ODEVC

UNIT-I : First Order ODE.

(i) Exact differential equations. $\left(\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}\right)$

(ii) Non-Exact differential equation $\left(\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}\right)$

→ {Method-I, Method-II, *Method-III
Method-IV, Method-V}.

*(iii) Linear differential eqn

(iv) Bernoulli's Equations.

**(V)* v.imp* Newton's law of cooling

(VI)* Law of Natural growth & decay.
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(VII) Equations Solvable for P.

(VIII) Orthogonal Trajectories,

UNIT-II : Linear differential Equations of Higher Order.

(i) To find the general solution (complementary fun)
$f(D)y=0$ {roots find, learn the formulaes}

(ii) General Solution of $f(D)y = Q(x)$.

$$\boxed{y = C.F + P.I}$$

Method 1:— P.I of $f(D)y=Q(x)$ where $Q(x) = e^{ax}$

$P.I = \frac{1}{f(D)} e^{ax}$ {learn the formulaes} & failure cases.

Method 2: P.I of $f(D)y = Q(x)$ where $Q(x) = \sin ax$ or $\cos ax$

$P.I = \frac{1}{f(D^2)} \cos ax$ or $\frac{1}{f(D^2)} \sin ax$ {formulae learn} → failure cases.

Method 3: P.I of $f(D)y = Q(x)$ where $Q(x) = x^K$

$P.I = \frac{1}{f(D)} x^K$ {learn Binomial series
Like $(1+D)^{-1} = 1-D+D^2-D^3 - - -$
$(1-D)^{-1} = 1+D+D^2+D^3 - - -$}

Method 4:— P.I of $f(D)y = Q(x)$ where $Q(x) = e^{ax}.V$.

$$P.I = \frac{1}{f(D)} e^{ax}.V \Rightarrow e^{ax}.\frac{1}{f(D+a)}.V$$ {V is a func of x}.

Method 5:— P.I of $f(D)y = \phi(x)$ where $\phi(x) = x^m V$.

$$P.I = \frac{1}{f(D)} x^m.V$$

{ Case I ⇒ $P.I = \frac{1}{f(D)} xV = \left[x - \frac{f'(D)}{f(D)}\right]\frac{1}{f(D)}V$
Case II ⇒ $P.I = \frac{1}{f(D)} x^m \sin ax$ or $\frac{1}{f(D)} x^m \cos ax$.

V.Imp
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Method of Variation of parameters:— (problems)

$$\frac{d^2y}{dx^2} + P.\frac{dy}{dx} + Qy = R \Rightarrow C.F = C_1 U(x) + C_2 V(x)$$

$$P.I = AU + B.V$$

$$A = -\int \frac{VR}{U\frac{dV}{dx} - V\frac{dU}{dx}}, \quad B = \int \frac{UR}{U.\frac{\partial V}{\partial x} - V\frac{\partial U}{\partial x}}.$$

⇒ Homogeneous linear Equations (Euler-Cauchy eqns)

UNIT - III Laplace Transform

{Very very Imp learn the formulaes first}

* first shifting theorem.
* Second shifting theorem.
* Unit step func.
* Change of scale property.
* Laplace Transform of derivative & Integration.
* Multiplication by t.
* division by t.
* Evaluation problems. like $\int_0^\infty t e^{-t} \sin t \, dt$
Imp ** Dirac delta func.
* Laplace Transform of periodic funcs.
* Inverse Laplace Transform.

Inverse Laplace Transform.

⇒ Use of partial fractions (Type I, II, III, IV)
→ first shifting theorem.
** Convolution Theorem :—
* Imp
** Applications of ordinary differential equations.

UNIT- IV :- Vector Differentiation & vector operator

* Directional derivative. (Type I, II, III, IV). (gradiant)
* Unit normal vector problems
* Angle b/w two surfaces.
* Divergence of a vector : (Solenoidal vector) ($\operatorname{div} F = 0$)
* Curl of a vector : (Irrotational ($\operatorname{curl} F = 0$))
  * Scalar potential func problems.
  * proof that problems,
  * Operators : proof that problems & Theorems.

UNIT- V : Vector Integration

1. Line Integral (work done by force problems)
2. Surface Integral
3. Volume Integral.

Vector Integration Theorems

1. Green's Theorem < Evalution problems / Verification problems.
2. Gauss divergence Theorem < Evaluation / Verification.
3. Stoke's Theorem < Evaluation / Verification.