

# Critique on Reiner Czerwinski "A Polynomial Time Algorithm for Graph Isomorphism"

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## Abstract

In the paper "A Polynomial Time Algorithm for Graph Isomorphism" we claimed, that there is a polynomial algorithm to test if two graphs are isomorphic. But the algorithm is wrong. It only tests if the adjacency matrices of two graphs have the same eigenvalues. There is a counterexample of two non-isomorphic graphs with the same eigenvalues.

## 1 Introduction

Let  $A$  the adjacency matrix of  $G$  and  $A'$  the adjacency matrix of  $G$ .  $G$  and  $G'$  are isomorphic if there is a permutation matrix  $P$  with  $A' = P * A * P^T$ . The adjacency matrices of isomorphic graphs have equal eigenvalues.

the algorithm described in [1] only tests if the graphs have the same eigenvalues. But unfortunately, there are non-isomorphic graphs with the same eigenvalue. In the next section we will show how to construct them.

## 2 Strongly Regular Graphs

Let  $G$  be a Graph.  $G \in \text{SRG}(n, k, a, c)$  if  $G$  is a  $k$  connected graph with  $n$  vertices, where adjacent vertices have  $a$  common neighbours and non-adjacent has  $c$  common neighbours. For further information see [2, chapter 10].  $G$  is strongly regular if there are non-negative numbers  $n, k, a, c$  with  $G \in \text{SRG}(n, k, a, c)$ .

**Theorem 1.** *If  $G$  and  $G'$  are in  $\text{SRG}(n, k, a, c)$  then  $G$  and  $G'$  have the same eigenvalues.*

A proof of this is shown in [2, page 219f].

### 2.1 Counterexample

There are non-isomorphic graphs with the same eigenvalues. E.g. there are 180 pairwise non-isomorphic graphs in  $\text{SRG}(36, 14, 4, 6)$  [3].

## References

- [1] Reiner Czerwinski. A polynomial time algorithm for graph isomorphism, 2008.
- [2] Chris Godsil and Gordon F Royle. *Algebraic graph theory*, volume 207. Springer Science & Business Media, 2001.
- [3] Brendan D McKay and Edward Spence. Classification of regular two-graphs on 36 and 38 vertices. *Australasian Journal of Combinatorics*, 24:293–300, 2001.