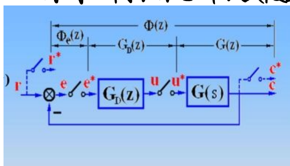
一、数字控制系统冲激传函



$\Phi_e(z)=1-\Phi(z),\ E(z)=\Phi_e(z)R(z)$

$$\begin{cases}\Phi(z)=\dfrac{G_D(z)G(z)}{1+G_D(z)G(z)}\\ \Phi_e(z)=\dfrac{1}{1+G_D(z)G(z)}=1-\Phi(z)\\ G_D(z)\cdot G(z)=\dfrac{\Phi(z)}{1-\Phi(z)}=\dfrac{\Phi(z)}{\Phi_e(z)}\end{cases}$$

$$\Rightarrow G_D(z)=\frac{\Phi(z)}{\Phi_e(z)G(z)}$$

二、最少拍控制 典型输入作用下，在有限拍内结束响应且在采样点上无稳态误差

1、典型测试输入的统一形式

$$r(t)=\begin{cases}1(t)\\ t\\ \frac{t^2}{2}\end{cases}\Rightarrow R(z)=\begin{cases}\dfrac{z}{z-1}=\dfrac{1}{1-z^{-1}}\\ \dfrac{Tz}{(z-1)^2}=\dfrac{Tz^{-1}}{(1-z^{-1})^2}\\ \dfrac{T^2z(z+1)}{2(z-1)^3}=\dfrac{T^2z^{-1}(1+z^{-1})}{2(1-z^{-1})^3}\end{cases}$$

$1,\ Tz^{-1},\ \dfrac{T^2z^{-1}(1+z^{-1})}{2} \rightarrow \dfrac{A(z)}{(1-z^{-1})^{\nu}} \rightarrow 1,\ 2,\ 3$

故设计思路：构造Φ(z)使输出可在最小步骤内匹配测试输入，再算出 $G_D(z)$

(I) 在除 (1,0j) 以外，G(z) 单位圆上/单位圆外无零极点前提下

$R(z)=\dfrac{A(z)}{(1-z^{-1})^{\nu}}$

$E(z)=\Phi_e(z)R(z),\quad \Phi_e(z)=1-\Phi(z)$

$e(\infty)=\lim\limits_{z\to1}(z-1)\,\Phi_e(z)\,R(z)\quad\Rightarrow\Phi_e(z)=(1-z^{-1})^{\nu}F(z^{-1})$

To make the D(z) simplest and of the lowest-order, we can choose $F(z^{-1})$ as 1.

$\Phi(z)=1-\Phi_e(z)=1-(1-z^{-1})^{\nu}$

而 $e(\infty T)=0$

$E(z)=\Phi_e(z)\cdot R(z)=\dfrac{A(z)}{(1-z^{-1})^{\nu}}\Phi_e(z)$

$e(\infty T)=\lim\limits_{z\to1}(1-z^{-1})\dfrac{A(z)}{(1-z^{-1})^{\nu}}\Phi_e(z)=0$

$\Phi_e(z)=(1-z^{-1})^{\nu}F(z)\overset{F(z)=1}{=}(1-z^{-1})^{\nu}$

$\Rightarrow \Phi(z)=1-\Phi_e(z)=1-(1-z^{-1})^{\nu}=\dfrac{b_1z^{\nu-1}+b_2z^{\nu-2}+\cdots+b_{\nu}}{z^{\nu}}$

Φ(z) 所有极点都位于z平面原点上

2、典型测试输入下的Φ(z)

① $r(t)=1(t)$

$\nu=1 \Rightarrow \Phi(z)=z^{-1}$，$E(z)=1$ 系统仅需一步即能实现跟踪

② $r(t)=t\cdot 1(t)$ 斜坡输入

$\nu=2 \Rightarrow \Phi(z)=2z^{-1}-z^{-2}$，$E(z)=Tz^{-1}$ 两步跟踪

③ $r(t)=\frac{1}{2}t^2\cdot 1(t)$

$\nu=3 \Rightarrow \Phi(z)=3z^{-1}-3z^{-2}+z^{-3}$，$E(z)=\frac{1}{2}T^2z^{-1}+\frac{1}{2}T^2z^{-2}$ 三步跟踪

