一、状态观测器

利用 y=cx 重新编码状态 x

1、全阶状态观测器 ⇒重编码状态向量与原状态向量同维度

观测器模型

The linear time-invariant system

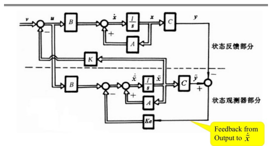
$$\dot{x} = Ax + Bu \qquad y = Cx$$

Compose a simulation system:

$$\dot{\tilde{x}} = A\tilde{x} + Bu \qquad \tilde{y} = c\tilde{x}$$

If $\dot{x}$ and $\dot{\tilde{x}}$ has the same initial value, we can use simulated state variables $\dot{\tilde{x}}$ to estimate $\dot{x}$.

When $\dot{x}$ and $\dot{\tilde{x}}$ are varying, the output y and $\tilde{y}$ will be different as well. We can use $y-\tilde{y}$ to revise the observer model $\dot{\tilde{x}}$, by the feedback from $y-\tilde{y}$ to $\dot{\tilde{x}}$, to achieve $x-\tilde{x} \to 0$



Feedback from Output to $\dot{\tilde{x}}$

The observer model is: $\dot{\tilde{x}} = A\tilde{x} + Bu + K_e c(x-\tilde{x})$

If it is satisfied for any initial value that: $\lim_{t\to\infty}[\tilde{x}(t) - x(t)] = 0$

Then, the state observer is: $\dot{\tilde{x}} = A\tilde{x} + Bu + K_e(y-\tilde{y})$

If the system (A,B,C) is observable, its state can be estimated by the full order state observer:

$$\dot{\tilde{x}} = A\tilde{x} + Bu + K_e c(x-\tilde{x}) = (A-K_e c)\tilde{x} + Bu + K_e y$$

In which, matrix K_e is selected as the condition of $\boxed{\text{pole assignment}}$

线性反馈矩阵Ke的选择

① ke存在的条件

Step 1: Establish the differential equation of $\tilde{x} - x$

Step 2: Consider the state equations of system and observer:

$$\dot{x} = Ax + Bu$$

$$\dot{\tilde{x}} = A\tilde{x} + Bu + K_e c(x-\tilde{x})$$

Calculate the minus of above equations: $\dot{\tilde{x}} - \dot{x} = (A-K_e c)(\tilde{x} - x)$

To satisfy: $\lim_{t\to\infty}[\tilde{x}(t) - x(t)] = 0$?

$$\dot{\tilde{x}} - \dot{x} = (A-K_e c)(\tilde{x} - x)$$

To satisfy: $\lim_{t\to\infty}[\tilde{x}(t) - x(t)] = 0$?

Solve the differential equation: $\dot{\tilde{x}} - \dot{x} = (A-K_e c)(\tilde{x} - x)$

We have: $x(t) - \tilde{x}(t) = e^{(A-K_eC)(t-t_0)}[x(t_0) - \tilde{x}(t_0)]$

If and only if the eigenvalue of $(\boldsymbol{A}\text{-}\boldsymbol{K_e c})$ are all in the left s plane, $\tilde{x} - x$ will tend to 0 by the exponential law of time t.

② ke选择

The decay rate of $\lim_{t\to\infty}[\tilde{x}(t) - x(t)] = 0$ is decided by the assigned poles of the estimator.

To satisfy the rapidity of $\tilde{x}$ approaching to x, the poles of state estimator should be allocated to the place far away to the imaginary axis in the ***s plane*** (much more minus than the poles of estimated system). However, the parameters of matrix $\mathbf{K_e}$ should not be too large.

1) the selection of $\mathbf{K_e}$ will be restricted by the real condition of the equipment (such as capability, saturation, heat, and stress, etc.)

2) Since the frequency band of observer is broadened, the noise of input u and output y will cause larger noise of $\tilde{x}$, which is sensitive to other kind of noise as well.

Therefore, in practical applications, the selection of $\mathbf{K_e}$ should not be too large.

