

$$\delta_j^{(2)} = \sum_k (\delta_k^{(3)})(w_{jk}^{(3)}) \llbracket s_j^{(2)} \geq 0 \rrbracket = \delta_1^{(3)} w_{j1}^{(3)} \llbracket s_j^{(2)} \geq 0 \rrbracket$$

$$\text{即: } \vec{\delta}^{(2)} = \begin{pmatrix} \delta_1^{(2)} \\ \delta_2^{(2)} \\ \delta_3^{(2)} \end{pmatrix} = \begin{pmatrix} \delta_1^{(3)} w_{11}^{(3)} \llbracket s_1^{(2)} \geq 0 \rrbracket \\ \delta_1^{(3)} w_{21}^{(3)} \llbracket s_2^{(2)} \geq 0 \rrbracket \\ \delta_1^{(3)} w_{31}^{(3)} \llbracket s_3^{(2)} \geq 0 \rrbracket \end{pmatrix} = \begin{pmatrix} -5.96 * (-1.94) * 1 \\ -5.96 * (-1.94) * 1 \\ -5.96 * (-1.94) * 1 \end{pmatrix} = \begin{pmatrix} 11.56 \\ 11.56 \\ 11.56 \end{pmatrix}$$

继续运用反向传播法，于是： $\delta_j^{(1)} = \sum_k (\delta_k^{(2)})(w_{jk}^{(2)})(x_j^{(1)})'$ ，所以：

$$\delta_j^{(1)} = \sum_k (\delta_k^{(2)})(w_{jk}^{(2)}) \llbracket s_j^{(1)} \geq 0 \rrbracket = (\delta_1^{(2)} w_{j1}^{(2)} + \delta_2^{(2)} w_{j2}^{(2)} + \delta_3^{(2)} w_{j3}^{(2)}) \llbracket s_j^{(1)} \geq 0 \rrbracket$$

由此可以得到：

$$\vec{\delta}^{(1)} = \begin{pmatrix} \delta_1^{(1)} \\ \delta_2^{(1)} \end{pmatrix} = \begin{pmatrix} \llbracket s_1^{(1)} \geq 0 \rrbracket & 0 \\ 0 & \llbracket s_2^{(1)} \geq 0 \rrbracket \end{pmatrix} \begin{pmatrix} w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} \\ w_{21}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} \end{pmatrix} \begin{pmatrix} \delta_1^{(2)} \\ \delta_2^{(2)} \\ \delta_3^{(2)} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -0.26 & -0.26 & -0.26 \\ -0.26 & -0.26 & -0.26 \end{pmatrix} \begin{pmatrix} 11.56 \\ 11.56 \\ 11.56 \end{pmatrix} = \begin{pmatrix} -9.02 \\ -9.02 \end{pmatrix}$$

令 $\eta = 0.01$ ，利用梯度下降法进行权系数更新：

$$\mathbf{w}_2^{(1)} = \mathbf{w}_1^{(1)} - \eta \vec{x}_n^{(0)} (\vec{\delta}^{(1)})^T = \mathbf{w}_1^{(1)} - \eta \begin{pmatrix} 1 \\ x_{n1}^{(0)} \\ x_{n2}^{(0)} \end{pmatrix} (\delta_1^{(1)}, \delta_2^{(1)})$$

$$= \begin{pmatrix} w_{01}^{(1)} & w_{02}^{(1)} \\ w_{11}^{(1)} & w_{12}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} \end{pmatrix} - \eta \begin{pmatrix} \delta_1^{(1)} & \delta_2^{(1)} \\ x_{n1}^{(0)} \delta_1^{(1)} & x_{n1}^{(0)} \delta_2^{(1)} \\ x_{n2}^{(0)} \delta_1^{(1)} & x_{n2}^{(0)} \delta_2^{(1)} \end{pmatrix}$$

$$= \begin{pmatrix} -0.26 & -0.26 \\ -0.26 & -0.26 \\ -0.26 & -0.26 \end{pmatrix} - 0.01 \begin{pmatrix} -9.02 & -9.02 \\ (-1) * (-9.02) & (-1) * (-9.02) \\ (-1) * (-9.02) & (-1) * (-9.02) \end{pmatrix}$$

$$= \begin{pmatrix} -0.17 & -0.17 \\ -0.35 & -0.35 \\ -0.35 & -0.35 \end{pmatrix}$$