$H_a(s)=\frac{1}{2s+1}+\frac{1}{s+1}=\frac{1}{2}\frac{1}{s-(-\frac{1}{2})}+\frac{1}{s-(-1)}$

$h(t)=\frac{1}{2}e^{-\frac{1}{2}t}u(t)+e^{-t}u(t)$

$h(n)=\frac{1}{2}e^{-\frac{1}{4}n}u(n)+e^{-\frac{1}{2}n}u(n)$

$H(z)=\frac{\frac{1}{2}}{1-e^{-\frac{1}{4}}z^{-1}}+\frac{1}{1-e^{-\frac{1}{2}}z^{-1}}$

$H(s)=\frac{1}{2s+1}+\frac{1}{s+1}=\frac{1}{2}\frac{1}{s-(-\frac{1}{2})}+\frac{1}{s-(-1)}$

$h(t)=\mathcal{L}^{-1}[H(s)]=\frac{1}{2}e^{-\frac{1}{2}t}u(t)+e^{-t}u(t)$

$\mathcal{L}[e^{at}]=\frac{1}{s-a}$

4.8 已知一模拟滤波器的传递函数为

$$H_a(s)=\frac{3s+2}{2s^2+3s+1}$$

试分别用冲激响应不变法和双线性变换法将它转换成数字滤波器的系统函数 $H(z)$，设 $T=0.5$。

$t=nT$

$t=\frac{n}{2}$

解 冲激响应不变 $\because H(s)=\frac{1}{2s+1}+\frac{1}{s+1}=\frac{1}{2}\frac{1}{s-(-\frac{1}{2})}+\frac{1}{s-(-1)}$

$\mathcal{L}[e^{at}]=\frac{1}{s-a}$

$\therefore h(t)=\mathcal{L}^{-1}[H(s)]=\frac{1}{2}e^{-\frac{1}{2}t}u(t)+e^{-t}u(t)$

$\therefore h(n)=\frac{1}{2}(e^{-\frac{1}{4}})^n u(n)+(e^{-\frac{1}{2}})^n u(n)$ 又采样

$\therefore H(z)=Z[h(n)]=\frac{1}{2}\frac{1}{1-e^{-\frac{1}{4}}z^{-1}}+\frac{1}{1-e^{-\frac{1}{2}}z^{-1}}$

$\frac{1}{4}$ $\frac{1}{2}$ 无所谓

又修正 $h(n)=Th(t)|_{t=nT}$

$=\frac{1}{2}[\frac{1}{2}e^{-\frac{1}{4}n}u(n)+e^{-\frac{1}{2}n}u(n)]$

双线性变换：令 $s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}=4\frac{1-z^{-1}}{1+z^{-1}}$

$s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

$\therefore H(z)=\frac{14+4z^{-1}-10z^{-2}}{45-62z^{-1}+21z^{-2}}$

$\boxed{s=\frac{1-z^{-1}}{1+z^{-1}}}$

则归一化 $s=\frac{s}{\Omega_c}$

$\Omega_c=\tan(\frac{w_c}{2})$

$\frac{1-z^{-1}}{1+z^{-1}}$

4.13 用冲激不变法设计一个数字巴特沃斯低通滤波器。这个滤波器的幅度响应在通带截止频率 $\omega_p=0.2613\pi$ 处的衰减不大于 0.75 dB，在阻带截止频率 $\omega_s=0.4018\pi$ 处的衰减不小于 20 dB。

Ω_p　A_p　Ω_s　A_s

4.14 使用双线性变换法设计一个巴特沃斯低通滤波器。假定取样频率为 10 kHz，在通带截止频率 $f_p=1$ kHz 处衰减不大于 1.8 dB，在阻带截止频率 $f_s=1.5$ kHz 处衰减不小于 12 dB。

4.13解 $\therefore N\geq\frac{\lg(\lambda/\varepsilon)}{\lg(\Omega_s/\Omega_c)}=\frac{1}{2}\frac{\lg[(10^{0.1A_s}-1)/(10^{0.1A_p}-1)]}{\lg(\Omega_s/\Omega_p)}=7.28$

取 $N=8$

而 $\begin{cases}1+(\frac{0.2613\pi}{\Omega_c})^{2N}\leq10^{0.075}\\1+(\frac{0.4018\pi}{\Omega_c})^{2N}\geq10^{2}\end{cases}\Rightarrow 0.9111\leq\Omega_c\leq0.9472$

取 $\Omega_c=0.9111$

查表有

$H_a(s)=\frac{1}{s^8+5.1258s^7+13.137s^6+21.846s^5+25.688s^4+21.846s^3+13.137s^2+5.1528s+1}$

令 $s=\frac{s}{\Omega_c}$ 有

$H_a(s)=\frac{0.4748}{s^8+4.67s^7+10.91s^6+16.54s^5+17.7s^4+13.72s^3+7.52s^2+2.67s+0.475}$

$=\frac{0.4748}{(s^2+2.63s+1.87)(s^2+1.67s+1.64)(s^2+0.40s+1.15)(s^2-0.03s+0.28)}$

