wedge r)$

Sol:- $\underset{A}{p} \wedge (\underset{B}{\sim q} \vee \underset{C}{\sim r})$ {De-Morgan's}

$\because \{A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)\}$

$(p \wedge \sim q) \underset{som}{\vee} (p \wedge \sim r)$

---

3) Find DNF for $\sim (\underset{P}{p} \to \underset{Q}{(q \wedge r)})$

Sol $\sim (\overset{P}{\sim p} \vee \overset{Q}{(q \wedge r)})$

$\because [p \to q \equiv \sim p \vee q]$

$\sim (\sim p) \wedge \sim (q \wedge r)$ $\because [\sim (p \vee q) \equiv \sim p \wedge \sim q]$

$p \wedge \sim (q \wedge r)$ $[\because \sim (\sim p) \equiv p]$

$\underset{A}{p} \wedge (\underset{B}{\sim q} \vee \underset{C}{\sim r})$ [De-Morgan's]

$(p \wedge \sim q) \underset{\text{Sum}}{\overset{+}{\vee}} (p \wedge \sim r)$ [Distributive]

2) Conjunctive Normal form

Elementary sums

[sum of variables] and their negation

Ex: $p \vee q$, $p \vee \sim q$, $p \vee \sim q \vee p \vee \sim q$

CNF

It is a product of elementary sums.

$\boxed{\vee \wedge \vee}$

$(p \vee q) \wedge (p \vee \sim q) \wedge (p \vee \sim p \vee p \vee \sim q)$

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1) Find CNF for: $p \wedge (p \rightarrow q)$

Sol. $p \wedge (\sim p \vee q)$ $[\because p \rightarrow q \equiv \sim p \vee q]$

$(p \vee p) \wedge (\sim p \vee q)$ $[\because p \vee p \equiv p$ Idempotent$]$

3) Find CNF for

$\sim(p \vee q) \Leftrightarrow (p \wedge q)$

Sol :- Note: $\boxed{p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)}$

$$\underset{p}{\sim(p \vee q)} \longleftrightarrow \underset{q}{(p \wedge q)}$$

$$\underset{p}{\sim(p \vee q)} \rightarrow \underset{q}{(p \wedge q)} \wedge \underset{p}{(p \wedge q)} \rightarrow \underset{q}{\sim(p \vee q)}$$

$\because [p \rightarrow q \equiv \sim p \vee q]$

$\Rightarrow \sim(\sim(p \vee q)) \vee (p \wedge q) \wedge \sim(p \wedge q) \vee \sim(p \vee q)$

$\Rightarrow \underset{A}{(p \vee q)} \vee (\underset{B}{p} \wedge \underset{C}{q}) \wedge \underset{A}{(\sim p \vee \sim q)} \vee (\underset{B}{\sim p} \wedge \underset{C}{\sim q})$

$[\because \sim(\sim p) \equiv p$, De-Morgan's$]$

$\because [A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)]$

$\Rightarrow (p \vee q \vee p) \wedge (p \vee q \vee q)$

$\wedge (\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q)$

convert $(p \to q) \wedge (\sim p \wedge q)$ to CNF

DNF

Sol $(\sim p \vee q) \wedge (\sim p \wedge q)$

$\quad$ B $\quad$ C $\quad\quad$ A

$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$

$\Rightarrow (\sim p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge q)$

$\Rightarrow (\sim p \wedge q) \vee (\sim p \wedge q)$

$\because [p \wedge p \equiv p]$

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$\vee (q \wedge \sim p)$ ...

CNF $[\vee \wedge \vee]$

$(p \to q) \wedge (\sim p \wedge q)$

$\Rightarrow (\sim p \vee q) \wedge (\sim p \wedge q)$

$\Rightarrow (\sim p \vee q) \wedge (\sim p \vee F) \wedge (q \vee F)$

$\Rightarrow (\sim p \vee q) \wedge (\sim p \vee (q \wedge \sim q)) \wedge (q \vee (p \wedge \sim p))$

A B C

$\Rightarrow (\sim p \vee q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

$\wedge (q \vee p) \wedge (q \vee \sim p)$

$\Rightarrow (\sim p \vee q) \wedge (\sim p \vee \sim q) \wedge (q \vee p)$

$\wedge (q \vee \sim p)$

No need to write repeated terms.

