

Problem 1

6/6 points (graded)

Consider the following simple data set of four points (x,y):

$$(1,1),(1,3),(4,4),(4,6)$$
.

a) Suppose you had to predict y without knowledge of x. What value would you predict?



Answer

Correct: This is simply the mean value of y: the average of the four observed values (1,3,4,6).

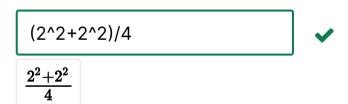
b) Continuing from part (a), what is the **mean squared error** (MSE) of your prediction, on the given four points?



Answer

Correct: This is the variance of the four observed values of y.

c) Now let's say you want to predict y based on x. Your initial choice of prediction rule is y=x. What is the MSE of the linear function y=x on the four given points?

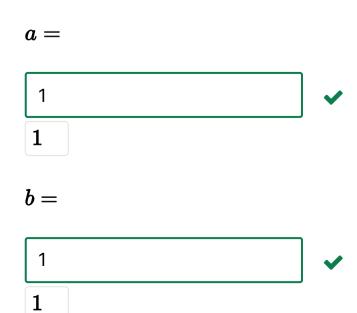


Answer

Correct:

This is not a good prediction function, as you can see if you plot it. The MSE is obtained by computing the (squared) error on each of the four points and averaging them.

d) Finally, you want to find the **best** prediction rule of the form y=ax+b. That is, you want to find the parameters $a,b\in\mathbb{R}$ such that this rule has the smallest possible mean squared error on the four training points. What are a and b?



e) Continuing from	part (d), what is	s the MSE of this c	optimal linear predictor?

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Problem 2

4/4 points (graded)

Suppose that we have data points $(x^{(1)},y^{(1)}),\ldots,(x^{(n)},y^{(n)})$, where $x^{(i)},y^{(i)}\in\mathbb{R}$, and that we want to fit them with a line that passes through the origin. The general form of such a line is y=ax: that is, the sole parameter is $a\in\mathbb{R}$.

a) In this setting, what are the **predictor** and **response** variables?

predictor x and response y ~



b) The goal is to find the value of a that minimizes the squared error on the data. We will do this by first writing down a **loss function** $L(\cdot)$. Which of the following statements is an accurate description of the loss function? Select all that apply.

lacksquare It takes a parameter $m{a}$ and returns a real number.

 \square It takes a data set and returns a parameter a.

✓ It is based on the given data set.

lt is the same regardless of the data set.



c) Using calculus, find the optimal setting of a. The answer is of the form a=N/D where the numerator N and the denominator D can be found in the following list.

$$\sum_{i=1}^{n} \, (y^{(i)} - x^{(i)}) \, x^{(i)}$$

$$\sum_{i=1}^n x^{(i)}y^{(i)}$$

$$\sum_{i=1}^n y^{(i)^2}$$

$$\sum_{i=1}^n x^{(i)^2}$$

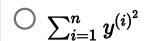
$$\sum_{i=1}^n \left(y^{(i)}-x^{(i)}
ight)^2$$

Which of these is N and which is D?

N =

$$igcap \sum_{i=1}^n \left(y^{(i)} - x^{(i)}
ight) x^{(i)}$$

$$lackbox{0} \sum_{i=1}^{n} x^{(i)} y^{(i)}$$



$$\bigcirc \sum_{i=1}^n x^{(i)^2}$$

$$igcirc$$
 $\sum_{i=1}^n \left(y^{(i)}-x^{(i)}
ight)^2$

~

D =

$$igcirc$$
 $\sum_{i=1}^n \left(y^{(i)}-x^{(i)}
ight)x^{(i)}$

$$\bigcirc \; \sum_{i=1}^n x^{(i)} y^{(i)}$$

$$\bigcirc \sum_{i=1}^n y^{(i)^2}$$

$$igcirc$$
 $\sum_{i=1}^n \left(y^{(i)}-x^{(i)}
ight)^2$



Submit

Problem 3

3/3 points (graded)

One fact that we used implicitly in the lecture is the following:

If we want to summarize a bunch of numbers x_1, \ldots, x_n by a single number s, the best choice for s, the one that minimizes the average squared error, is the **mean** of the x_i 's.

