# Contents

# 1 Reminder

## 1.1 Bug List

- 沒開 long long
- 陣列戳出界／陣列開不夠大
- 寫好的函式忘記呼叫
- 變數打錯
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- <= 打成 <+
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case
- 線段樹改值懶標初始值不能設為 0
- DFS 的時候不小心覆寫到全域變數

## 1.2 OwO

- Enjoy The Game!

# 2 Basic

## 2.1 Default

```cpp
#include <bits/stdc++.h>

using namespace std;
using ll = long long;
using pii = pair<int, int>;
using pll = pair<ll, ll>;

#define endl '\n'

#define F first
#define S second
#define ep emplace
#define pb push_back
#define eb emplace_back
#define ALL(x) x.begin(), x.end()
#define SZ(x) (int)x.size()

namespace{
const int INF = 0x3f3f3f3f;
const ll LINF = 0x3f3f3f3f3f3f3f3f;

template<typename T> using V=vector<T>;
template<typename T1,typename T2=T1> using P = pair<T1, T2>;

void _debug() {}
template<typename A,typename... B> void _debug(A a,B... b){
    cerr<<a<<' ',_debug(b...);
}
#define debug(...) cerr<<#__VA_ARGS__<<": ",_debug(__VA_ARGS__),cerr<<endl;
template<typename T>
ostream& operator<<(ostream& os,const vector<T>& v){
    for(const auto& i:v)
        os<<i<<' ';
    return os;
}
}
/*-----------------------------------------------------*/

const ll MOD = 1e9 + 7;
const int maxn = 2e5 + 5;


void init() {
    ;
}

void solve() {
    ;
}

/*


*/

signed main() {
    cin.tie(0), ios::sync_with_stdio(0);

int T = 1;
// cin >> T;
while (T--) {
    init();
    solve();
}

    return 0;
}
```

## 2.2 Vimrc

```vim
syn on
se ai nu rnu ru cul mouse=a
se cin et ts=4 sw=4 sts=4
colo desert
set autochdir
no <F5> :!./a.out<CR>
no <F9> :!~/run.sh %:p:h %:p:t<CR>
```

## 2.3 Run.sh

```bash
clear
echo File Location: $1
echo File Name: $2
echo ============================================
echo Start compiling \"$2\"...
echo
g++ $1/$2 -std=c++20 -Ofast -Wall -Wextra -g -fsanitize=address,undefined -o$1/a.out
if [ "$?" -ne 0 ]
then
        exit 1
fi
echo
echo Done compiling...
echo ============================================
echo Input file:
echo --------------------------------------------
cat $1/input
echo ============================================
declare startTime=`date +%s%N`
$1/a.out < $1/input > $1/output
declare endTime=`date +%s%N`
delta=`expr $endTime - $startTime`
delta=`expr $delta / 1000000`
echo "Program ended in $delta ms with the return value $?"
cat $1/output
```

## 2.4 Stress

```bash
g++ gen.cpp -o gen.out
g++ ac.cpp -o ac.out
g++ wa.cpp -o wa.out
for ((i=0;;i++))
do
    echo "$i"
    ./gen.out > in.txt
    ./ac.out < in.txt > ac.txt
    ./wa.out < in.txt > wa.txt
    diff ac.txt wa.txt || break
done
```

## 2.5 PBDS

```cpp
#include <bits/extc++.h>
using namespace __gnu_pbds;

// map
tree<int, int, less<>, rb_tree_tag, tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// set
tree<int, null_type, less<>, rb_tree_tag, tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// priority queue
__gnu_pbds::priority_queue<int, less<int> > big_q;  // Big First
__gnu_pbds::priority_queue<int, greater<int> > small_q; // Small First
q1.join(q2);  // join
```

## 2.6 Random

```cpp
mt19937 gen(chrono::steady_clock::now().time_since_epoch().count());
uniform_int_distribution<int> dis(1, 100);
cout << dis(gen) << endl;
shuffle(v.begin(), v.end(), gen);
```

# 3 Python

## 3.1 I/O

```python
import sys
input = sys.stdin.readline

# Input
def readInt():
    return int(input())
def readList():
    return list(map(int,input().split()))
def readStr():
    s = input()
    return list(s[:len(s) - 1])
def readVars():
    return map(int,input().split())

# Output
sys.stdout.write(string)

# faster
def main():
    pass
main()
```

## 3.2 Decimal

```python
from decimal import *
getcontext().prec = 2500000
getcontext().Emax = 2500000
a,b = Decimal(input()),Decimal(input())
a*=b
print(a)
```

# 4 Data Structure

## 4.1 Heavy Light Decomposition

```cpp
constexpr int maxn=2e5+5;
int arr[(maxn+1)<<2];
#define m ((l+r)>>1)
void build(V<int>& v,int i=1,int l=0,int r=maxn){
    if((int)v.size()<=l) return;
    if(r-l==1){arr[i]=v[l];return;}
    build(v,i<<1,l,m),build(v,i<<1|1,m,r);
    arr[i]=max(arr[i<<1],arr[i<<1|1]);
}
```

```
void modify(int p,int k,int i=1,int l=0,int r=maxn){
    if(p<l||r<=p) return;
    if(r-l==1){arr[i]=k;return;}
    if(p<m) modify(p,k,i<<1,l,m);
    else modify(p,k,i<<1|1,m,r);
    arr[i]=max(arr[i<<1],arr[i<<1|1]);
}
int query(int ql,int qr,int i=1,int l=0,int r=maxn){
    if(qr<=l||r<=ql) return 0;
    if(ql<=l&&r<=qr) return arr[i];
    if(qr<=m) return query(ql,qr,i<<1,l,m);
    if(m<=ql) return query(ql,qr,i<<1|1,m,r);
    return max(query(ql,qr,i<<1,l,m),query(ql,qr,i<<1|1,m,r));
}
#undef m
inline void solve(){
    int n,q;cin>>n>>q;
    V<int> v(n);
    for(auto& i:v)
        cin>>i;
    V<V<int>> e(n);
    for(int i=1;i<n;i++){
        int a,b;cin>>a>>b,a--,b--;
        e[a].emplace_back(b);
        e[b].emplace_back(a);
    }
    V<int> d(n,0),f(n,0),sz(n,1),son(n,-1);
    F<void(int,int)> dfs1=
    [&](int x,int pre){
        for(auto i:e[x]) if(i!=pre){
            d[i]=d[x]+1,f[i]=x;
            dfs1(i,x),sz[x]+=sz[i];
            if(!~son[x]||sz[son[x]]<sz[i])
                son[x]=i;
        }
    };dfs1(0,0);
    V<int> top(n,0),dfn(n,-1),rnk(n,0);
    F<void(int,int)> dfs2=
    [&](int x,int t){
        static int cnt=0;
        dfn[x]=cnt++,rnk[dfn[x]]=x,top[x]=t;
        if(!~son[x]) return;
        dfs2(son[x],t);
        for(auto i:e[x])
            if(!~dfn[i]) dfs2(i,i);
    };dfs2(0,0);
    V<int> dfnv(n);
    for(int i=0;i<n;i++)
        dfnv[dfn[i]]=v[i];
    build(dfnv);
    while(q--){
        int op,a,b;cin>>op>>a>>b;
        switch(op){
        case 1:{
            modify(dfn[a-1],b);
        }break;
        case 2:{
            a--,b--;
            int ans=0;
            while(top[a]!=top[b]){
                if(d[top[a]]>d[top[b]]) swap(a,b);
                ans=max(ans,query(dfn[top[b]],dfn[b]+1));
                b=f[top[b]];
            }
            if(dfn[a]>dfn[b]) swap(a,b);
            ans=max(ans,query(dfn[a],dfn[b]+1));
            cout<<ans<<endl;
        }break;
        }
    }
}
```