3) Find CNF for

$\sim(p \vee q) \Leftrightarrow (p \wedge q)$

Sol :- Note: $\boxed{p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$\sim(p \vee q) \leftrightarrow (p \wedge q)$
$\quad\; (p) \qquad\qquad (q)$

$\sim(p \vee q) \to (p \wedge q) \wedge (p \wedge q) \to \sim(p \vee q)$
$\quad (p) \qquad\qquad (q) \qquad\qquad (p) \qquad\qquad (q)$

$\because [p \to q \equiv \sim p \vee q]$

$\Rightarrow \sim(\sim(p \vee q)) \vee (p \wedge q) \wedge \sim(p \wedge q) \vee \sim(p \vee q)$

$\Rightarrow (p \vee q) \vee (p \wedge q) \wedge (\sim p \vee \sim q) \vee (\sim p \wedge \sim q)$
$\quad (A) \qquad (B) \quad (C) \qquad\qquad (A) \qquad\qquad (B) \quad (C)$

$[\because \sim(\sim p) \equiv p$, De-Morgan's$]$

$\because [A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)]$

$\Rightarrow (p \vee q \vee p) \wedge (p \vee q \vee q)$

$\wedge (\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q)$

Working Rule for both DNF/CNF

1) Remove $\rightarrow, \leftrightarrow$ symbols by using equivalence formulas mentioned below.

2) Eliminate '$\sim$' by using De-morgan's law

before sums & products.

3) Apply Distributive law.

Formulas

1) $p \rightarrow q \equiv \sim p \vee q$

2) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

(or)

3) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$.

De-morgan's

$\sim (p \wedge q) \equiv \sim p \vee \sim q$

$\sim (p \vee q) \equiv \sim p \wedge \sim q$

Distributive

$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$ → CNF → PCNF

$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$.

→ DNF

Minterms:-

The minterms consists of '∧' conjunctions in which each variable or its negation; but not both appears

p&p:-

p	q	minterms(∧)
T	T	$p \wedge q$
T	F	$p \wedge \sim q$
F	T	$\sim p \wedge q$
F	F	$\sim p \wedge \sim q$

$\boxed{F \rightarrow \sim}$

Maxterms:

The maxterms consists of disjunctions 'v' in which each variable or its negations, but not both appears

$\boxed{T \rightarrow \sim}$

p	q	maxterms(v)
T	T	$\sim p \vee \sim q$
T	F	$\sim p \vee q$
F	T	$p \vee \sim q$
F	F	$p \vee q$

Minterms and Maxterms for 3 propositions

p	q	r	Minterms (∧) [F → ~]	Maxterms (∨) [T → ~]
T	T	T	$p \wedge q \wedge r$	$\sim p \vee \sim q \vee \sim r$
T	T	F	$p \wedge q \wedge \sim r$	$\sim p \vee \sim q \vee r$
T	F	T	$p \wedge \sim q \wedge r$	$\sim p \vee q \vee \sim r$
T	F	F	$p \wedge \sim q \wedge \sim r$	$\sim p \vee q \vee r$
F	T	T	$\sim p \wedge q \wedge r$	$p \vee \sim q \vee \sim r$
F	T	F	$\sim p \wedge q \wedge \sim r$	$p \vee \sim q \vee r$
F	F	T	$\sim p \wedge \sim q \wedge r$	$p \vee q \vee \sim r$
F	F	F	$\sim p \wedge \sim q \wedge \sim r$	$p \vee q \vee r$

PDNF :- sum of minterms [only T]

PCNF :- product of maxterms. [only F]

Principal Disjunction Normal form [PDNF]

Sum of all minterms is called PDNF [only T values] → A

working rule of PDNF without truth table –

1) Replace →, ↔ by ∨, ∧, ~

2) Apply "∧T" for missing term

[$P \wedge T \equiv P$] ⇒ Identity law.

3) Instead of T, apply $P \vee \sim P$

[$P \vee \sim P \equiv T$] ⇒ Negation law.

4) Apply Distributive law

5) If there are Identical minterms [means repeated] need to be written only once.