Let's see why this is true. We begin by defining a suitable loss function. Any value $s \in \mathbb{R}$ induces a mean squared loss (MSE) given by:

$$L\left(s
ight) =rac{1}{n}\sum_{i=1}^{n}\left(x_{i}-s
ight) ^{2}.$$

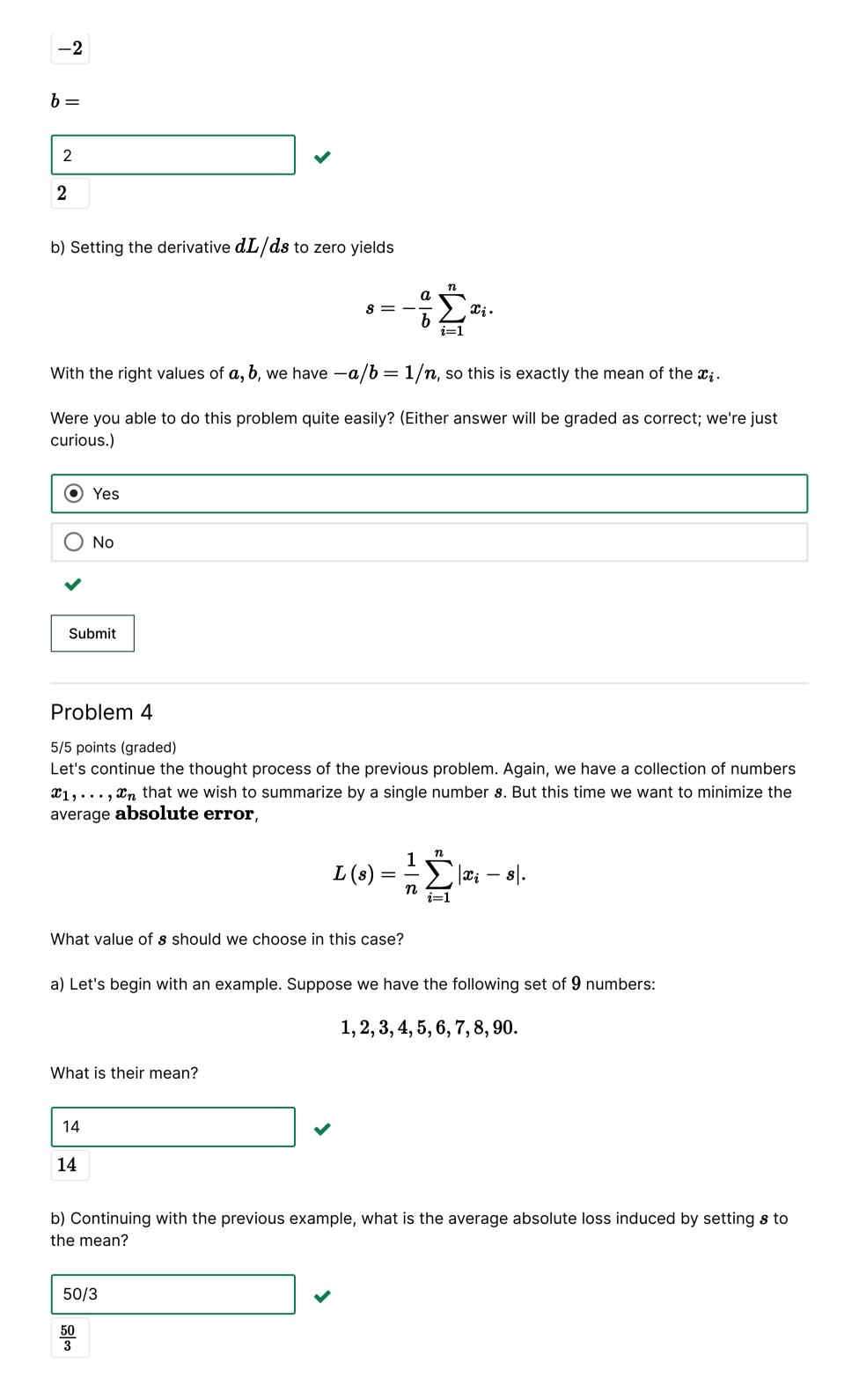
We want to find the \boldsymbol{s} that minimizes this function.

a) Compute the derivative of $L\left(s
ight)$. The answer is of the form

$$rac{dL}{ds}=ax_1+\cdots+ax_n+bs,$$

where a, b are some constants. What are a and b?

