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Assignment # 2 | Problem Set 1

1) Show $A^TA \neq AA^T$ in general. (Proof and demonstration)

We will show this by contradiction

assume: $A^TA = AA^T$

let A be a $m \times n$ matrix w/ $n \neq m$

A^T by definition is a $n \times m$ matrix

columns of A^T are taken directly from rows of A (rows $\rightarrow$ columns)

$A^T \rightarrow n \times m$

$A \rightarrow m \times n$ so when we multiply a $n \times m$ with $m \times n$ our resulting matrix is $n \times n$ as is the case with A^TA.

$AA^T \Rightarrow m \times n \cdot n \times m \Rightarrow m \times m$ matrix.

since we stated $n \neq m$ a $m \times m$ matrix CANNOT equal $n \times n$ matrix.

~~demonstrate:~~

~~let~~ $\cancel{A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ C & F \end{bmatrix}}$ ~~then~~ $\cancel{A^T = \begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}}$

$\cancel{A^TA = \begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \cdot \begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}}$

demonstrate:

let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{m \times n}$ $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{n \times m}$

$A^TA \Rightarrow n \times m \cdot m \times n = n \times n$

$= \begin{bmatrix} 17 & 22 & 27 \\ 22 & 29 & 36 \\ 27 & 36 & 45 \end{bmatrix}$

$AA^T \Rightarrow m \times n \cdot n \times m = m \times m$

$= \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}$

$A^TA \neq AA^T$ $\triangle$

2) For a special type of square matrix A, we get $A^TA = AA^T$. Under what conditions is this true? (Hint: Identity Matrix is one)

2x2 identity matrix: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{\text{trans.}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^TA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

we have checked that the identity matrix is an exception.

$A^TA = AA^T$ when A is a <u>diagonal matrix</u>: a matrix where all entries are 0's except the diagonal