

# TOPOLOGICAL APPROACH TO SOLVE P VERSUS NP

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## 1. OVERVIEW

This paper talks about difference between P and NP by using topological space that mean resolution principle. I pay attention to restrictions of antecedent and consequent in resolution, and show what kind of influence the restrictions have for difference of structure between P and NP regarding relations of relation.

First, I show the restrictions of antecedent and consequent in resolution principle. Antecedents connect each other, and consequent become a linkage between these antecedents. And we can make consequent as antecedents product by using some resolutions which have same joint variable. We can determine these consequents reducible and irreducible.

Second, I introduce RCNF that mean topology of resolution principle in CNF. RCNF is HornCNF and that variable values are presence of restrictions of CNF formula clauses. RCNF is P-Complete.

Last, I introduce TCNF that have 3CNF's character which relate 2 variables relations with 1 variable. I show CNF complexity by using CCNF that combine some TCNF. TCNF is NP-Complete and product irreducible. I introduce CCNF that connect TCNF like Moore graph. We cannot reduce CCNF to RCNF with polynomial size. Therefore, TCNF is not in P.

## 2. PREPARATION

In this paper, I use CNF description like this.

**Definition 1.** About  $F \in CNF$ , set of truth value assignment that  $F$  become true is  $[F]$ ,  $F$  become false is  $\overline{[F]}$ , only clauses  $c \in F$  become false is  $\widehat{[c]}$ . Number of variables of  $c$  is  $|c|$ , size of  $F$  is  $|F|$ . Number of truth value assignments of  $[F]$ ,  $\overline{[F]}$  and  $\widehat{[c]}$  is  $|[F]|$ ,  $|\overline{[F]}|$  and  $|\widehat{[c]}|$ .

The composition of the clauses  $c \in F$  may be denoted by a subscript. That is,  $c_{i\ldots j\ldots} = (x_i \vee \cdots \vee \overline{x_j} \vee \cdots)$ . The subscript of a capital letter shall be either positive or negative of a variable. For examples,  $c_I, c_{\overline{I}}$  means  $c_I, c_{\overline{I}} \in \{c_i, \overline{c_i}\}, c_I \neq c_{\overline{I}}$ .

About resolution, I will use the term “Joint Variable” as variables that positive and negative variable which are included in each antecedents and not included in consequent, and “Positive Antecedent” as antecedent that have positive joint variable, “Negative Antecedent” as antecedent that have negative joint variable. We treat some resolution that have same joint variable. Such case, positive antecedent, negative antecedent and consequent become set of clauses.

In this paper, I use metric space of fomula like this.

**Definition 2.** A metric space of fomula is the metric space of the set of a truth value assignment becoming false in a fomula.

### 3. RESOLUTION

We arrange structure of the resolution.

**Theorem 3.** *Antecedents of a resolution connect each others.*

*Proof.* I prove it using reduction to absurdity. We assume that some resolution have 0 or over 2 joint variable.

The case that resolution have 0 joint variable contradicts a condition of the resolution clearly. The case that resolution have 2 joint variable contradicts a condition of the resolution because  $c_{IJp\dots} \vee c_{\overline{IJ}q\dots} \rightarrow c_{Jp\dots\overline{J}q\dots} = \top$ .

Therefore, this theorem was shown than reduction to absurdity.  $\square$

**Theorem 4.** *In  $c_{ip\dots}, c_{iq\dots} \in F \in CNF$  and resolution  $c_{ip\dots} \wedge c_{iq\dots} \rightarrow c_{p\dots q\dots}$ ,  $\overline{[c_{p\dots q\dots}]} become linkage of  $\overline{[c_{ip\dots}]}$ ,  $\overline{[c_{iq\dots}]}$  at the intersection of antecedent and consequent  $\overline{[c_{ip\dots q\dots}]} \cap \overline{[c_{iq\dots p\dots}]}.$$*

*Proof.* It is self-evident than a definition.  $\square$

**Definition 5.** I will use the term “Clauses product” like following.

$$(c_{P\dots} \wedge c_{Q\dots}) \times (c_{R\dots} \wedge c_{S\dots}) = c_{P\dots R\dots} \wedge c_{Q\dots R\dots} \wedge c_{P\dots S\dots} \wedge c_{Q\dots S\dots}$$

