

Method of resolution of 3SAT in polynomial time

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Abstract

Presentation of a Method for determining whether a problem 3Sat has solution, and if yes to find one, in time max $O(n^{15})$. Is thus proved that the problem 3Sat is fully resolved in polynomial time and therefore that it is in P, by the work of Cook and Levin, and can transform a SAT problem in a 3Sat in polynomial time (ref. Karp), it follows that P = NP. Open Source program is available at <http://www.visainformatica.it/3sat>

Abstract (in Italiano)

Presentazione di un Metodo per determinare se un problema 3Sat ha soluzione, e se si trovarne una, che richiede un tempo non superiore a $O(n^{15})$. Viene così provato che il problema 3Sat è risolto in un tempo polinomiale e quindi che lo stesso è in P, dai lavori di Cook e Levin, e dal poter trasformare un problema SAT in uno 3Sat in un tempo polinomiale (rif. Karp), ne segue che P = NP. E' disponibile, Open Source, un programma che usa il Metodo per risolvere un problema 3Sat su sito <http://www.visainformatica.it/3sat>

My English is bad, so this work is essential. I hope my page is enough clear, I hope someone wants to rewrite in true English.

Introduction

Everything comes from intuition and coincidence

Intuition: 3Sat problem is research of True Value that making TRUE all Clauses of problem. With n Variables we find $8n^*(n-1)^*(n-2)/6$ Clauses, this Clauses are not all in 3Sat [max 7/8 are in 3Sat]. We post in ~3Sat Clauses that not is in 3Sat. We move one Clauses from ~3Sat to 3Sat, if number of solutions not decrease we leave Clause in 3Sat else no. We end if not is possible move Clause from ~3Sat to 3Sat because we lost solutions. Now we can find one solution, n-tuple of Literal.

This the intuition: minimize Clauses in ~3Sat.

Coincidence: One operation move, in polynomial time, Clauses from ~3Sat to 3Sat. When it end we have in ~3Sat Clauses and in all solution triad of True Value. **This the coincidence: number of Clauses in ~3Sat is equal number of tried of True Values, also when we not have Clauses because not have tried [not have solution].** Really funny, and I not believe in coincidence..

One example

3Sat initial [12 Clauses]: (A1 or A2 or A3) and (A1 or A2 or ~A3) and (~A1 or A2 or A3) and (~A1 or A2 or ~A3) and (~A1 or ~A2 or A3) and (~A1 or ~A2 or ~A3) and (A1 or A2 or ~A4) and (A1 or ~A2 or A4) and (~A1 or A2 or A4) and (~A1 or A2 or ~A4) and (~A1 or A3 or ~A4) and (A2 or A3 or ~A4)

~3Sat initial [20 Clauses]: (A1 or ~A2 or A3) (A1 or ~A2 or ~A3) (A1 or A2 or A4) (A1 or ~A2 or ~A4) (~A1 or ~A2 or A4) (~A1 or ~A2 or ~A4) (A1 or A3 or A4) (A1 or A3 or ~A4) (A1 or ~A3 or A4) (A1 or ~A3 or ~A4) (~A1 or A3 or A4) (~A1 or A3 or ~A4) (~A1 or ~A3 or A4) (~A1 or ~A3 or ~A4) (A2 or A3 or A4) (A2 or A3 or ~A4) (A2 or ~A3 or A4) (A2 or ~A3 or ~A4) (~A2 or A3 or A4) (~A2 or A3 or ~A4) (~A2 or ~A3 or A4) (~A2 or ~A3 or ~A4)

~3Sat reduced (insert in 3Sat all Clauses possible that not lost solutions) [7 Clauses]: (~A1 or A2 or A3) (~A1 or A2 or ~A3) (~A1 or A2 or A4) (~A1 or A3 or A4) (~A1 or ~A3 or A4) (A2 or A3 or A4) (A2 or ~A3 or A4)

3Sat after move Clauses [25 Clauses]: (A1 or A2 or A3) and (A1 or A2 or ~A3) and (A1 or ~A2 or A3) and (A1 or ~A2 or ~A3) and (~A1 or ~A2 or A3) and (~A1 or ~A2 or ~A3) and (A1 or A2 or A4) and (A1 or A2 or ~A4) and (A1 or ~A2 or A4) and (A1 or ~A2 or ~A4) and (~A1 or A2 or A4)

$\sim A_4$) and ($\sim A_1$ or $\sim A_2$ or A_4) and ($\sim A_1$ or $\sim A_2$ or $\sim A_4$) and (A_1 or A_3 or A_4) and (A_1 or A_3 or $\sim A_4$) and (A_1 or $\sim A_3$ or A_4) and ($\sim A_1$ or A_3 or $\sim A_4$) and ($\sim A_1$ or $\sim A_3$ or $\sim A_4$) and (A_2 or A_3 or $\sim A_4$) and (A_2 or $\sim A_3$ or $\sim A_4$) and ($\sim A_2$ or A_3 or A_4) and ($\sim A_2$ or A_3 or $\sim A_4$) and ($\sim A_2$ or $\sim A_3$ or A_4) and ($\sim A_2$ or $\sim A_3$ or $\sim A_4$)

Initial 3Sat with 12 Clauses and initial \sim 3Sat with 20 Clauses. After operation [of Method] \sim 3Sat have 7 Clauses and 3Sat have 25 Clauses.

