例: 已知两类样本

$$\omega_1:\{(-5,-5)',(-5,-4)',(-4,-5)',(-5,-6)',(-6,-5)'\}$$

$$\omega_2:\{(5,5)',(5,6)',(6,5)',(5,4)',(4,5)'\}$$

试用PCA变换做一维特征提取。

解： $\because \hat{P}(\omega_1)=\hat{P}(\omega_2)=5/10=1/2$

（1） $\therefore R=E[xx']=\sum_{i=1}^{2}\ \hat{P}(\omega_i)E[x^{(i)}x^{(i)}\,']=\frac{1}{2}[\frac{1}{5}\sum_{i=1}^{5}x_i^{(1)}x_i^{(1)}\,']+\frac{1}{2}[\frac{1}{5}\sum_{i=1}^{5}x_i^{(2)}x_i^{(2)}\,']$

$$=\begin{pmatrix}25.4 & 25\\ 25 & 25.4\end{pmatrix}$$

（2）求R的特征值、特征矢量

$$|R-\lambda I|=(25.4-\lambda)^2-25^2=0\ \Rightarrow\ \lambda_1=50.4,\ \ \lambda_2=0.4$$

$$Rt_j=\lambda_j t_j, j=1,2\ \ \Rightarrow\ \ t_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\quad t_2=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

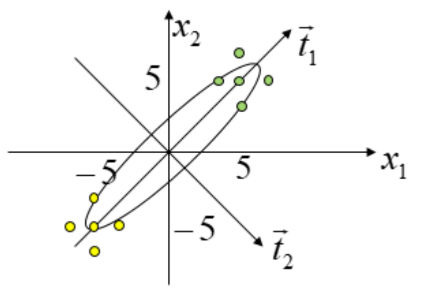
(4) 选λ_1对应的 $\vec{t}_1$作为变换矩阵 $U=[\vec{t}_1]=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

由 $y=T'\vec{x}$ 得变换后的一维模式特征为

$$\vec{y}_1^{(1)}=U^T\vec{x}_1^{(1)}=\frac{1}{\sqrt{2}}(1,1)\begin{pmatrix}-5\\-5\end{pmatrix}=-\frac{10}{\sqrt{2}}$$

$$\vdots$$

$$y_5^{(1)}=U^T\vec{x}_5^{(1)}=-\frac{11}{\sqrt{2}}$$



得 $\omega_1:\{-\frac{10}{\sqrt{2}},-\frac{9}{\sqrt{2}},-\frac{9}{\sqrt{2}},-\frac{11}{\sqrt{2}},-\frac{11}{\sqrt{2}}\}$

$\omega_2:\{\frac{10}{\sqrt{2}},\frac{11}{\sqrt{2}},\frac{11}{\sqrt{2}},\frac{9}{\sqrt{2}},\frac{9}{\sqrt{2}}\}$