**Theorem 6.** *Consequent is the Clauses product with positive antecedent and negative antecedent that remove joint variables.*

$$\begin{aligned} & (c_{ip\dots} \wedge c_{iq\dots} \wedge \dots) \wedge (c_{ir\dots} \wedge c_{is\dots} \wedge \dots) \\ & \rightarrow c_{p\dots r\dots} \wedge c_{p\dots s\dots} \wedge \dots \wedge c_{q\dots r\dots} \wedge c_{q\dots s\dots} \wedge \dots = (c_{p\dots} \wedge c_{q\dots} \wedge \dots) \times (c_{r\dots} \wedge c_{s\dots} \wedge \dots) \end{aligned}$$

*Proof.* It is self-evident than below5.  $\square$

**Definition 7.** About  $X \in CNF$ , I will use the term “Direct sum of Clauses” like that  $X = Y \times Z$ ,  $Y, Z \in CNF$  and there is no injection  $X \ni c \mapsto f(c) \in Y \cup Z$ .

Therefore, Direct sum of Clauses  $Y \times Z$  is clauses product as  $X$  and  $Y, Z$  representation does not include  $X$ . We can introduce reducible and irreducible by using direct sum of clauses.

**Definition 8.** I will use the term “Product Reducible” like that if direct sum of clauses  $x = y \times z \in CNF$  exists in  $x \subset X \in CNF$ . I will use the term “Product Irreducible” like that if  $X$  is not product reducible.

### 4. RCNF

I introduce topology of deduction system to formula. For simplification, I treat topology as formula.

**Definition 9.** About  $F \in CNF$ , I will use the term “DCNF(Deduction CNF)” as formula that variables value are presence of restrictions of CNF formula clauses. Especially, I will use the term “RCNF(Resolution CNF)” and “ $RCNF(F)$ ” as DCNF that deduction system is resolution principle. Clauses become variables and resolution become clauses in  $RCNF(F)$ . Antecedent become negative variables and consequent become positive variables. And furthermore, RCNF does not include variable that correspond to empty clause.

That is, if

$$F \supset c_{ip\dots} \wedge c_{iq\dots},$$

then

$$RCNF(F) \supset (c_{ip\dots}) \wedge (c_{iq\dots}) \wedge (\overline{c_{ip\dots}} \vee \overline{c_{iq\dots}} \vee c_{p\dots q\dots}).$$

And RCNF does not include variable correspond to empty clause, therefore sufficiency of  $F$  accords with  $RCNF(F)$ . Resolution consequent is 1 or less, therefore  $RCNF(F) \in HornCNF$ .

RCNF is P-Complete.

**Theorem 10.** *RCNF is P-Complete.*

*Proof.* Clearly RCNF is HornCNF and  $RCNF \in P$ , I should show that we can reduce HornCNF to RCNF in logarithm space.

To reduce HornCNF to RCNF, I show 2-step procedures.

First, I reduce HornCNF to at most 3 variables clauses HornCNF. We can reduce by using same way to reduce CNF to 3CNF. That is, each clauses change follows with new variables.

$$c_{I\overline{j}\overline{k}\overline{l}\dots} \rightarrow c_{I\overline{j}\overline{0}} \wedge c_{0\overline{k}\overline{1}} \wedge c_{1\overline{l}\overline{2}} \wedge \dots$$

We can execute this reduction with logarithm space, pointer to consequent, pointer to variable, counter that show already used variables.

Second, I reduce this HornCNF to RCNF. We can reduce by adding resolution formula for each clauses. We can reduce HornCNF with unit resolution, therefore it is enough to keep SAT by using resolution formula that variables of antecedent decreases. That is;

$$\begin{aligned} c_R &\rightarrow (c_R) \wedge (\overline{c_R} \vee \overline{c_R}) \\ c_{P\overline{q}} &\rightarrow (c_{P\overline{q}}) \wedge (c_P \vee \overline{c_{P\overline{q}}} \vee \overline{c_q}) \wedge (\overline{c_P} \vee \overline{c_{P\overline{q}}}) \\ c_{I\overline{j}\overline{k}} &\rightarrow (c_{I\overline{j}\overline{k}}) \wedge (c_{I\overline{k}} \vee \overline{c_{I\overline{j}\overline{k}}} \vee \overline{c_j}) \wedge (c_{I\overline{j}} \vee \overline{c_{I\overline{j}\overline{k}}} \vee \overline{c_k}) \\ &\wedge (c_I \vee \overline{c_{I\overline{j}}} \vee \overline{c_j}) \wedge (c_I \vee \overline{c_{I\overline{k}}} \vee \overline{c_k}) \wedge (\overline{c_I} \vee \overline{c_{I\overline{j}}}) \end{aligned}$$

We can execute this reduction with logarithm space, pointer to consequent, pointer to variable.