Initial 3Sat with 10 Clauses and final 3Sat with 25 Clauses have 2 solutions: FTTT e FTFT, The final 3Sat is largest [with max number of Clauses] of all 3Sat with this solutions.

By FTTT we have 4 tried of True Values [index is position]: $F_1 T_2 T_3 - F_1 T_2 T_4 - F_1 T_3 T_4 - T_2 T_3 T_4$

By FTFT we have 4 tried of True Values: $F_1 T_2 F_3 - F_1 T_2 T_4 - F_1 F_3 T_4 - T_2 F_3 T_4$

Tried $F_1 T_2 T_4$ is in both, then distinct tried are 7; WHAT COINCIDENCE 7 is number of Clauses in \sim 3Sat reduced.

Now start.

Definitions

Denote by A_1, A_2, \dots, A_n Boolean Variables and by $\sim A_1, \sim A_2, \dots, \sim A_n$ their negation. Each Variable can have value "TRUE" or "FALSE", sometimes shorten the Value of Variables in "T" and "F".

We use the letters i, j, k, f, g, h, m as integer indices in the interval $[1 \dots n]$.

Denote by V_1, V_2, \dots, V_n assigning of Values to the aforementioned Variables where can be $V_i = T$ or $V_i = F$.

Denote by L_1, L_2, \dots, L_n the Literal of Variables. Each Literal can be a Variable or its negation, then $L_i = A_i$ or $L_i = \sim A_i$.

Call "Pair of Literal (L_i, L_j) " or simply "Pair (L_i, L_j) " or even more simply if there is no ambiguity "Pair" the set of 2 literals L_i, L_j with $i < j$.

We are $2n^*(2n-2)/2=2n^*(n-1)$ possible Pairs

Order by Pairs: $(L_i, L_j) < (L_k, L_h)$ if $i < k$; or $i = k, j < h$; or $i = k, j = h, L_i = A_i, L_k = \sim A_i$; or $i = k, j = h, L_i = L_k, L_j = A_j, L_h = \sim A_j$.

Is called "Clause" disjunction of 3 Literal [ex.: $(L_i \text{ or } L_j \text{ or } L_k)$]. Clause is TRUE if is TRUE at least one Literal also is FALSE. Clause is "Described Sorted" if $i < j < k$.

We call "AClausola" conjunction of 3 Literal [ex.: $(L_i \text{ and } L_j \text{ and } L_k)$]. AClausola is TRUE if are TRUE all Literal also is FALSE. AClausola is "Described Sorted" if $i < j < k$. Sometimes we write AClausola $[L_i L_j L_k]$ for save space.

Any Clause $(L_i \text{ or } L_j \text{ or } L_k)$ "Described Sorted" corresponds to AClausola "Described Sorted" $(L_i \text{ and } L_j \text{ and } L_k)$ with same Literal and vice versa.

If A_i, A_j are 2 Variables distinct $[i > j]$ then $A_i < A_j$ if $i < j$.

We call "Tried of Variables (A_i, A_j, A_k) " o simply "Tried (A_i, A_j, A_k) " o simply "Tried" the set of 3 Variables A_i, A_j, A_k with $i < j < k$.

One Tried (A_i, A_j, A_k) have 8 Clause "Described Sorted" e 8 AClausola "Described Sorted" with Literal of Variables.

We are $n^*(n-1)^*(n-2)/6$ Tried possible, then we are $8*n^*(n-1)^*(n-2)/6$ Clause "Described Sorted" and $8*n^*(n-1)^*(n-2)/6$ AClausole "Described Ordinate" possible

We call "Row of Variables (A_i, A_j, A_k) " o simply "Row (A_i, A_j, A_k) " o simply "Row" the set of 0 or more AClausola "Described Sorted" all of one Tried (A_i, A_j, A_k) .

Max number of AClausola of one Row is 8

Max number of Row is $n^*(n-1)*(n-2)/6$ (one for Tried)

If Row (Ai, Aj, Ak) contains 0 AClausola is called “empty Row”.

Order of Rows: Row (Ai, Aj, Ak) < Row (Af, Ag, Ah) if i < f, or i = f e j < g, or i = f, j = g e k < h.

