$$(\vec{x}_3, y_3) \rightarrow (\vec{z}_3, y_3)\colon \{(0,1)^T, -1\} \rightarrow \{(1,0,1,0,0,1)^T, -1\}$$

$$(\vec{x}_4, y_4) \rightarrow (\vec{z}_4, y_4)\colon \{(0,-1)^T, -1\} \rightarrow \{(1,0,-1,0,0,1)^T, -1\}$$

令$\alpha_1 \geq 0$、$\alpha_2 \geq 0$、$\alpha_3 \geq 0$、$\alpha_4 \geq 0$

由 SVM 对偶模型得到：

$$\begin{cases} L(\vec{w}, b, \alpha) = \dfrac{1}{2}\sum_{n=1}^{4}\sum_{m=1}^{4}\alpha_n\alpha_m y_n y_m \vec{z}_n^T\vec{z}_m - \sum_{n=1}^{4}\alpha_n \\ \sum_{n=1}^{4} y_n\,\alpha_n = 0 \end{cases}$$

求$L(\vec{w}, b, \alpha)$对α的梯度：$\frac{\partial L}{\partial \alpha_n} = \sum_{m=1}^{4}\alpha_m y_n y_m \vec{z}_n^T\vec{z}_m - 1$

且：$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 = 0$

代入训练样本，

$$\frac{\partial L}{\partial \alpha_1} = 3\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4 - 1 = 0 \rightarrow 2\alpha_1 - 1 = 0$$
$$\frac{\partial L}{\partial \alpha_2} = 3\alpha_2 + \alpha_1 - \alpha_3 - \alpha_4 - 1 = 0 \rightarrow 2\alpha_2 - 1 = 0$$
$$\frac{\partial L}{\partial \alpha_3} = 3\alpha_3 - \alpha_1 - \alpha_2 + \alpha_4 - 1 = 0 \rightarrow 2\alpha_3 - 1 = 0$$
$$\frac{\partial L}{\partial \alpha_4} = \alpha_4 - \alpha_1 - \alpha_2 + \alpha_3 - 1 = 0 \rightarrow 2\alpha_4 - 1 = 0$$

求解得到：$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \frac{1}{2}$

$$\therefore\ \vec{w} = \sum_{n=1}^{4}\alpha_n y_n \vec{z}_n = \frac{1}{2}(\vec{z}_1 + \vec{z}_2 - \vec{z}_3 - \vec{z}_4) = (0,0,0,0,1,-1)^T$$

$$b = y_1 - \vec{w}^T\vec{z}_1 = 1 - (0,0,0,0,1,-1)(1,1,0,0,1,0)^T = 0$$

$$\therefore\ \ g_{SVM} = sign(\vec{w}^T\phi_2(\vec{x}) + b) = sign(x_1^2 - x_2^2)$$

且四个样本均为支撑向量。