DISCRETRE MATHEMATICS

UNIT 1

1. Define Negation, Disjunction, conjunction, Implication and Biimplication with example.
2. Construct the truth table of the compound proposition $(p \vee \sim q) \rightarrow p \wedge q$.
3. Construct truth table for $p \vee \sim q$
4. Construct truth table for $(p \vee q) \vee \sim p$
5. Construct truth table for $\sim (p \wedge q) \leftrightarrow (\sim p \vee \sim q)$
6. Show that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
7. Show that $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is a tautology
8. Prove that $p \vee \sim(q \wedge r)$ and $((p \vee \sim q) \vee (\sim r))$ are equivalent.
9. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent
10. Write converse, inverse, contrapositive for the given statement

    "If it rains then the ground is wet"
11. Write the symbolic form for given statement

    "If either Ram takes C++ or Kumar takes pascal then Latha will take java"
12. Find DNF for $p \wedge (p \rightarrow q)$
13. Find DNF for $\neg (p \rightarrow (q \wedge r))$
14. Find CNF for $(p \rightarrow q) \wedge (\neg p \wedge q)$
15. Find the PDNF & PCNF by constructing the truth table: $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.
16. Obtain PDNF of $(p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$ without using truth table
17. Obtain PCNF of $p \leftrightarrow q$
    (i) With using truth table
    (ii) Without using truth table
18. Given the following statements as premises, all referring to an arbitrary meal: i) If he takes coffee, he doesn't drink milk ii) He eats crackers only if he drinks milk iii) He does not take soup unless he eats crackers iv) At noon today, he had coffee Whether he took soup at noon today? If so, what is the correct conclusion?
19. Check the validity using truth table

    $H1 : p \rightarrow q$, $H2 : \neg p$, $C : q$
20. Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$
21. Prove by indirect method $\neg Q$, $P \rightarrow Q$, $P \vee R \rightarrow Q$
22. Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$,

    $\neg R \vee P$, Q
23. Prove that $R \wedge (P \vee Q)$ is a valid conclusion from the premises PVQ , $Q \rightarrow R$, $P \rightarrow M$ and $7M$.
24. Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $7R \vee P$, and Q
25. .Show that $P \rightarrow S$ can be derived from the premises, $7P \vee Q$, $7Q \vee R$, and $R \rightarrow S$.
26. Consider these statements "All lions are fierce", "Some lions do not drink coffee", "Some fierce creatures do not drink coffee"

    Let P(x),Q(x), and R(x) be the statements "x is a lion", "x is fierce" and "x drinks coffee" respectively. Assuming that the domain consists of all creatures express the statement in the argument using quantifiers and P(x),Q(x) and R(x).

27. Show that $\neg\forall x\,(P(x) \rightarrow Q(x))$ and $\exists\, x\,(P(x) \wedge \neg Q(x))$ are logically equivalent.
28. Show that $(x)\,(P(x) \rightarrow Q(x)) \wedge (x)\,(Q(x) \rightarrow R(x)) \rightarrow (x)\,(P(x) \rightarrow R(x))$

