$$\text{III: } \begin{pmatrix} x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} \max(0, s_1^{(1)}) \\ \max(0, s_2^{(1)}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

第二层神经元的输入为:

$$\begin{pmatrix} s_1^{(2)} \\ s_2^{(2)} \\ s_2^{(2)} \end{pmatrix} = (\mathbf{w}^{(2)})^T \begin{pmatrix} 1 \\ x_1^{(1)} \\ x_2^{(1)} \end{pmatrix} = \begin{pmatrix} 0.42 & -0.29 & -0.29 \\ 0.42 & -0.29 & -0.29 \\ 0.42 & -0.29 & -0.29 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.42 \\ 0.42 \\ 0.42 \end{pmatrix}$$

则:
$$\begin{pmatrix} x_1^{(2)} \\ x_2^{(2)} \\ x_3^{(2)} \end{pmatrix} = \begin{pmatrix} 0.42 \\ 0.42 \\ 0.42 \end{pmatrix}$$

则第三层的输入为:

$$\mathbf{s}_{1}^{(3)} = (\mathbf{w}^{(3)})^{T} \begin{pmatrix} 1 \\ x_{1}^{(2)} \\ x_{2}^{(2)} \\ x_{3}^{(2)} \end{pmatrix} = (0.66 -1.90 -1.90 -1.90) \begin{pmatrix} 1 \\ 0.42 \\ 0.42 \\ 0.42 \end{pmatrix} = -1.73$$

即输出 $\hat{y} = s_1^{(3)} = -1.73$

对于样本 \vec{x}_4 ,其标签为-1,采用平方误差函数: $e_n = (y_n - \hat{y}_n)^2$,则:

$$\delta_1^{(3)} = -2(y_n - s_1^{(3)}) = -2(-1 - (-1.73)) = -1.46$$

运用反向传播法,于是:

$$\delta_{j}^{(2)} = \sum_{k} (\delta_{k}^{(3)})(w_{jk}^{(3)}) \left[s_{j}^{(2)} \ge 0 \right] = \delta_{1}^{(3)} w_{j1}^{(3)} \left[s_{j}^{(2)} \ge 0 \right]$$

$$\mathbb{R} : \vec{\delta}^{(2)} = \begin{pmatrix} \delta_1^{(2)} \\ \delta_2^{(2)} \\ \delta_3^{(2)} \end{pmatrix} = \begin{pmatrix} \delta_1^{(3)} w_{11}^{(3)} \begin{bmatrix} s_1^{(2)} \geq 0 \end{bmatrix} \\ \delta_1^{(3)} w_{21}^{(3)} \begin{bmatrix} s_2^{(2)} \geq 0 \end{bmatrix} \\ \delta_1^{(3)} w_{31}^{(3)} \begin{bmatrix} s_2^{(2)} \geq 0 \end{bmatrix} \end{pmatrix} = \begin{pmatrix} (-1.46) * (-1.90) * 1 \\ (-1.46) * (-1.90) * 1 \\ (-1.46) * (-1.90) * 1 \end{pmatrix} = \begin{pmatrix} 2.77 \\ 2.77 \end{pmatrix}$$

继续运用反向传播法,于是: $\delta_j^{(1)} = \sum_k (\delta_k^{(2)})(w_{jk}^{(2)})(x_j^{(1)})'$,所以:

$$\delta_j^{(1)} = \sum\nolimits_k (\delta_k^{(2)}) (w_{jk}^{(2)}) \left[\! \left[s_j^{(1)} \geq 0 \right] \! \right] = (\delta_1^{(2)} w_{j1}^{(2)} + \delta_2^{(2)} w_{j2}^{(2)} + \delta_3^{(2)} w_{j3}^{(2)}) \left[\! \left[s_j^{(1)} \geq 0 \right] \! \right]$$

由此可以得到: