$$\mathbf{w}^{(1)} = \begin{pmatrix} w_{01}^{(1)} & w_{02}^{(1)} \\ w_{11}^{(1)} & w_{12}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} \end{pmatrix}, \quad \mathbf{w}^{(2)} = \begin{pmatrix} w_{01}^{(2)} & w_{02}^{(2)} & w_{03}^{(2)} \\ w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} \\ w_{21}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} \end{pmatrix}, \quad \mathbf{w}^{(3)} = \begin{pmatrix} w_{01}^{(3)} \\ w_{01}^{(3)} \\ w_{11}^{(3)} \\ w_{21}^{(3)} \\ w_{31}^{(3)} \end{pmatrix}$$

则第一层神经元的输入为:

$$\begin{pmatrix} s_1^{(1)} \\ s_2^{(1)} \end{pmatrix} = (\mathbf{w}^{(1)})^T \vec{\mathbf{x}}_n^{(0)}$$

假设第一层神经元的激活函数为ReLU,即: $x^{(1)} = \max(0, s^{(1)})$,则:

$$\begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} \max(0, s_1^{(1)}) \\ \max(0, s_2^{(1)}) \end{pmatrix}$$

第二层神经元的输入为:

$$\begin{pmatrix} s_1^{(2)} \\ s_2^{(2)} \\ s_3^{(2)} \end{pmatrix} = (\mathbf{w}^{(2)})^T \begin{pmatrix} 1 \\ x_1^{(1)} \\ x_2^{(1)} \end{pmatrix}$$

假设第二层神经元的激活函数为ReLU, 即: $x^{(2)} = \max(0, s^{(2)})$, 则:

$$\begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{pmatrix} = \begin{pmatrix} \max(0, s_1^{(2)}) \\ \max(0, s_2^{(2)}) \\ \max(0, s_3^{(2)}) \end{pmatrix}$$

则第三层的输入为:

$$\mathbf{s}_{1}^{(3)} = (\mathbf{w}^{(3)})^{T} \begin{pmatrix} 1 \\ x_{1}^{(2)} \\ x_{2}^{(2)} \\ x_{3}^{(2)} \end{pmatrix}$$

因为第三层是线性操作,即输出 $\hat{y} = s_1^{(3)}$

对于输入样本 $ec{x}_n$,假设其标签为 y_n ,采用平方误差函数。即: $e_n = (y_n - \hat{y}_n)^2$

$$\delta_1^{(3)} = -2(y_n - s_1^{(3)})$$

运用反向传播法,于是: $\delta_j^{(2)} = \sum_k (\delta_k^{(3)}) (w_{jk}^{(3)}) (x_j^{(2)})'$

对于ReLU来说,其导数为: $(x_i^{(L)})' = [s_i^{(L-1)} \ge 0]$