## 4.2 Skew Heap

```
struct node{
    node *l,*r;
    int v;
    node(int x):v(x){
        l=r=nullptr;
    }
};
node* merge(node* a,node* b){
    if(!a||!b) return a?:b;
//  min heap
    if(a->v>b->v) swap(a,b);
    a->r=merge(a->r,b);
    swap(a->l,a->r);
    return a;
}
```

## 4.3 Leftist Heap

```
struct node{
    node *l,*r;
    int d, v;
    node(int x):d(1),v(x){
        l=r=nullptr;
    }
};
static inline int d(node* x){return x?x->d:0;}
node* merge(node* a,node* b){
    if(!a||!b) return a?:b;
//  min heap
    if(a->v>b->v) swap(a,b);
    a->r=merge(a->r,b);
    if(d(a->l)<d(a->r))
        swap(a->l,a->r);
    a->d=d(a->r)+1;
    return a;
}
```

## 4.4 Persistent Treap

```
struct node {
  node *l, *r;
  char c; int v, sz;
  node(char x = '$'): c(x), v(mt()), sz(1) {
    l = r = nullptr;
  }
  node(node* p) {*this = *p;}
  void pull() {
    sz = 1;
    for (auto i : {l, r})
      if (i) sz += i->sz;
  }
} arr[maxn], *ptr = arr;
inline int size(node* p) {return p ? p->sz : 0;}
node* merge(node* a, node* b) {
  if (!a || !b) return a ? : b;
  if (a->v < b->v) {
    node* ret = new(ptr++) node(a);
    ret->r = merge(ret->r, b), ret->pull();
    return ret;
  }
  else {
    node* ret = new(ptr++) node(b);
    ret->l = merge(a, ret->l), ret->pull();
    return ret;
  }
}
P<node*> split(node* p, int k) {
  if (!p) return {nullptr, nullptr};
  if (k >= size(p->l) + 1) {
    auto [a, b] = split(p->r, k - size(p->l) - 1);
    node* ret = new(ptr++) node(p);
    ret->r = a, ret->pull();
    return {ret, b};
  }
  else {
    auto [a, b] = split(p->l, k);
    node* ret = new(ptr++) node(p);
    ret->l = b, ret->pull();
    return {a, ret};
  }
}
```

## 4.5 Li Chao Tree

```
constexpr int maxn = 5e4 + 5;
struct line {
  ld a, b;
  ld operator()(ld x) {return a * x + b;}
} arr[(maxn + 1) << 2];
bool operator<(line a, line b) {return a.a < b.a;}
#define m ((l+r)>>1)
void insert(line x, int i = 1, int l = 0, int r = maxn) {
  if (r - l == 1) {
    if (x(l) > arr[i](l))
      arr[i] = x;
    return;
  }
  line a = max(arr[i], x), b = min(arr[i], x);
  if (a(m) > b(m))
    arr[i] = a, insert(b, i << 1, l, m);
  else
    arr[i] = b, insert(a, i << 1 | 1, m, r);
}
ld query(int x, int i = 1, int l = 0, int r = maxn) {
  if (x < l || r <= x) return -numeric_limits<ld>::max();
  if (r - l == 1) return arr[i](x);
  return max({arr[i](x), query(x, i << 1, l, m), query(x, i << 1 | 1, m, r)});
}
#undef m
```

# 5 DP

## 5.1 Aliens

```
int n; ll k;
vector<ll> a;
vector<pll> dp[2];
void init() {
  cin >> n >> k;
  Each(i, dp) i.clear(), i.resize(n);
  a.clear(); a.resize(n);
  Each(i, a) cin >> i;
}
pll calc(ll p) {
  dp[0][0] = mp(0, 0);
  dp[1][0] = mp(-a[0], 0);
  FOR(i, 1, n, 1) {
    if (dp[0][i-1].F > dp[1][i-1].F + a[i] - p) {
      dp[0][i] = dp[0][i-1];
    } else if (dp[0][i-1].F < dp[1][i-1].F + a[i] - p) {
      dp[0][i] = mp(dp[1][i-1].F + a[i] - p, dp[1][i-1].S+1);
    } else {
      dp[0][i] = mp(dp[0][i-1].F, min(dp[0][i-1].S, dp[1][i-1].S+1));
    }
    if (dp[0][i-1].F - a[i] > dp[1][i-1].F) {
      dp[1][i] = mp(dp[0][i-1].F - a[i], dp[0][i-1].S);
    } else if (dp[0][i-1].F - a[i] < dp[1][i-1].F) {
      dp[1][i] = dp[1][i-1];
    } else {
      dp[1][i] = mp(dp[1][i-1].F, min(dp[0][i-1].S, dp[1][i-1].S));
    }
  }
  return dp[0][n-1];
}
void solve() {
  ll l = 0, r = 1e7;
  pll res = calc(0);
  if (res.S <= k) return cout << res.F << endl, void();
  while (l < r) {
    ll mid = (l+r)>>1;
    res = calc(mid);
    if (res.S <= k) r = mid;
    else l = mid+1;
  }
  res = calc(l);
  cout << res.F + k*l << endl;
}
```