1) Obtain PDNF for $p \to q$

i) with truth table

2) without using truth table

(1)

p	q	$p \to q$	Minterms [F→~] (∧)
T	T	T	$p \wedge q$
T	F	F	
F	T	T	$\sim p \wedge q$
F	F	T	$\sim p \wedge \sim q$

$\because$ sum of all minterms ⇒ PDNF

$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

(2) without truth table

⇒ $p \to q$

⇒ $\sim p \vee q$ $\quad [\because p \to q \equiv \sim p \vee q]$

⇒ $(\sim p \wedge \overset{\text{q is miss}}{T}) \vee (q \wedge \overset{\text{p is miss}}{T})$

⇒ $(\underset{A}{\sim p} \wedge (\underset{B}{q} \vee \underset{C}{\sim q})) \vee (\underset{A}{q} \wedge (\underset{B}{p} \vee \underset{C}{\sim p}))$

⇒ $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge p)$
$\vee (q \wedge \sim p)$ ⇒ repeated

$\boxed{\because A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)}$

$\Rightarrow (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge p)$ //

obtain PDNF of

$(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$

1) with truth table

2) without truth table.

1)

p	q	r	$p \wedge q$	$\sim p$	$\sim p \wedge r$	$q \wedge r$	Q(x)	min term ($\wedge$) [F→~]
T	T	T	T	F	F	T	T	$p \wedge q \wedge r$
T	T	F	T	F	F	F	T	$p \wedge q \wedge \sim r$
T	F	T	F	F	F	F	F	
T	F	F	F	F	F	F	F	
F	T	T	F	T	T	T	T	$\sim p \wedge q \wedge r$
F	T	F	F	T	F	F	F	
F	F	T	F	T	T	F	T	$\sim p \wedge \sim q \wedge r$
F	F	F	F	T	F	F	F	

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r)$

$\vee (\sim p \wedge \sim q \wedge r)$

2) without truth table

$$(p\wedge q)\vee(\sim p\wedge r)\vee(q\wedge r)$$

Sol: $(p\wedge q\wedge T)\vee(\sim p\wedge r\wedge T)\vee(q\wedge r\wedge T)$

$\Rightarrow (\overset{A}{p\wedge q}\wedge(\overset{B}{r}\vee\overset{C}{\sim r}))\vee(\overset{A}{\sim p\wedge r}\wedge(\overset{B}{q}\vee\overset{C}{\sim q}))$

$\vee(\overset{A}{q\wedge r}\wedge(\overset{B}{p}\vee\overset{C}{\sim p}))$

$\Rightarrow$

$\boxed{\because A\wedge(B\vee C)\equiv(A\wedge B)\vee(A\wedge C)}$ $\left[\begin{array}{l} p\vee\sim p\equiv T \\ q\vee\sim q\equiv T \\ r\vee\sim r\equiv T \end{array}\right]$

$\Rightarrow (p\wedge q\wedge r)\vee(p\wedge q\wedge\sim r)\vee(\sim p\wedge r\wedge q)$

$\vee(\sim p\wedge r\wedge\sim q)$

No repeated terms

PDNF	PCNF
→ $\wedge T$ missing term	$\vee F$ missing term
→ $p\vee\sim p$ instead of T	$p\wedge\sim p$ instead of F
→ Σ sum of minterms ($\wedge$) [only T] F→~	Π product of maxterms ($\vee$) T→~ [only F]

4) Principal conjunctive Normal form [PCNF]

$\wedge$ of maxterms [only F]

working Rule for without truthtable

1) Replace $\rightarrow, \leftrightarrow$ by $\vee, \wedge, \sim$

2) Apply "$\vee F$" for missing term

[$\because P \vee F \equiv P$] $\rightarrow$ Identity law.

3) Instead of F, apply $P \wedge \sim P$

$\because [P \wedge \sim P \equiv F] \rightarrow$ Negation law

4) Apply Distributive law.

5) If there are identical maxterms, need to be written only once.

---

1) obtain PCNF for $P \wedge q$

(i) with truth table

(ii) without truthtable

$T \rightarrow \sim$

P	q	$P \wedge q$	maxterms ($\vee$)
T	T	T	
T	F	F	$\sim p \vee q$
F	T	F	$p \vee \sim q$.
F	F	F	$p \vee q$

$\Rightarrow (\sim p \vee q) \wedge (p \vee \sim q) \wedge (p \vee q)$

(ii) $p \wedge q$

$\Rightarrow (p \vee F) \wedge (q \vee F)$

$\Rightarrow (p \vee (q \wedge \sim q)) \wedge (q \vee (p \wedge \sim p))$

A B C A B C

$\Rightarrow (\underline{p \vee q}) \wedge (p \vee \sim q) \wedge (\underline{q \vee p}) \wedge (q \vee \sim p)$

$\Rightarrow (p \vee q) \wedge (p \vee \sim q) \wedge (q \vee \sim p)$ //

Same Answers !