Above two reduction, we can reduce HornCNF to RCNF. Both reductions use only logarithm space, we can execute all reduction in logarithm space.

Therefor, RCNF is P-Complete.  $\square$

RCNF have structure that correspond to resolution restrictions.

**Theorem 11.** *If  $F \in MUC$  have  $\widehat{[c]}$  that is product irreducible,  $RCNF(F)$  have antecedent variables which correspond to  $\widehat{[c]}$  connected component.*

*Proof.* Mentioned above 4,  $RCNF(F)$  have consequents that correspond to each clauses linkages, and clauses that is resolved from another clauses also have consequents. Therefore, to repeating resolution we can resolve the consequents that correspond to  $\widehat{[c]}$  linkages.

From assumptions that  $F \in MUC$ , we must resolve these consequents to empty clause. Therefore,  $RCNF(F)$  have these consequents as antecedents.

Therefore, we can say the theorem is correct.  $\square$

### 5. 3CNF

I show 3CNF complexity. I introduce TCNF that keep 3CNF complexity and become product irreducible. And I show that we cannot reduce CCNF that made by TCNF to RCNF with polynomial size.

**Theorem 12.** *3CNF associate 2 variables relations with other one variable.*

*HornCNF associate some true variables relations with other one true variable.*

*2CNF associate one variable with other one variable.*

*Proof.* About 3CNF, it is clearer than the next relations;

$$(x_P \vee x_Q \vee x_R) \Leftrightarrow (\overline{x_P} \wedge \overline{x_Q} \rightarrow x_R) \Leftrightarrow (\overline{x_P} \rightarrow x_Q \vee x_R)$$

About HornCNF, it is clearer than the next relations;

$$(\overline{x_p} \vee \overline{x_q} \vee \dots \vee x_r) \Leftrightarrow (x_p \wedge x_q \rightarrow x_r)$$

About 2CNF, it is clearer than the next relations;

$$(x_P \vee x_Q) \Leftrightarrow (\overline{x_P} \rightarrow x_Q)$$

□

Next, I introduce some CNF that we can treat easily with 3CNF's character (association 2 variables relation with 1 variable).

**Definition 13.** I will use the term “Triangular CNF” and “TCNF” like that combine formulas

$$[T_{PQR}] = \overline{T_{PQR}} = c_{PQ\overline{R}} \wedge c_{P\overline{Q}R} \wedge c_{\overline{P}QR}$$

therefore

$$T_{PQR} = c_{\overline{P}\overline{Q}} \wedge c_{\overline{Q}\overline{R}} \wedge c_{\overline{P}\overline{R}} \wedge c_{PQR}.$$

And I will use the term “Triangular Minimal Unsatisfiable Core” and “TMUC” like that is MUC of TCNF.

I show some TCNF's characters.

**Theorem 14.** *TCNF is NP-Complete.*

*Proof.* Clearly TCNF is 3CNF and  $TCNF \in NP$ , I should show that we can reduce 3CNF to TCNF in polynomial size.

To reduce 3CNF to TCNF, I reduce  $C_{\overline{X}\overline{Y}\overline{Z}} \in 3CNF$  to TCNF.

$T_{STX} \wedge T_{TUY} \wedge T_{UVZ} \in TCNF$  is

$$(x_X, x_Y, x_Z) = (\top, \perp, \top) \leftrightarrow \forall x_S, x_T, x_U, x_V (T_{STX} \wedge T_{TUY} \wedge T_{UVZ} = \perp), \text{ then}$$

$$C_{\overline{X}\overline{Y}\overline{Z}} \equiv T_{STX} \wedge T_{TUY} \wedge T_{UVZ}.$$

And  $T_{STX} \wedge T_{TUY} \wedge T_{UVZ}$  size is atmost polynomial size of  $C_{\overline{X}\overline{Y}\overline{Z}}$ , therefore TCNF that reduce from 3CNF become atmost polynomial size.