# 6 Graph

## 6.1 Bellman-Ford + SPFA

```
int n, m;

// Graph
vector<vector<pair<int, ll> > > g;
vector<ll> dis;
vector<bool> negCycle;

// SPFA
vector<int> rlx;
queue<int> q;
vector<bool> inq;
vector<int> pa;
void SPFA(vector<int>& src) {
    dis.assign(n+1, LINF);
    negCycle.assign(n+1, false);
    rlx.assign(n+1, 0);
    while (!q.empty()) q.pop();
    inq.assign(n+1, false);
    pa.assign(n+1, -1);

    for (auto& s : src) {
        dis[s] = 0;
        q.push(s); inq[s] = true;
    }

    while (!q.empty()) {
        int u = q.front();
        q.pop(); inq[u] = false;
        if (rlx[u] >= n) {
            negCycle[u] = true;
        }
        else for (auto& e : g[u]) {
            int v = e.first;
            ll w = e.second;
            if (dis[v] > dis[u] + w) {
                dis[v] = dis[u] + w;
                rlx[v] = rlx[u] + 1;
                pa[v] = u;
                if (!inq[v]) {
                    q.push(v);
                    inq[v] = true;
} } } } }


// Bellman-Ford
queue<int> q;
vector<int> pa;
void BellmanFord(vector<int>& src) {
    dis.assign(n+1, LINF);
    negCycle.assign(n+1, false);
    pa.assign(n+1, -1);

    for (auto& s : src) dis[s] = 0;

    for (int rlx = 1; rlx <= n; rlx++) {
        for (int u = 1; u <= n; u++) {
            if (dis[u] == LINF) continue;  // Important !!
            for (auto& e : g[u]) {
                int v = e.first; ll w = e.second;
                if (dis[v] > dis[u] + w) {
                    dis[v] = dis[u] + w;
                    pa[v] = u;
                    if (rlx == n) negCycle[v] = true;
} } } } }


// Negative Cycle Detection
void NegCycleDetect() {
/* No Neg Cycle: NO
Exist Any Neg Cycle:
YES
v0 v1 v2 ... vk v0 */

    vector<int> src;
    for (int i = 1; i <= n; i++)
        src.emplace_back(i);
```

```cpp

    SPFA(src);
    // BellmanFord(src);

    int ptr = -1;
    for (int i = 1; i <= n; i++) if (negCycle[i])
        { ptr = i; break; }

    if (ptr == -1) { return cout << "NO" << endl, void(); }

    cout << "YES\n";
    vector<int> ans;
    vector<bool> vis(n+1, false);

    while (true) {
        ans.emplace_back(ptr);
        if (vis[ptr]) break;
        vis[ptr] = true;
        ptr = pa[ptr];
    }
    reverse(ans.begin(), ans.end());

    vis.assign(n+1, false);
    for (auto& x : ans) {
        cout << x << ' ';
        if (vis[x]) break;
        vis[x] = true;
    }
    cout << endl;
}

// Distance Calculation
void calcDis(int s) {
    vector<int> src;
    src.emplace_back(s);
    SPFA(src);
    // BellmanFord(src);

    while (!q.empty()) q.pop();
    for (int i = 1; i <= n; i++)
        if (negCycle[i]) q.push(i);

    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (auto& e : g[u]) {
            int v = e.first;
            if (!negCycle[v]) {
                q.push(v);
                negCycle[v] = true;
} } } }
```

## 6.2 BCC - AP

```cpp
int n, m;
int low[maxn], dfn[maxn], instp;
vector<int> E, g[maxn];
bitset<maxn> isap;
bitset<maxm> vis;
stack<int> stk;
int bccnt;
vector<int> bcc[maxn];
inline void popout(int u) {
  bccnt++;
  bcc[bccnt].emplace_back(u);
  while (!stk.empty()) {
    int v = stk.top();
    if (u == v) break;
    stk.pop();
    bcc[bccnt].emplace_back(v);
  }
}
void dfs(int u, bool rt = 0) {
  stk.push(u);
  low[u] = dfn[u] = ++instp;
  int kid = 0;
  Each(e, g[u]) {
    if (vis[e]) continue;
    vis[e] = true;
    int v = E[e]^u;
    if (!dfn[v]) {
      // tree edge
      kid++; dfs(v);
      low[u] = min(low[u], low[v]);
      if (!rt && low[v] >= dfn[u]) {
        // bcc found: u is ap
        isap[u] = true;
        popout(u);
      }
    } else {
      // back edge
      low[u] = min(low[u], dfn[v]);
    }
  }
  // special case: root
  if (rt) {
    if (kid > 1) isap[u] = true;
    popout(u);
  }
}
void init() {
  cin >> n >> m;
  fill(low, low+maxn, INF);
  REP(i, m) {
    int u, v;
    cin >> u >> v;
    g[u].emplace_back(i);
    g[v].emplace_back(i);
    E.emplace_back(u^v);
  }
}
void solve() {
  FOR(i, 1, n+1, 1) {
    if (!dfn[i]) dfs(i, true);
  }
  vector<int> ans;
  int cnt = 0;
  FOR(i, 1, n+1, 1) {
    if (isap[i]) cnt++, ans.emplace_back(i);
  }
  cout << cnt << endl;
  Each(i, ans) cout << i << ' ';
  cout << endl;
}
```

## 6.3 BCC - Bridge

```cpp
int n, m;
vector<int> g[maxn], E;
int low[maxn], dfn[maxn], instp;
int bccnt, bccid[maxn];
stack<int> stk;
bitset<maxm> vis, isbrg;
void init() {
  cin >> n >> m;
  REP(i, m) {
    int u, v;
    cin >> u >> v;
    E.emplace_back(u^v);
    g[u].emplace_back(i);
    g[v].emplace_back(i);
  }
  fill(low, low+maxn, INF);
}
void popout(int u) {
  bccnt++;
  while (!stk.empty()) {
    int v = stk.top();
    if (v == u) break;
    stk.pop();
    bccid[v] = bccnt;
  }
}
void dfs(int u) {
  stk.push(u);
  low[u] = dfn[u] = ++instp;

  Each(e, g[u]) {
    if (vis[e]) continue;
    vis[e] = true;

    int v = E[e]^u;
    if (dfn[v]) {
      // back edge
      low[u] = min(low[u], dfn[v]);
    } else {
      // tree edge
      dfs(v);
      low[u] = min(low[u], low[v]);
      if (low[v] == dfn[v]) {
        isbrg[e] = true;
        popout(u);
      }
    }
  }
}
void solve() {
  FOR(i, 1, n+1, 1) {
    if (!dfn[i]) dfs(i);
  }
  vector<pii> ans;
  vis.reset();
  FOR(u, 1, n+1, 1) {
    Each(e, g[u]) {
      if (!isbrg[e] || vis[e]) continue;
      vis[e] = true;
      int v = E[e]^u;
      ans.emplace_back(mp(u, v));
    }
  }
  cout << (int)ans.size() << endl;
  Each(e, ans) cout << e.F << ' ' << e.S << endl;
}
```

