一、线性时不变系统常见反馈

1、状态反馈

An n-dimension linear time-invariant system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

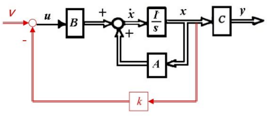
In which, x,u,y are n-dimension, p-dimension and q-dimension vectors;

A,B,C are $n\times n$, $n\times p$ and $q\times n$ real matrix.

If we choose the linear function of state variable as the control value u.

$$u = v - Kx$$

it is called linear state feedback, or state feedback in short .(all states can be the feedback).



The system with state feedback

The dynamic of the state feedback system.

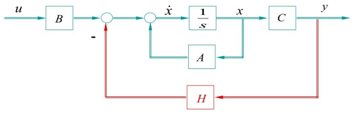
$$\dot{x} = (A - BK)x + Bv$$
$$y = Cx \quad \text{(unchanged)}$$

Transfer function matrix is:

$$G_K(s) = C(sI - A + BK)^{-1}B$$

2、输出反馈

① 反馈至状态微分

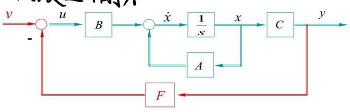


State space : $\dot{x} = Ax + Bu - Hy = (A - HC)x + Bu$

$y = Cx$

Transfer function: $G_H(s) = C(sI - A + HC)^{-1}B$

② 反馈至输入



Choose the linear function of output y to be the control value u

$$u = v - Fy$$

State space expression:

$$\dot{x} = (A - BFC)x + Bv$$
$$y = Cx$$

Transfer function matrix is:

$$H_F = C(sI - A + BFC)^{-1}B$$

二、状态反馈与极点设计

A SISO linear time-invariant system:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu$$
$$x(t) \in R^n, u(t) \in R^1, A \in R^{n\times n}, B \in R^{n\times 1} \qquad K \in R^{1\times n}$$

Select linear feedback controller: $u = \boldsymbol{v} - K\boldsymbol{x}$

In which, $K \in R^{1\times n}$ is called the state feedback gain matrix or linear state feedback matrix.

$$\dot{x}(t) = (A - BK)x + Bv$$

The eigenvalues of the matrix *A-BK* are the poles of the closed-loop system. Choose the value of feedback matrix *K*, we can construct the matrix *A-BK* to an asymptotically stable system, and allocate the closed poles to the expected place, which is called **Poles assignment/poles allocation**.

Assume the control input u is unrestricted, if we choose following control input: $u=v-Kx$, with K is the linear state feedback matrix.

Consider the assignment conditions, we have the poles assignment theorem:

Theorem (n.s. condition): if a linear time-invariant system is completely controllable, all poles can be allocated by the linear state feedback. (Such condition will be available in both SISO systems and MIMO systems.)

设计

Obtain the transforming matrix P

If system is completely controllable, there exists nonsingular transformation, which can transform the system to the controllable canonical form.

Define the transforming matrix: **P=QW**

$$Q = [B \quad AB \quad \cdots \quad A^{n-1}B]$$

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

In which, a_i are coefficients of the eigenpolynomials.

Poles assignment algorithm for SISO system

For system (A,b) and expected eigenvalues $(\lambda_1,\lambda_2,\ldots,\lambda_n)$

We can obtain $1\times n$ dimension feedback gain vector $\boldsymbol{k}$ by following steps:

Step 1: Analyze the controllability of the system, then continue the following steps if system is states completely controllable;

Step 2: Calculate the eigenpolynomial of A;

$$\det[sI - A] = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1}s + a_n$$

Step 3: Calculate the expected eigenpolynomial from closed-loop eigenvalues

$$a^*(s) = (s-\lambda_1)(s-\lambda_2)\cdots(s-\lambda_n)$$
$$= s^n + a_1^* s^{n-1} + \cdots + a_{n-1}^* s + a_n^*$$

Step 4: Calculate $\bar{\mathbf{k}} = [a_n^* - a_n \quad a_{n-1}^* - a_{n-1} \quad \cdots \quad a_1^* - a_1]$

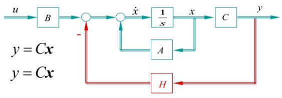
Step 5: Calculate transforming matrix **P=QW**, if the given state equation is controllable canonical, the **P=I**.

Step 6: The state feedback matrix is: $\mathbf{K} = \bar{\mathbf{k}}\mathbf{P}^{-1}$

顺序进行/设矩阵K

$|sI-A+BK|$后与$a^*(s)$比较系数

三、输出反馈与极点设计（PPT例题）

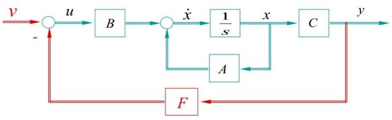


$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} - \boldsymbol{h}y \qquad y = C\boldsymbol{x}$$
$$\dot{\boldsymbol{x}} = (A - \boldsymbol{h}C)\boldsymbol{x} + B\boldsymbol{u} \qquad y = C\boldsymbol{x}$$

Theorem (n.s. condition): if the system is completely observable, all poles can be allocated by the feedback of the output to the differential of the state.

To design the output feedback matrix h from expected closed-loop poles, we should compare the eigenpolynomials of expected system with the output feedback system $|\lambda I - (A - hC)|$.

If we use Output to Input feedback (in figure), using MISO system e.g., the feedback matrix $F \in R^{p\times l}$,



$$u = v - Fy$$
$$\dot{x} = (A - BFc)x + Bv$$

Assume the output feedback Fc=K is equal to the state feedback, the poles can be allocated freely by selecting the proper F.

四、反馈对系统性能的影响

- By feedback, the system matrices will be changed, which will influent the system performance.
- Controllability, Observability, Stability, other performance？
- ✓ The State feedback and Output feedback can stabilize the system
- ✓ The State feedback won't affect the system controllability, but might change its observability
- ✓ The output to state differential feedback won't affect system observability, but might change its controllability
- ✓ The Output to input feedback won't affect system controllability and observability
- ✓ The State feedback and Output feedback won't change the zeros of the system.