We call “3Sat” problem to find solution at conjunction of more Clauses (ex.: (A1 or ~A2 or A3) and (A2 or A3 or ~A4) and ..) where the solution, if exists, if a set of True Value the makes TRUE any Clause, then makes TRUE formula. If set of True Value not exists then 3Sat not have solutions. We suppose that Clauses are “Described Sorted”

Now we see the Method for find solutions if exists

Method

We call “I3Sat” (reverse of 3Sat) set of Clauses get with substitution of any Variables with its negation (then any negation is substituted with Variable).

Create of I3Sat have time $O(n^3)$

Theorem 1

3Sat have solution IFF I3Sat have solution and any solution of one is solution other with substitution T with F and F with T

Proof

Let V1, V2, ..., Vn n-tupla of True Value that resolve 3Sat and (Li or Lj or Lk) one Clause of I3Sat. Then Clause (~Li or ~Lj or ~Lk) is in 3Sat and is TRUE for Values Vi, Vj e Vk, then Values ~Vi, ~Vj and ~Vk make TRUE corresponding Clause in I3Sat. And vice versa.

Let 3Sat and Tried (Ai, Aj, Ak) we call “Complementation” to find of AClausole “Described Sorted”, relatively Tried, that Clause “Described Sorted” corresponding NOT is in 3Sat.

We call “C3Sat” (Complementation 3Sat) set of $n^*(n-1)*(n-2)/6$ order Rows, one for any Tried, where any Row have complementary AClausole [AClausola (Li and Lj and Lk) is in Row if Clause (Li or Lj or Lk) not is in 3Sat].

Create of C3Sat have time $O(n^3)$

n-tupla of True Values V1, V2, ..., Vn solving C3Sat if make TRUE one AClausola in any Row.

Then C3Sat with one or more empty Rows NOT have solutions

If no n-tupla of True Values V1, V2, ..., Vn solve C3Sat, then C3Sat not have solutions

Theorem 2

3Sat have solution IFF C3Sat have solutions, from any solution of one we make solution of other simple exchange T with F and F with T .

Proof

Let V1, V2, ..., Vn one solution of 3Sat then extract Vi, Vj and Vk the Clause (Li or Lj or Lk) (Literal “L” equal negation of Variable “A” if Value “V” is TRUE or equal Variable “A” if Value “V” is FALSE) not is in 3Sat, then AClausola (Li and Lj and Lk) is in C3Sat and is TRUE by Values ~Vi, ~Vj e ~Vk. And vice versa.

C3Sat with less $n^*(n-1)*(n-2)/6$ AClausola not have solutions, because at least one Row is empty

We call “CI3Sat” complementation of reverse of 3Sat

Create CI3Sat have time $O(n^3)$, we create I3Sat and CI3Sat, but $O(n^3) + O(n^3) = O(n^3)$

Theorem 3

3Sat have solution IFF CI3Sat have solution and any solution of one is solution of other.

Proof

For solution of CI3Sat we have, with inversion of True Values, solution of I3sat and, with new inversion of True Values [then we have original True Values], solution of 3Sat. And vice versa

We call “Imposition Li” the elimination, in CI3Sat, all AClause with Literal $\sim L_i$. Then we leave, if they are, only solution where $L_i = \text{TRUE}$.

Theorem 4

Imposition A_i not decrease and not increase number of solution of CI3Sat where $V_i = \text{TRUE}$.

Proof

To remove AClause never increase solution.

If we have in CI3SAT solution with $V_i = \text{TRUE}$ this make TRUE one AClause for each Row, but in Rows with A_i and $\sim A_i$ make TRUE any AClause with A_i [not with $\sim A_i$]. To remove AClause with $\sim A_i$ not decrease number of solution with $A_i = \text{TRUE}$.

Theorem 5

If Imposition A_i make empty one or more Rows then we not have solution of CI3Sat with $V_i = \text{TRUE}$.

Proof

We have at least one Row of CI3Sat with AClause with only $\sim A_i$, then none set of True Values with $A_i = \text{TRUE}$ make TRUE one AClause in this Row.

Similar for Imposition $\sim A_i$

Imposition L_i have time $O(n^3)$

In not empty Row of CI3Sat we have from 3 [if only one AClause in Row] to 12 Pairs of Literal. Of the 12 possible Pairs we have some present other absent.

We call “Reduction” to find Pair of Literal (L_i, L_j) absent in one Row and the remove all AClasola with this Pair in any Rows of CI3Sat.

Any Pair of Literal (L_i, L_j) absent in Row [ex.: Row (A_i, A_j, A_k)] limit number of solution, that is any n-tupla of True Values that make TRUE formula (L_i and L_j) not is solution of CI3Sat. In fact is missing assign of Values for Row (A_i, A_j, A_k) [in Row is missing AClasola (L_i and L_j and A_k) and (L_i and L_j and $\sim A_k$)].

Then any AClause with Pair of Literal (L_i, L_j) absent in other Row can remove from CI3Sat, this non decrease solution.

We can execute Reduction more time [removed Pair we can make new Row without new Pair]. It end if

Case 1) at least one Row is empty.