UNIT 2

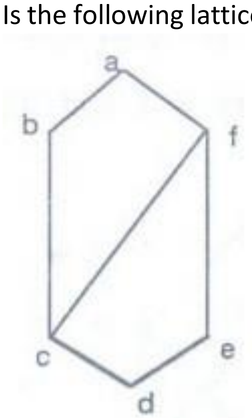
1. Define set, powerset, cardinality of set
2. For sets P = {a, b, c, d} and Q = {c, d, e, f, g}, find P ∪ Q and P - Q.
3. List and explain various operations performed on sets.
4. Set X contains all even numbers between 1 and 20, and Set Y contains all multiples of 3 between 1 and 20. Find X ∩ Y.
5. If Set A = {2, 4, 6, 8} and Set B = {1, 3, 5, 7, 9}, find the Cartesian product A × B and B × A.
6. Let R and S be the following relations $A = \{a, b, c, d\}$ defined
   $R = \{(a, a), (a, c), (c, b), (c, d), (d, b)\}$ and $S = \{(b, a), (c, c), (c, d), (d, a)\}$
   Find i) RoS (ii) SoR iii) RoR.
7. Define Equivalence relation. Show that the “divides” relation is the set of positive integers is not an equivalence relation.
8. Let $X = \{1,2,3,4,5,6,7\}$ and $R = \{(x, y)/x - y \textit{ is divisible by } 3\} \textit{ in } X$. Show that R is an equivalence relation.
9. Define Hasse diagram.
10. Draw the Hasse diagram representing the partial ordering.
    $\{(a,b)\,/\,a \text{ divides } b\}$ on $\{1,2,3,4,6,8,12\}$.
11. If D(n) denotes the lattice of all the divisors of the integer n draw the Hasse diagrams of D(10), D(15), D(32) and D(45).
12. Define totally ordered set with an example.
13. Define minimal and maximal elements of a poset.
14. Give an example of a set X such that (P(X), ≤) is a totally ordered set.
15. Draw the Hasse diagrams of the following sets under the partial ordering relation "divides" and indicate those which are totally ordered. {2, 6, 24}, {3, 5, 15}, {1, 2, 3, 6, 12}, {2, 4, 8, 16}, {3, 9, 27, 54}
16. Check whether the posets {(1, 3, 6, 9), D} and {(1, 5, 25, 125), D} are lattices or not. Justify your claim.
17. Consider the set D_{50} = {1, 2, 5, 10, 25, 50} and the relation divides () be a partial ordering relation on D_{50}

    (1) Draw the Hasse diagram of D_{50} with relation divides.

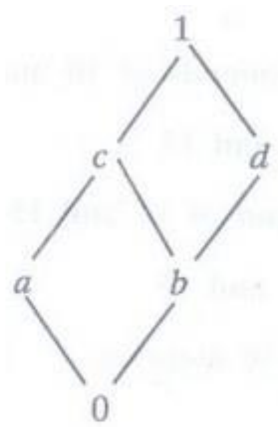
    (2) Determine all upper bounds of 5 and 10.
    (3) Determine all lower bounds of 5 and 10.
    (4) Determine LUB of 5 and 10.
    (5) Determine GLB of 5 and 10.
18. Let D_{100} = {1, 2, 4, 5, 10, 20, 25, 50, 100} be the divisions of 100. Draw the Hasse diagram of (D_{100}, l) where l is the relation "division".
    Find (1) glb {10, 20} (II) lub {10, 20} (III) glb {5, 10, 20, 25} (IV) lub {5, 10, 20, 25}.

UNIT 3

1. Show that the operation * defined by a * b = a + b - ab is a semigroup.
2. Show that Q with * defined by a * b = a - b + ab is not a semigroup.
3. Show that R with * defined by $x * y = x^y$ is not a semigroup.
4. Show that (N, -) is not a semigroup.
5. Show that G = {1, -1, i, -i} is a group with respect to multiplication.
6. Show that G = $\{1, w, w^2\}$ is a group with respect to multiplication where 1, w, w^2 are cube roots of unity.
7. Prove that Z with * defined by a * b = a+b+1 is an abelian group.
8. Show that Q_+ with * defined by $a * b = \frac{ab}{3}$ is an abelian group.
9. Show that G = {0,1,2,3,4} is an abelian group with respect to addition modulo 5.
10. Show that G = {1,2,3,4} is an abelian group with respect to multiplication modulo 5.
11. Cancellation law holds in G for every a,b,c ∈ G then a * b = a * c ⇒ b = c,
    b * a = c * a ⇒ b = c.
12. If (G, *) is a group then $(a * b)^{-1} = b^{-1} * a^{-1}$ for every a,b ∈ G.
13. If every element of a group G, is its own inverse Show that G is an abelian group.
14. Show that in a group G, for a,b ∈ G $(ab)^2 = a^2 b^2$.
15. Prove that if $a^2 = a$ then a = e for every a ∈ G.
16. Show that G = { x / x = $2^a \cdot 3^b$ for some a,b ∈ Z } is a group under multiplication.
17. Find the complements of every element of the lattice (S_n, D) for n = 75.
18. Which of the two lattices (S_n, 3) for n =30 and n = 45 are complemented? Are these lattices distributive?
19. In S_{42} is the set all divisors of 42 and D is the relation "divisor of" on S_{42}, prove that { S_{42}, D} is a complemented lattice.
20. Is the following lattice a complemented lattice?



