Tansman Scheduler: Model #1

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LP Formulation

Indices $i \in \{0, 1, \ldots, n_{items} - 1\}$; $t \in \{0, 1, \ldots, n_{slots} - 1\}$; $t_\delta \in \{0, 1, \ldots, WINDOWSIZE\}$

See below for the use of t_δ and $WINDOWSIZE$ to express limits on practice "density".

Data

n_{items}	Number of items to schedule
n_{slots}	Number of time slots available
$ENERGYUSED_i$	Amount of mental "energy" required per chunk of practice time for task i
$TIMEAVAILABLE_t$	Time available during a training slot t (chunks per day)
$TIMEPERITEM_i$	Practice time needed to master item i
$MINPERWINDOW$	Minimum number of training slots for item i within a window (days)
$MAXPERWINDOW$	Maximum number of training slots for item i within a window (days)
$WINDOWSIZE$	Used to express minimum and maximum "density" of practice days (days)

Variables

$P_{i,t}$	Amount of practice in time slot t for item i (chunks of study)
$ENERGY_t$	Energy expended on a given day (really a macro not a variable)
$ENERGYDECREASE_t$	Amount by which energy expenditure decreased at t from $t-1$
$ENERGYINCREASE_t$	Amount by which energy expenditure increased at t from $t-1$
$TIMESOFAR_{i,t}$	Amount of practice for item i up to and including time t

Objective

Minimize

$\sum_{i,t} TIMEPERITEM_i P_{i,t} + \sum_{t=1} (ENERGYINCREASE_t - ENERGYDECREASE_t) + \sum_{i,t} TIMESOFAR_{i,t}$

Constraints

$$\sum_i P_{i,t} \leq TIMEAVAILABLE_t \; \forall \; t$$ (C1: No more time allocated to time slot than available)

$$\sum_t P_{i,t} \geq TIMEPERITEM_i \; \forall \; i$$ (C2: Practice time greater than required to learn item)

$$\sum_i P_{i,t+t_\delta} \geq MINPERWINDOW \; \forall i, t \forall t_\delta$$ (C3: Practice spacing - minimum amount of practice in window)

$$\sum_i P_{i,t+t_\delta} \leq MAXPERWINDOW \; \forall i, t$$ (C4: Practice spacing for rest - maximum amount of practice in window)

$$TIMESOFAR_{i,t} = \sum_i \sum_{t0=0\ldots t} P_{i,t0} \; \forall i, t$$ (C5: Definition. Used in objective to bias toward finishing earlier)

$$ENERGY_t = \sum_i ENERGYUSED_i \times P_{i,t}$$ (C6: Definition. Used in definition of increase/decrease for smoothing)

$$ENERGY_{t+1} - ENERGY_t = ENERGYINCREASE_{t+1} - ENERGYDECREASE_{t+1}$$ (C7: Change in energy to smooth. Minimization means only one non-zero)

Note I had to introduce the $WINDOWSIZE$ parameter in order to have a way to express (C3) and (C4). Right now this is something that needs to be specified as part of the configuration of the model. It's a bit unnatural.

When I first started experimenting I noticed that these constraints could result in practice schedules "pushed to the right" with low density slots to the left of the schedule because I had not expressed the idea of optimizing the amount of calendar time spent. As a heuristic, I decided to incorporate another variable: $TIMESOFAR_{i,t}$ defined as the sum of practice time for each item i up to time t. The idea was to maximize the sum of these times as a "timeliness" metric. It ends up penalizing late completion for each task.

The time window is sort of awkward and creates some odd scheduling artifacts. Next I'm going to try different sorts of energy surpluses/deficits. where there are constraints that prevent it from going too high or too low. For each practice item a day of rest will deplete the "learning" banked, while a day of practice will increase it, but only within a threshold. Equivalent to production smoothing in other problems I've seen.