$r(t)$	$R(z)$	$\Phi_e(z)=(1-z^{-1})^{\nu}$	$\Phi(z)=1-\Phi_e(z)$	$G_D(z)$	t_s
$1(t)$	$\dfrac{1}{1-z^{-1}}$	$1-z^{-1}$	z^{-1}	$\dfrac{z^{-1}}{(1-z^{-1})\cdot G(z)}$	T
t	$\dfrac{Tz^{-1}}{(1-z^{-1})^2}$	$(1-z^{-1})^2$	$2z^{-1}-z^{-2}$	$\dfrac{z^{-1}(2-z^{-1})}{(1-z^{-1})^2G(z)}$	$2T$
$\dfrac{t^2}{2}$	$\dfrac{T^2z^{-1}(1+z^{-1})}{2(1-z^{-1})^3}$	$(1-z^{-1})^3$	$3z^{-1}-3z^{-2}+z^{-3}$	$\dfrac{z^{-1}(3-3z^{-1}+z^{-2})}{(1-z^{-1})^3G(z)}$	$3T$

3、最少拍控制系统设计

① 计算 G(z)：G(z) 单位圆上/外无零极点，

② 据典型测试输入确定 $\Phi_e(z)$　$r(t)\Rightarrow R(z)=\dfrac{A(z)}{(1-z^{-1})^{\nu}}\Rightarrow\Phi_e(z)=(1-z^{-1})^{\nu}F(z)$ 取为 1

③ 计算 Φ(z)　$\Phi(z)=1-\Phi_e(z)$

④ 得 $G_D(z)$　$G_D(z)=\dfrac{\Phi(z)}{\Phi_e(z)G(z)}$

(II) G(z) 在单位圆上/外在零极点时

假设 $G(z)=\dfrac{z^{-\nu}\prod\limits_{i=1}(1-z_iz^{-1})}{\prod\limits_{i=1}(1-p_iz^{-1})}$ 零点，极点

进一步有 $G_D(z)=\dfrac{\Phi(z)}{\Phi_e(z)G(z)}=\dfrac{z^{\nu}\prod\limits_{i=1}(1-p_iz^{-1})\Phi(z)}{\prod\limits_{i=1}(1-z_iz^{-1})\Phi_e(z)}$

若存在，物理不可实现

必须含 $z^{-\nu}$ 以保证 $G_D(z)$ 可实现

零点为 z_i 以与分母抵消

若 p_i 在单位圆上/外，Φ(z) 不稳定

若 z_i 在单位圆上/外，$G_D(z)$ 不稳定

而 $\Phi(z)=G_D(z)G(z)\Phi_e(z)$

零点为 p_i 以保证Φ(z)稳定

Although the deadbeat control system tracks a particular test input accurately within a number of steps, it has the following disadvantages:

(1) It is designed only for a particular input.

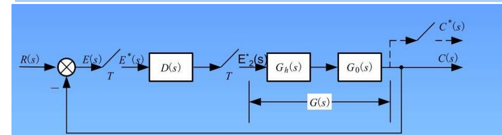
(2) The output has ripples although there are no errors on the sampling points.

(3) The control input changes drastically.

5. Ripple-free deadbeat control design

Ripple: though the system outputs are stable at the sampling time, they are varying between two sampling time, See p230, Fig. 7-50。

Objective: Not only tracking the input at the sampling time, the outputs are ripple-free.



$E_2(z)=D(z)E(z)$

solution: ensure $E_2(z)$ being a polynomial on z^{-1} of a finite order.

Condition: $E_2(z)$ is a polynomial on z^{-1} of finite order。

$E_2(z)=D(z)E(z)=D(z)\varphi_e(z)R(z),\quad D(z)\varphi_e(z)=\dfrac{\varphi(z)}{G(z)}$

➔ $D(z)\Phi_e(z)$=（*） $/z^r$, that is the zero of G(z) must be a zero of Φ(z)

最少拍设计中，Φ(z) 和 $\Phi_e(z)$ 选取时应遵循的原则：

1。D(z)零点的数目不能大于极点的数目；

2。 $\Phi_e(z)$ 应把G(z)在单位圆上及单位圆外的极点作为自己的零点；

3。 Φ(z)应把G(z)在单位圆上及单位圆外的零点作为自己的零点；

4。当G(z)含有 z^{-1} 因子时，要求Φ(z)也含有 z^{-1} 的因子；

5。 因为Φ(z)=1- $\Phi_e(z)$，他们应该是关于 z^{-1} 同样阶次的多项式，而且 $\Phi_e(z)$ 还应包含常数项1。

6。当最小拍系统还有无纹波要求时，闭环脉冲传函Φ(z)的零点应抵消G(z)的全部零点（因为最少拍系统设计中G（z）单位圆上及单位圆外的零极点已经被补偿，因此在无纹波的设计中只需抵消G(z)单位圆内的零点）。