Problem Set 3

Problem Sets due Jul 12, 2022 06:35 PKT Completed

Problems 1-7 correspond to "Linear algebra I: basic notation and dot products"

Problem 1

1/1 point (graded)

A data set consists of 200 points in $\mathbb{R}^{80}$. If we store these in a matrix, with one point per row, what is the dimension of the matrix?

- (selected) 200×80
- 80×200
- 200×1
- 1×80

✔

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Problem 2

3/3 points (graded)

For $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$, compute

a) $A^T =$

- $\begin{pmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$
- $\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$
- (selected) $\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$
- $\begin{pmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{pmatrix}$

✔

b) $A + B =$

- [ ] $\begin{pmatrix} 2 & 2 & 4 \\ 4 & 6 & 6 \end{pmatrix}$
- [x] $\begin{pmatrix} 0 & 2 & 4 \\ 5 & 4 & 6 \end{pmatrix}$
- [ ] $\begin{pmatrix} 0 & 0 & 2 \\ 5 & 5 & 6 \end{pmatrix}$
- [ ] $\begin{pmatrix} 6 & 3 & 1 \\ 2 & 4 & 7 \end{pmatrix}$

✔

c) $A - B =$

- [ ] $\begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$
- [x] $\begin{pmatrix} 2 & 2 & 2 \\ 3 & 6 & 6 \end{pmatrix}$
- [ ] $\begin{pmatrix} 2 & 2 & 2 \\ 1 & 3 & 6 \end{pmatrix}$
- [ ] $\begin{pmatrix} 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$

✔

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Problem 3

2/2 points (graded)

Let $x = (1, 0, -1)$ and $y = (0, 1, -1)$.

a) What is $x \cdot y$?

1 ✔

b) What is the angle between these two vectors, in degrees (give a number in the range 0 to 180)?

60 ✔

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Problem 4

2/2 points (graded)

For each pair of vectors below, say whether or not they are orthogonal.

a) $(1, 3, 0, 1)$ and $(-1, -3, 0, -1)$

not orthogonal ✓

b) $(1, 3, 0, 1)$ and $(1, 3, 0, -10)$

orthogonal ✓

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Problem 5

1/1 point (graded)

Find the unit vector in the same direction as $x = (1, 2, 3)$.

- ○ $(1, 2, 3)/6$
- ○ $(1, 2, 3)/14$
- ○ $(1, 2, 3)/\sqrt{7}$
- ◉ $(1, 2, 3)/\sqrt{14}$

✓

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Problem 6

1/1 point (graded)

Find all unit vectors in $\mathbb{R}^2$ that are orthogonal to $(1, 1)$.

- [ ] $(1, 1)/\sqrt{2}$ and $(-1, -1)/\sqrt{2}$
- [ ] $(1, -1)$ and $(-1, 1)$
- [ ] $(1, 1)/2$ and $(-1, -1)/2$
- [x] $(1, -1)/\sqrt{2}$ and $(-1, 1)/\sqrt{2}$

✓

Submit

Problem 7

1/1 point (graded)

How would you describe the set of all points $x \in \mathbb{R}^d$ with $x \cdot x = 25$? Select all that apply.

- [x] All points of ℓ_2 length 5.
- [ ] The surface of a sphere that is centered at the origin, of radius 25.
- [ ] All points of ℓ_2 length 25.
- [x] The surface of a sphere that is centered at the origin, of radius 5.

Submit

Problems 8-17 correspond to "Linear algebra II: matrix products and linear functions"

Problem 8

1/1 point (graded)

Which of the following is a linear function of $x \in \mathbb{R}^3$? Select all that apply.

- [ ] $x_1^2 + 3x_2 + x_3$
- [x] $2x_1 - 3x_2$
- [ ] $x_1 x_2 - x_3$

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Problem 9

1/1 point (graded)

True or false: the function $f(x) = 2x_1 - x_2 + 6x_3$ can be written as $w \cdot x$ for $x \in \mathbb{R}^3$, where $w = (2, -1, 6)$.

- (•) True
- ( ) False

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Problem 10

3/3 points (graded)

Consider the linear function that is expressed by the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 0 & -1 \end{pmatrix}$.

This function maps vectors in $\mathbb{R}^p$ to $\mathbb{R}^q$.

a) What is p?

3

b) What is q?

2

c) Which of the following vectors are mapped to zero?

- [x] $(2, -1, 6)$
- [x] $(-4, 2, -12)$
- [ ] $(1, 4, -1)$
- [ ] $(4, -2, 1)$

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Problem 11

3/3 points (graded)

Compute the product: $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 0 & 1 \\ 0 & 0 & 1 \\ 2 & 6 & 0 \end{pmatrix}$:

$$= \begin{pmatrix} 1 & a & 1 \\ 14 & b & c \end{pmatrix}$$

$a =$

-6

$b =$

24

$c =$

5

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Problem 12

4/4 points (graded)

For a certain pair of matrices A, B, the product AB has dimension 10×20. Suppose A has 30 columns.

a) $A \in \mathbb{R}^{m \times n}$

$m =$

10

$n =$

30

b) $B \in \mathbb{R}^{r \times s}$

$r =$

30

$s =$

20

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Problem 13

3/3 points (graded)

We have n data points $x^{(1)}, \ldots, x^{(n)} \in \mathbb{R}^d$ and we store them in a matrix X, one point per row.

a) True or false: X has dimension $d \times n$.

- [ ] True
- [x] False

b) True or false: $X^T X$ has dimension $d \times d$.

- [x] True
- [ ] False

c) Which of the following is a matrix with (i, j) entry $x^{(i)} \cdot x^{(j)}$?

- [ ] XX
- [ ] X^TX
- [x] XX^T
- [ ] X^TX^T

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Problem 14

1/1 point (graded)
Vector x has length 10. What is $x^Txx^Txx^Tx$?

100^3

Submit

Problem 15

5/5 points (graded)

Suppose $x = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$.

a) What is x^Tx?

35

b) What is xx^T?

$$xx^T = \begin{pmatrix} 1 & a & b \\ 3 & 9 & c \\ 5 & 15 & d \end{pmatrix}$$

$a =$

3

$b =$

5

$c =$

15

$d =$

25

Submit

Problem 16

1/1 point (graded)

Vectors $x, y \in \mathbb{R}^d$ both have length 2. If $x^T y = 2$, what is the angle between x and y, in degrees (the answer is an integer in the range 0 to 180)?

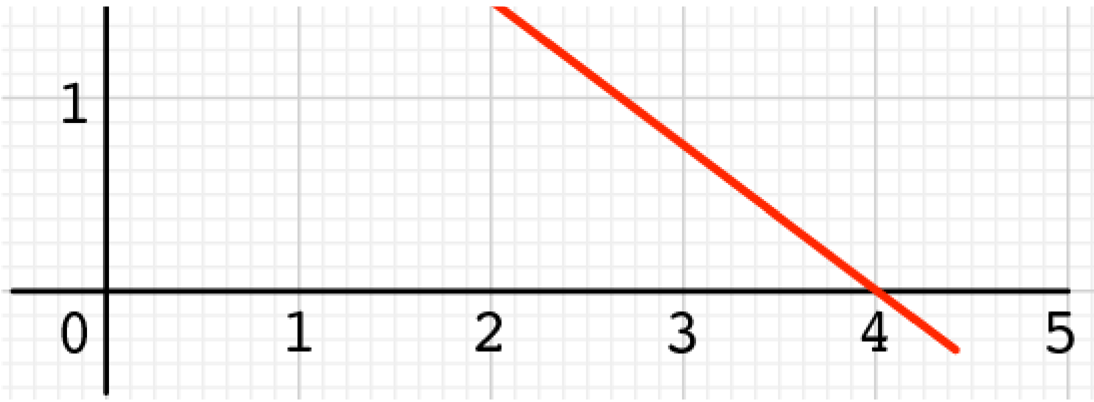
60

Submit

Problem 17

2/2 points (graded)

The line shown below can be expressed in the form $w \cdot x = 12$ for $x \in \mathbb{R}^2$. What is w?



$w = (w_1, w_2)$

$w_1 =$

3

$w_2 =$

4

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Problems 18-24 correspond to "Linear algebra III: square matrices as quadratic functions"

Problem 18

4/4 points (graded)

The quadratic function $f: \mathbb{R}^3 \to \mathbb{R}$ given by

$$f(x) = 3x_1^2 + 2x_1x_2 - 4x_1x_3 + 6x_3^2$$

can be written in the form $x^T M x$ for some **symmetric** matrix M. What are the missing entries in M?

$$M = \begin{pmatrix} a & 1 & b \\ 1 & c & 0 \\ -2 & d & 6 \end{pmatrix}$$

$a =$

3

$b =$

-2

$c =$

0

$d =$

0

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Problem 19

7/7 points (graded)

Answer the following questions about the quadratic function $f : \mathbb{R}^3 \to \mathbb{R}$ associated with the matrices A.

a) True or false: the quadratic function associated with $A = \mathrm{diag}(6, 2, -1)$ is $f(x_1, x_2, x_3) = 6x_1^2 + 2x_2^2 - x_3^2$.

- (selected) True
- False

b) $A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & 4 \\ 2 & -2 & 1 \end{pmatrix}$

Find the coefficients of the function $f(x_1, x_2, x_3) = ax_1^2 + bx_1x_2 + cx_1x_3 + dx_2^2 + ex_2x_3 + fx_3^2$ generated by this matrix.

$a =$

1

$b =$

4

$c =$

6

$d =$

-1

$e =$

2

$f =$

1

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Problem 20

1/1 point (graded)

Which of the following matrices is necessarily symmetric? Select all that apply.