Case 2) Reduction not remove more AClause.

Reduction find Pair of Literal in any Row [$O(n^3)$] and for each find AClause with this Pair in all Rows [$O(n^3)$]. If not end we make new research many times how many are AClause [$O(n^3)$]. Then work in time $O(n^9)$

If CI3Sat have empty Row we call that it is empty.

Theorem 6

Reduction not decrease and not increase number of solution of CI3Sat

Proof

Reduction only can remove AClusola, then never increase solution

All AClusola removed have one Pair [ex.: (Li, Lj)] that is absent in other Row, than not is none n-tupla of True Values that make TRUE la formula (Li and Lj) and make TRUE one AClusola in any Row. Remove this AClusole not decrease solution.

Remove AClusola is equal to move Clause from ~3Sat to 3Sat. Reduction make this, but is sure that not lost solutions

Theorem 7

After Reduction if Row have Literal then any Row with Variables of Literal have same Literal

Proof

Absurd: if not then in Row (Ai, Aj, Ak) we have AClusola (Ai and Lj and Lk) and in other Row (Ai, Af, Ag) we have only AClusole (~Ai and Lf And Lg), then is missing any AClusola (Ai And Lf And Lg) for each Literal of Af and Ag. Then is missing Pair (Ai, Lf) for each Literal of Af, then is missing any AClusola type (Ai, Lf, Lj) in Row (Ai, Af, Aj), then is missing any Pair (Ai, Lj) for any Literal of Aj in any Row, also in Row (Ai, Aj, Ak). This is absurd.

Then after Reduction if one Variable is present with both Literal, both Literal are in any Row reference this Variables

Trivial if Variables is absent in one Row refer Variable CI33Sat is empty.

We call “Saturation” this operation:

- We extract all AClusole (Li and Lj and Lk) of CI3Sat
- For each AClusola we make: Imposition Ai, Imposition Aj, Imposition Ak and Reduction [for test]. If Reduction [of test] make empty CI3Sat we delete AClusola of CI3Sat and make Reduction [ultimate], else cancel Reduction [of test] and to the next AClusola.
- We Repeat 2 step previous until CI3Sat is empty [at least one Row is empty] or we cannot delete AClusole

*Saturation work in max time $O[n^{15}]$, in fact for each AClusola ($\max O[n^3]$) we make 3 Imposition and 2 Reduction, then $\max O[3n^3] + O[2n^9] = O[n^9]$ operation and $O[n^3]*O[n^9] = O[n^{12}]$. We repeat $\max O[n^3]$ time the operation [for each AClusola] then $O[n^{15}]$.*

Theorem 8

Saturation non decrease and not increase solution of CI3Sat

Proof

Not increase because Saturation can only remove AClusola

Not decrease because any AClusola deleted is refer to True Values that cannot solve CI3Sat (Rif. Theorem 4), and Reduction not remove solution (Rif. Theorem 7)

From CI3Sat of n Variable we can extract CI3Sat(m) [con m < n] with AClusole with only m Variables .

Theorem 9

Let CI3Sat Saturated of n Variable any CI3Sat(m) [con m < n] of m Variable extract from CI3Sat is Saturated

Proof

If CI3Sat(m) not is Saturated than exist in this AClusola (L_i and L_j and L_k) that Imposition L_i , Imposition L_j , Imposition L_k and Reduction make empty [make empty one Row]. But then make empty same Row in CI3Sat, but CI3Sat is Saturated, then CI3Sat(m) is Saturated

Theorem 10

If CI3Sat Saturated is empty then not have solutions

Proof

CI3Sat have at least one Row empty, then no solutions

Theorem 11

CI3Sat Saturated and not empty have solutions

Proof (with building of solution)

Choice of positive Literal A_k is “Consistent Choice” if any AClusola (A_i and A_j and A_k) con $i < j < k$ is in CI3Sat. Positive Literal not is needful, but is great simplification, However we can always get positive Literal.

We get first AClusola in CI3Sat, let (L_1 and L_2 and L_3). If Literals are positive confirm else if someone is negative replace [in all CI3Sat] with positive [ex.: if $L_1 = \sim A_1$ put $A_1 = \sim A_1$ then L_1 equal A_1]. First choice is A_1, A_2, A_3 ; choice Consistent because AClusola (A_1 and A_2 and A_3) is in CI3Sat [with possible replace].

We Impose now A_1, A_2 and A_3 and Reduction, we have CI3Sat_new [CI3Sat is instead initial first Imposition A_1, A_2, A_3 and Reduction] not empty because CI3Sat è Saturated [remember: for each AClusola we can Impose of Literal of AClusola and Reduce].

