Overall Concept

The decision variables $P_{i,j}$, the amount of practice of task i during time slot j (e.g., a day in a sequence of days), are organized into a tableau like the following:

	1	2	...	m
$item_1$	$P_{1,1}$	$P_{1,2}$		$P_{1,m}$
$item_2$	$P_{2,1}$	$P_{2,2}$		$P_{2,1}$
$\vdots$			$\ddots$	
$item_n$	$P_{n,1}$	$P_{n,2}$		$P_{n,m}$

where $n = NITEMS$, and $m = NSLOTS$.

Rows are the items to be studied and columns are time slots (e.g., days). Constraints are expressed in terms of sums over rows and columns, or in the case of the density constraint, over a series of adjacent columns.

LP Formulation

Counts and Indices

$NITEMS$	Number of items to schedule
$NSLOTS$	Number of time slots available
$i \in \{0, 1, \ldots, NITEMS - 1\}$	Index over items
$t \in \{0, 1, \ldots, NSLOTS - 1\}$	Index over time slots
$t_\delta \in \{0, 1, \ldots, WINSZ - 1\}$	Index over time window

See below for the use of t_δ and $WINSZ$ to express limits on practice "density".

Data

$ERGUSED_i$	Amount of mental "energy" required per unit of practice time for task i ("ergs" of energy)
$ERGEVAIL_t$	Amount of mental "energy" available per day ("ergs"/day)
$TIMEAVAIL_t$	Amount of time resource available during a training slot t (time units per day)
$TIMEPER_i$	Practice time needed to master item i
$MINPERWIN_i$	Minimum number of training slots for item i within a window (days)
$MAXPERWIN_i$	Maximum number of training slots for item i within a window (days)
$WINSZ$	Used to express minimum and maximum "density" of practice days (days)

Variables

$P_{i,t}$	Amount of practice in time slot t for item i (number of units of time study) **NB: This is an integer.**
ERG_t	Energy expended on a given day (macro over P)
$ERGDEC_t$	Amount by which energy expenditure decreased at t from $t-1$
$ERGINC_t$	Amount by which energy expenditure increased at t from $t-1$
$TIMESOFAR_{i,t}$	Amount of practice for item i up to and including time t (Macro over P)

Objective and Constraints

$$\text{Minimize } \underbrace{\sum_{i,t} TIMEPER_i P_{i,t}}_{\text{(1) Overall time spent}} + \underbrace{\sum_{t=1} (ERGINC_t - ERGDEC_t)}_{\text{(2) Energy smoothing}} + \underbrace{\sum_{i,t} TIMESOFAR_{i,t}}_{\text{(3) Calendar time}}$$

s.t.

$$\sum_i P_{i,t} \leq TIMEAVAIL_t \ \forall\, t \qquad \text{C1: No more time allocated to time slot than available}$$

$$\sum_t P_{i,t} \geq TIMEPER_i \ \forall\, i \qquad \text{C2: Practice time greater than required to learn item}$$

$$\sum_i P_{i,t+t_\delta} \geq MINPERWIN \ \forall i, t \forall t_\delta \qquad \text{C3: Practice spacing - min. amount in window}$$

$$\sum_i P_{i,t+t_\delta} \leq MAXPERWIN \ \forall i, t \qquad \text{C4: Practice spacing for rest - max. amount in window}$$

$$TIMESOFAR_{i,t} = \sum_i \sum_{t0=0\ldots t} P_{i,t0} \ \forall i, t \qquad \text{C5: Defn. Used in objective to bias toward finishing earlier}$$

$$ERG_t = \sum_i ERGUSED_i \times P_{i,t} \ \forall t \qquad \text{C6: Defn. Used to define increase/decrease for smoothign.}$$

$$ERGINC_{t+1} - ERGDEC_{t+1} = ERG_{t+1} - ERG_t \ \forall t \qquad \text{C7: Defn. Change in either direction. One always zero.}$$

where $P_{i,t} \in \mathbb{Z}_{\geq 0}$; $ERG_t, ERGINC_t, ERGDEC_t \geq 0$ C8: Variable domains

Discussion

The objective function is composed of three positive terms to be minimized. The first term (1) captures the amount the sum of the total amount of practice time across all items. The second term (2) is the relative energy expenditure on practice from day to day. This smoothes energy expenditure, which has the effect of preferring spreading around the more energy-sapping work, and increasing the diversity with respect to intensity on any given day.

The first two constraints are essentially supply and demand constraints. The time spent practicing cannot exceed the time available for practice (C1) and each item requires that a minimum amount of practice is done to master the item (C2).

The constraints (C3) and (C4) are contraints that limit the number of consecutive days spent practicing. They are expressed in terms of a time window parameter. The goal is to ensure an adequate number of days spent resting while ensuring that practice is resumed before too long passes.

The constraints (C5), (C6) and C(7) relate to energy expenditure. The equality constraints define the difference in energy expenditure from day to day. The difference $ERGINC_{t+1} - ERGDEC_{t+1}$ of two positive variables captures this change. Either $ERGINC_{t+1}$ or $ERGDEC_{t+1}$ will always be driven to zero by the minimization.

Finally, with (C8) we require that the amount of practice of a given item on a given day $P_{i,t}$ is a non-negative *integer* to enforce the fixed-size practice sessions, and that all other variables are non-negative real numbers.

There are several known issues with this formulation that I will need to address:

- Incommensurability of the three terms in the objective function. A common metric that captures utility with appropriate scaling factors is needed. This should correlate with the expected "satisfaction" experience by the practicer. I'll need to add coefficients to the required Data that encode this.
- It's not clear that $TIMESOFAR_{i,t}$ is needed in the model. It's not clear that preference should be given to completing tasks as early as possible. That is, it's not clear a priori that given excess days that "packing" the schedule is best. It may be that this is needs to be scaled in in some way (see above).
- There are no constraints related to energy except for the smoothing constraint. It is probably desirable to specify minimum and maximum expenditures in some way.

Diagnostic Metrics

The slacks and excesses are useful information for a user of the scheduling tool if presented properly. In addition, some efficiency-related metrics are probably going to be useful:

- Re-practice ratio for an item i. Amount of practice beyond required beyond $TIMEPER_i$ over required practice (i.e., excess for the constraint). This is:

$$R_i = \frac{\sum_t P_{i,t} - TIMEPER_i}{TIMEPER_i}$$

- The most and least efficient tasks are then $arg\,min_i\ r$ and $arg\,imax_i\ r$
- Daily intensity is:

$$I_t = \frac{ERG_t}{TIMEAVAIL_t}$$

The sums of these over all items may also be useful to present to the user.