线性反馈矩阵ke计算

Transform the system to the observable canonical form to design $\mathbf{K_e}$ by the following steps:

Define the transforming matrix P: $P = (WR)^{-1}$

Here, R is the observable matrix:

$$R^T = [C^T \vdots A^T C^T \vdots \cdots \vdots (A^T)^{n-1} C^T]$$

The symmetrical matrix **W** is defined as follow:

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

a_i is the coefficients of following eigenequation (i=1,2,…,n):

$$|sI - A| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n = 0$$

Since the system is supposed to be completely observable, the inverse of matrix WR is existed. Under the effect of the linear transformation $x = P\xi$, system can be transformed to the observable canonical form.

$$\tilde{A} = P^{-1}AP = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix}$$

$$\tilde{B} = P^{-1}B = \begin{bmatrix} b_n - a_n b_o \\ b_{n-1} - a_{n-1} b_o \\ \vdots \\ b_1 - a_1 b_o \end{bmatrix} \qquad \tilde{C} = CP = [0 \ 0 \ \cdots \ 0 \ 1]$$

Follow the method to obtain state feedback matrix **K** in the poles assignment of state feedback.

$$K_e = P\begin{bmatrix} a_n^* - a_n \\ a_{n-1}^* - a_{n-1} \\ \vdots \\ a_1^* - a_1 \end{bmatrix} = (WR)^{-1}\begin{bmatrix} a_n^* - a_n \\ a_{n-1}^* - a_{n-1} \\ \vdots \\ a_1^* - a_1 \end{bmatrix}$$

In which, a_i and a_i^*, (i=1,2,…,n) are the eigenequations coefficients of the original system and expected state observer. Above equation achieves the desired state observer gain matrix.

If the expected eigenvalues are selected (or expected eigenequation), and the system is completely observable, the full dimensional observer can be designed.

观测器对闭环系统影响

The state function of the state completely controllable and observerable linear time-invariant system:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Import the state observer, achieve the feedback from observed state $\tilde{x}(t)$ to the input: $\boldsymbol{u = v - K\tilde{x}}$

$$\dot{x} = Ax - BK\tilde{x} + Bv = (A-BK)x + BK(x-\tilde{x}) + Bv$$

The difference between real state and estimate state of the system is:

$$\dot{\tilde{x}} - \dot{x} = (A-K_e c)(\tilde{x} - x)$$

$$\dot{x} = Ax - BK\tilde{x} + Bv = (A-BK)x + BK(x-\tilde{x}) + Bv$$

Combine 2 equations above, we have:

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} - \dot{x} \end{bmatrix} = \begin{bmatrix} A-BK & BK \\ 0 & A-K_eC \end{bmatrix}\begin{bmatrix} x \\ \tilde{x} - x \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v$$

which describe the dynamic character of the observe-state feedback control system, whose eigenequation is:

$$\begin{vmatrix} sI - A + BK & -BK \\ 0 & sI - A + K_eC \end{vmatrix} = 0$$

therefore $|sI - A + BK|\,|sI - A + K_eC| = 0$

The Poles from pure poles allocation

The Poles from pure observer design

可观状态反馈控制系统包含纯极点分配与纯观测设计

若系统和观测器均n阶(全阶观测)，整个闭环系统的特征方程为2n阶

Separation Theorem:

If the control system (A,B,C) is controllable and observable, the pole allocation (matrix **K**) and observer design (matrix $\mathbf{K_e}$) of the system can be calculated separately to derive the state feedback by the estimate value of the state observer.

➤ Transfer function description of the State Controllability and Observability Conditions

State controllability and Observability conditions can be described by transfer function, as well.

The n.s. condition of state controllable and observable is No Cancellation Appeared in the Transfer Function.

If there is cancellation in the transfer function, the system is uncontrollable or unobservable, or even uncontrollable and unobservable simultaneously.

Attention that: there is reducible factor in the transfer function of the system.

The transfer function between $X_1(s)$ and $U(s)$ is:

$$\frac{X_1(s)}{U(s)} = \frac{1}{(s+1)(s+2)(s+3)}$$

The one between $Y(s)$ and $X_1(s)$ is:

$$\frac{Y(s)}{X_1(s)} = (s+1)(s+4) \qquad \text{then} \qquad \frac{Y(s)}{U(s)} = \frac{(s+1)(s+4)}{(s+1)(s+2)(s+3)}$$

The factor (s+1) in the numerator and denominator polynomial can be reduced. Therefore, the system is unobservable, or some nonzero initial state x(0) cannot be measured by y(t).

If and only if, system is controllable and observable, its transfer function has no reducible factor. That is to say, the reducible transfer function doesn't have complete information to describe the dynamic system.