Therefor, TCNF is NP-Complete. □

**Theorem 15.** *About  $T_{PQR} \in TCNF$ ,  $T_{PQR}$  and  $[T_{PQR}]$  is Product Irreducible.*

*Proof.* I prove it using reduction to absurdity. We assume that  $T_{PQR}$  or  $[T_{PQR}]$  have Direct sum of Clauses  $A \times B$ .

About  $A \times B$ ,  $[a]$  is equivalence relation that  $a \in A$  is representative,  $B_{[a]}$  is projection map of  $[a]$  to  $B$ .

We treat the relation between  $B_{[a]}$  and  $A \times B$ . If  $|B_{[a_p]} \cap B_{[a_q]}| \leq 1$  at all  $a_p, a_q \in A$ , we can decide  $t \in A \times B$  by using  $a_p, a_q$  when  $b \subset t$  is  $b \in B_{[a_p]} \cap B_{[a_q]}$ , or by using  $b$  when  $b \notin B_{[a_p]} \cap B_{[a_q]}$ . And because mentioned above 5,  $A \times B$  have clauses that correspond to  $a \in A$ ,  $b \in B$ . And  $A \times B$  have injection  $A \times B \ni c \mapsto f(c) \in A \cup B$  and contradicts a condition of clauses product. Therefore  $|B_{[a_p]} \cap B_{[a_q]}| > 1$ .

Calculate  $|B_{[a_p]} \cap B_{[a_q]}|$  by using  $T_{PQR}$  structure. The pair  $(a, B_{[a]})$  are;

$$(c_P, \{c_{QR}\}), (c_{\overline{P}}, \{c_{\overline{Q}}, c_{\overline{R}}\}), (c_Q, \{c_{PR}\}), (c_{\overline{Q}}, \{c_{\overline{P}}, c_{\overline{R}}\}), \\ (c_R, \{c_{PQ}\}), (c_{\overline{R}}, \{c_{\overline{P}}, c_{\overline{Q}}\}), (c_{PQ}, \{c_R\}), (c_{QR}, \{c_P\}), (c_{PR}, \{c_Q\}).$$

$|B_{[a_p]} \cap B_{[a_q]}| \leq 1$  at all  $a_p, a_q \in A$  and contradicts a condition that  $T_{PQR} = A \times B$ .

Calculate  $|B_{[a_p]} \cap B_{[a_q]}|$  by using  $[T_{PQR}]$  structure. The pair  $(a, B_{[a]})$  are;

$$(c_P, \{c_{QR}, c_{\overline{QR}}\}), (c_{\overline{P}}, \{c_{QR}\}), \\ (c_Q, \{c_{P\overline{R}}, c_{\overline{P}R}\}), (c_{\overline{Q}}, \{c_{PR}\}), \\ (c_R, \{c_{P\overline{Q}}, c_{\overline{P}Q}\}), (c_{\overline{R}}, \{c_{PQ}\}), \\ (c_{PQ}, \{c_{\overline{R}}\}), (c_{\overline{P}Q}, \{c_R\}), (c_{P\overline{Q}}, \{c_R\}), \\ (c_{QR}, \{c_{\overline{P}}\}), (c_{\overline{Q}R}, \{c_P\}), (c_{Q\overline{R}}, \{c_P\}), \\ (c_{PR}, \{c_{\overline{Q}}\}), (c_{\overline{P}R}, \{c_Q\}), (c_{P\overline{R}}, \{c_Q\}).$$

$|B_{[a_p]} \cap B_{[a_q]}| \leq 1$  at all  $a_p, a_q \in A$  and contradicts a condition that  $[T_{PQR}] = A \times B$ .

Therefore, this theorem was shown than reduction to absurdity.  $\square$

We can make large CNF by combine some TCNF.

**Definition 16.** I will use the term “Chaotic CNF” and “CCNF” like that TCNF combine Moore graph(diameter k, degree 3) as  $T \in TCNF$  are nodes and variables are edges, and  $M_k \in CCNF$ . I will use the term “Chaotic MUC” and “CMUC” like that CCNF of MUC.

If we decide a  $t_0 \subset M_k$ , I will use the term “ $t_m$ ” like that distance from  $t_0$  become m. If  $x_P, x_Q, x_R \in t_m$  and all  $t_{m-1}$  are  $x_Q, x_R \notin t_m \ni x_P$ , I will use the term “ $t_{P-QR}$ ”.