2) PCNF

$(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

Sol:- (i) with truth table.

p	q	r	$\sim p$	$\sim p \rightarrow r$	$q \leftrightarrow p$	Question	maxterms
T	T	T	F	T	T	T	
T	T	F	F	T	T	T	
T	F	T	F	T	F	F	$\sim p \vee q \vee \sim r$
T	F	F	F	T	F	F	$\sim p \vee q \vee r$
F	T	T	T	T	F	F	$p \vee \sim q \vee \sim r$
F	T	F	T	F	F	F	$p \vee \sim q \vee r$
F	F	T	T	T	T	T	
F	F	F	T	F	T	F	$p \vee q \vee r$

$\Rightarrow (\sim p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r)$

$\wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r)$

(ii) without truth table

$(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

Sol:- $(\sim(\sim p) \vee r) \wedge (q \rightarrow p) \wedge (p \rightarrow q)$

$\Rightarrow (p \vee r) \wedge (\sim q \vee p) \wedge (\sim p \vee q)$

$\left[\begin{array}{l} \because \sim(\sim p) \equiv p \\ p \rightarrow q \equiv \sim p \vee q \end{array} \right]$

$\Rightarrow (p \vee r \vee F) \wedge (\sim q \vee p \vee F) \wedge (\sim p \vee q \vee F)$

$\Rightarrow (p \vee r \vee (q \wedge \sim q)) \wedge (\sim q \vee p \vee (r \wedge \sim r))$

A B C A B C

$\wedge (\sim p \vee q \vee (r \wedge \sim r))$

A B C

$\Rightarrow (p \vee r \vee q) \wedge (p \vee r \vee \sim q) \wedge (\sim q \vee p \vee r)$

$\wedge (\sim q \vee p \vee \sim r) \wedge (\sim p \vee q \vee r)$

$\wedge (\sim p \vee q \vee \sim r)$

$\Rightarrow (p \vee q \vee r) \wedge (p \vee r \vee \sim q) \wedge (\sim q \vee p \vee \sim r)$

$\wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

Both same answers

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H.W

→ obtain PDNF of $q \vee (p \vee \sim q)$

→ PDNF of $\sim p \vee q$

→ Find PCNF & PDNF of

$(\sim p \rightarrow q) \wedge (p \leftrightarrow q)$ with &

without Truth tables

The theory of Inference for Statement calculus

Inference means deriving conclusions from evidences.
"premises."

Premises: It is a proposition on the basis of which would be able to draw a conclusion.

[You can think premise as an evidence or assumption].

conclusion: It is a proposition that is reached from the given set of premises.

[If premise then conclusion].

Argument: It is a set of one or more premises and conclusion

consistent: A set of formulas $H_1, H_2, \ldots, H_n$ is said to be consistent if ~~there~~ $H_1 \wedge H_2 \wedge H_3 \ldots \wedge H_n$ conclusion is tautology.

If it is contradiction, it is Inconsistent

Validity using Truth Tables

Let A and B be 2 statements formulas, we say that "B logically follows from A" (or) "B is a valid conclusion from the premise A" iff $A \rightarrow B$ is a tautology.]

Note: consider a set of premises $H_1, H_2, \ldots, H_n$ and conclusion C, then compound proposition of form

$$H_1 \wedge H_2 \wedge H_3 \wedge \ldots \wedge H_n \rightarrow C \text{ tautology}$$

Is called an Argument where $H_1, H_2, \ldots, H_n$ are called premises & C is called conclusion of Argument.

$$\underbrace{H_1, H_2, \ldots, H_n}_{\text{Premises}} \rightarrow \underbrace{C}_{\text{conclusion}}$$

$$\underbrace{H_1 \wedge H_2 \wedge H_3 \wedge \ldots \wedge H_n \rightarrow C}_{\text{Argument}}$$

check the validity using truth tables

(1) $H_1 : p \to q$, $H_2 : p$, $c : q$

p (H_2)	q (c)	$p \to q$ (H_1)	$H_1 \land H_2$	$H_1 \land H_2 \to c$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$\because H_1 \land H_2 \to c$ is a tautology.