a =



c) What is the average absolute loss induced by setting s=5? 101/9 <u>101</u> d) From parts (b) and (c), we see that the value of $m{s}$ that minimizes absolute loss is $m{not}$ the mean. In fact, it is the **median**: if you arrange the set of numbers in order, the median is the number right in the middle (if the set has odd size) or any number between the two middle numbers (if the set has even size). What is the median in the example above? 5 5 e) To see why the median is the solution in general (not just for the specific numbers in the example, but always, for any numbers), we could try to use calculus, as we did in the case of squared loss. But this is tricky, because the absolute value function |x| is not differentiable (at x=0). A related approach is to reason that if s is less than the median, then the loss function gets lower when you increase s; and if s is more than the median, then the loss function gets lower when you decrease s. Work through this reasoning on your own, and then select one of the following. (You'll be marked correct either way.) I was able to work through the entire argument. I get the general idea, but got stuck on some details. I don't know how to start. Submit Problems 5-7 correspond to "Linear regression"

Problem 5

1/1 point (graded)

Let's say we have a regression problem with predictor variables x_1, \ldots, x_d and response variable y. Sometimes, we are interested in solutions that do not necessarily use all the predictor variables. For instance, the linear function

$$f\left(x\right)=3x_{2}-7x_{9}$$

uses just two of the features. We call this a *sparse* solution.

Here is a question: does adding more features always help? That is, let S be some subset of the features and define LOSS(S) to be the loss obtained by doing least-squares regression using just these features. Now let's say we add another feature to S, to get S'. How does LOSS(S') compare to LOSS(S)? Select all that apply.

$oxed{oxed} \operatorname{LOSS}\left(S' ight) \leq \operatorname{LOSS}\left(S ight)$ always					
\square $\mathrm{LOSS}\left(S' ight) \leq \mathrm{LOSS}\left(S ight)$ sometimes, but not always					
$\square \; \mathrm{LOSS}\left(S' ight) < \mathrm{LOSS}\left(S ight)$ always					
$oxed{oxed} extbf{LOSS}\left(S' ight) < extbf{LOSS}\left(S ight) ext{ sometimes, but not always.}$					
Submit					
Problem 6					
1/1 point (graded) We have a data set $(x^{(1)},y^{(1)}),\ldots,(x^{(n)},y^{(n)})$, where $x^{(i)}\in\mathbb{R}^d$ and $y^{(i)}\in\mathbb{R}$. We want to express y as a linear function of x , of the form $w\cdot x+b$, but the error penalty we have in mind is not the usual squared loss: if we predict \hat{y} and the true value is y , then the penalty should be the absolute difference, $ y-\hat{y} $. Write down the loss function that corresponds to the total penalty on the training set.					
Click here where you think you have the correct loss function.					
Answer Correct: Your loss function should be $L\left(w,b\right)=\sum_{i=1}^{n} y^{(i)}-(w\cdot x^{(i)}+b) $. Check to ensure that your loss function is correct, and let us know on the forums what you came up with.					
Problem 7					
4/4 points (graded) Let $x^{(1)},\ldots,x^{(n)}$ be a set of n data points in \mathbb{R}^d , and let $y^{(1)},\ldots,y^{(n)}\in\mathbb{R}$ be corresponding response values. In this problem, we will see how to rewrite several basic functions of the data using matrix-vector calculations. To this end, define:					
X , the $n imes d$ matrix whose rows are the $x^{(i)}$					
y , the n -dimensional vector with entries $y^{(i)}$					
$oldsymbol{1}$, the $oldsymbol{n}$ -dimensional vector whose entries are all 1					
and consider the matrix-vector expressions:					
i) XX^T					
ii) $(1/n)1^T y$					
iii) $(1/n)X^TX$					

Each of the following quantities is equivalent to one of the expressions (i) (iv) shows. In each case