- [x] AA^T for arbitrary matrix A.
- [x] A^TA for arbitrary matrix A.
- [x] $A + A^T$ for arbitrary square matrix A.
- [ ] $A - A^T$ for arbitrary square matrix A.

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Problem 21

2/2 points (graded)

Let $A = \text{diag}(1, 2, 3, 4, 5, 6, 7, 8)$.

a) What is $|A|$?

1*2*3*4*5*6*7*8

b) True or false: $A^{-1} = \text{diag}(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8})$

- (•) True
- ( ) False

Submit

Problem 22

2/2 points (graded)

Vectors $u_1, \ldots, u_d \in \mathbb{R}^d$ all have unit length and are orthogonal to each other. Let U be the $d \times d$ matrix whose rows are the u_i.

a) What is UU^T?

- ○ U
- ○ U^T
- ○ U^{-1}
- ◉ I_d

✔

b) What is U^{-1}?

- ○ U
- ◉ U^T
- ○ U^{-1}
- ○ I_d

✔

Submit

Problem 23

1/1 point (graded)

Matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & z \end{pmatrix}$ is singular. What is z?

6 ✔

Submit

Problem 24

1/1 point (graded)

The *trace* of a $d \times d$ matrix A is defined to be $\text{tr}(A) = \sum_{i=1}^{d} A_{ii}$. Which of the following statements is true, for arbitrary $d \times d$ matrices A, B? Select all that apply.

- [x] $\text{tr}(A) = \text{tr}(A^T)$.
- [x] $\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$.
- [ ] $\text{tr}(AB) = \text{tr}(A)\text{tr}(B)$.
- [x] $\text{tr}(AB) = \text{tr}(BA)$.

✔

Submit

Problems 25-27 correspond to "The multivariate Gaussian"

Problem 25

1/1 point (graded)

A spherical Gaussian has mean $\mu = (1, 0, 0)$. At which of the following points will the density be the same as at $(1, 1, 0)$? Select all that apply.

- [x] $(0, 0, 0)$
- [ ] $(1, 1, 1)$
- [x] $(2, 0, 0)$
- [x] $(1, 0, 1)$

✔

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Problem 26

1/1 point (graded)

How many real-valued parameters are needed to specify a diagonal Gaussian in $\mathbb{R}^d$?

- ○ d
- ◉ $2d$
- ○ $\frac{1}{2}d^2$
- ○ d^2

✔

Submit

Problems 28-29 correspond to "Gaussian generative models"

Problem 28

3/3 points (graded)

Suppose we solve a classification problem with k classes by using a Gaussian generative model in which the jth class is specified by parameters π_j, μ_j, Σ_j. In each of the following situations, say whether the decision boundary is $\text{linear, spherical,}$ or other quadratic.

a) We compute the empirical covariance matrices of each of the k classes, and then set $\Sigma_1 = \Sigma_2 = \cdots = \Sigma_k$ to the $\textbf{average}$ of these matrices.

linear ✔

b) The covariance matrices Σ_j are all $\textbf{diagonal}$, but no two of them are the same.

other quadratic ✔

c) There are two classes (that is, $k = 2$) and the covariance matrices Σ_1 and Σ_2 are multiples of the identity matrix.

spherical ✔

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Problem 29

2/2 points (graded)

Consider a binary classification problem in which we fit a Gaussian to each class and find that they are both centered at the origin but have different covariances: $\mu_1 = \mu_2 = 0$ and $\Sigma_1 \neq \Sigma_2$. Derive the precise form of the $\textbf{decision boundary}$, that is, the points x for which the two classes are equally likely. You will find that it is

$$x^T(\Sigma_2^{-1} - \Sigma_1^{-1})x = a \ln \frac{|\Sigma_1|}{|\Sigma_2|} + b \ln \frac{\pi_1}{\pi_2}.$$

What are a and b?

$a =$

1 ✔

$b =$

-2 ✔

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Problem 30 corresponds to "More generative modeling"

Problem 30

5/5 points (graded)

For each of the situations below, say which of the following distributions would be the best model for the data: Gaussian, gamma, beta, Poisson, or categorical.

a) You collect the number of airplane landings at Los Angeles International Airport during each one hour interval over the course of a week (thus, a total of 168 data points).

Poisson ✔

b) For your favorite sports team, you compute the fraction of games they won each year, during the period 1980-2015 (thus, a total of 36 data points).

beta ✔

c) Your local pet store has mammals, reptiles, birds, amphibians, and fish. You measure the fraction of each (thus, a total of five numbers).

categorical ✔

d) You collect the pollution levels (positive real numbers reflecting concentrations of particulate matter) recorded in your city over the past year (thus, a total of 365 numbers).

gamma ✔

e) Like (d), but instead you use the $\log$ of these values.

Gaussian ✔

Submit