1. 对于两分类问题，证明最小风险贝叶斯规则可表示为

若 $l(x)=\dfrac{p(X|\omega_1)}{p(X|\omega_2)}>\dfrac{\lambda_{12}-\lambda_{22}}{\lambda_{21}-\lambda_{11}}\dfrac{P(\omega_2)}{P(\omega_1)}$，则决策$X\in\omega_1$；否则$X\in\omega_2$。

解：计算条件风险

$$\begin{aligned}R(\alpha_1|x)&=\sum_{j=1}^{2}\lambda_{1j}P(w_j|x)\\&=\lambda_{11}P(w_1|x)+\lambda_{12}P(w_2|x)\end{aligned}$$

$$\begin{aligned}R(\alpha_2|x)&=\sum_{j=1}^{2}\lambda_{2j}P(w_j|x)\\&=\lambda_{21}P(w_1|x)+\lambda_{22}P(w_2|x)\end{aligned}$$

如果$R(\alpha_1|x)<R(\alpha_2|x)$，则$x\in w_1$。

$$\begin{aligned}\lambda_{11}P(w_1|x)+\lambda_{12}P(w_2|x)&<\lambda_{21}P(w_1|x)+\lambda_{22}P(w_2|x)\\(\lambda_{21}-\lambda_{11})P(w_1|x)&>(\lambda_{12}-\lambda_{22})P(w_2|x)\\(\lambda_{21}-\lambda_{11})P(w_1)p(x|w_1)&>(\lambda_{12}-\lambda_{22})P(w_2)p(x|w_2)\\\frac{p(x|w_1)}{p(x|w_2)}&>\frac{(\lambda_{12}-\lambda_{22})P(w_2)}{(\lambda_{21}-\lambda_{11})P(w_1)}\end{aligned}$$

所以，如果$\dfrac{p(x|w_1)}{p(x|w_2)}>\dfrac{(\lambda_{12}-\lambda_{22})P(w_2)}{(\lambda_{21}-\lambda_{11})P(w_1)}$，则$x\in w_1$。反之则$x\in w_2$。