

一、DFT计算量
 $X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk}$

$\int N^2$ 个复数乘法, $\int 2N(N-1)$ 实数加法

$$W_N^{nk} = e^{-j\frac{2\pi}{N}nk}$$

$$\text{周期性: } W_N^{k+N} = W_N^k$$

$$\text{对称性: } (W_N^{nk})^* = W_N^{-nk} = W_N^{n-nk}$$

$$\text{特殊点: } W_N^0 = 1, W_N^{\frac{N}{2}} = -1, W_N^{\frac{N}{2}+k} = W_N^{\frac{N}{2}-nk} = -W_N^k, W_N^{\frac{N}{2}} = W_N^{\frac{1}{2}}$$

\Rightarrow FFT思想: 利用DFT系数特性合并FFT运算中部分项, 将DFT \rightarrow 短DFT

$\int DFT$ 时间抽选

DIF 频率抽选

二、DIT基2-FFT算法

1、算法原理

设序列点数 $N = 2^L$, L 为整数。

若不满足, 则补零。

N 为2的整数幂的FFT算法称基-2FFT算法。

将序列 $x(n)$ 按 n 的奇偶分成两组:

$$x(2r) = x_1(r) \quad r = 0, 1, \dots, N/2-1$$

$$x(2r+1) = x_2(r)$$

则 $x(n)$ 的DFT:

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n)W_N^{nk} = \sum_{\text{偶数} n} x(n)W_N^{nk} + \sum_{\text{奇数} n} x(n)W_N^{nk} \\ &= \sum_{r=0}^{N/2-1} x(2r)W_N^{2rk} + \sum_{r=0}^{N/2-1} x(2r+1)W_N^{(2r+1)k} \\ &= \sum_{r=0}^{N/2-1} x_1(r)(W_N^2)^{rk} + W_N^k \sum_{r=0}^{N/2-1} x_2(r)(W_N^2)^{rk} \end{aligned}$$

因为: $W_N^2 = W_{N/2}^1$, 所以:

$$= \sum_{r=0}^{N/2-1} x_1(r)W_{N/2}^{rk} + W_N^k \sum_{r=0}^{N/2-1} x_2(r)W_{N/2}^{rk}$$

$$= X_1(k) + W_N^k X_2(k) \quad k = 0, 1, \dots, N/2-1$$

再利用周期性求 $X(k)$ 的后半部分

$\because X_1(k), X_2(k)$ 是以 $N/2$ 为周期的

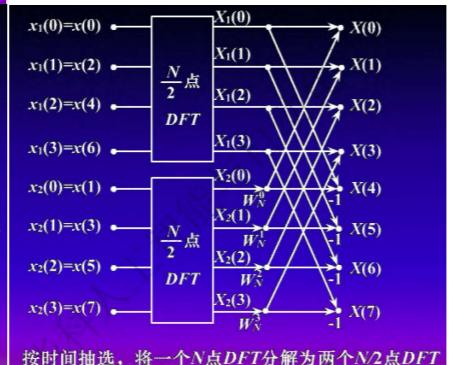
$$\therefore X_1\left(k + \frac{N}{2}\right) = X_1(k) \quad X_2\left(k + \frac{N}{2}\right) = X_2(k)$$

$$\text{又 } W_N^{\frac{N}{2}} = W_N^{N/2}W_N^k = -W_N^k$$

$$\begin{cases} X(k) = X_1(k) + W_N^k X_2(k) \\ X(k + \frac{N}{2}) = X_1(k) - W_N^k X_2(k) \end{cases} \quad k = 0, 1, \dots, N/2-1$$

时间抽取算法蝶形运算流程图

$$X(k) \xrightarrow{W_N^k} X(k) - W_N^k X_2(k)$$



按时间抽选, 将一个 N 点 DFT 分解为两个 $N/2$ 点 DFT

	复数乘法	复数加法
一个 $N/2$ 点 DFT	$(N/2)^2$	$N/2(N/2-1)$
两个 $N/2$ 点 DFT	$N^2/2$	$N(N/2-1)$
一个蝶形	1	2
$N/2$ 个蝶形	$N/2$	N
总计	$N^2/2 + N/2$ $\approx N^2/2$	$N(N/2-1) + N$ $\approx N^2/2$

$\frac{N}{2}$ 仍为偶数, 可进一步分解

$$\begin{cases} x_1(2l) = x_3(l) & l = 0, 1, \dots, N/4-1 \\ x_1(2l+1) = x_4(l) & \end{cases}$$

$$\begin{cases} X_3(k) = X_3(k) + W_{N/2}^k X_4(k) \\ X_3(k + \frac{N}{4}) = X_3(k) - W_{N/2}^k X_4(k) \end{cases} \quad k = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = X_4(k) + W_{N/2}^k X_3(k) \\ X_4(k + \frac{N}{4}) = X_4(k) - W_{N/2}^k X_3(k) \end{cases} \quad k = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = X_1(k) + W_{N/2}^k X_2(k) \\ X_1(k + \frac{N}{4}) = X_1(k) - W_{N/2}^k X_2(k) \end{cases} \quad k = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = X_2(k) + X_3(k) \\ X_2(k + \frac{N}{4}) = X_2(k) - X_3(k) \end{cases} \quad k = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) = DFT[x_2(l)] = DFT[x_2(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_3(k) = DFT[x_3(l)] = DFT[x_3(2l)] \\ X_3(k + \frac{N}{4}) = DFT[x_3(l)] = DFT[x_3(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_4(k) = DFT[x_4(l)] = DFT[x_4(2l)] \\ X_4(k + \frac{N}{4}) = DFT[x_4(l)] = DFT[x_4(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_1(k) = DFT[x_1(l)] = DFT[x_1(2l)] \\ X_1(k + \frac{N}{4}) = DFT[x_1(l)] = DFT[x_1(2l+1)] \end{cases} \quad l = 0, 1, \dots, N/4-1$$

$$\begin{cases} X_2(k) = DFT[x_2(l)] = DFT[x_2(2l)] \\ X_2(k + \frac{N}{4}) =$$