iv) $(1/n)\,X^T 1$

Each of the following quantities is equivalent to one of the expressions (i)-(iv) above. In each case, choose the correct match (i,ii,iii,iv) from the list. a) The average of the $y^{(i)}$ values, that is, $(y^{(1)} + \cdots + y^{(n)}) / n$. ii b) The n imes n matrix whose (i,j) entry is the dot product $x^{(i)} \cdot x^{(j)}$. c) The average of the $x^{(i)}$ vectors, that is, $(x^{(1)}+\cdots+x^{(n)})/n$. İ۷ d) The empirical covariance matrix, assuming the points $x^{(i)}$ are centered (that is, assuming the average of the $x^{(i)}$ vectors is zero). This is the d imes d matrix whose (i,j) entry is $rac{1}{n} \sum_{k=1}^n x_i^{(k)} x_j^{(k)}.$ iii Submit Problems 8-11 correspond to "Regularized linear regression" Problem 8 1/1 point (graded) Suppose we want to predict a response value $m{y}$ using $m{d}$ predictor variables. The way we will do this is to use training data, consisting of n points $(x^{(i)},y^{(i)})$, to fit a linear function. In general, how would the dimension of the data (d) influence the number of training points (n) that we need to fit a good model? lacktriangle For larger d, we need more data points. igcup For larger $oldsymbol{d}$, we need fewer data points. The number of data points needed is unrelated to $oldsymbol{d}$. **Submit**

Problem 9

2/2 points (graded)

In lecture, we asserted that in d-dimensional space, it is possible to perfectly fit (almost) any set of d+1 points $(x^{(0)},y^{(0)}),(x^{(1)},y^{(1)}),\dots,(x^{(d)},y^{(d)})$. Let's see how this works in the specific case where:

$$\bullet \ x^{(0)} = 0$$

- $ullet x^{(i)}$ is the ith coordinate vector (the vector that has a 1 in position i, and zeros everywhere else), for $i=1,\ldots,d$
- $ullet y^{(i)} = c_i$, where c_0, c_1, \ldots, c_d are arbitrary constants.

Find $w\in\mathbb{R}^d$ and $b\in\mathbb{R}$ such that $w\cdot x^{(i)}+b=y^{(i)}$ for all i. You should express your answer in terms of c_0,c_1,\ldots,c_d .

a) What is b?

$$\bigcirc \ b = c_0 + c_1 + \ldots + c_d$$

 $leftondownburge b = c_0$

$$\bigcirc b = c_d$$

$$\bigcirc b = (1/d)(c_0 + c_1 + \ldots + c_d)$$

~

b) Write $w=(w_1,\ldots,w_d)$. What is w_i ?

- $\bigcirc \ w_i = c_i$
- $\bigcirc \ w_i = c_0$
- $\bigcirc \ w_i = c_i = c_0$

/

Submit

Problem 10

4/4 points (graded)

Continuing from the previous problem, let's keep the same set of d+1 points $(x^{(0)},y^{(0)}),(x^{(1)},y^{(1)}),\dots,(x^{(d)},y^{(d)})$. As we saw, we can find w,b that perfectly fit these points; hence least-squares regression would find this perfect solution and have zero loss on the training set.

Now, let us instead use ridge regression, with parameter $\lambda \geq 0$, to obtain a solution. We can denote this solution by w_λ,b_λ . Also define the squared training loss associated with this solution,

$$L\left(\lambda
ight) = rac{1}{n} \sum_{i=1}^n \left(y^{(i)} - \left(w_\lambda \cdot x^{(i)} + b_\lambda
ight)
ight)^2.$$

a) What is $L\left(0\right)$?



Correct: This is just the least-squares solution					
b) As $\pmb{\lambda}$ increases, how does $\ \pmb{w}_{\pmb{\lambda}}\ $ behave?					
O It increases					
It decreases					
O It initially decreases, then increases					
O It doesn't necessarily follow any of these trends					
c) As $\pmb{\lambda}$ increases, how does $\pmb{L}\left(\pmb{\lambda}\right)$ behave?					
It increases					
O It decreases					
O It initially increases, then decreases					
O It doesn't necessarily follow any of these trends					
d) As $\pmb{\lambda}$ goes to infinity, what value does $L\left(\pmb{\lambda}\right)$ approach?					
○ Zero					
Infinity					
$lacktriangle$ The variance of the $m{y}$ -values					
\bigcirc The average of the \emph{y} -values					
Submit					

Problem 11

1/1 point (graded)

We saw in lecture that the Lasso tends to produce **sparse** solutions to regression problems. We will now look at an alternative strategy for doing this.

Suppose we have d predictor variables. For any subset of features $S \subset \{1,2,\ldots,d\}$ define $\mathbf{LOSS}(S)$ to be the loss obtained by doing least-squares regression using just the features S. Given a sparsity level k, we would ideally like to find the subset S of size S such that S is as small as possible. Unfortunately, this problem is NP-hard, which means that there is unlikely to be an efficient algorithm for it (the brute-force strategy, trying all possible subsets of S features, is inefficient when S is large).