$\therefore$ 'c' is a valid conclusion

$\therefore$ ~~It is consistent.~~

(2) $H_1 : p \to q$, $H_2 : \sim p$, $c : q$

$\sim p$ (H_2)	p	q (c)	$p \to q$ (H_1)	$H_1 \land H_2$	$H_1 \land H_2 \to c$
F	T	T	T	F	T
F	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F

$\because H_1 \land H_2 \to c$ is not a tautology

$\therefore$ c is not a valid conclusion

$H_1: \sim p$, $H_2: p \leftrightarrow q$, $C: \sim(p \wedge q)$

Sol: p q $\sim p$ $p \leftrightarrow q$ $p \wedge q$ $\sim(p \wedge q)$

		H_1	H_2		C		
p	q	$\sim p$	$p \leftrightarrow q$	$p \wedge q$	$\sim(p \wedge q)$	$H_1 \wedge H_2$	$H_1 \wedge H_2 \to C$
T	T	F	T	T	F	F	T
T	F	F	F	F	T	F	T
F	T	T	F	F	T	F	T
F	F	T	T	F	T	T	T

valid

Rule P: A premise may be introduced at any step in the derivation

Rule T: A formula may be introduced in the derivation

Rules of Inference

Modus Ponens

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

Modus Tollens

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

Hypothetical syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Disjunctive syllogism

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Disjunction $\dfrac{\begin{array}{c}p\\q\end{array}}{\therefore p \vee q}$

Addition $\dfrac{p}{\therefore p \vee q}$ (or) $\dfrac{q}{p \vee q}$

Simplification $\dfrac{p \wedge q}{\therefore p}$ (or) $\dfrac{p \wedge q}{\therefore q}$

Dilemma

$$\dfrac{\begin{array}{c}p \vee q\\p \to r\\q \to r\end{array}}{\therefore r}$$

Constructive Dilemma

$$\dfrac{\begin{array}{c}(p \to q) \wedge (r \to s)\\p \vee r\end{array}}{\therefore q \vee s}$$

Destructive Dilemma

$$\dfrac{\begin{array}{c}(p \to q) \wedge (r \to s)\\\sim q \vee \sim s\end{array}}{\therefore \sim p \vee \sim r}$$

Resolution

$$\dfrac{\begin{array}{c}p \vee q\\\sim p \vee r\end{array}}{\therefore q \vee r}$$

3 types → Direct Proof
→ Indirect proof
→ conditional proof.

1) Direct Proof :- when a conclusion is derived from a set of premises by using the equivalence and implication rules, then the process of derivation is called direct proof.

H.W. $H_1 : p \vee q$, $H_2 : p \to r$, $H_3 : q \to r$

$C : r$

p q r $p \vee q$ $p \to r$ $q \to r$

p	q	C r	H_1 $p \vee q$	H_2 $p \to r$	H_3 $q \to r$	$H_1 \wedge H_2 \wedge H_3$	$H_1 \wedge H_2 \wedge H_3 \to C$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

∴ tautology.

∴ C is a valid

S.T $H_1: p \to q$, $H_2: q \to \sim r$, $H_3: p \to r$, $p: H_4$

is consistent.

p (H_4)	q	r	$p \to q$ (H_1)	$\sim r$	$q \to \sim r$ (H_2)	$p \to r$ (H_3)	$H_1 \wedge H_2 \wedge H_3 \wedge H_4$
T	T	T	T	F	F	T	F
T	T	F	T	T	T	F	F
T	F	T	F	F	T	T	F
T	F	F	F	T	T	F	F
F	T	T	T	F	F	T	F
F	T	F	T	T	T	T	F
F	F	T	T	F	T	T	F
F	F	F	T	T	T	T	F

Inconsistent.

⇒ $H_1 \wedge H_2 \to C$ is tautology then 'c' is valid otherwise Invalid

⇒ [No conditional $\to c$] only if $H_1 \wedge H_2 \wedge H_3 \wedge H_4$ is tautology means consistent, otherwise Inconsistent

Demonstrate that R is a valid inference from the premises $p \to q$, $q \to r$, p

Sol:- $p \to q$ (Rule P)

$q \to r$ (Rule P)

$p \to r$ (Rule T) (Hypothetical Syllogism)

p Rule P.

r (Rule T) Modus Ponens.

1) Demonstrate that 'r' is a valid inference from the premises $p \to q$, $q \to r$, p

Sol: (1) $p \to q$ Rule P

(2) $q \to r$ Rule P

(3) $p \to r$ Rule T [Hypothetical syllogism]

(4) p Rule P

(5) r Rule T [Modus Ponens].

$\therefore$ r is a valid inference.

