Lecture 9 习题作业

1，假设有如下训练样本：$\vec{x}_1 = (0,0)^T$属于第一类，$\vec{x}_2 = (1,1)^T$属于第二类，$\vec{x}_3 = (-1,1)^T$属于第三类，请用多类分类中的 OVO（One-versus-one）策略，设计上述三类别的两两分类器，并分析测试样本$\vec{\mathrm{x}} = (1,-2)^T$属于那个类别。

解：利用 OVO 策略，对三个类别两两求分类面：

（1）用感知器算法求第一类和第二类之间的分类面

样本增广后为：$\vec{x}_1 = (1,0,0)^T$，$y_1 = 1$，$\vec{x}_2 = (1,1,1)^T$，$y_2 = -1$，初始化权重：$\vec{w}_{[1,2]}^{(0)} = (0,0,0)^T$

$sign\left(\vec{w}_{[1,2]}^{(0)T}\vec{x}_1\right) = 0 \neq y_1$，$\therefore\ \vec{w}_{[1,2]}^{(1)} = \vec{w}_{[1,2]}^{(0)} + y_1\vec{x}_1 = (1,0,0)^T$，

$sign\left(\vec{w}_{[1,2]}^{(1)T}\vec{x}_2\right) = 1 \neq y_2$，$\therefore\ \vec{w}_{[1,2]}^{(2)} = \vec{w}_{[1,2]}^{(1)} + y_2\vec{x}_2 = (0,-1,-1)^T$

$sign\left(\vec{w}_{[1,2]}^{(2)T}\vec{x}_1\right) = 0 \neq y_1$，$\therefore\ \vec{w}_{[1,2]}^{(3)} = \vec{w}_{[1,2]}^{(2)} + y_1\vec{x}_1 = (1,-1,-1)^T$

$sign\left(\vec{w}_{[1,2]}^{(3)T}\vec{x}_2\right) = -1 = y_2$，且 $sign\left(\vec{w}_{[1,2]}^{(3)T}\vec{x}_1\right) = 1 = y_1$

$\therefore\ \vec{w}_{[1,2]} = (1,-1,-1)^T$，分类面为：$1 - x_1 - x_2 = 0$

（2）用感知器算法求第一类和第三类之间的分类面

样本增广后为：$\vec{x}_1 = (1,0,0)^T$，$y_1 = 1$，$\vec{x}_3 = (1,-1,1)^T$，$y_3 = -1$，初始化权重：$\vec{w}_{[1,3]}^{(0)} = (0,0,0)^T$

$sign\left(\vec{w}_{[1,3]}^{(0)T}\vec{x}_1\right) = 0 \neq y_1$，$\therefore\ \vec{w}_{[1,3]}^{(1)} = \vec{w}_{[1,3]}^{(0)} + y_1\vec{x}_1 = (1,0,0)^T$，

$sign\left(\vec{w}_{[1,3]}^{(1)T}\vec{x}_3\right) = 1 \neq y_3$，$\therefore\ \vec{w}_{[1,3]}^{(2)} = \vec{w}_{[1,3]}^{(1)} + y_3\vec{x}_3 = (0,1,-1)^T$

$sign\left(\vec{w}_{[1,3]}^{(2)T}\vec{x}_1\right) = 0 \neq y_1$，$\therefore\ \vec{w}_{[1,3]}^{(3)} = \vec{w}_{[1,3]}^{(2)} + y_1\vec{x}_1 = (1,1,-1)^T$

$sign\left(\vec{w}_{[1,3]}^{(3)T}\vec{x}_3\right) = -1 = y_3$，且 $sign\left(\vec{w}_{[1,3]}^{(3)T}\vec{x}_1\right) = 1 = y_1$