## 6.4 SCC - Tarjan

```
// 2-SAT
vector<int> E, g[maxn];  // 1~n, n+1~2n
int low[maxn], in[maxn], instp;
int sccnt, sccid[maxn];

stack<int> stk;
bitset<maxn> ins, vis;

int n, m;

void init() {
    cin >> m >> n;
    E.clear();
    fill(g, g+maxn, vector<int>());
    fill(low, low+maxn, INF);
    memset(in, 0, sizeof(in));
    instp = 1;
    sccnt = 0;
    memset(sccid, 0, sizeof(sccid));
    ins.reset();
    vis.reset();
}

inline int no(int u) {
    return (u > n ? u-n : u+n);
}

int ecnt = 0;
inline void clause(int u, int v) {
    E.eb(no(u)^v);
    g[no(u)].eb(ecnt++);
    E.eb(no(v)^u);
    g[no(v)].eb(ecnt++);
}

void dfs(int u) {
    in[u] = instp++;
    low[u] = in[u];
    stk.push(u);
    ins[u] = true;

    Each(e, g[u]) {
        if (vis[e]) continue;
        vis[e] = true;

        int v = E[e]^u;
        if (ins[v]) low[u] = min(low[u], in[v]);
        else if (!in[v]) {
            dfs(v);
            low[u] = min(low[u], low[v]);
        }
    }

    if (low[u] == in[u]) {
        sccnt++;
        while (!stk.empty()) {
            int v = stk.top();
            stk.pop();
            ins[v] = false;
            sccid[v] = sccnt;
            if (u == v) break;
        }
    }
}


int main() {
    WiwiHorz
    init();

    REP(i, m) {
        char su, sv;
        int u, v;
        cin >> su >> u >> sv >> v;
        if (su == '-') u = no(u);
        if (sv == '-') v = no(v);
        clause(u, v);
    }

    FOR(i, 1, 2*n+1, 1) {
        if (!in[i]) dfs(i);
    }

    FOR(u, 1, n+1, 1) {
        int du = no(u);
        if (sccid[u] == sccid[du]) {
            return cout << "IMPOSSIBLE\n", 0;
        }
    }

    FOR(u, 1, n+1, 1) {
        int du = no(u);
        cout << (sccid[u] < sccid[du] ? '+' : '-') << ' ';
    }
    cout << endl;

    return 0;
}
```

## 6.5 Eulerian Path - Undir

```
// from 1 to n
#define gg return cout << "IMPOSSIBLE\n", void();

int n, m;
vector<int> g[maxn];
bitset<maxn> inodd;

void init() {
cin >> n >> m;
inodd.reset();
for (int i = 0; i < m; i++) {
  int u, v; cin >> u >> v;
  inodd[u] = inodd[u] ^ true;
  inodd[v] = inodd[v] ^ true;
  g[u].emplace_back(v);
  g[v].emplace_back(u);
} }
stack<int> stk;
void dfs(int u) {
    while (!g[u].empty()) {
        int v = g[u].back();
        g[u].pop_back();
        dfs(v);
    }
stk.push(u);}
```

## 6.6 Eulerian Path - Dir

```
// from node 1 to node n
#define gg return cout << "IMPOSSIBLE\n", 0

int n, m;
vector<int> g[maxn];
stack<int> stk;
int in[maxn], out[maxn];

void init() {
cin >> n >> m;
for (int i = 0; i < m; i++) {
  int u, v; cin >> u >> v;
  g[u].emplace_back(v);
  out[u]++, in[v]++;
}
for (int i = 1; i <= n; i++) {
  if (i == 1 && out[i]-in[i] != 1) gg;
  if (i == n && in[i]-out[i] != 1) gg;
  if (i != 1 && i != n && in[i] != out[i]) gg;
} }
void dfs(int u) {
    while (!g[u].empty()) {
        int v = g[u].back();
        g[u].pop_back();
        dfs(v);
    }
    stk.push(u);
}
void solve() {
  dfs(1)
    for (int i = 1; i <= n; i++)
        if ((int)g[i].size()) gg;
    while (!stk.empty()) {
        int u = stk.top();
        stk.pop();
        cout << u << ' ';
} }
```

## 6.7 Hamilton Path

```
// top down DP
// Be Aware Of Multiple Edges
int n, m;
ll dp[maxn][1<<maxn];
int adj[maxn][maxn];

void init() {
    cin >> n >> m;
    fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
}

void DP(int i, int msk) {
    if (dp[i][msk] != -1) return;
    dp[i][msk] = 0;
    REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i]) {
        int sub = msk ^ (1<<i);
        if (dp[j][sub] == -1) DP(j, sub);
        dp[i][msk] += dp[j][sub] * adj[j][i];
        if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
    }
}


int main() {
    WiwiHorz
    init();

    REP(i, m) {
        int u, v;
        cin >> u >> v;
        if (u == v) continue;
        adj[--u][--v]++;
    }

    dp[0][1] = 1;
    FOR(i, 1, n, 1) {
        dp[i][1] = 0;
        dp[i][1|(1<<i)] = adj[0][i];
    }
    FOR(msk, 1, (1<<n), 1) {
        if (msk == 1) continue;
        dp[0][msk] = 0;
    }


    DP(n-1, (1<<n)-1);
    cout << dp[n-1][(1<<n)-1] << endl;

    return 0;
}
```

## 6.8 Kth Shortest Path

```
// time: O(|E| \lg |E|+|V| \lg |V|+K)
// memory: O(|E| \lg |E|+|V|)
struct KSP{ // 1-base
  struct nd{
    int u,v; ll d;
    nd(int ui=0,int vi=0,ll di=INF){ u=ui; v=vi; d=di; }
  };
  struct heap{ nd* edge; int dep; heap* chd[4]; };
  static int cmp(heap* a,heap* b)
  { return a->edge->d > b->edge->d; }
  struct node{
    int v; ll d; heap* H; nd* E;
    node(){}
    node(ll _d,int _v,nd* _E){ d =_d; v=_v; E=_E; }
    node(heap* _H,ll _d){ H=_H; d=_d; }
    friend bool operator<(node a,node b)
    { return a.d>b.d; }
  };
  int n,k,s,t,dst[N]; nd *nxt[N];
  vector<nd*> g[N],rg[N]; heap *nullNd,*head[N];
  void init(int _n,int _k,int _s,int _t){
    n=_n; k=_k; s=_s; t=_t;
    for(int i=1;i<=n;i++){
      g[i].clear(); rg[i].clear();
      nxt[i]=NULL; head[i]=NULL; dst[i]=-1;
    }
  }
  void addEdge(int ui,int vi,ll di){
    nd* e=new nd(ui,vi,di);
    g[ui].push_back(e); rg[vi].push_back(e);
  }
  queue<int> dfsQ;
  void dijkstra(){
    while(dfsQ.size()) dfsQ.pop();
    priority_queue<node> Q; Q.push(node(0,t,NULL));
    while (!Q.empty()){
      node p=Q.top(); Q.pop(); if(dst[p.v]!=-1)continue;
      dst[p.v]=p.d; nxt[p.v]=p.E; dfsQ.push(p.v);
      for(auto e:rg[p.v]) Q.push(node(p.d+e->d,e->u,e));
    }
  }
  heap* merge(heap* curNd,heap* newNd){
    if(curNd==nullNd) return newNd;
    heap* root=new heap;memcpy(root,curNd,sizeof(heap));
    if(newNd->edge->d<curNd->edge->d){
      root->edge=newNd->edge;
      root->chd[2]=newNd->chd[2];
      root->chd[3]=newNd->chd[3];
      newNd->edge=curNd->edge;
      newNd->chd[2]=curNd->chd[2];
      newNd->chd[3]=curNd->chd[3];
    }
    if(root->chd[0]->dep<root->chd[1]->dep)
      root->chd[0]=merge(root->chd[0],newNd);
    else root->chd[1]=merge(root->chd[1],newNd);
    root->dep=max(root->chd[0]->dep,
              root->chd[1]->dep)+1;
    return root;
  }
  vector<heap*> V;
  void build(){
    nullNd=new heap; nullNd->dep=0; nullNd->edge=new nd;
    fill(nullNd->chd,nullNd->chd+4,nullNd);
    while(not dfsQ.empty()){
      int u=dfsQ.front(); dfsQ.pop();
      if(!nxt[u]) head[u]=nullNd;
      else head[u]=head[nxt[u]->v];
      V.clear();
      for(auto&& e:g[u]){
        int v=e->v;
        if(dst[v]==-1) continue;
        e->d+=dst[v]-dst[u];
        if(nxt[u]!=e){
          heap* p=new heap;fill(p->chd,p->chd+4,nullNd);
          p->dep=1; p->edge=e; V.push_back(p);
        }
      }
      if(V.empty()) continue;
      make_heap(V.begin(),V.end(),cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
      for(size_t i=0;i<V.size();i++){
        if(L(i)<V.size()) V[i]->chd[2]=V[L(i)];
        else V[i]->chd[2]=nullNd;
        if(R(i)<V.size()) V[i]->chd[3]=V[R(i)];
        else V[i]->chd[3]=nullNd;
      }
      head[u]=merge(head[u],V.front());
    }
  }
  vector<ll> ans;
  void first_K(){
    ans.clear(); priority_queue<node> Q;
    if(dst[s]==-1) return;
    ans.push_back(dst[s]);
    if(head[s]!=nullNd)
      Q.push(node(head[s],dst[s]+head[s]->edge->d));
    for(int _=1;_<k and not Q.empty();_++){
      node p=Q.top(),q; Q.pop(); ans.push_back(p.d);
      if(head[p.H->edge->v]!=nullNd){
        q.H=head[p.H->edge->v]; q.d=p.d+q.H->edge->d;
        Q.push(q);
      }
      for(int i=0;i<4;i++)
        if(p.H->chd[i]!=nullNd){
          q.H=p.H->chd[i];
          q.d=p.d-p.H->edge->d+p.H->chd[i]->edge->d;
          Q.push(q);
  } }   }
  void solve(){ // ans[i] stores the i-th shortest path
    dijkstra(); build();
    first_K(); // ans.size() might less than k
  }
} solver;
```