---

2) S.T $R \vee S$ can be derived from the premises $C \vee D$, $(C \vee D) \to \sim H$, $(A \wedge \sim B) \to R \vee S$, $\sim H \to (A \wedge \sim B)$

Sol: (1) $C \vee D$ Rule P

(2) $(C \vee D) \to \sim H$ Rule P

(3) $\sim H$ Rule T [Modus Ponens]

(4) $\sim H \to (A \wedge \sim B)$ Rule P

(5) $(A \wedge \sim B)$ Rule T [Modus Ponens]

(6) $(A \wedge \sim B) \to R \vee S$ Rule P

(7) $R \vee S$ Rule T [Modus Po...

1) S.T $T \wedge S$ can be derived from the premises $P \to q$, $q \to \sim r$, r, $P \vee (T \wedge S)$

Sol:- (1) $P \to Q$ Rule P

(2) $Q \to \sim R$ Rule P

(3) $P \to \sim R$ Rule T [Hypothetical syllogism]

(4) R Rule P

(5) $\sim(\sim R)$ Rule T [Double Negation]

(6) $\sim P$ Rule T [Modus Tollens] [using 3 & 5.]

(7) $P \vee (T \wedge S)$ Rule P

(8) $T \wedge S$ Rule T [Disjunctive syllogism]

3) S.T $R \wedge (P \vee Q)$ from $P \vee Q$, $Q \rightarrow R$; $P \rightarrow M$, $\sim M$.

Sol:

(1)	$P \rightarrow M$	Rule P
(2)	$\sim M$	Rule P
(3)	$\sim P$	Rule T [Modus Tollens]
(4)	$P \vee Q$	Rule P
(5)	Q	Rule T [Disjunctive syllogism]
(6)	$Q \rightarrow R$	Rule P
(7)	R	Rule T [Modus Ponens]
(8)	$R \wedge (P \vee Q)$	Rule T [Using 4 & 5 conjunction.

* S.T $S \vee R$ is tautologically implied by $P \vee Q$, $P \to R$, $Q \to S$.

Sol:

(1) $P \vee Q$ Rule P

(2) $P \to R$ Rule P

(3) $\sim P \vee R$ Rule T [$\because p \to q \equiv \sim p \vee q$]

(4) $Q \vee R$ Rule T [using 1 & 3 Resolution]

(5) $Q \to S$ Rule P

(6) $\sim Q \vee S$ Rule T [$\because p \to q \equiv \sim p \vee q$]

(7) $R \vee S$ Rule T [using 4 & 6 Resolution]

---

3) Conditional Proof [Rule CP]

1) S.T $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\sim R \vee P$, Q

Sol: (1) R Rule P [Assumed premise]

(2) $\sim R \vee P$ Rule P

(3) $R \to P$ Rule T [$\because P \to q \equiv \sim p \vee q$]

(4) P Rule T [using 1 & 3 Modus Pollens]

(5) $P \to (Q \to S)$ Rule P

(6) $Q \to S$ Rule T [using 4 & 5 Modus Pollens]

(7) Q. Rule P

(8) S Rule T [using 6 & 7 Modus Pollens]

(9) $R \to S$ Rule CP

$\therefore$ $R \to S$ is a valid conclusion

2) Derive the following using CP rule if necessary $\sim P \vee Q, \sim Q \vee R, R \rightarrow S \Rightarrow P \rightarrow S$

Sol

(1)	P	Rule P [Assumed premise]
(2)	$\sim P \vee Q$	Rule P
(3)	$P \rightarrow Q$	Rule T [$\because P \rightarrow q \equiv \sim p \vee q$]
(4)	$\sim Q \vee R$	Rule P
(5)	$Q \rightarrow R$	Rule T
(6)	$R \rightarrow S$	Rule P
(7)	$Q \rightarrow S$	Rule T {hypothetical syllogism}
(8)	$P \rightarrow S$	Rule T [using 3 & 7 hypothetical syllogism]
(9)	S	Rule T [using 1 & 8] [Modus pollens]

Another way

$\sim P \vee Q, \sim Q \vee R, R \to S \Rightarrow P \to S$

Sol:-

(1) P	Rule P	
(2) $\sim P \vee Q$	Rule P	
(3) $P \to Q$	Rule T	
(4) Q	Rule T	[using 1 & 3 Modus Pollens]
(5) $\sim Q \vee R$	Rule P	
(6) $Q \to R$	Rule T	
(7) $R \to S$	Rule P	
(8) $Q \to S$	Rule T	[hypothetical]
(9) S	Rule T	[using 4 & 8]
(10) $P \to S$	Rule CP	

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Indirect Method

For doing indirect method introduce negation ($\sim$) to the conclusion as a additional premise and together with the given premises derive conclusion.

