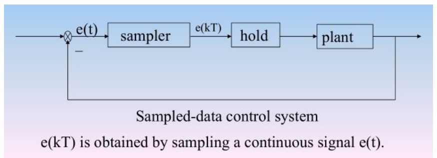
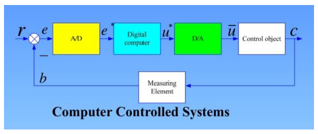
7.1 Discrete-Time System 概述

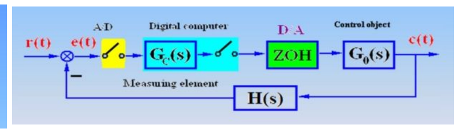
{ sample system: continuous value 除少数采样操作，系统连续
{ digital system: quantized value 引脉冲序列/数字信号



A→D: 采样＋量化 ; D→A: 保持＋平滑

一般计算机系统





7.2 采样系统

sample process: continuous → discrete

holding process: discrete → continous

sampling period

Sampler 采样器: switch which close every T seconds for one instant time $e(t)$ ⟋ $e^*(t)$

ideal sample process: ① $t \ll T$ sample process is completed instantaneously 采样瞬时完成 $\tau \to 0$

② word length is enough, $e^*(kt) = e(kt)$ 采样点无误差

e^* 在采样时间整数倍点上

Unit Impulse signal $\delta(t) = \begin{cases} 0, & t \neq 0 \\ 1, & t = 0 \end{cases}$

Unit Impulse Sequence 单位脉冲序列 $\delta_T = \sum_{k=0}^{\infty} \delta(t-kT)$

Sampling signal $e^*(t) = \sum_{k=0}^{\infty} e(t)\delta(t-kT) = e(t)\sum_{k=0}^{\infty}\delta(t-kT) = e(t)\delta_T(t)$

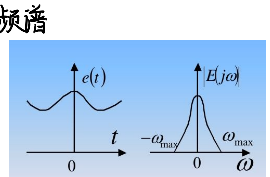
Laplace Transformation $e^*(t) = \sum_{k=0}^{\infty} e(kT)\delta(t-kT)$

$E^*(s) = \mathcal{L}[e^*(t)] = \sum_{k=0}^{\infty} e(kT)e^{-kTs}$

*位移定理 $\begin{cases} \text{实域位移: } \mathcal{L}[f(t-\tau)] = e^{-\tau s}F(s) \\ \text{复域位移: } \mathcal{L}^{-1}[F(s-a)] = e^{at}f(t) \end{cases}$

采样信号频域分析

考虑如下连续信号及频谱



对采样信号 $\delta_T(s)$ 有傅氏展开 傅立叶级数

$\delta_T(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$

采样得信号

$e^*(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e(t)e^{jk\omega_s t}$

有拉氏变换

$E^*(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} E(s+jk\omega_s)$

$E^*(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} E[j(\omega+k\omega_s)]$

$E^*(s) = \sum_{n=0}^{\infty} e(nT)\cdot e^{-nTs}$

① shows the value relation of E*(s) and e(t) on the sampling point;

② can be written into the closed form;

③ can be used to obtain the time response and the Z transform of e*(t)

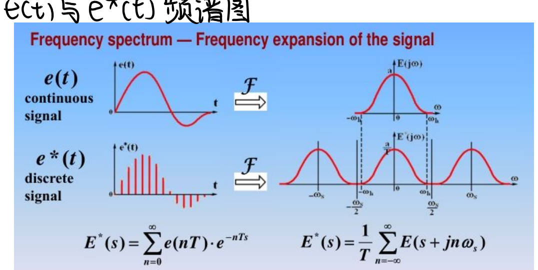
$E^*(s) = \frac{1}{T}\sum_{n=-\infty}^{\infty} E(s+jn\omega_s)$

① shows relationship of E*(s) and E(s);

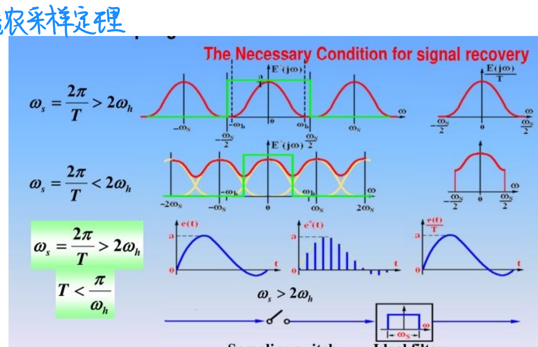
② can not be written as close form;

③ can be used for the frequency spectrum analysis of e*(t).

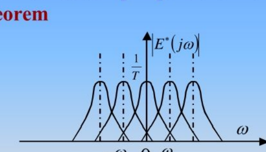
可画出 $e(t)$ 与 $e^*(t)$ 频谱图



引出香农采样定理



there are no overlap of each component, so the input signal can be recovered approximately. The is called sampling theorem or Shannon's Theorem

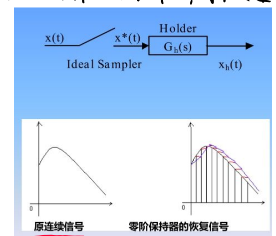


In the figure the input signal can't be recovered.

7.3 信号恢复与零阶保持

1、信号恢复

设采样信号由理想采样环节获取，有恢复信号



2、零阶保持 → $\boxed{ZOH}$ →

$k(t) = 1(t) - 1(t-T)$

$\Rightarrow x_h(t) = \sum_{k=0}^{\infty} x(kT)[1(t-kT) - 1(t-kT-T)]$

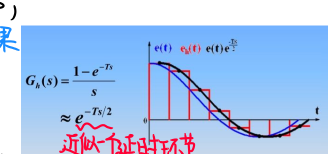
进行拉氏变化 $x^*(s)$

$x_h(s) = \underbrace{\sum_{k=0}^{\infty} x(kT)e^{-kTs}}_{x^*(s)}\left(\frac{1}{s} - \frac{1}{s}e^{-Ts}\right)$

故

$G_h(s) = \frac{x_h(s)}{x^*(s)} = \frac{1-e^{-Ts}}{s}$

及其恢复效果

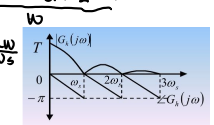


近似一个延时环节

频域分析

$G_h(j\omega) = \frac{1-e^{-j\omega T}}{j\omega} = \frac{e^{-\frac{1}{2}j\omega T}(e^{\frac{1}{2}j\omega T} - e^{-\frac{1}{2}j\omega T})}{j\omega} = \frac{2e^{-\frac{1}{2}j\omega T}\sin(\frac{1}{2}\omega T)}{\omega}$

$= T \cdot \underbrace{\frac{\sin(\frac{\omega T}{2})}{\frac{\omega T}{2}}}_{Sa(\frac{\omega T}{2})} e^{-\frac{1}{2}j\omega T} = \frac{2\pi}{\omega_s} Sa\left(\frac{\pi\omega}{\omega_s}\right) e^{-j\frac{\pi\omega}{\omega_s}}$ ($T = \frac{2\pi}{\omega_s}$)



{ 幅值 $|G_h(j\omega)| = \frac{2\pi}{\omega_s}\left|Sa\left(\frac{\pi\omega}{\omega_s}\right)\right|$

{ 相角 $\angle G_h(j\omega) = -\frac{2\pi}{\omega_s} + \angle Sa\left(\frac{\pi\omega}{\omega_s}\right)$ * $\angle Sa\left(\frac{\pi\omega}{\omega_s}\right) = \begin{cases} 0, & 2n\omega_s < \omega < (2n+1)\omega_s \\ \pi, & (2n+1)\omega_s < \omega < (2n+2)\omega_s \end{cases}$