Now in CI3Sat_new we are 2 type of Variables: those with one Literal e those with both Literals. If unique Literal of Variable with one Literal is negative we replace them [if $L_i = \sim A_i$ put $A_i = \sim A_i$] then unique Literal of Variables is positive. If index of Variable with one Literal is greater of index of Variable with 2 Literal exchange index [ex.: Variable A_i have only Literal A_i and Variable A_{i-1} have both Literal then exchange index “ i ” with index “ $i-1$ ” e vice versa]. Not is needful, but is great simplification

Then Variable with index $\leq m$ have only one positive Literal and Variable with index $> m$ have both Literal. For simply we suppose that Variable with one Literal are only A_1, A_2, A_3 . Now we choice one positive Literal from any other Variable check that choice is Consistent.

We choice A_4 . They are (A_1 and A_2 and A_4), (A_1 and A_3 and A_4) and (A_2 and A_3 and A_4) because A_1, A_2 and A_3 are present only with positive Literal and in any Row of CI3Sat_new have any Literal of A_4 , Consistent choice

From Row (A_1, A_4, A_5) we choice A_5 if AClusola (A_1 and A_4 and A_5) is present, otherwise (A_1 and A_4 and $\sim A_5$) is present and we choice $\sim A_5$; in last case we replace $\sim A_5$ with A_5 [substitution are ALWAYS make in CI3Sat_new and CI3Sat] so we get A_5 . They are (A_1 and A_2 and A_5), (A_1 and A_3 and A_5) and (A_2 and A_3 and A_5) because is present A_5 ; (A_1 and A_4 and A_5) for choice, (A_2 and A_4 and A_5) and (A_3 and A_4 and A_5) for presence of Pair (A_4, A_5) [Reduction ensure presence of one Pair of one Row in any Row of CI3Sat_new]. Consistent choice.

From Row (A_1, A_4, A_6) we choice A_6 if AClusola (A_1 and A_4 and A_6) is present and (A_1 and A_4 and $\sim A_6$) is missing; we choice $\sim A_6$ if (A_1 and A_4 and $\sim A_6$) is present and (A_1 and A_4 and A_6) is missing [but put $\sim A_6 = A_6$ so we choice A_6], if are both AClusole move to Row (A_1, A_5, A_6) [equal reasoning] if still both AClusole move to Row (A_4, A_5, A_6) [equal reasoning]. Finally if both AClusola choice A_6 .

We suppose choice from Row (A_1, A_4, A_6) [$(A_1$ and A_4 and $\sim A_6$) is missing], check Consistence: They are (A_1 and A_2 and A_6), (A_1 and A_3 and A_6) and (A_2 and A_3 and A_6) because is present A_6 ;

$(A_1 \text{ and } A_4 \text{ and } A_6)$ for choice. They are $(A_2 \text{ and } A_4 \text{ and } A_6)$ and $(A_3 \text{ and } A_4 \text{ and } A_6)$ because is present Pair (A_4, A_6)

Pair $(A_4, \sim A_6)$ is missing, then $(A_4 \text{ and } A_5 \text{ and } \sim A_6)$ is missing, but Pair (A_4, A_5) is present then $(A_4 \text{ and } A_5 \text{ and } A_6)$ is present. Then they are $(A_1 \text{ and } A_5 \text{ and } A_6)$, $(A_2 \text{ and } A_5 \text{ and } A_6)$ and $(A_3 \text{ and } A_5 \text{ and } A_6)$ [because Pair (A_5, A_6) is present]. Consistent choice.

We suppose choice from Row (A_1, A_5, A_6) [$(A_1 \text{ and } A_5 \text{ and } \sim A_6)$ is missing], check Consistence: They are $(A_1 \text{ and } A_2 \text{ and } A_6)$, $(A_1 \text{ and } A_3 \text{ and } A_6)$ and $(A_2 \text{ and } A_3 \text{ and } A_6)$ because A_6 is present. $(A_1 \text{ and } A_4 \text{ and } A_6)$ and $(A_1 \text{ and } A_5 \text{ and } A_6)$ for choice. They are $(A_2 \text{ and } A_5 \text{ and } A_6)$ and $(A_3 \text{ and } A_5 \text{ and } A_6)$ because Pair (A_5, A_6) is present

Pair $(A_5, \sim A_6)$ is missing, then $(A_4 \text{ and } A_5 \text{ and } \sim A_6)$ is missing, but Pair (A_4, A_5) is present then $(A_4 \text{ and } A_5 \text{ and } A_6)$ is present. Then $(A_2, \text{ and } A_5 \text{ and } A_6)$ and $(A_3 \text{ and } A_5 \text{ and } A_6)$ are present. Consistent choice.

If we choice from (A_4, A_5, A_6) Consistence is trivial.

Now we see the general criterion for choice neatly one Literal A_k from any Variable A_k with both Literal.