[conjunction must be false.

$$p \wedge \sim p \equiv F$$

$$q \wedge \sim q \equiv F, \quad r \wedge \sim r \equiv F$$

---

1) using Indirect method P.T

$p \to q, q \to r, p \vee r \Rightarrow r$

Sol:-

(1)	$\sim r$	Rule P	[Assumed]
(2)	$p \to q$	Rule P	
(3)	$q \to r$	Rule P	
(4)	$p \to r$	Rule T	[hypothetical 2 & 3]
(5)	$\sim p$	Rule T	[using 1 & 4 Modus Tollens]
(6)	$p \vee r$	Rule P	
(7)	r	Rule T	[5 & 6 Disjunctive]
(8)	$r \wedge \sim r$	Rule T	[7 & 1 conjunction]

→ P.T $P \to Q, Q \to S, P \vee Q \Rightarrow S \vee R$

Sol:- (1) $\sim(S \vee R)$ Rule P [Assumed]

(2) $\sim S \wedge \sim R$ [Rule T [De-Morgans]]

(3) $\sim S$ Rule T [simplification]

(4) $Q \to S$ Rule P

(5) $\sim Q$ Rule T [3 & 4 Modus Tollens]

(6) $P \to Q$ Rule P

(7) $\sim P$ Rule T [5 & 6 Modus Tollens]

(8) $P \vee Q$ Rule P

(9) Q Rule T [7 & 8 Disjunctive]

(10) $Q \wedge \sim Q$ Rule T [conjunction 5 & 9]

$\perp$

False.

H.W Using Indirect method S.T

$R \to \sim Q, R \vee S, S \to \sim Q, P \to Q \Rightarrow \sim P$

Sol

(1) $\sim(\sim P)$ Rule P

(2) P Rule T [$\because \sim(\sim P) \equiv P$]

(3) $P \to Q$ Rule P

(4) $Q \to \sim(\sim Q)$ Rule T [2 & 3 Modus Ponens]

(5) $R \to \sim Q$ Rule P

(6) $\sim R$ Rule T [4 & 5 Modus Tollens]

(7) $R \vee S$ Rule P.

(8) S Rule T [6 & 7 Disjunctive]

(9) $S \to \sim Q$ Rule P

(10) $\sim Q$ Rule T [8 & 9 Modus Ponens]

(11) $Q \wedge \sim Q$ Rule T [conjunction 4 & 10]

$\downarrow$

False.

check validity through statements

1) If Ram works hard, he will get a job. Ram works hard therefore he will get a job check validity

Sol:- p : Ram works hard

q : He will get a job.

premises : $p \to q$, p

Conclusion $\therefore q$

$$\frac{\begin{matrix} p \to q \\ p \end{matrix}}{q}$$ → Modus Ponens.

∴ The argument is valid.

If a man is a bachelor, he is unhappy, If a man is unhappy, he dies young. Therefore if bachelors die young.

check validity.

Sol:- p : A man is a bachelor.

q : He is unhappy.

r : He dies young.

Premises: $p \to q$, $q \to r$

Conclusion: $p \to r$

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$ → hypothetical syllogism

$\therefore$ valid.

If this number is divisible by 6, then it is divisible by 3. This number is not divisible by 3. Therefore this number is not divisible by 6

Sol: p : This no. is divisible by 6

q : It is divisible by 3.

Premises: $p \to q$, $\sim q$

Conclusion: $\sim p$.

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \sim p \end{array}$$ → Modus Tollen's

$\therefore$ Argument is valid.

Predicate calculus

predicate :- It describes something about one or more subjects.

Ex:- Ram is a good boy

Ram → subject; good boy → predicate

→ Generally predicates are denoted by upper case letters and subjects are denoted by lower case.

Ex:- 1) Sita is an Intelligent girl : I(s)

2) Anita is a student : S(a)

3) x is a Man : M(x)

4) Suresh is taller than Mahesh : T(s,m)

Quantifiers

There are 2 types of quantifiers

1) Universal Quantifier [$\forall$]

Sentence contains words such as for all, forevery, for each, for any, Everything, Anything.

2) Existential Quantifiers [$\exists$]

Sentence contains words such as there exists, for some, there is, atleast one, there exists some.