# 7 String

## 7.1 Rolling Hash

```
const ll C = 27;
inline int id(char c) {return c-'a'+1;}
struct RollingHash {
    string s; int n; ll mod;
    vector<ll> Cexp, hs;
    RollingHash(string& _s, ll _mod):
        s(_s), n((int)_s.size()), mod(_mod)
    {
        Cexp.assign(n, 0);
        hs.assign(n, 0);
        Cexp[0] = 1;
        for (int i = 1; i < n; i++) {
            Cexp[i] = Cexp[i-1] * C;
            if (Cexp[i] >= mod) Cexp[i] %= mod;
        }
        hs[0] = id(s[0]);
        for (int i = 1; i < n; i++) {
            hs[i] = hs[i-1] * C + id(s[i]);
            if (hs[i] >= mod) hs[i] %= mod;
    } }
    inline ll query(int l, int r) {
        ll res = hs[r] - (l ? hs[l-1] * Cexp[r-l+1] : 0);
        res = (res % mod + mod) % mod;
        return res; }
};
```

## 7.2 Trie

```
struct node {
    int c[26]; ll cnt;
    node(): cnt(0) {memset(c, 0, sizeof(c));}
    node(ll x): cnt(x) {memset(c, 0, sizeof(c));}
};
struct Trie {
    vector<node> t;
    void init() {
        t.clear();
        t.emplace_back(node());
    }
    void insert(string s) { int ptr = 0;
        for (auto& i : s) {
            if (!t[ptr].c[i-'a']) {
                t.emplace_back(node());
                t[ptr].c[i-'a'] = (int)t.size()-1; }
            ptr = t[ptr].c[i-'a']; }
        t[ptr].cnt++; }
} trie;
```

## 7.3 KMP

```
int n, m;
string s, p;
vector<int> f;
void build() {
  f.clear(); f.resize(m, 0);
  int ptr = 0; for (int i = 1; i < m; i++) {
    while (ptr && p[i] != p[ptr]) ptr = f[ptr-1];
    if (p[i] == p[ptr]) ptr++;
    f[i] = ptr;
}}
void init() {
  cin >> s >> p;
  n = (int)s.size();
  m = (int)p.size();
  build(); }
void solve() {
  int ans = 0, pi = 0;
  for (int si = 0; si < n; si++) {
    while (pi && s[si] != p[pi]) pi = f[pi-1];
    if (s[si] == p[pi]) pi++;
    if (pi == m) ans++, pi = f[pi-1];
  }
cout << ans << endl; }
```

## 7.4 Z Value

```
string is, it, s;
int n; vector<int> z;
void init() {
    cin >> is >> it;
    s = it+'0'+is;
    n = (int)s.size();
    z.resize(n, 0); }
void solve() {
    int ans = 0; z[0] = n;
    for (int i = 1, l = 0, r = 0; i < n; i++) {
        if (i <= r) z[i] = min(z[i-l], r-i+1);
        while (i+z[i] < n && s[z[i]] == s[i+z[i]]) z[i]++;
        if (i+z[i]-1 > r) l = i, r = i+z[i]-1;
        if (z[i] == (int)it.size()) ans++;
    }
    cout << ans << endl; }
```

## 7.5 Manacher

```
int n; string S, s;
```

```
vector<int> m;
void manacher() {
s.clear(); s.resize(2*n+1, '.');
for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
m.clear(); m.resize(2*n+1, 0);
// m[i] := max k such that s[i-k, i+k] is palindrome
int mx = 0, mxk = 0;
for (int i = 1; i < 2*n+1; i++) {
  if (mx-(i-mx) >= 0) m[i] = min(m[mx-(i-mx)], mx+mxk-i);
  while (0 <= i-m[i]-1 && i+m[i]+1 < 2*n+1 &&
      s[i-m[i]-1] == s[i+m[i]+1]) m[i]++;
  if (i+m[i] > mx+mxk) mx = i, mxk = m[i];
} }
void init() { cin >> S; n = (int)S.size(); }
void solve() {
  manacher();
  int mx = 0, ptr = 0;
  for (int i = 0; i < 2*n+1; i++) if (mx < m[i])
    { mx = m[i]; ptr = i; }
  for (int i = ptr-mx; i <= ptr+mx; i++)
    if (s[i] != '.') cout << s[i];
cout << endl; }
```