21. Examine whether the lattice given in the following Hasse diagram is distributive or not.



UNIT 4

1.In the no of 3 digit even nos with no repeated digits. npr :- permutations. Arranging r object in a line is npr arranging n objects in a line is n! arranging n objects, out of which n1 are one type, n2 are 2nd type ....nx are kth type with $n+n_2++nk=n$ then the no. of permutations of n objects is n!/ n1!*n2!...nk!1.

2.Find the no. of permutations of the letters of the word (1) Engineering (2)Structures (3) Mathematics.
3. In how many ways we can distribute 12 identical pencils to 5 children so that every child atleast one pencil
4. In how many way's can 4 questions be selected from 7 questions?
5. Suppose there are 16 boys and 18 girls in a class and how many ways we can select one of these students (either a boy or a girl) as a class representative
6.Suppose a hostel library has 12 books on mathematics,10 books on physics,16 books on computer science and 11 books on electronics .How many ways we can choose a book
7.Suppose a person has 3 shirts and 5 ties.Then in how many ways a person can choose a shirt and a tie?
8. Find the number of binary sequences of length n that contain an even number of 1^r

9. How many numbers x can be formed by using the digits 3,4,4,5,5,6 If we want x to exceed 5,000,000?

10.How many 4 digit numbers can be formed by using the digits 2,4,6,8 when repitition of digits is allowed

11.In how many ways 10 boys and 10 girls can sit around a circular table?

12. In how many ways 3 persons can sit around a circular table?

13.How many different arrangements of letters in the word 'BOUGHT' can be formed?

14.How many different strings of length 4 can be formed by using the letters of the word INTRODUCE

15.How many words of 3 distinct letters can be formed from the letters of the word PASCAL

16.Prove that (2n) ! = 2^n . n ! (1.3.5…..2n-1)

17.If n_2^p = 72, find n?

18. If $2n + 1_{n-1}^p : 2n - 1_n^p$ = 3 : 5 find n?

19. In how many ways the letters of the word DISCRETE can be formed so that the vowels may come together?

20.In how many ways can 7 boys and 5 girls be seated in a row so that no 2 girls sit together?

21. In how many ways 20 similar books be placed on 5 different shelves?

22. In how many ways can we distribute 10 identical marbles among 6 distinct containers?

23. . In how many ways can we distribute 12 identical pencils to 5 children so that every child get atleast 1 pencil?

24. . In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child get an apple?

25.Find the number of positive integer solutions and of the equation $x_1 + x_2 + x_3 = 17$ where

$x_1 \geq 1$, $x_2 \geq 1$, $x_3 \geq 1$

26. Find the number of integer solutions and of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where

$x_1 \geq 1$, $x_2 \geq 3$, $x_3 \geq 4$, $x_4 \geq 2$, $x_5 \geq 0$

27. Properties (i) $n_r^c = n_{n-r}^c$

(ii) $n_r^c + n_{r-1}^c = n + 1_r^c$

(iii) nc_x = nc_y => x = y or x+y = n

29. Evaluate the following

(i) C(6,3) (II) C (10,8)

(iii)P(8,5) (IV) P(6,3)

30. If 18 c_r = 18 c_{r+2} find the value of 18 c_5

31. If nc_x = 56 and np_x = 336 find n and x

32. If 1000c_{98} = 999c_{99} + xc_{901} find x?

33. In how many ways can you select atleast 1 king if you choose 5 cards from a deck of 52 cards?

34. Pigeon hole principle

Prove that in any set of 29 persons atleast 5 persons must have been born on the same day of the month?