$=\frac{1.2839+1.3901i}{s-(-0.5678+0.7639i)}+\frac{1.2839-1.3901i}{s-(-0.5678-0.7639i)}+\frac{2.4647-8.0352i}{s-(-0.9337+0.0812i)}+\frac{2.4647+8.0352i}{s-(-0.9337-0.0812i)}$
$+\frac{0.2585-0.1455i}{s-(-0.1698+0.8908i)}+\frac{0.2585+0.1455i}{s-(-0.1698-0.8908i)}+\frac{-4.0071-1.3007i}{s-(-0.6636+0.5343i)}+\frac{-4.0071+1.3007i}{s-(-0.6636-0.5343i)}$

$h(t)=\mathcal{L}^{-1}[H_a(s)]=(1.2839+1.3901i)e^{(-0.5678+0.7639i)}u(t)+(1.2839-1.3901i)e^{(-0.5678-0.7639i)}u(t)$
$+(2.4647-8.0352i)e^{(-0.9337+0.0812i)}u(t)+(2.4647+8.0352i)e^{(-0.9337-0.0812i)}u(t)$
$+(0.2585-0.1455i)e^{(-0.1698+0.8908i)}u(t)+(0.2585+0.1455i)e^{(-0.1698-0.8908i)}u(t)$
$+(-4.0071-1.3007i)e^{(-0.6636+0.5343i)}u(t)+(-4.0071+1.3007i)e^{(-0.6636-0.5343i)}u(t)$

取 $t=nT$ 再z变换有

$H(z)=\frac{1.2839+1.3901i}{1-e^{(-0.5678+0.7639i)}z^{-1}}+\frac{1.2839-1.3901i}{1-e^{(-0.5678+0.7639i)}z^{-1}}+\frac{2.4647-8.0352i}{1-e^{(-0.9337+0.0812i)}z^{-1}}+\frac{2.4647+8.0352i}{1-e^{(-0.9337-0.0812i)}z^{-1}}$
$+\frac{0.2585-0.1455i}{1-e^{(-0.1698+0.8908i)}z^{-1}}+\frac{0.2585+0.1455i}{1-e^{(-0.1698-0.8908i)}z^{-1}}+\frac{-4.0071-1.3007i}{1-e^{(-0.6636+0.5343i)}z^{-1}}+\frac{-4.0071+1.3007i}{1-e^{(-0.6636-0.5343i)}z^{-1}}$

4.14解 $\therefore \Omega_p=2000\pi$ rad/s，$\Omega_s=3000\pi$ rad/s，$T=\frac{1}{f}=10^{-4}$

$\therefore w_p=T\Omega_p=0.2\pi$，$w_s=0.3\pi$

进行预畸变，取 $T=1$

$\Omega_p=2\tan\frac{0.2\pi}{2}=0.65$，$\Omega_s=2\tan\frac{0.3\pi}{2}=1.02$

$\therefore N\geq\frac{1}{2}\frac{\lg[(10^{0.1A_s}-1)/(10^{0.1A_p}-1)]}{\lg(\Omega_s/\Omega_p)}=3.74$　$\therefore N$ 取 4

$\therefore\begin{cases}1+(\frac{0.65}{\Omega_c})^8\leq10^{0.18}\\1+(\frac{1.02}{\Omega_c})^8\geq10^{1.2}\end{cases}\Rightarrow 0.7063\leq\Omega_c\leq0.7274$

取 $\Omega_c=0.7169$

查表有

$H_a(s)=\frac{1}{s^4+2.6131s^3+3.4142s^2+2.6131s+1}$

令 $s=\frac{s}{\Omega_c}$ 有

$H(s)=\frac{0.2641}{s^4+1.8733s^3+1.7547s^2+0.9628+0.2641}$

令 $s=\frac{1-z^{-1}}{1+z^{-1}}$

$H(z)=\frac{0.2641(1+z^{-1})^4}{(1-z^{-1})^4+1.8733(1-z^{-1})^3(1+z^{-1})+1.7647(1-z^{-1})^2(1+z^{-1})^2+0.9628(1-z^{-1})(1+z^{-1})^3+0.2641(1+z^{-1})^4}$

4.15 用双线性变换法设计一个数字切比雪夫低通滤波器，各指标与习题 4.14 相同。

$f_a=10kHz$，$f_p=1kHz$，$A_p\geq1.8dB$；$f_s=1.5kHz$，$A_s\leq12dB$

解：$\therefore \varepsilon=\sqrt{10^{0.1A_p}-1}=0.717$

同4.14取 $\Omega_c=0.7169$，$\Omega_s=1.07$，$\Omega_p=0.65$

$\therefore N\geq\frac{\operatorname{arccosh}[\frac{1}{0.717}\sqrt{10^{1.2}-1}]}{\operatorname{arccosh}(1.07/0.7169)}=2.48$

取 $N=3$

$\therefore \alpha=3.11$，$a=0.387$，$b=1.07$

$\therefore s_1=-0.28$

$s_2=-0.14-0.66j$

$s_3=0.14-0.66j$

$\therefore H(s)=\frac{0.7169^3}{0.717\cdot2^2\cdot(s+0.2)(s+0.1+0.47j)(s-0.1+0.47j)}$

$=0.13\cdot\frac{1}{(s+0.2)(s+0.1+0.47j)(s-0.1+0.47j)}$

令 $s=\frac{1-z^{-1}}{1+z^{-1}}$

$H(z)=0.13\frac{(1-z^{-1})^3}{(1.2-0.8z^{-1})[(1.1+0.47j)+(-0.9+0.47j)z^{-1}][(0.9+0.47j)+(-1.1+0.47j)z^{-1}]}$