## 7.6 Suffix Array

```
#define F first
#define S second
struct SuffixArray {  // don't forget s += "$";
    int n; string s;
    vector<int> suf, lcp, rk;
    vector<int> cnt, pos;
    vector<pair<pii, int> > buc[2];
    void init(string _s) {
        s = _s; n = (int)s.size();
// resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
    }
    void radix_sort() {
        for (int t : {0, 1}) {
            fill(cnt.begin(), cnt.end(), 0);
            for (auto& i : buc[t]) cnt[ (t ? i.F.F : i.F.S) ]++;
            for (int i = 0; i < n; i++)
                pos[i] = (!i ? 0 : pos[i-1] + cnt[i-1]);
            for (auto& i : buc[t])
                buc[t^1][pos[ (t ? i.F.F : i.F.S) ]++] = i;
    }}
    bool fill_suf() {
        bool end = true;
        for (int i = 0; i < n; i++) suf[i] = buc[0][i].S;
        rk[suf[0]] = 0;
        for (int i = 1; i < n; i++) {
            int dif = (buc[0][i].F != buc[0][i-1].F);
            end &= dif;
            rk[suf[i]] = rk[suf[i-1]] + dif;
        } return end;
    }
    void sa() {
        for (int i = 0; i < n; i++)
            buc[0][i] = make_pair(make_pair(s[i], s[i]), i);
        sort(buc[0].begin(), buc[0].end());
        if (fill_suf()) return;
        for (int k = 0; (1<<k) < n; k++) {
            for (int i = 0; i < n; i++)
                buc[0][i] = make_pair(make_pair(rk[i], rk[(i + (1<<k)) % n]), i);
            radix_sort();
            if (fill_suf()) return;
    }}
    void LCP() {  int k = 0;
        for (int i = 0; i < n-1; i++) {
            if (rk[i] == 0) continue;
            int pi = rk[i];
            int j = suf[pi-1];
            while (i+k < n && j+k < n && s[i+k] == s[j+k]) k++;
            lcp[pi] = k;
            k = max(k-1, 0);
    }}
};
SuffixArray suffixarray;
```

## 7.7 SA-IS

```
const int N=300010;
struct SA{
#define REP(i,n) for(int i=0;i<int(n);i++)
#define REP1(i,a,b) for(int i=(a);i<=int(b);i++)
  bool _t[N*2]; int _s[N*2],_sa[N*2];
  int _c[N*2],x[N],_p[N],_q[N*2],hei[N],r[N];
  int operator [](int i){ return _sa[i]; }
  void build(int *s,int n,int m){
    memcpy(_s,s,sizeof(int)*n);
    sais(_s,_sa,_p,_q,_t,_c,n,m); mkhei(n);
  }
  void mkhei(int n){
    REP(i,n) r[_sa[i]]=i;
    hei[0]=0;
    REP(i,n) if(r[i]) {
      int ans=i>0?max(hei[r[i-1]]-1,0):0;
      while(_s[i+ans]==_s[_sa[r[i]-1]+ans]) ans++;
      hei[r[i]]=ans;
    }
  }
  void sais(int *s,int *sa,int *p,int *q,bool *t,int *c,int n,int z){
    bool uniq=t[n-1]=true,neq;
    int nn=0,nmxz=-1,*nsa=sa+n,*ns=s+n,lst=-1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa,n);\
memcpy(x,c,sizeof(int)*z); XD;\
memcpy(x+1,c,sizeof(int)*(z-1));\
REP(i,n) if(sa[i]&&!t[sa[i]-1]) sa[x[s[sa[i]-1]]++]=sa[i]-1;\
memcpy(x,c,sizeof(int)*z);\
for(int i=n-1;i>=0;i--) if(sa[i]&&t[sa[i]-1]) sa[--x[s[sa[i]-1]]]=sa[i]-1;
    MS0(c,z); REP(i,n) uniq&=++c[s[i]]<2;
    REP(i,z-1) c[i+1]+=c[i];
    if(uniq) { REP(i,n) sa[--c[s[i]]]=i; return; }
    for(int i=n-2;i>=0;i--)
      t[i]=(s[i]==s[i+1]?t[i+1]:s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i]&&!t[i-1]) sa[--x[s[i]]]=p[q[i]=nn++]=i);
    REP(i,n) if(sa[i]&&t[sa[i]]&&!t[sa[i]-1]){
      neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
      ns[q[lst=sa[i]]]=nmxz+=neq;
    }
    sais(ns,nsa,p+nn,q+n,t+n,c+z,nn,nmxz+1);
    MAGIC(for(int i=nn-1;i>=0;i--) sa[--x[s[p[nsa[i]]]]]=p[nsa[i]]);
  }
}sa;
int H[N],SA[N],RA[N];
void suffix_array(int* ip,int len){
  // should padding a zero in the back
  // ip is int array, len is array length
  // ip[0..n-1] != 0, and ip[len]=0
  ip[len++]=0; sa.build(ip,len,128);
  memcpy(H,sa.hei+1,len<<2); memcpy(SA,sa._sa+1,len<<2);
  for(int i=0;i<len;i++) RA[i]=sa.r[i]-1;
  // resulting height, sa array \in [0,len)
}
```

## 7.8 Minimum Rotation

```
//rotate(begin(s), begin(s)+minRotation(s), end(s))
int minRotation(string s) {
int a = 0, n = s.size(); s += s;
for(int b = 0; b < n; b++) for(int k = 0; k < n; k++) {
    if(a + k == b ||| s[a + k] < s[b + k]) {
        b += max(0, k - 1);
        break; }
    if(s[a + k] > s[b + k]) {
        a = b;
        break;
    } }
return a; }
```

## 7.9 Aho Corasick

```
struct ACautomata{
  struct Node{
    int cnt;
    Node *go[26], *fail, *dic;
    Node (){
      cnt = 0; fail = 0; dic=0;
      memset(go,0,sizeof(go));
    }
  }pool[1048576],*root;
  int nMem;
  Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
  }
  void init() { nMem = 0; root = new_Node(); }
  void add(const string &str) { insert(root,str,0); }
  void insert(Node *cur, const string &str, int pos){
    for(int i=pos;i<str.size();i++){
      if(!cur->go[str[i]-'a'])
        cur->go[str[i]-'a'] = new_Node();
      cur=cur->go[str[i]-'a'];
    }
    cur->cnt++;
  }
  void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
      Node* fr=que.front(); que.pop();
      for (int i=0; i<26; i++){
        if (fr->go[i]){
          Node *ptr = fr->fail;
          while (ptr && !ptr->go[i]) ptr = ptr->fail;
          fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
          fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
          que.push(fr->go[i]);
  } } } }
}AC;
```