35. I f 9 books are to be kept in 4 shelves there must be atleast 1 shelf which contain atleast 3 books

36. Principle of inclusion-exclusion

In a sample of 200 logic chips,46 have a defect D1,52 have a defect D2, 60 have a defect D3,14 have defects D1 and D2 ,16 have defects D1 and D3 ,20 have defects D2 and D3,and 3 have all the 3 defects. Find the number of chips having (i) atleast 1 defect (ii) no defect

37. From a group of 10 professors ,how many ways can committee of members are formed so that atleast 1 professor A and professor B will be included

38. How many integers between 1 and 300 are divisible by atleast one of 5,6 and 8

39. Find out the coefficient of $x^9\ y^3$ in $(\ x + 2y)^{12}$

40. Find out the coefficient of $x^5\ y^2$ in the expansion of $(\ 2x - 3y)^7$

41. What is the coefficient of $x^2\ y^4$ in $(\ x + y)^6$

42. What is the coefficient of $x^{101}\ y^{99}$ in the expansion of $(\ 2x - 3y)^{200}$

43. Compute (i) $\left(\frac{7}{2,3,2}\right)$ (ii) $\left(\frac{4}{1,1,2}\right)$ (iii) $\left(\frac{12}{5,3,2,2}\right)$

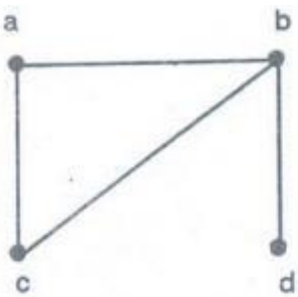
44. Determine the coefficient of xyz^2 in the expansion of $(2x - y - z)^4$

45. Determine the coefficient of $x^3y^3z^2$ in $(2x - 3y - 5z)^8$

UNIT - 5

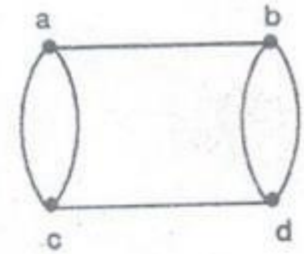
1. What type of graphs are the following.

(a)



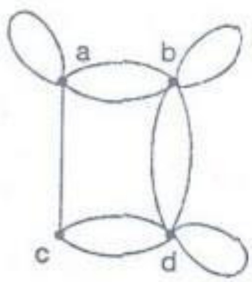
Ans. Simple graph

(b)



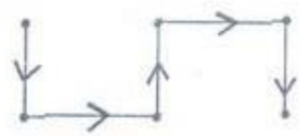
Ans. Multi graph

(c)



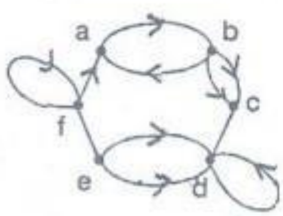
Ans. Pseudo graph

(d)



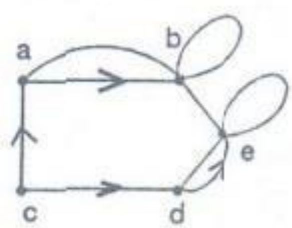
Ans. Simple directed graph

(c)



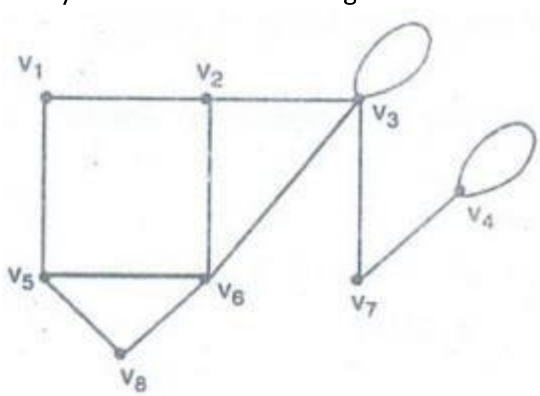
Ans. Directed multi graph

(f)