Negation of Quantifier

Statement	Negation
$(\forall x).P(x)$	$(\exists x).\sim P(x)$
$(\forall x).\sim P(x)$	$(\exists x).P(x)$
$(\exists x).P(x)$	$(\forall x).\sim P(x)$
$(\exists x)\sim .P(x)$	$(\forall x).P(x)$

write down the Quantifier statement to symbolic form.

a) All monkeys have tails

b) No monkey has a tail.

c) some monkeys have tails.

d) some monkeys has no tails.

Sol:- Let us consider.

$M(x)$: x is a Monkey

$T(x)$: x has a tail.

a) All monkeys have tails.

ex:- If there are 10 monkeys then 10 monkeys have tails.

$$(\forall x)[M(x) \rightarrow T(x)].$$

---

b) No monkey has a tail.

Sol:- All monkeys has no tails

If there are 10 monkeys then 10 has no tails

$$(\forall x)(M(x) \rightarrow \sim T(x))$$

---

d) Some monkeys have no tails.

Sol:- out of 10 monkeys 6 haves tails and 4 doesn't have.

$$\exists x\,(M(x) \wedge \sim T(x))$$

c) Some monkeys have ~~bar~~ tails

Sol: out of 10, 6 ~~are~~ monkeys and they have tails.

$\exists x\,(M(x) \wedge T(x))$

---

2) a) All men are good

b) No men are good

c) Some men are good

d) Some men are not good

Sol: a) let $M(x)$: x is a Man

$G(x)$: x is Good

a) $(\forall x)(M(x) \rightarrow G(x))$

b) $(\forall x)(M(x) \rightarrow \sim G(x))$

c) $(\exists x)(M(x) \wedge G(x))$

d) $(\exists x)(M(x) \wedge \sim G(x))$

$\boxed{\begin{array}{l}\text{All} \Rightarrow \forall x \\ \text{Some} \Rightarrow \exists x\end{array}}$

Inference theory for Predicate calculus

Rule US [Universal specification]

from $(\forall x)\, p(x)$ one may
(or)
$(x)\, p(x)$

concluded that $p(y)$.

Rule ES [Existential specification]

from $(\exists x)\, p(x)$ one may conclude that $p(y)$.

Rule UG [Universal Generalisation]

from $p(y)$ one may conclude that $(x).p(x)$

Rule EG [Existential Generalisati[on]

from $p(y)$ one may conclude that $(\exists x).p(x)$.

US, ES } delete

UG, EG } add.

S.T $(x)(H(x) \to M(x)) \land H(3) \Rightarrow M(3)$

Sol:

(1) $(x)(H(x) \to M(x))$ Rule P.

(2) $H(3) \to M(3)$ Rule US (1)

(3) $H(3)$ Rule P.

(4) $M(3)$ Rule T [Modus Ponens 2 & 3].

S.T $(x)(P(x) \to Q(x)) \land (x)(Q(x) \to R(x)) \Rightarrow (x)(P(x) \to R(x))$

Sol:

(1) $(x)(P(x) \to Q(x))$ Rule P.

(2) $(P(y) \to Q(y))$ Rule US (1)

(3) $(x)(Q(x) \to R(x))$ Rule P

(4) $(Q(y) \to R(y))$ Rule US (3)

(5) $P(y) \to R(y)$ Rule T [2 & 4]

(6) $(x)(P(x) \to R(x))$ Rule UG (5)

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S.T $(\exists x) M(x)$ follows logically from the premises.

$(x)(H(x) \to M(x))$ and $(\exists x) H(x)$

Sol:- (1) $(x)(H(x) \to M(x))$ Rule P

(2) $H(y) \to M(y)$ Rule US (1)

(3) $(\exists x) H(x)$ Rule P

(4) $H(y)$ Rule ES (3)

(5) $M(y)$ Rule T [Modus Ponens 2 & 4]

(6) $(\exists x) M(x)$ Rule EG (5).

P.T $(\exists x)(P(x) \land Q(x)) \Rightarrow (\exists x) P(x) \land (\exists x). Q(x)$

Sol:- (1) $(\exists x)(P(x) \land Q(x))$ Rule P

(2) $P(y) \land Q(y)$ Rule ES (1)

(3) $P(y)$ Rule T [simplification]

(4) $(\exists x). P(x)$ Rule EG (3)

(5) $Q(y)$ Rule T [simplification]

(6) $(\exists x) Q(x)$ Rule EG (5)

(7) $(\exists x) P(x) \land (\exists x) Q(x)$ [Rule T Conjunction rule]