# 8 Geometry

## 8.1 Basic Operations

```
typedef long long T;
// typedef long double T;
const long double eps = 1e-8;

short sgn(T x) {
    if (abs(x) < eps) return 0;
    return x < 0 ? -1 : 1;
}

struct Pt {
T x, y;
Pt(T _x=0, T _y=0):x(_x), y(_y) {}
Pt operator+(Pt a) { return Pt(x+a.x, y+a.y); }
Pt operator-(Pt a) { return Pt(x-a.x, y-a.y); }
Pt operator*(T a)  { return Pt(x*a, y*a); }
Pt operator/(T a)  { return Pt(x/a, y/a); }
T operator*(Pt a)  { return x*a.x + y*a.y; }
T operator^(Pt a)  { return x*a.y - y*a.x; }
bool operator<(Pt a)
    { return x < a.x || (x == a.x && y < a.y); }
//return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn(y-a.y) < 0); }
bool operator==(Pt a)
    { return sgn(x-a.x) == 0 && sgn(y-a.y) == 0; }
};

Pt mv(Pt a, Pt b) { return b-a; }
T len2(Pt a) { return a*a; }
T dis2(Pt a, Pt b) { return len2(b-a); }

short ori(Pt a, Pt b) { return ((a^b)>0) - ((a^b)<0); }
bool onseg(Pt p, Pt l1, Pt l2) {
    Pt a = mv(p, l1), b = mv(p, l2);
    return ((a^b) == 0) && ((a*b) <= 0);
}
```

## 8.2 InPoly

```
short inPoly(Pt p) {
// 0=Bound 1=In -1=Out
REP(i, n) if (onseg(p, E[i], E[(i+1)%n])) return 0;
int cnt = 0;
REP(i, n) if (banana(p, Pt(p.x+1, p.y+2e9),
                     E[i], E[(i+1)%n])) cnt ^= 1;
return (cnt ? 1 : -1);
}
```

## 8.3 Sort by Angle

```
int ud(Pt a) {  // up or down half plane
    if (a.y > 0) return 0;
    if (a.y < 0) return 1;
    return (a.x >= 0 ? 0 : 1);
}
sort(ALL(E), [&](const Pt& a, const Pt& b){
    if (ud(a) != ud(b)) return ud(a) < ud(b);
    return (a^b) > 0;
});
```

## 8.4 Line Intersect Check

```
inline bool banana(Pt p1, Pt p2, Pt q1, Pt q2) {
if (onseg(p1, q1, q2) || onseg(p2, q1, q2) ||
    onseg(q1, p1, p2) || onseg(q2, p1, p2)) {
    return true;
}
Pt p = mv(p1, p2), q = mv(q1, q2);
return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2)) < 0 &&
        ori(q, mv(q1, p1)) * ori(q, mv(q1, p2)) < 0);
}
```

## 8.5 Line Intersection

```
// T: long double
Pt bananaPoint(Pt p1, Pt p2, Pt q1, Pt q2) {
if (onseg(q1, p1, p2)) return q1;
if (onseg(q2, p1, p2)) return q2;
if (onseg(p1, q1, q2)) return p1;
if (onseg(p2, q1, q2)) return p2;
double s = abs(mv(p1, p2) ^ mv(p1, q1));
double t = abs(mv(p1, p2) ^ mv(p1, q2));
return q2 * (s/(s+t)) + q1 * (t/(s+t));
}
```

## 8.6 Convex Hull

```
vector<Pt> hull;
void convexHull() {
hull.clear(); sort(ALL(E));
REP(t, 2) {
    int b = SZ(hull);
    Each(ei, E) {
        while (SZ(hull) - b >= 2 &&
               ori(mv(hull[SZ(hull)-2], hull.back()),
                   mv(hull[SZ(hull)-2], ei)) == -1) {
            hull.pop_back();
        }
        hull.eb(ei);
    }
    hull.pop_back();
    reverse(ALL(E));
} }
```

## 8.7 Polygon Area

```cpp
T dbarea(vector<Pt>& e) {
ll res = 0;
REP(i, SZ(e)) res += e[i]^e[(i+1)%SZ(e)];
return abs(res);
}
```

## 8.8 Pick's Theorem

Consider a polygon which vertices are all lattice points.

Let $i$ = number of points inside the polygon.

Let $b$ = number of points on the boundary of the polygon.

Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

## 8.9 Minimum Enclosing Circle

```cpp
Pt circumcenter(Pt A, Pt B, Pt C) {
// a1(x-A.x) + b1(y-A.y) = c1
// a2(x-A.x) + b2(y-A.y) = c2
// solve using Cramer's rule
T a1 = B.x-A.x, b1 = B.y-A.y, c1 = dis2(A, B)/2.0;
T a2 = C.x-A.x, b2 = C.y-A.y, c2 = dis2(A, C)/2.0;
T D  = Pt(a1, b1) ^ Pt(a2, b2);
T Dx = Pt(c1, b1) ^ Pt(c2, b2);
T Dy = Pt(a1, c1) ^ Pt(a2, c2);
if (D == 0) return Pt(-INF, -INF);
return A + Pt(Dx/D, Dy/D);
}
Pt center; T r2;
void minEncloseCircle() {
mt19937 gen(chrono::steady_clock::now().time_since_epoch().count());
shuffle(ALL(E), gen);
center = E[0], r2 = 0;

for (int i = 0; i < n; i++) {
    if (dis2(center, E[i]) <= r2) continue;
    center = E[i], r2 = 0;
    for (int j = 0; j < i; j++) {
        if (dis2(center, E[j]) <= r2) continue;
        center = (E[i] + E[j]) / 2.0;
        r2 = dis2(center, E[i]);
        for (int k = 0; k < j; k++) {
            if (dis2(center, E[k]) <= r2) continue;
            center = circumcenter(E[i], E[j], E[k]);
            r2 = dis2(center, E[i]);
        }
    }
} }
```

## 8.10 PolyUnion

```cpp
struct PY{
  int n; Pt pt[5]; double area;
  Pt& operator[](const int x){ return pt[x]; }
  void init(){ //n,pt[0~n-1] must be filled
    area=pt[n-1]^pt[0];
    for(int i=0;i<n-1;i++) area+=pt[i]^pt[i+1];
    if((area/=2)<0)reverse(pt,pt+n),area=-area;
  }
};
PY py[500]; pair<double,int> c[5000];
inline double segP(Pt &p,Pt &p1,Pt &p2){
  if(dcmp(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
  return (p.x-p1.x)/(p2.x-p1.x);
}
double polyUnion(int n){ //py[0~n-1] must be filled
  int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
  for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
  for(i=0;i<n;i++){
    for(ii=0;ii<py[i].n;ii++){
      r=0;
      c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
      for(j=0;j<n;j++){
        if(i==j) continue;
        for(jj=0;jj<py[j].n;jj++){
          ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]));
          tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj+1]));
          if(ta==0 && tb==0){
            if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[i][ii])>0&&j<i){
              c[r++]=make_pair(segP(py[j][jj],py[i][ii],py[i][ii+1]),1);
              c[r++]=make_pair(segP(py[j][jj+1],py[i][ii],py[i][ii+1]),-1);
            }
          }else if(ta>=0 && tb<0){
            tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
            td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
            c[r++]=make_pair(tc/(tc-td),1);
          }else if(ta<0 && tb>=0){
            tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
            td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
            c[r++]=make_pair(tc/(tc-td),-1);
      } } }
      sort(c,c+r);
      z=min(max(c[0].first,0.0),1.0); d=c[0].second; s=0;
      for(j=1;j<r;j++){
        w=min(max(c[j].first,0.0),1.0);
        if(!d) s+=w-z;
        d+=c[j].second; z=w;
      }
      sum+=(py[i][ii]^py[i][ii+1])*s;
    }
  }
  return sum/2;
}
```