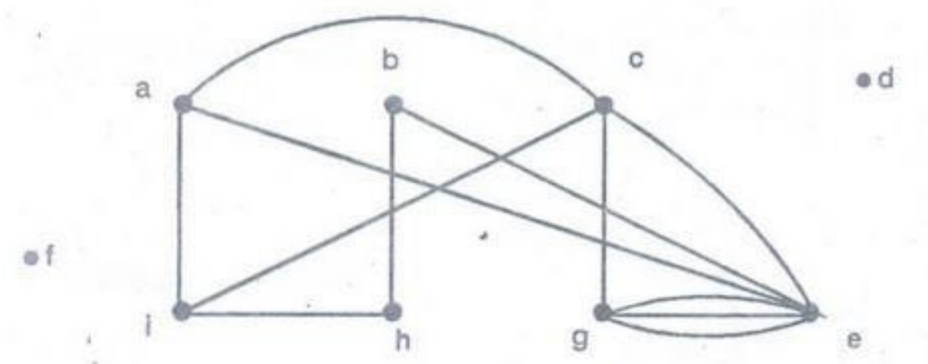


Ans. Mixed graph

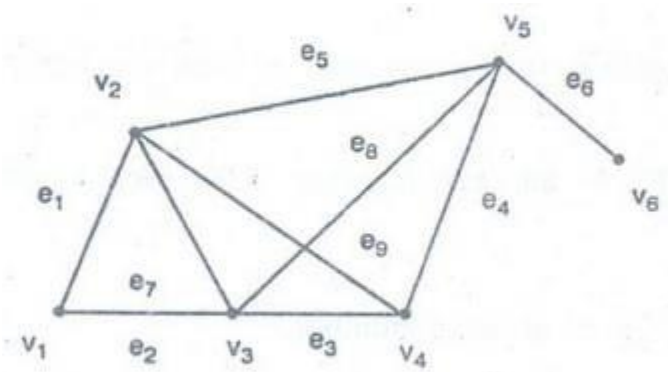
2. State and prove Hand shaking Theorem.
3. S.T the number of odd vertices in a graph is always even.
4. Verify that the sum of the degree of all the vertices is even for the graph.



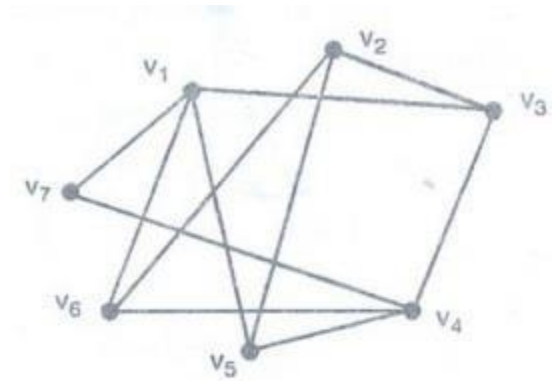
5. Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



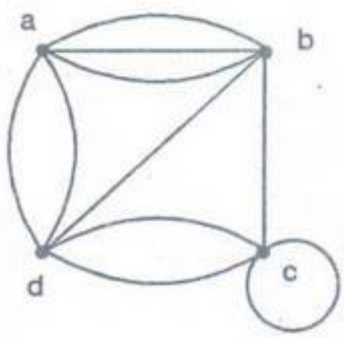
6. Verify the handshaking theorem for the graph.