**Theorem 17.**  $M_k \in CMUC$  exists.

*Proof.* I prove it using mathematical induction.

Case  $k = 1$ .

$$t_0 = T_{PQR}$$

and

$$M_1 = T_{PQR} \wedge T_{P-S\overline{T}} \wedge T_{Q-T\overline{U}} \wedge T_{R-U\overline{S}}$$

is MUC and CCNF. Therefore CMUC exists at  $k = 1$ .

In  $k = n$  then  $M_n \in MUC$ .

$$T_{M-PQ} \wedge T_{N-\overline{Q}R} = O_n \subset M_n, T_{M-PQ} \wedge T_{N-\overline{Q}R} \text{ is}$$

$$X_M = X_N \rightarrow X_P \neq X_R \text{ and}$$

$$X_M \neq X_N \rightarrow X_P = X_R.$$

To construct  $M_{n+1}$  by using  $M_n$ , we rename  $X_Q$  to new variable  $X_S$  and  $X_Q$  to independ each other, and we add  $T_{n+1}$  with  $X_Q, X_S$  keeping same relations. For examples,

$$O_{n,n+1} = T_{M-PQ} \wedge T_{N-RS} \wedge T_{P-UV} \wedge T_{R-VW} \wedge T_{\overline{Q}-XY} \wedge T_{\overline{S}-\overline{YZ}} \text{ is}$$

$$X_M = X_N \neq \top \rightarrow (X_U = X_W) \neq (X_X = X_Z) \text{ and}$$

$$X_M \neq X_N \rightarrow (X_U = X_W) = (X_X = X_Z).$$

In the same way,

$$T_{M-PQ} \wedge T_{N-QR} = A_l \subset M_l, T_{M-PQ} \wedge T_{N-QR} \text{ is}$$

$$X_M = X_N \rightarrow X_P = X_R \text{ and}$$

$$X_M \neq X_N \rightarrow X_P \neq X_R.$$

And we can construct  $M_{n+1}$  same way. For examples,

$$A_{n,n+1} = T_{M-PQ} \wedge T_{N-RS} \wedge T_{P-UV} \wedge T_{R-VW} \wedge T_{Q-XY} \wedge T_{S-YZ} \text{ is}$$

$$X_M = X_N \rightarrow (X_U = X_W) = (X_X = X_Z) \text{ and}$$

$$X_M \neq X_N \rightarrow (X_U = X_W) \neq (X_X = X_Z).$$

Now we think structure of  $T_n \subset M_n$ . From Moor Graph structure, all  $T_n$  adjoin each other and make one circle.  $X_P, X_R$  also become part of this circle. And 3 degree moore graph's girth is even, therefore we can treat  $T_n$  divide two set of arches that is adjoin each other. In this circle,  $(X_M, X_N)$  relation decide  $(X_P, X_R)$  relation, and  $X_Q$  that shares these adjoin  $T$  decides these relation.

In the same way, we think structure of  $T_{n+1} \subset M_{n+1}$ .

From Moor Graph structure, all  $T_{n+1}$  adjoin each other and make one circle.  $X_U, X_W, X_X, X_Z$  also become part of this circle. And 3 degree moore graph's girth is even, therefore we can treat  $T_{n+1}$  divide two set of arches that is adjoin each other. In this circle,  $(X_M, X_N)$  relation decide  $(X_U, X_W), (X_X, X_Z)$  relation, and  $(X_V, X_Y)$  that shares these adjoin  $T$  decides these relation.

Thus, we can make  $M_{n+1} \in MUC$  by keeping  $T_{n+1}$  relation in  $M_{n+1}$  to same as  $T_n$  relation in  $M_n$ .

Therefore, this theorem was shown than mathematical induction.  $\square$

**Theorem 18.**  $F \in CMUC$  exist that  $\left| \widehat{[c]} \right|$  of all  $c \in F$  is not polynomial size.

*Proof.* I proof by  $T_n \subset M_n$  and  $T_n \wedge T_{n+1} \subset M_{n+1}$  in mentioned above 17 become

$$\frac{|[T_{n+1}]|}{|[T_n]|} > \frac{|M_{n+1}|}{|M_n|} \rightarrow \frac{|[T_{n+1}]|}{|[T_n]|} \times \frac{|M_n|}{|M_{n+1}|} > 1.$$