Instead, an approximate solution can be obtained using a \mathbf{greedy} algorithm called

forward stepwise regression. It chooses one feature at a time, as follows:

- ullet Let S be the set of features chosen so far. Initially S is empty.
- ullet Repeat while |S| < k: Find the feature x_i such that $\mathrm{LOSS}\left(S \cup \{i\}
 ight)$ is as small as possible, and add i to S

Does this make sense? (Just give an honest answer; you'll get marked correct either way.)

This makes perfect sense to me.
 I am somewhat hazy on what is going on here.
 Submit

Problems 12-13 correspond to "Linear models for conditional probability estimation"

Problem 12

1/1 point (graded)

We identified *inherent uncertainty* as one reason why it might be difficult to get perfect classifiers, even with a lot of training data. In which of the following situations is there likely to be a significant amount of inherent uncertainty? Select two of the four below.

 $oxedsymbol{\square}$ $oldsymbol{x}$ is a picture of an animal and $oldsymbol{y}$ is the name of the animal

extstyle z consists of the dating profiles of two people and y is whether they will be interested in each other

 $oxedsymbol{\square}$ $oldsymbol{x}$ is a speech recording and $oldsymbol{y}$ is the transcription of the speech into words

 $oxedsymbol{arphi}$ $oldsymbol{x}$ is the recording of a new song and $oldsymbol{y}$ is whether it will be a big hit



Submit

Problem 13

3/3 points (graded)

A logistic regression model given by parameters $w \in \mathbb{R}^d$ and $b \in \mathbb{R}$ is fit to a data set of points $x \in \mathbb{R}^d$ with binary labels $y \in \{-1,1\}$. For any constant c, the model assigns the same conditional probability $\Pr(y=1|x)$ to all points x that satisfy $x \cdot x + b = c$.

What is the value of $w \cdot x + b$ for which the following conditional probabilities are assigned? In each case, just give a single real number, to two decimal places.

a)
$$\Pr\left(y=1|x
ight)=1/2$$

0

b) $\Pr\left(y=1 x\right)$	(x)=3/4
1.10	
1.10	
c) $\Pr\left(y=1 x ight)$	c)=1/4
-1.10	✓
-1.10	
Submit	
Problems 14-16	6 correspond to "Logistic regression"
Problem 14	
1/1 point (graded	
	a logistic regression model from training data $(x^{(1)},y^{(1)}),\dots,(x^{(n)},y^{(n)})$, which of o we try to do? Select all that apply.
☐ Maximize	the probabilities of the $x^{(i)}$
✓ Maximize	the conditional probabilities of the $y^{(i)}$ given $x^{(i)}$
	the conditional probabilities of the $y^{(i)}$ given $x^{(i)}$ the joint probabilities of $x^{(i)}$ and $y^{(i)}$
☐ Maximize ✓ Submit	the joint probabilities of $x^{(i)}$ and $y^{(i)}$
Submit Problem 15 1/1 point (graded)	the joint probabilities of $x^{(i)}$ and $y^{(i)}$
Submit Problem 15 1/1 point (graded Which of the fothat apply.	the joint probabilities of $x^{(i)}$ and $y^{(i)}$
Submit Problem 15 1/1 point (graded Which of the forthat apply. There is a	the joint probabilities of $x^{(i)}$ and $y^{(i)}$
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interpret, for reasonable s	instance, one that	nelpful to have a conditional probability function that is eadepends on just a few features. Which of the following a ng a sparse logistic regression model (that is, one in which all that apply.	re
$igcap {igwedge} {igwedge} {igwedge} {oxed{oxedge} { ho} { $		or logistic regression by adding a regularization term of t	ne form
$oxed{arphi}_{oxed{\lambda} \ w\ _1}$		or logistic regression by adding a regularization term of tl	ne form
	-	trategy to select one feature at a time: the feature that (we most reduces the loss function.	/hen added
First fit		on model, and then retain the features with the highest-v	alued
Submit			
Problem 17 c	orresponds to "Log	gistic regression in use"	
• •	ed) t in a bag-of-words	s representation, we decide to use the following vocabuland. In. What is the vector form of the following sentence: <i>A r</i> eads.	•
(2,3,3)			
(2,0,3,3	3,0)		
(1,0,1,1,0	0)		
(3,3,2,0),0)		
~			
Submit			
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Problem 16

1/1 point (graded)



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