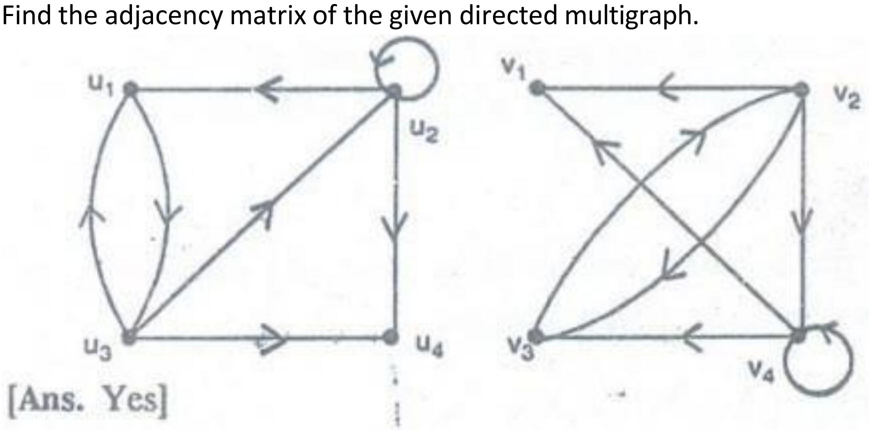
7. Is there a simple graph corresponding to the following degree sequences? (i) (1, 1, 2, 3) (ii) (2, 2, 4, 6)
8. Show that the maximum number of edges in a simple graph with n vertices is n (n - 1) / 2
9. Draw the graphs (a) K_5 (b) K_6 (c) K_7 (d) Kn for $1 \le n \le 4$.
10. How many vertices and how many edges the graph Kn has? Also Find the degree sequence of each of the following graphs (a) K4 (b) K5 (c) K2
11. Draw the graphs (a) C_3 (b) C_4 (c) C_5 (a) C_6, (e) C_8
12. How many vertices and how many edges do these graphs have (a) Cn (b) C8 (c) Also find the degree sequence of C4
13. Draw the graphs (a) W3, (b) W4 (c) W5, (d) W6, (e) W7
14. How many vertices and how many edges do these graph have (a) Wn (b) W5 also find the degree sequence of W4
15. Does there exists a 4-regular graph on 6 vertices if so construct a graph.
16. How many vertices does a regular graph of degree four with 10 edges have?
17. Define Bipartite Graph and Show that C6 is a bipartite graph ?
18. Is K3 is bipartite?
19. How many vertices and how many edges $K_{m,n}$ graph have?
20. Show that the following graph G is bipartite.



21. Draw the complete bipartite graphs K2,3, K3,3 K3,5 and K2,6
22. Define a Planar Graph also S.T. If G is a connected planar graph then prove that $|V| - |E| + |R| = 2$. (Euler's Formula).
23. Define Subgraph and Draw two subgraph of K5.
24. Define Adjacency matrix of a graph and represent the following graph with an adjacency matrix.



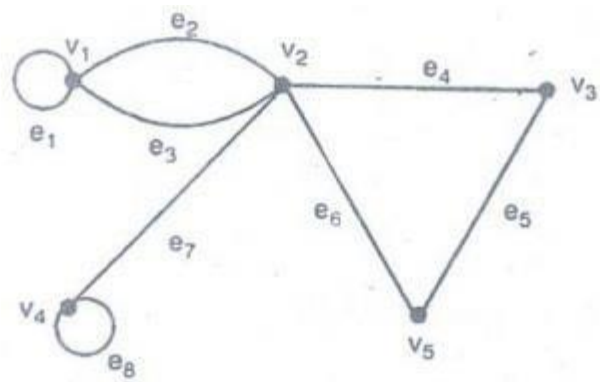
25. Write the adjacency matrix of K4, k2,3 w4.
26. Find the adjacency matrix of the given directed multigraph.



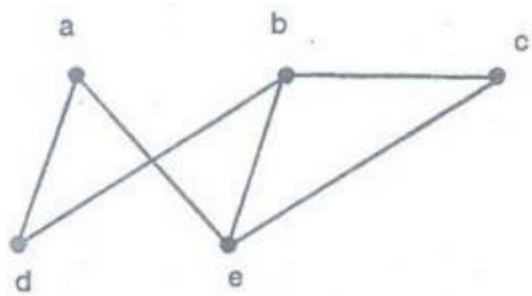
27. Draw the graph represented by the given adjacency matrix.

$$\begin{matrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{matrix}$$

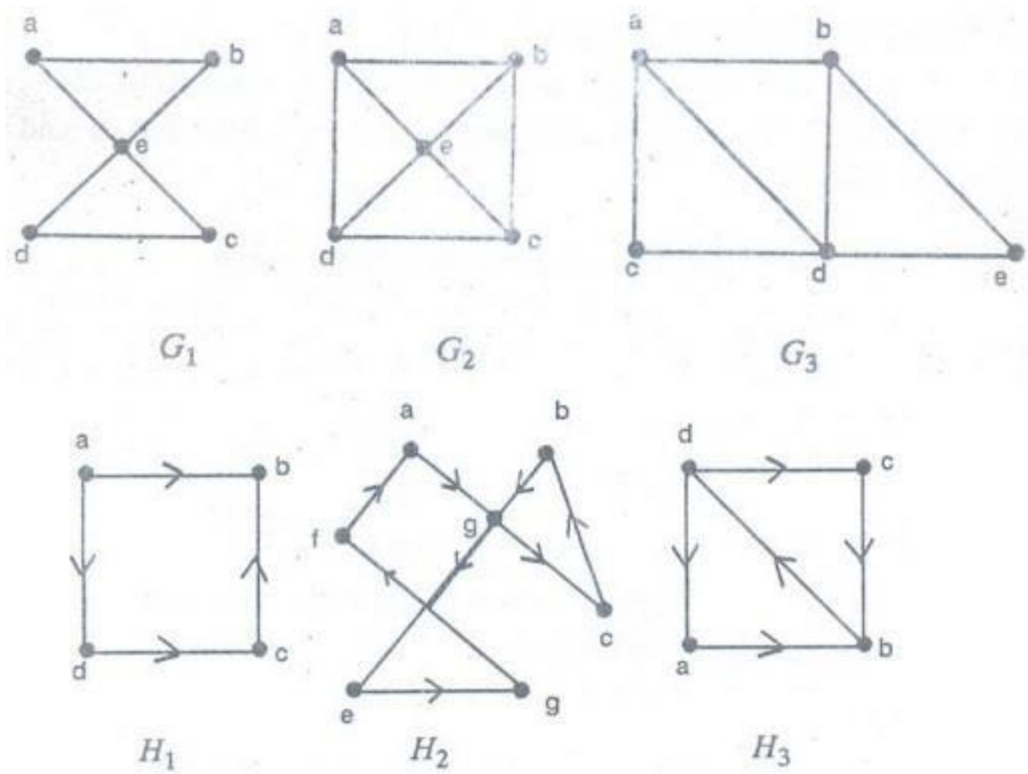
28. Represent pseudograph shown in figure using an incidence matrix.



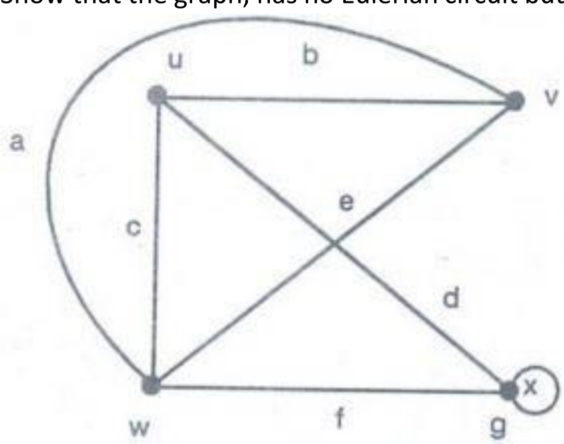
29. Does each of these lists of vertices from a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?
    (a) a, e, b, c, b (b) e, b, a, d, b, e
    (c) a, e, a, d, b, c, a (d) c, b, d, a, e, c



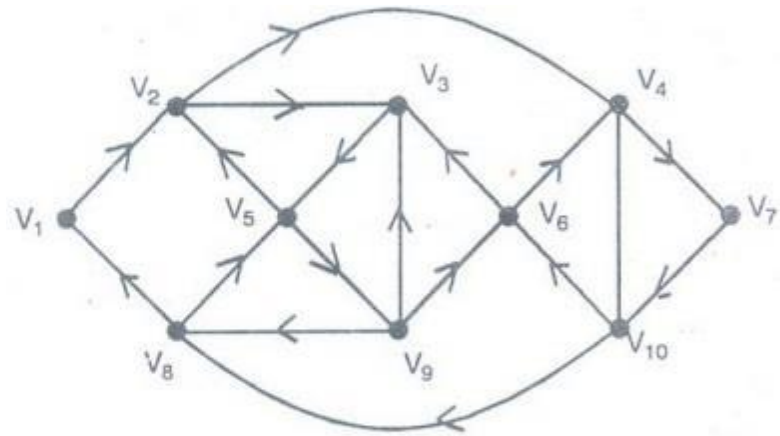
30. Which of the graphs in Figures has a Euler circuit? Of those that do not, which have a Euler path?