- We see Row (A_1, A_4, A_k) , if only one AClausola with Pair (A_1, A_4) get Literal L_k in this AClausola [if negative put $\sim L_k = L_k$ and get positive] if both AClausole with Pair (A_1, A_4) go to next step
- We see Row (A_1, A_5, A_k) and we stop if only one Literal L_k , otherwise we see Row (A_1, A_6, A_k) and so on to Row (A_1, A_{k-1}, A_k) , if not only one Literal go to next step
- We see Row (A_4, A_5, A_k) and we stop if only one Literal L_k , otherwise we see Row (A_4, A_6, A_k) and so on neatly to Row (A_{k-2}, A_{k-1}, A_k) , if not only one Liter go to next step
- Get A_k

If Literal L_k negative put $\sim L_k = L_k$ and get positive.

For A_7 is more complicated

From Row (A_1, A_4, A_7) or (A_1, A_5, A_7) or (A_1, A_6, A_7) or (A_4, A_5, A_7) or (A_4, A_6, A_7) or (A_5, A_6, A_7) we choice A_7 .

We suppose choice from Row (A_1, A_4, A_7) [$(A_1 \text{ and } A_4 \text{ and } \sim A_7)$ is missing] and check Consistence

They are $(A_1 \text{ and } A_2 \text{ and } A_7)$, $(A_1 \text{ and } A_3 \text{ and } A_7)$ and $(A_2 \text{ and } A_3 \text{ and } A_7)$ because A_7 is present. $(A_1 \text{ and } A_4 \text{ and } A_7)$ for choice. They are $(A_2 \text{ and } A_4 \text{ and } A_7)$ and $(A_3 \text{ and } A_4 \text{ and } A_7)$ because Pair (A_4, A_7) is present

Pair $(A_4, \sim A_7)$ is missing , then $(A_4 \text{ and } A_5 \text{ and } \sim A_7)$ and $(A_4 \text{ and } A_6 \text{ and } \sim A_7)$ is missing, but Pair (A_4, A_5) and $(A_4 A_6)$ are present [A_4, A_5 and A_6 are Consistent] then $(A_4 \text{ and } A_5 \text{ and } A_7)$ and $(A_4 \text{ and } A_6 \text{ and } A_7)$ are present. Then Pairs (A_5, A_7) and (A_6, A_7) are present and then $(A_1 \text{ and } A_5 \text{ and } A_7)$, $(A_1 \text{ and } A_6 \text{ and } A_7)$, $(A_2 \text{ and } A_5 \text{ and } A_7)$, $(A_2 \text{ and } A_6 \text{ and } A_7)$, $(A_3 \text{ and } A_5 \text{ and } A_7)$ and $(A_3 \text{ and } A_6 \text{ and } A_7)$ are present.

To proof than $(A_5 \text{ and } A_6 \text{ and } A_7)$ is present we reason for absurd. We suppose it is missing then the CI3Sat(4) for Variables A_4, A_5, A_6 and A_7 maximum is

