求解其它指标之前应先判断

稳定性

一、引入

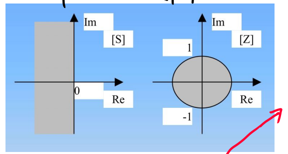
对于一连续系统稳定的充要条件：传函极点实部为负

离散系统引入 $z=e^{Ts}$，$s=\sigma+j\omega$，故对上述条件进行延拓

二、s域到z域的映射情况

$z=e^{sT}\xrightarrow{s=\sigma+j\omega} z=e^{\sigma T}e^{j\omega T}\Rightarrow |z|=e^{\sigma T}$，$\angle z=\omega T$

$\Rightarrow\begin{cases}\sigma>0, & |z|>1\\ \sigma<0, & |z|<1\\ \sigma=0, & |z|=1\end{cases}$



$\Phi(z)=\dfrac{G(z)}{1+G(z)}$

特征方程 $1+G(z)=0$

※线性离散系统稳定充要条件 ⇒ $\Phi(z)$ 极点均在z域的单位圆中

Prove: $\Phi(z)=\dfrac{M(z)}{D(z)}=\dfrac{\prod_{i=1}^{m}(z-\alpha_i)}{\prod_{j=1}^{n}(z-\beta_j)}=\sum_{j=1}^{n}\dfrac{C_j z}{z-\beta_j}=K(z)$

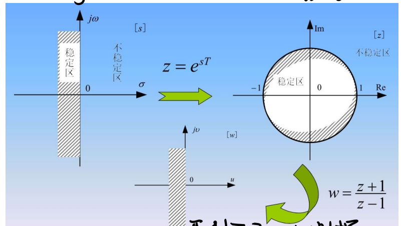
$c(k)=\sum_{j=1}^{n}C_j\beta_j^k\xrightarrow{k\to\infty}0$ $\quad|\beta_j|<1$ — Necessity

$c^*(t)=\sum_{k=0}^{\infty}\left(\sum_{j=1}^{n}C_j\beta_j^k\right)\cdot\delta(t-kT)$ — Sufficiency

三、离散时间系统稳定性判据 ⇒ Routh表 仅适用于s域

⇒ 进行映射 将圆内映射到左半平面 ⇒ 双线性变换 $z=\dfrac{w+1}{w-1}$，$w=\dfrac{z+1}{z-1}$

z-domain $\begin{cases}x^2+y^2<1\\ x^2+y^2>1\end{cases}\xrightarrow{z=\frac{w+1}{w-1}}$ z-domain $\begin{cases}w<0\\ w>0\end{cases}$



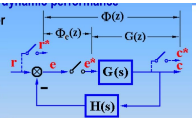
再利用Routh判据

动态性能

一、一般方法

(1) Obtain the impulse transfer function

Let $\begin{cases}GH(z)=Z[G(s)H(s)]\\ \Phi(z)=\dfrac{G(z)}{1+GH(z)}=\dfrac{M(z)}{D(z)}\end{cases}$

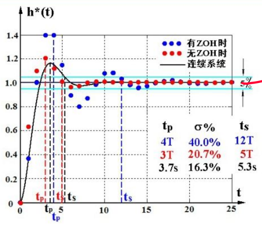


(2) Obtain $C(z)=\Phi(z)R(z)=\dfrac{M(z)}{D(z)}\cdot\dfrac{z}{z-1}$

$=c(0)+c(T)z^{-1}+c(2T)z^{-2}+\cdots$

(3) $c^*(t)=c(0)\delta(t)+c(T)\delta(t-T)+c(2T)\delta(t-2T)+\cdots$

(4) Determine the specifications $\sigma\%$, t_s .



二、动态响应与闭环极点

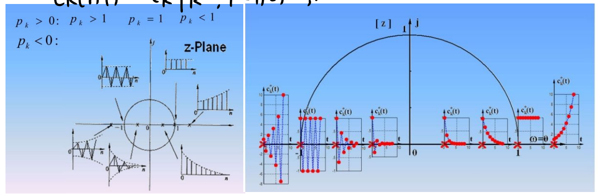
$\Phi(z)=\dfrac{M(z)}{D(z)}=\dfrac{b_m}{a_n}\dfrac{\prod_{i=1}^{m}(z-z_i)}{\prod_{k=1}^{n}(z-p_k)}$，$m\le n$

$C(z)=\Phi(z)R(z)=\dfrac{M(z)}{D(z)}\cdot\dfrac{z}{z-1}=\dfrac{M(1)}{D(1)}\cdot\dfrac{z}{z-1}+\sum_{k=1}^{n}\dfrac{C_k z}{z-p_k}$

1.实轴上单闭环极点

$c_k^*(t)=Z^{-1}\left[\dfrac{C_k z}{z-p_k}\right]$，$k=1,2,\cdots,n$

$C_k(nT)=C_k p_k^n$，$k=1,2,\cdots,n$



2.复共轭闭环极点

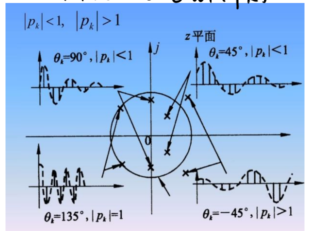
$p_k=|p_k|e^{j\theta_k}$ $\quad\bar{p}_k=|p_k|e^{-j\theta_k}$ $\quad\begin{cases}a=\frac{1}{T}\ln|p_k|\\ \omega=\frac{\theta_k}{T}\\ 0<\theta_k<\pi\end{cases}$

$C_{k,\bar{k}}^*(k)=Z^{-1}\left[\dfrac{C_k z}{z-p_k}+\dfrac{\bar{C}_k z}{z-\bar{p}_k}\right]$

$=C_k p_k^n+\bar{C}_k\bar{p}_k^n$

$=C_k e^{a_k nT}+\bar{C}_k e^{\bar{a}_k nT}=|C_k|e^{j\varphi_k}e^{(a+j\omega)nT}+|C_k|e^{-j\varphi_k}e^{(a-j\omega)nT}$

$=2|C_k|e^{anT}\cos(n\omega T+\varphi_k)$



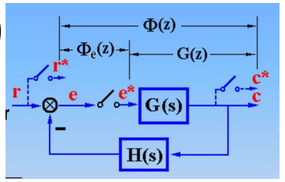
稳态误差

一、一般方法

二、终值定理

令 $GH(z)=Z[G(s)H(s)]=\dfrac{1}{(z-1)^v}GH_0(z)$ v阶系统

$\lim_{z\to1}GH_0(z)=K$



步骤

① 稳定性判断

② 计算冲激响应传函

$\Phi_e(z)=\dfrac{E(z)}{R(z)}=\dfrac{1}{1+GH(z)}$

③ 计算 $e(\infty)$

$e(\infty)=\lim_{z\to1}(z-1)\Phi_e(z)R(z)=\lim_{z\to1}(z-1)R(z)\cdot\dfrac{1}{1+GH(z)}$

三、稳态误差常数法

$e(\infty T)=\lim_{z\to1}(z-1)\Phi_e(z)R(z)=\lim_{z\to1}(z-1)\cdot R(z)\cdot\dfrac{1}{1+GH(z)}$

$r(t)=A\cdot1(t)$ $\quad e(\infty T)=\lim_{z\to1}(z-1)\cdot\dfrac{Az}{z-1}\cdot\dfrac{1}{1+GH(z)}=\dfrac{A}{1+\lim_{z\to1}GH(z)}=\dfrac{A}{K_p}$

Static position error constant $K_p=1+\lim_{z\to1}GH(z)$

$K_p=1+\lim_{z\to1}GH(z)$ $\begin{cases}\text{Type 0: } k_p=\text{constant}\\ \text{Type}>=1:\ k_p=\infty\end{cases}$

$r(t)=A\cdot t$ $\quad e(\infty T)=\lim_{z\to1}(z-1)\cdot\dfrac{ATz}{(z-1)^2}\cdot\dfrac{1}{1+GH(z)}=\dfrac{AT}{\lim_{z\to1}(z-1)GH(z)}=\dfrac{AT}{K_v}$

Static velocity error constant $K_v=\lim_{z\to1}(z-1)GH(z)$

$K_v=\lim_{z\to1}(z-1)GH(z)$ $\begin{cases}\text{Type 0: } k_v=0\\ \text{Type 1: } k_v=\text{constant}\\ \text{Type}>=2:\ k_v=\infty\end{cases}$

$r(t)=\dfrac{A}{2}t^2$ $\quad e(\infty T)=\lim_{z\to1}(z-1)\cdot\dfrac{AT^2z(z+1)}{2(z-1)^3}\cdot\dfrac{1}{1+GH(z)}=\dfrac{AT^2}{\lim_{z\to1}(z-1)^2GH(z)}=\dfrac{AT^2}{K_a}$

Static acceleration error constant $K_a=\lim_{z\to1}(z-1)^2GH(z)$

$K_a=\lim_{z\to1}(z-1)^2GH(z)$ $\begin{cases}\text{Type 0,1: } k_a=0\\ \text{Type 2: } k_a=\text{constant}\\ \text{Type}>=3:\ k_a=\infty\end{cases}$

型别	Static Error Constant			Steady-State Error		
V	Kp=limGH(z)	Kv=lim(z-1)GH(z)	Ka=lim(z-1)²GH(z)	r=A·1(t) e(∞)=A/Kp	r=A·t e(∞)=AT/Kv	r=A·t²/2 e(∞)=AT²/Ka
0	Kp	0	0	A/Kp	∞	∞
I	∞	Kv	0	0	AT/Kv	∞
II	∞	∞	Ka	0	0	AT²/Ka