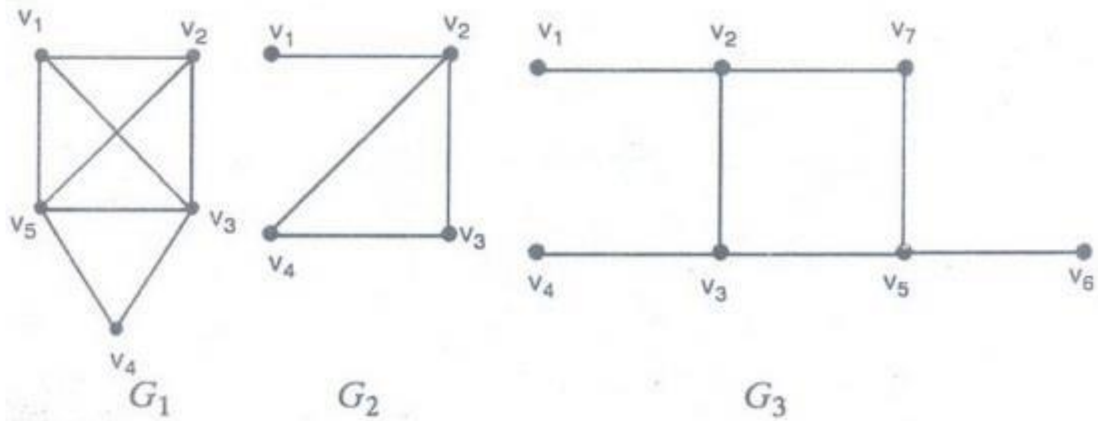
31. Define Euler Graph and Give two examples of Euler graph.
32. Show that the graph, has no Eulerian circuit but has a Eulerian path.



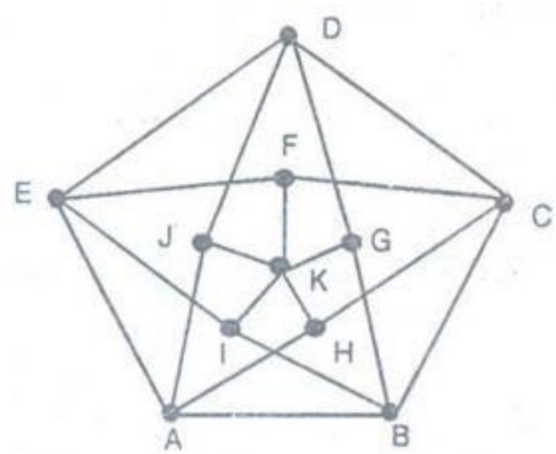
33. S.T. the graph shown in figure is a Euler graph. Determine Euler circuit for this graph.



34. Explain Konisberg bridge problem. Represent the problem by means of graph. Does the problem have a solution?
35. Define Hamiltonian Graph and Which of the simple graphs given below have a Hamilton cycle or, if not, a Hamilton path?



36. Show that the graph is Hamiltonian



37. Give an example of a graph that has a Euler circuit and a Hamiltonian cycle, which are distinct.
38. Give an example of a graph which has a Euler circuit but not a Hamiltonian cycle.
39. Define a tree. What are its key properties?

40. What is a rooted tree? Give an example. Define ancestor, descendant, siblings, and subtree in a rooted tree.
41. Define Spanning tree and what are its properties. Explain with an example to find all spanning trees of a graph.
42. Write the algorithm for BFS and DFS. Illustrate BFS and DFS traversal on a given undirected graph. Or Differentiate between Breadth-First Search (BFS) and Depth-First Search (DFS).
43. What is a minimum spanning tree (MST)? Give an example with Kruskal's Algorithm.
44. Define colouring of a graph. Define Chromatic number. Find the Chromatic number of K5, C5, W5, K3,4.
45. Describe Four colouring problem.