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[A4 A5 A6] [A4 A5 ~A6] [A4 ~A5 A6] [A4 ~A5 ~A6] [~A4 A5 A6] [~A4 A5 ~A6] [~A4 ~A5 A6]
[A4 A5 A7] [A4 A5 ~A7] [A4 ~A5 A7] [A4 ~A5 ~A7] [~A4 A5 A7] [~A4 A5 ~A7] [~A4 ~A5 ~A7]
[A4 A6 A7] [A4 A6 ~A7] [A4 ~A6 A7] [A4 ~A6 ~A7] [~A4 A6 A7] [~A4 A6 ~A7] [~A4 ~A6 A7]
[~A5 A6 A7] [A5 A6 ~A7] [A5 ~A6 A7] [A5 ~A6 ~A7] [~A5 A6 A7] [~A5 A6 ~A7] [~A5 ~A6 A7]
```

In Red AClausole missing: $(A_4 \text{ and } A_5 \text{ and } \sim A_7)$ and $(A_4 \text{ and } A_6 \text{ and } \sim A_7)$ are missing because Pair $(A_4, \sim A_7)$ is missing [for choice of A_7] and $(A_5 \text{ and } A_6 \text{ and } A_7)$ is missing for hypothesis.

IMPORTANT (A_5 and A_6 and A_7) not deleted by Imposition $A_1, A_2, A_3 + Reduction$ because Pairs (A_5, A_6) , (A_5, A_7) and (A_6, A_7) are present in CI3Sat_new and then not removed this AClausola with Imposition + Reduction, then this is missing in CI3Sat Saturated. But this prevent AClausola $(A_4 \text{ and } A_5 \text{ and } A_6)$ in CI3Sat because Imposition A_4, A_5 and $A_6 + Reduction$ put

CI3Sat(4) empty and then put empty also CI3Sat. For see we Impose AClusola [in yellow] then remain AClusole [in Green]

[A4 A5 A6]	[A4 A5 ~A6]	[A4 ~A5 A6]	[A4 ~A5 ~A6]	[~A4 A5 A6]	[~A4 A5 ~A6]	[~A4 ~A5 A6]	[~A4 ~A5 ~A6]
[A4 A5 A7]	[A4 A5 ~A7]	[A4 ~A5 A7]	[A4 ~A5 ~A7]	[~A4 A5 A7]	[~A4 A5 ~A7]	[~A4 ~A5 A7]	[~A4 ~A5 ~A7]
[A4 A6 A7]	[A4 A6 ~A7]	[A4 ~A6 A7]	[A4 ~A6 ~A7]	[~A4 A6 A7]	[~A4 A6 ~A7]	[~A4 ~A6 A7]	[~A4 ~A6 ~A7]
[A5 A6 A7]	[A5 A6 ~A7]	[A5 ~A6 A7]	[A5 ~A6 ~A7]	[~A5 A6 A7]	[~A5 A6 ~A7]	[~A5 ~A6 A7]	[~A5 ~A6 ~A7]

But this CI3Sat(4) is empty after Reduction because Variable A7 not is present with equal Literal in latest 3 Rows. (Rif. Theorem 7).

AClusola (A4 and A5 and A6) is not missing because choice A6 is Consistent, then AClusola (A5 and A6 and A7) is present and choice of A7 is Consistent.

Similarly to proof Consistent for Literal A7 choice other Row.

Focus of proof is than if we suppose absence of AClusola (A5 and A6 and A7) we have absurd [absence other AClusola than is sure present] in equal mode for any other Variable, example A8

From Row (A1, A4, A8) or (A1, A5, A8) or (A1, A6, A8) or (A1, A7, A8) or (A4, A5, A8) or (A4, A6, A8) or (A4, A7, A8) or (A5, A6, A8) or (A5, A7, A8) or (A6, A7, A8) we choice A8.

We suppose choice from Row (A1, A4, A8) [(A1 and A4 and ~A8) is missing] and check Consistence

They are (A1 and A2 and A8), (A1 and A3 and A8) and (A2 and A3 and A8) because A8 is present. (A1 and A4 and A8) for choice. They are (A2 and A4 and A8) and (A3 and A4 and A8) because Pair (A4, A8) is present

Pair (A4, ~A8) is missing, then (A4 and A5 and ~A8), (A4 and A6 and ~A8) and (A4 and A7 and ~A8) are missing, but Pairs (A4, A5), (A4, A6) and (A4, A7) are present then (A4 and A5 and A8), (A4 and A6 and A8) and (A4 and A7 and A8) are present. Then Pairs (A5, A8), (A6, A8) and (A7, A8) are present, then (A1 and A5 and A8), (A1 and A6 and A8), (A1 and A7 and A8), (A2 and A5 and A8), (A2 and A6 and A8), (A2 and A7 and A8), (A3 and A5 and A8), (A3 and A6 and A8) and (A3 and A7 and A8) are present.

To proof than (A5 and A6 and A8) is present we reason for absurd. We suppose it is missing then the CI3Sat(4) for Variables A4, A5, A6 and A8 maximum is

[A4 A5 A6]	[A4 A5 ~A6]	[A4 ~A5 A6]	[A4 ~A5 ~A6]	[~A4 A5 A6]	[~A4 A5 ~A6]	[~A4 ~A5 A6]	[~A4 ~A5 ~A6]
[A4 A5 A8]	[A4 A5 ~A8]	[A4 ~A5 A8]	[A4 ~A5 ~A8]	[~A4 A5 A8]	[~A4 A5 ~A8]	[~A4 ~A5 A8]	[~A4 ~A5 ~A8]
[A4 A6 A8]	[A4 A6 ~A8]	[A4 ~A6 A8]	[A4 ~A6 ~A8]	[~A4 A6 A8]	[~A4 A6 ~A8]	[~A4 ~A6 A8]	[~A4 ~A6 ~A8]
[A5 A6 A8]	[A5 A6 ~A8]	[A5 ~A6 A8]	[A5 ~A6 ~A8]	[~A5 A6 A8]	[~A5 A6 ~A8]	[~A5 ~A6 A8]	[~A5 ~A6 ~A8]

In Red AClusole missing: (A4 and A5 and ~A8) and (A4 and A6 and ~A8) are missing because Pair (A4, ~A8) is missing [for choice of A8] and (A5 and A6 and A8) is missing for hypothesis.

IMPORTANT (A5 and A6 and A8) not deleted by Imposition A1, A2, A3 + Reduction because Pairs (A5, A6), (A5, A8) and (A6, A8) are present in CI3Sat_new and then not removed this AClusola with Imposition + Reduction, then this is missing in CI3Sat Saturated. But this prevent AClusola (A4 and A5 and A6) in CI3Sat because Imposition A4, A5 and A6 + Reduction put CI3Sat(4) empty and then put empty also CI3Sat. For see we Impose AClusola [in yellow] then remain AClusole [in Green]

