

所以:  $\delta_j^{(2)} = \sum_k (\delta_k^{(3)})(w_{jk}^{(3)}) \llbracket s_j^{(2)} \geq 0 \rrbracket = \delta_1^{(3)} w_{j1}^{(3)} \llbracket s_j^{(2)} \geq 0 \rrbracket$

$$\text{即: } \vec{\delta}^{(2)} = \begin{pmatrix} \delta_1^{(2)} \\ \delta_2^{(2)} \\ \delta_3^{(2)} \end{pmatrix} = \begin{pmatrix} \delta_1^{(3)} w_{11}^{(3)} \llbracket s_1^{(2)} \geq 0 \rrbracket \\ \delta_1^{(3)} w_{21}^{(3)} \llbracket s_2^{(2)} \geq 0 \rrbracket \\ \delta_1^{(3)} w_{31}^{(3)} \llbracket s_3^{(2)} \geq 0 \rrbracket \end{pmatrix}$$

继续运用反向传播法, 于是:  $\delta_j^{(1)} = \sum_k (\delta_k^{(2)})(w_{jk}^{(2)})(x_j^{(1)})'$ , 所以:

$$\delta_j^{(1)} = \sum_k (\delta_k^{(2)})(w_{jk}^{(2)}) \llbracket s_j^{(1)} \geq 0 \rrbracket = (\delta_1^{(2)} w_{j1}^{(2)} + \delta_2^{(2)} w_{j2}^{(2)} + \delta_3^{(2)} w_{j3}^{(2)}) \llbracket s_j^{(1)} \geq 0 \rrbracket$$

由此可以得到:

$$\vec{\delta}^{(1)} = \begin{pmatrix} \delta_1^{(1)} \\ \delta_2^{(1)} \end{pmatrix} = \begin{pmatrix} \llbracket s_1^{(1)} \geq 0 \rrbracket & 0 \\ 0 & \llbracket s_2^{(1)} \geq 0 \rrbracket \end{pmatrix} \begin{pmatrix} w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} \\ w_{21}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} \end{pmatrix} \begin{pmatrix} \delta_1^{(2)} \\ \delta_2^{(2)} \\ \delta_3^{(2)} \end{pmatrix}$$

假定t表示迭代次数,  $\eta$ 为学习步长, 利用梯度下降法进行权系数更新:

$$\begin{aligned} \mathbf{w}_{t+1}^{(1)} &= \mathbf{w}_t^{(1)} - \eta \vec{x}_n^{(0)} (\vec{\delta}^{(1)})^T = \mathbf{w}_t^{(1)} - \eta \begin{pmatrix} 1 \\ x_{n1}^{(0)} \\ x_{n2}^{(0)} \end{pmatrix} (\delta_1^{(1)}, \delta_2^{(1)}) \\ &= \begin{pmatrix} w_{01}^{(1)} & w_{02}^{(1)} \\ w_{11}^{(1)} & w_{12}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} \end{pmatrix} - \eta \begin{pmatrix} \delta_1^{(1)} & \delta_2^{(1)} \\ x_{n1}^{(0)} \delta_1^{(1)} & x_{n1}^{(0)} \delta_2^{(1)} \\ x_{n2}^{(0)} \delta_1^{(1)} & x_{n2}^{(0)} \delta_2^{(1)} \end{pmatrix} \\ \mathbf{w}_{t+1}^{(2)} &= \mathbf{w}_t^{(2)} - \eta \vec{x}_n^{(1)} (\vec{\delta}^{(2)})^T = \mathbf{w}_t^{(2)} - \eta \begin{pmatrix} 1 \\ x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} (\delta_1^{(2)}, \delta_2^{(2)}, \delta_3^{(2)}) \\ &= \begin{pmatrix} w_{01}^{(2)} & w_{02}^{(2)} & w_{03}^{(2)} \\ w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} \\ w_{21}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} \end{pmatrix} - \eta \begin{pmatrix} \delta_1^{(2)} & \delta_2^{(2)} & \delta_3^{(2)} \\ x_1^{(1)} \delta_1^{(2)} & x_1^{(1)} \delta_2^{(2)} & x_1^{(1)} \delta_3^{(2)} \\ x_2^{(1)} \delta_1^{(2)} & x_2^{(1)} \delta_2^{(2)} & x_2^{(1)} \delta_3^{(2)} \end{pmatrix} \\ \mathbf{w}_{t+1}^{(3)} &= \mathbf{w}_t^{(3)} - \eta \vec{x}_n^{(2)} (\vec{\delta}^{(3)})^T = \mathbf{w}_t^{(3)} - \eta \begin{pmatrix} 1 \\ x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{pmatrix} \delta_1^{(3)} = \begin{pmatrix} w_{01}^{(3)} \\ w_{11}^{(3)} \\ w_{21}^{(3)} \\ w_{31}^{(3)} \end{pmatrix} - \eta \begin{pmatrix} \delta_1^{(3)} \\ x_1^{(2)} \delta_1^{(3)} \\ x_2^{(2)} \delta_1^{(3)} \\ x_3^{(2)} \delta_1^{(3)} \end{pmatrix} \end{aligned}$$

反复迭代至T次。

(2) 代入习题数据的解答流程: