Biscuit Dunking Analysis

Konstantin Nomerotski

May 2024

Abstract

Biscuit dunking experiments were analysed to provide insight into the differences between different types of biscuits. The relationship between the pore radius and the length travelled up the biscuits upon dunking was investigated for three biscuits: Hobnob, Digestive and Rich Tea. The pore radius was found to be the largest differentiating factor among the different biscuits, and the Washburn model was evaluated as a means of predicting the pore radius, finding an R^2 of 0.99738. The time resolved absorption was investigated to find a positive correlation between biscuit pore size and capillary flow rate, with average capillary flow rates calculated for each biscuit type. A machine learning classifier was trained to predict the biscuit type, providing a fast and accurate method for identifying biscuits based on their dunking data.

1 Introduction

McVities has commissioned the investigation of biscuit dunking data collected as part of an initiative to explore the physical differences in biscuits. Three seperate experiments have been performed, see Table 1, producing five empirical databases: The dunking, microscopy and time resolved datasets. The Washburn model, Equation 1 has been proposed as a starting point for describing the capillary flow in biscuits as they are dunked in tea[2, 1].

$$L^2 = \frac{\gamma r t \cos\phi}{2\eta} \tag{1}$$

The microscopy data is obtained for a subset of dunking data, as this is too costly and time consuming an experiment to run for the full sample of data, and is provided unlabeled. Similarly, the time resolved datasets are limited to a single biscuit dunk for each type. This project aims to identify the differences across biscuit types, and investigate the time dependent nature of capillary action in different types of biscuits. Machine learning solutions will be explored to predict the biscuit type and reduce the reliance on expensive empirical big data collections for the determination of biscuit physical properties.

Datasets	Dunking	Microscopy	Time resolved
Biscuit type	Yes	No	No
Pore radius (m)	No	Yes	No
Time range (s)	10-30	10-30	30-300
Size	3000	500	100

Table 1: Summary of provided datasets

2 Analysis and Discussion

2.1 Data wrangling

The microscopy data is a subset of the dunking data with the biscuit type removed, therefore the corresponding entries were removed from the dunking data, and combined with the relevant biscuit names to give a fully labeled dataset containing the microscopy measurements. The pore radius was determined as the most differentiating feature, Figure 3, with Digestive biscuits having the largest pore radii, followed by Hobnobs and Rich Tea. The distributions of pore radius across the different biscuit types are separated enough to provide a basis for biscuit classification.

2.2 Evaluating the Washburn model

The Washburn equation can be used to obtain the pore radius from the data provided alongside the microscopy measurements, therefore the microscopy dataset was used to evaluate the Washburn model fit on the empirical pore radius values, Figure 1. The model provided extremely positive results, Table 2, with relative errors of around 0.05 %. A limitation arises at high pore radius, where the error is more pronounced. Overall, The Washburn model is a reliable predictor of pore radius.

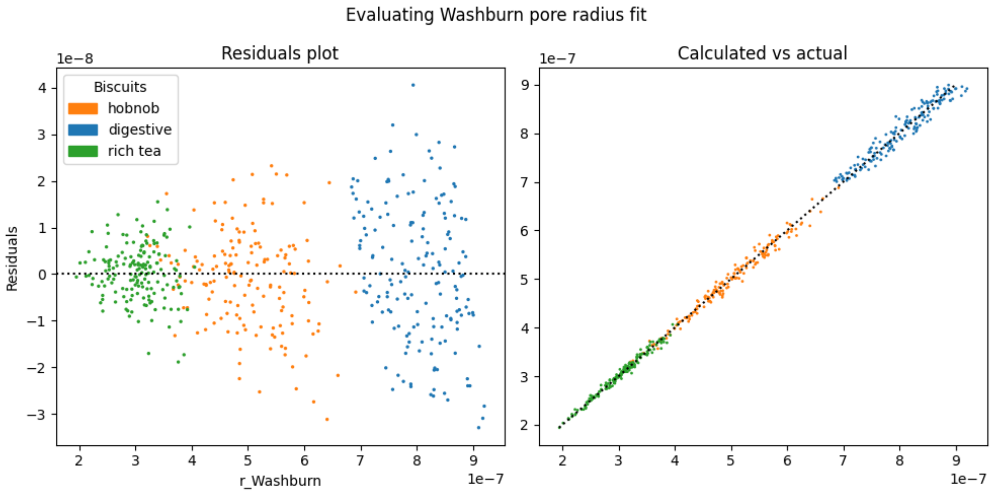

Figure 1: An evaluation of the Washburn model fit for calculating the pore radius

	R^2	$RMSE$	MAE	MSE	*Average residuals*	
					Digestive:	2.52e-10
Washburn	0.99738	1.10e-08	8.29e-09	1.21e-16	*Hobnob*:	-1.09e-09
					Rich Tea:	3.08e-10

Table 2: Evaluation metrics for the Washburn model fit

2.3 Training a classifier

Initially, K-means clustering was used to identify 3 clusters in the biscuits, Table 7. The confusion matrix, Figure 4, shows a miss-classification of Hobnobs into Rich Tea, an issue identified in the exploratory data analysis. This is due to the K-means algorithm being distance based, and the overlap of pore radius distributions of Hobnob and Rich Tea. Multiple supervised machine learning approaches were taken to improve the performance of classification, trained both with and without to find that the inclusion of the pore radius greatly improves the predictive power of the model, Table 4. A Random Forest classifier with scaled features provided the most accurate classification with the default hyper-parameters so this model was taken as a baseline, with an average accuracy of 96 %. Hyperparameter tuning was attempted using GridSearch cross-validation, although the default parameters performed better.

K-means	*Precision*	*Recall*	*F-1*
Hobnob	0.905	0.931	0.917
Rich Tea	0.944	0.910	0.926
Digestives	0.985	0.992	0.989

Table 3: Classification evaluation metrics for K-means clustering on pore radius

2.4 Attributing time-resolved datasets

We have successfully found a means of differentiating between different biscuits, the pore radius. The average pore radius was found for each time-resolved dataset, and compared to the distribution of empirical pore radius, Figure 6. Thereby, the time resolved datasets are attributed to their respective biscuits: TR1 - Hobnob, TR2 - Rich Tea, and TR3 - Digestive. The time resolved datasets do not contain information before 30 s, this was

RF clf.	*With pore radius*			*Without pore radius*		
Biscuit	*Precision*	*Recall*	*F-1*	*Precision*	*Recall*	*F-1*
Hobnob	0.935	0.935	0.935	0.520	0.419	0.464
Rich Tea	0.947	0.947	0.947	0.787	0.684	0.732
Digestive	1.00	1.00	1.00	0.659	0.900	0.761

Table 4: Classification metrics for Random Forest classifier with and without the pore radius training feature

taken from the dunking data, averaged for every second over each biscuit type. The time dependance of L for each biscuit type was investigated, Figure 7, to find the capillary flow rates of each biscuit type, Table 5.

This analysis clearly shows a correlation between the biscuit pore radius and the length that the tea goes up the biscuit, and therefore is an important factor in the length of time that you can dunk a biscuit, which could be of interest. A point to note is that the time resolved datasets go up to 300 s, with the length constantly increasing, however, from personal experience a biscuit breaks in the tea in less than a minute. The inclusion of biscuit breaking time in the data could aid in the analysis of an "optimal" dunking time for each biscuit.

Biscuit	Average pore radius (m^2)	Capillary flow rate ($m^2 s^-1$)
Digestive	1.00e-06	4.2e-06
Hobnob	2.79e-07	2.1e-06
Rich Tea	5.19e-07	1.1e-06

Table 5: Capillary flow rate calculated from the time-resolved datasets

2.5 Can we do better than the Washburn model?

From the data available, the pore radius is clearly the most important predictor of biscuit type, however there is some underlying error in the Washburn model used in our classification model. Machine learning regression models were tested in an attempt to improve upon the given model. Two approaches are presented: a univariate correction of the Washburn predicted pore radius using polynomial Kernel Ridge regression (KRR), and multivariate prediction of pore radius, using the data available in dunking data. The former is a technique using the kernel trick, fitting a polynomial kernel to account for non linear dependencies, KRR is commonly used as a robust tool for the correction calculated values, for example, in the correction of calculated electronic spectra [3]. It is a lightweight model that provides reliable results when trained on relatively small datasets (< 1000).

Of the models tested, the Random Forest regressor (RFR) was chosen as the best performing model, and subsequent optimization was focused on improving the RFR and KRR models. When the Washburn pore radius was included in models, it resulted in an extremely unbalanced model dependence on the pore radius as a predictor. Feature engineering was attempted, separating parts of the Washburn equation in an attempt to balance the feature contributions, however, the resulting model still relied disproportionately on a single term, $\frac{L^2}{Cos(\phi)}$. This suggests that the angle at which the biscuit is dunked is important in the determination of r, and therefore in turn related to the length of tea up the biscuit.

	R^2	$RMSE$	MAE	MSE	Cross-val R^2	Average residuals	
KRR	0.99737	1.11e-08	8.18e-09	1.23e-16	0.9971	*Digestive*:	2.11e-10
						Hobnob:	8.49e-10
						Rich Tea:	-4.30-11
RFR	0.99675	1.23e-08	9.61e-09	1.53e-16	0.9963	*Digestive*:	-1.27e-09
						Hobnob:	-9.76e-10
						Rich Tea:	-7.60e-10

Table 6: Evaluation metrics for the Washburn model fit

The Kernel Ridge regressor showed better performance than the Random Forest regressor across the evaluation metrics, moreover, the training and optimization time is much more favourable than that of the RFR. Although the fit is slightly worse than the Washburn model (-1.0e-5 ΔR^2) the residuals are lower, and the mean absolute error is also lower, while the other metrics have little difference. Overall, this suggests that the Kernel Ridge regression provides a suitable correction to the Washburn model, lowering the error in pore radius calculations. An graphical evaluation of the fit can be seen in Figure 8.

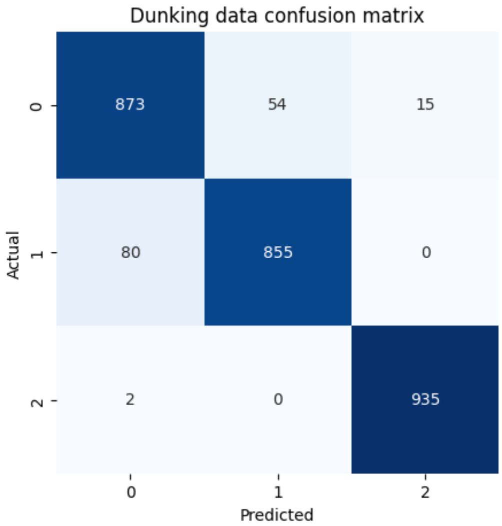

Figure 2: Confusion matrix of KRR pore radius and RF classifier

2.6 Biscuit classification with ML pore radius

The optimized KRR model was used to predict the pore radii for the dunking data, subsequently, the biscuit types were predicted using the Random Forest classifier introduced in Section 2.3. The resulting pipeline provides a means for the classification of biscuits automatically from the dunking data, saving the need for microscopy measurements. The resulting predictions provide an accurate determination of biscuit types with an accuracy of 94.6 %. The model provides almost perfect recall and precision for Digestive biscuits, 99.7 and 98.4 % respectively, however it confuses Hobnobs and Rich Tea biscuits, with slightly worse performance than the model based on real pore radii, see Figure 2. This suggests that our prediction of pore radius is lacking in accuracy.

	Precision	*Recall*	*F-1*
Hobnob	0.914	0.926	0.920
Rich Tea	0.945	0.914	0.927
Digestives	0.984	0.998	0.990

Table 7: Classification evaluation metrics for KRR pore radius with RF classifier

3 Conclusion

The dunking of biscuits is a controversial topic, with the British population standing by their individual dunking traditions. This project has shed some light on the relationship between the dunking time, the length of tea up the biscuit and the pore radius of three types of biscuits. We have found that the most important factor in differentiation of biscuit types is the pore radius of the biscuit, moreover, a positive relationship between the pore radius and the length of tea travelled up the biscuit has been identified. The Washburn model and machine learning approaches have been evaluated as a means of calculating the pore radius, finding the Washburn model to be accurate, yet limited. A machine learning approach for the prediction of biscuit types has been proposed, using a polynomial Kernel Ridge regression model to predict the pore radius, which is then used alongside existing data to classify the biscuits with 94.6 % accuracy. This presents an approach based on computations rather than empirical measurements, reducing the need for costly lab-based experiments. Future experiments could focus on the time taken for the biscuit to crumble in the tea, as an initial faster capillary flow rate was observed in the dunking data. Furthermore, the angle of dunking should be extended past 1.1 rad as the angle was found to negatively correlate with the length travelled up the biscuit, which could be prefered by some consumers.

References

[1] Len Fisher. Physics takes the biscuit. *Nature*, 397(6719):469–469, Feb 1999.

[2] Len Fisher. *How to dunk a doughnut: The science of Everyday Life.* Penguin, 2004.

[3] Bao-Xin Xue, Mario Barbatti, and Pavlo O. Dral. Machine learning for absorption cross sections. *The Journal of Physical Chemistry A*, 124(35):7199–7210, Aug 2020.