I pay attention to structure of  $T_n, T_{n+1}$ . Mentioned above 17,  $O_{n,n+1}$  construct from  $O_n$  have atmost one  $O_{n+1}$ , and another is  $A_{n+1}$ . If we extend  $M_n$  to  $M_{n+1}$ ,  $M_{n+1}$  have  $O_{n+1}$  same number of  $O_n$  include in  $M_n$ . Therefore

$$\frac{|[T_{n+1}]|}{|[T_n]|} \times \frac{|M_n|}{|M_{n+1}|} = \frac{|[A_{n,n+1}]|}{|[A_n]|} \times \frac{|A_n|}{|A_{n,n+1}|} \quad (\text{as } n \gg 0).$$

First, I calculate  $\frac{|A_n|}{|A_{n,n+1}|}$ . Because the fomula  $A_{n,n+1}$  and  $A_n$ ,  $\frac{|A_n|}{|A_{n,n+1}|} = \frac{1}{3}$ .

Second, I calculate  $\frac{|[A_{n,n+1}]|}{|[A_n]|}$ .

$[A_n]$  is

$(X_M, X_N, X_P, X_R) = (\perp, \perp, \perp, \perp), (\perp, \perp, \top, \top), (\perp, \top, \top, \perp), (\top, \perp, \perp, \top), (\top, \top, \perp, \perp)$  and  $[A_{n,n+1}]$  is

$(X_M, X_N, X_U, X_W, X_X, X_Z)$

$= (\perp, \perp, \perp, \perp, \perp, \perp), (\perp, \perp, \perp, \perp, \top, \top), (\perp, \perp, \top, \top, \perp, \perp),$

$(\perp, \perp, \perp, \top, \top, \perp), (\perp, \perp, \top, \perp, \perp, \top),$

$(\perp, \top, \perp, \perp, \perp, \top), (\perp, \top, \perp, \top, \perp, \perp), (\perp, \top, \top, \top, \perp, \top), (\perp, \top, \top, \top, \top, \perp),$

$(\top, \perp, \perp, \top, \top, \perp), (\top, \perp, \top, \perp, \perp, \top), (\top, \perp, \top, \top, \top, \top), (\top, \perp, \top, \top, \top, \perp),$

$(\top, \top, \perp, \perp, \perp, \top), (\top, \top, \perp, \top, \perp, \top), (\top, \top, \top, \top, \perp, \top), (\top, \top, \top, \top, \top, \top).$

Thus,  $\frac{|[A_{n,n+1}]|}{|[A_n]|} = \frac{17}{5}$ .

Therefore,

$$\frac{|[T_{n+1}]|}{|[T_n]|} \times \frac{|M_n|}{|M_{n+1}|} = \frac{|[A_{n,n+1}]|}{|[A_n]|} \times \frac{|A_n|}{|A_{n,n+1}|} = \frac{17}{5} \times \frac{1}{3} = \frac{17}{15} > 1 \quad (\text{as } n \gg 0).$$

And this relation holds in all  $T$  because  $T$  are symmetry in  $M_n$  structure.

Therefore, this theorem was shown.  $\square$

**Theorem 19.**  $F \in CNF : O(RCNF(F)) > O(|F|^m) \quad m : constant exists.$

*Proof.* Mentioned above 11, if  $\widehat{[c]}$  is product irreducible,  $RCNF(F)$  have antecedent variables which correspond to  $\widehat{[c]}$  connected component. And mentioned above 18,  $|\widehat{[c]}|$  exceed polynomial size of  $|F|$ . Therefore, to reduce CNF to RCNF, we cannot include within polynomial size.  $\square$

**Theorem 20.**  $CNF \not\leq_p RCNF \equiv_L HornCNF$

*Proof.* Mentioned above 10, RCNF is P-Complete. But mentioned above 19, we cannot reduce CNF to RCNF in polynomial size. Therefore,  $CNF \not\leq_p RCNF \equiv_L HornCNF$ .  $\square$

#### REFERENCES

- [1] Michael Sipser, (translation) OHTA Kazuo, TANAKA Keisuke, ABE Masayuki, UEDA Hiroki, FUJIOKA Atsushi, WATANABE Osamu, Introduction to the Theory of COMPUTATION Second Edition, 2008
- [2] HAGIYA Masami, NISHIZAKI Shinya, Mechanism of Logic and Calculation, 2007