[A4 A5 A6]	[A4 A5 ~A6]	[A4 ~A5 A6]	[A4 ~A5 ~A6]	[~A4 A5 A6]	[~A4 A5 ~A6]	[~A4 ~A5 A6]	[~A4 ~A5 ~A6]
[A4 A5 A8]	[A4 A5 ~A8]	[A4 ~A5 A8]	[A4 ~A5 ~A8]	[~A4 A5 A8]	[~A4 A5 ~A8]	[~A4 ~A5 A8]	[~A4 ~A5 ~A8]
[A4 A6 A8]	[A4 A6 ~A8]	[A4 ~A6 A8]	[A4 ~A6 ~A8]	[~A4 A6 A8]	[~A4 A6 ~A8]	[~A4 ~A6 A8]	[~A4 ~A6 ~A8]
[A5 A6 A8]	[A5 A6 ~A8]	[A5 ~A6 A8]	[A5 ~A6 ~A8]	[~A5 A6 A8]	[~A5 A6 ~A8]	[~A5 ~A6 A8]	[~A5 ~A6 ~A8]

But this CI3Sat(4) is empty after Reduction because Variable A8 not is present with equal Literal in latest 3 Rows. (Rif. Theorem 7).

AClusola (A4 and A5 and A6) is not missing because choice A6 is Consistent, then AClusola (A5 and A6 and A8) is present and choice of A8 is Consistent.

Similar to proof than $(A_5 \text{ and } A_7 \text{ and } A_8)$ is present [we suppose is missing and to proof than $(A_4 \text{ and } A_5 \text{ and } A_7)$ is absent] and $(A_6 \text{ and } A_7 \text{ and } A_8)$ [we suppose is missing and to proof than $(A_4 \text{ and } A_6 \text{ and } A_7)$ is absent]. Then choice of A_8 is Consistent.

Similar for A_9, A_{10}, \dots, A_n . Then CI3Sat_new [with substituting] have solution “All TRUE” because any AClausola $(A_i \text{ and } A_j \text{ and } A_k)$ with $i < j < k \leq n$ is present.

Now we remember substitution negative Literal with positive and substituting in n-tuple TRUE corresponding with FALSE. This is solution of initial CI3Sat and initial 3Sat.

In this proof number of Variable with only one Literal is insignificant [in proof we use only A_1 and easy we proof Consistence for any Variable with only one Literal]. The theorem is proof in any case.

Corollary 11.1

CI3Sat Saturated have at least one solution than make TRUE any AClausola contains

Proof

Result of Theorem 11. We get AClausola $(L_i \text{ and } L_j \text{ and } L_k)$, we impose $L_i, L_j \text{ e } L_k$, we reduce and we find, for building, solution

Group Clause of Solutions

If 3Sat have solution (V_1, V_2, \dots, V_n) then we get $n^*(n-1)^*(n-2)/6$ Clause that are identically FALSE [ex: $V_i = \text{TRUE}$ for any i , then any Clause $(\sim A_i, \sim A_j, \sim A_k)$ is FALSE]. We call $GCS_F(V_1, V_2, \dots, V_n)$ or simply GCS_F set of this Clause

Corollary 11.2

3Sat have solution IFF exists GCS_F entirely container in ~3Sat

Proof

If 3Sat have solution (V_1, V_2, \dots, V_n) then $GCS_F(V_1, V_2, \dots, V_n)$ is entirely in $\sim 3Sat$. If $GCS_F(V_1, V_2, \dots, V_n)$ is entirely in $\sim 3Sat$ then (V_1, V_2, \dots, V_n) is solution of 3Sat.

CI3Sat is equivalent to $\sim 3Sat$. Then redefine GCS_F like set of AClausole identically TRUE for any solution V_1, V_2, \dots, V_n . We call $GCS(V_1, V_2, \dots, V_n)$ or simply with GCS

Corollary 11.3

CI3Sat ha solution IFF exist one GCS entirely container in CI3Sat

Proof

Like previous case

Corollary 11.4

CI3Sat Saturated container only AClausole of one GCS entirely container in CI3Sat

Proof

For any AClausola we find one solution that put it TRUE, then exists at least one GCS container in CI3sat than container AClausola..

Corollary 11.5

The complement of CI3Sat Saturated is largest 3Sat with all solution of initial 3Sat

Proof

If we remove one AClausola from CI3Sat [we add one Clause to 3Sat] then at least one GCS first container now will not container, then we lost solution

Corollary 11.6

In CI3Sat Saturated number of tried of True Values of all solution is equal number of AClausola

Proof

For each AClausola is one tried of True Values because we can building at least one solution

For each tried of True Values in all solution is at least one solution [trivial], then al least one GCS is entirely container in CI3Sat and this container AClausola corresponding at tried

If CI3Sat is empty not have solution [zero AClausole zero tried]

WAS NOT A COINCIDENCE!

Theorem 12

CI3Sat Saturated have solution IFF not is empty

Proof

Theorem 10 + Theorem 11

Referenze

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