Appendices

A Code archive

The code archive can be found at https://github.com/kon-218/biscuit_analysis

B Selected figures (also available in notebook)

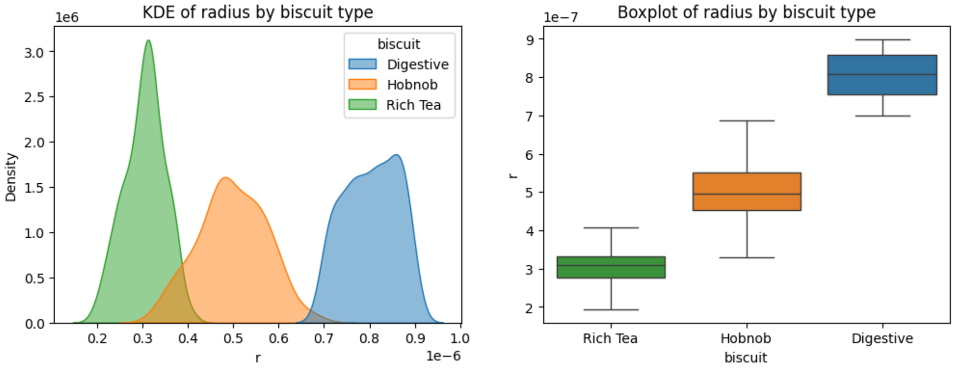

Figure 3: Investigatory analysis of the pore radius from the microscopy dataset

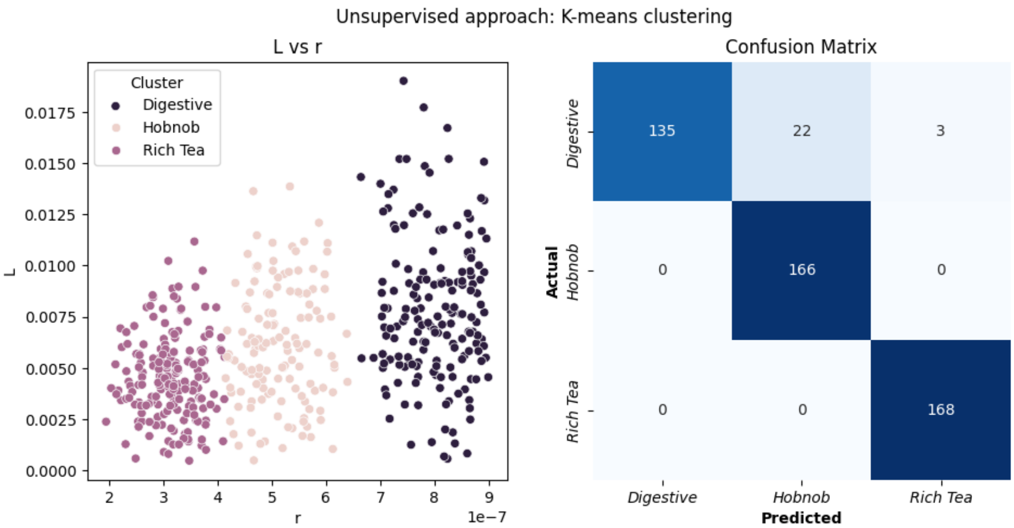

Figure 4: Unsupervised classification using k-means clustering with n=3 on the pore radius from the microscopy data.

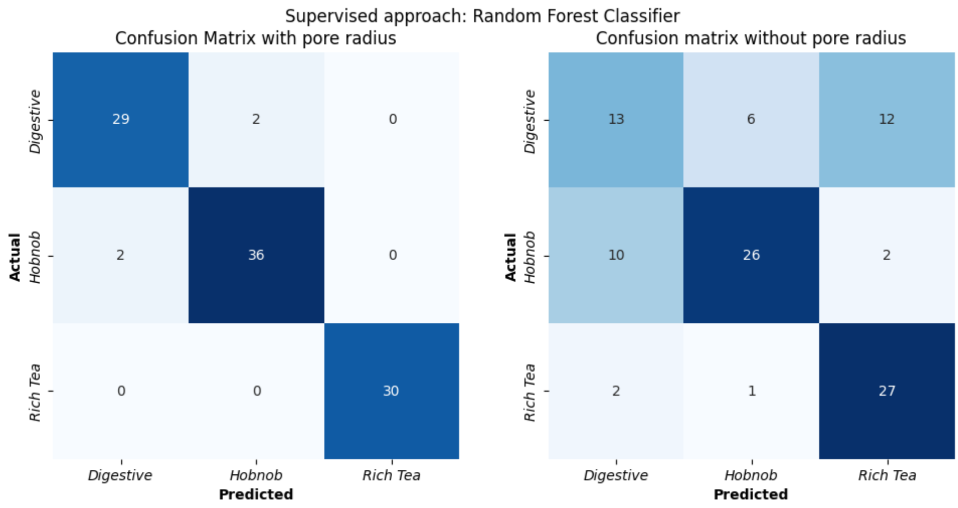

Figure 5: Supervised classification using a Random Forest classifier, trained with and without the pore radius

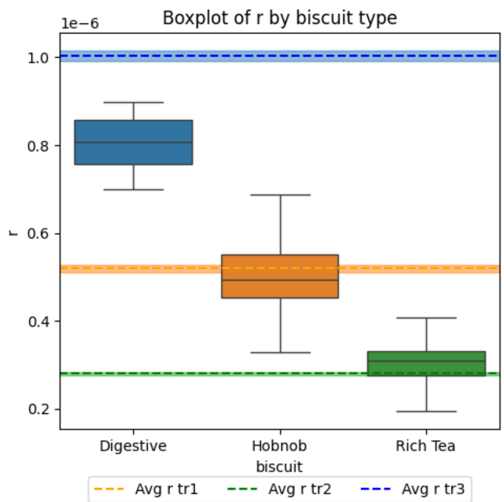

Figure 6: Average pore radius in the time resolved datasets calculated using the Washburn model, overlaid on the pore radius distributions of microscopy data

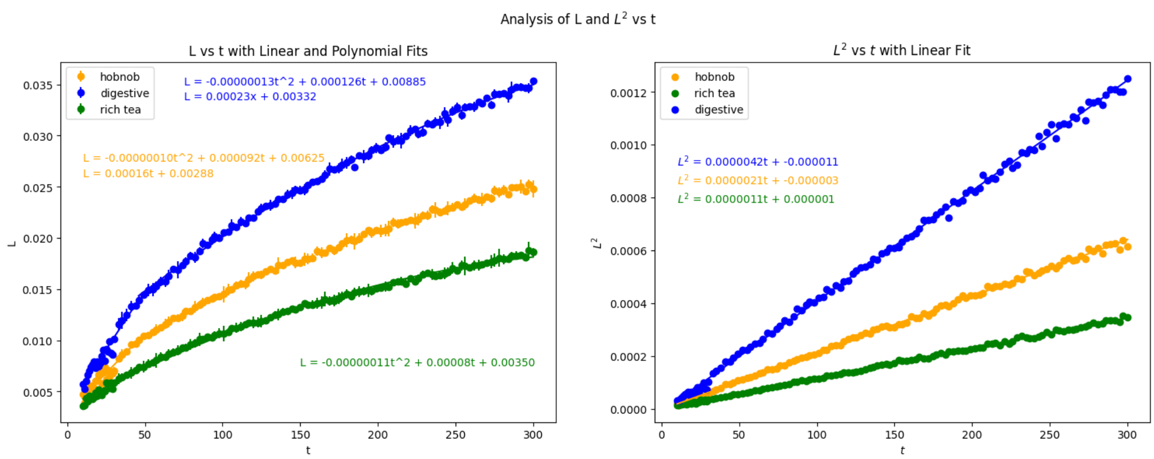

Figure 7: Linear and polynomial relationship of L over time for each biscuit type

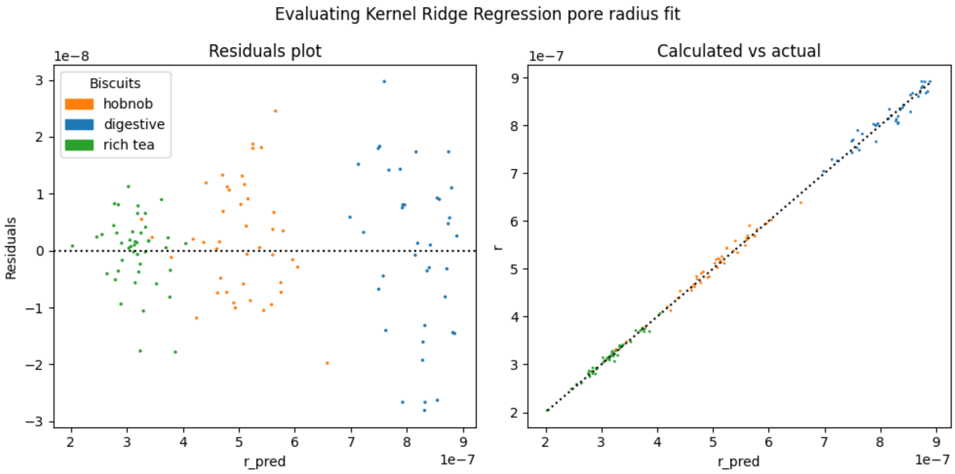

Figure 8: KRR optimised model regression fit