

$$d_3 = \frac{|\vec{w}^T \vec{x}_3 + b|}{\|\vec{w}\|} = \frac{|(1,1) \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 1|}{\sqrt{1^2 + 1^2}} = \frac{\sqrt{2}}{2}$$

3, 假设训练样本集为 $D = \{(\vec{x}_1, y_1) = ((0,0)^T, -1), (\vec{x}_2, y_2) = ((2,2)^T, -1), (\vec{x}_3, y_3) = ((2,0)^T, 1), (\vec{x}_4, y_4) = ((3,0)^T, 1)\}$, 使用QP求解器时, $\vec{a}_n^T (n=1,2,3,4)$ 分别为多少?

解: $\vec{a}_1^T = (-1, 0, 0)$, $\vec{a}_2^T = (-1, -2, -2)$, $\vec{a}_3^T = (1, 2, 0)$, $\vec{a}_4^T = (1, 3, 0)$

4, 假设训练样本集为: $D = \{(\vec{x}_1, y_1) = ((1,1)^T, 1), (\vec{x}_2, y_2) = ((2,2)^T, 1), (\vec{x}_3, y_3) = ((2,0)^T, 1), (\vec{x}_4, y_4) = ((0,0)^T, -1), (\vec{x}_5, y_5) = ((1,0)^T, -1), (\vec{x}_6, y_6) = ((0,1)^T, -1)\}$, 请分别在 $y_n(\vec{w}^T \vec{x}_n + b) \geq 1$ 和 $y_n(\vec{w}^T \vec{x}_n + b) \geq 5$ 的条件下用 Primal SVM 方法来设计最优分类面 $g(\vec{x})$, 判断两种情况下的分类面是否一致, 指出哪些是候选的支撑向量, 并回答如何确认哪些是支撑向量。

解: (1) 对于条件 $y_n(\vec{w}^T \vec{x}_n + b) \geq 1$, 可列出如下的式子

$$\begin{aligned} \min_{b, w} \quad & \frac{1}{2} \mathbf{w}^T \mathbf{w} \\ \text{s.t.} \quad & \begin{cases} w_1 + w_2 + b \geq 1 \\ 2w_1 + 2w_2 + b \geq 1 \\ 2w_1 + b \geq 1 \\ -b \geq 1 \\ -w_1 - b \geq 1 \\ -w_2 - b \geq 1 \end{cases} \implies \begin{cases} w_1 \geq 2 \\ w_2 \geq 2 \\ b \leq -3 \end{cases} \end{aligned}$$

当且仅当 $w_1 = 2, w_2 = 2, b = -3$,