## 8.11 Minkowski Sum

```cpp
/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
int pos( const Pt& tp ){
  if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
  return tp.Y > 0 ? 0 : 1;
}
#define N 300030
Pt pt[ N ], qt[ N ], rt[ N ];
LL Lx,Rx;
int dn,un;
inline bool cmp( Pt a, Pt b ){
  int pa=pos( a ),pb=pos( b );
  if(pa==pb) return (a^b)>0;
  return pa<pb;
}
int minkowskiSum(int n,int m){
  int i,j,r,p,q,fi,fj;
  for(i=1,p=0;i<n;i++){
    if( pt[i].Y<pt[p].Y ||
        (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
  for(i=1,q=0;i<m;i++){
    if( qt[i].Y<qt[q].Y ||
        (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
  rt[0]=pt[p]+qt[q];
  r=1; i=p; j=q; fi=fj=0;
  while(1){
    if((fj&&j==q) ||
       ( (!fi||i!=p) &&
         cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
      rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
      p=(p+1)%n;
      fi=1;
    }else{
      rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
      q=(q+1)%m;
      fj=1;
    }
    if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0) r++;
    else rt[r-1]=rt[r];
    if(i==p && j==q) break;
  }
  return r-1;
}
```

```
void initInConvex(int n){
  int i,p,q;
  LL Ly,Ry;
  Lx=INF; Rx=-INF;
  for(i=0;i<n;i++){
    if(pt[i].X<Lx) Lx=pt[i].X;
    if(pt[i].X>Rx) Rx=pt[i].X;
  }
  Ly=Ry=INF;
  for(i=0;i<n;i++){
    if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i; }
    if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }
  }
  for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
  qt[dn]=pt[q]; Ly=Ry=-INF;
  for(i=0;i<n;i++){
    if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i; }
    if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
  }
  for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
  rt[un]=pt[q];
}
inline int inConvex(Pt p){
  int L,R,M;
  if(p.X<Lx || p.X>Rx) return 0;
  L=0;R=dn;
  while(L<R-1){ M=(L+R)/2;
    if(p.X<qt[M].X) R=M; else L=M; }
    if(tri(qt[L],qt[R],p)<0) return 0;
    L=0;R=un;
    while(L<R-1){ M=(L+R)/2;
      if(p.X<rt[M].X) R=M; else L=M; }
      if(tri(rt[L],rt[R],p)>0) return 0;
      return 1;
}
int main(){
  int n,m,i;
  Pt p;
  scanf("%d",&n);
  for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y);
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];
  initInConvex(n);
  scanf("%d",&m);
  for(i=0;i<m;i++){
    scanf("%lld %lld",&p.X,&p.Y);
    p.X*=3; p.Y*=3;
    puts(inConvex(p)?"YES":"NO");
  }
}
```

# 9 Number Theory

## 9.1 Pollard's rho

```
from itertools import count
from math import gcd
from sys import stdin

for s in stdin:
    number, x = int(s), 2
    break2 = False
    for cycle in count(1):
        y = x
        if break2:
            break
        for i in range(1 << cycle):
            x = (x * x + 1) % number
            factor = gcd(x - y, number)
            if factor > 1:
                print(factor)
                break2 = True
                break
```

## 9.2 Miller Rabin

```
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383          6 :  pirmes <= 13
// n < 2^64                       7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool witness(ll a,ll n,ll u,int t){
  if(!(a%=n)) return 0;
  ll x=mypow(a,u,n);
  for(int i=0;i<t;i++) {
    ll nx=mul(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  }
  return x!=1;
}
bool miller_rabin(ll n,int s=100) {
  // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if(n<2) return 0;
  if(!(n&1)) return n == 2;
  ll u=n-1; int t=0;
  while(!(u&1)) u>>=1, t++;
  while(s--){
    ll a=randll()%(n-1)+1;
    if(witness(a,n,u,t)) return 0;
  }
  return 1;
}
```

## 9.3 Fast Power

Note: $a^n \equiv a^{(n\ mod\ (p-1))} (mod\ p)$

## 9.4 Extend GCD

```
ll GCD;
pll extgcd(ll a, ll b) {
    if (b == 0) {
        GCD = a;
        return pll{1, 0};
    }
    pll ans = extgcd(b, a % b);
    return pll{ans.S, ans.F - a/b * ans.S};
}
pll bezout(ll a, ll b, ll c) {
    bool negx = (a < 0), negy = (b < 0);
    pll ans = extgcd(abs(a), abs(b));
    if (c % GCD != 0) return pll{-LLINF, -LLINF};
    return pll{ans.F * c/GCD * (negx ? -1 : 1),
               ans.S * c/GCD * (negy ? -1 : 1)};
}
ll inv(ll a, ll p) {
    if (p == 1) return -1;
    pll ans = bezout(a % p, -p, 1);
    if (ans == pll{-LLINF, -LLINF}) return -1;
    return (ans.F % p + p) % p;
}
```

## 9.5 Mu + Phi

```
const int maxn = 1e6 + 5;
ll f[maxn];
vector<int> lpf, prime;
void build() {
lpf.clear(); lpf.resize(maxn, 1);
prime.clear();
f[1] = ...;  /* mu[1] = 1, phi[1] = 1 */
for (int i = 2; i < maxn; i++) {
    if (lpf[i] == 1) {
        lpf[i] = i; prime.emplace_back(i);
        f[i] = ...;  /* mu[i] = 1, phi[i] = i-1 */
    }
    for (auto& j : prime) {
        if (i*j >= maxn) break;
        lpf[i*j] = j;
        if (i % j == 0) f[i*j] = ...;  /* 0, phi[i]*j */
        else f[i*j] = ...;  /* -mu[i], phi[i]*phi[j] */
```

```cpp
        if (j >= lpf[i]) break;
} } }
```

## 9.6 Other Formulas

- Inversion:
  $aa^{-1} \equiv 1 \pmod{m}$. $a^{-1}$ exists iff $\gcd(a, m) = 1$.
- Linear inversion:
  $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod{m}$
- Fermat's little theorem:
  $a^p \equiv a \pmod{p}$ if $p$ is prime.
- Euler function:
  $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$
- Euler theorem:
  $a^{\phi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.
- Extended Euclidean algorithm:
  $ax + by = \gcd(a, b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$
- Divisor function:
  $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^r p_i^{a_i}$.
  $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x} - 1}{p_i^x - 1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^r (a_i + 1)$.
- Chinese remainder theorem (Coprime Moduli):
  $x \equiv a_i \pmod{m_i}$.
  $M = \prod m_i$. $M_i = M/m_i$. $t_i = M_i^{-1}$.
  $x = kM + \sum a_i t_i M_i$, $k \in \mathbb{Z}$.
- Chinese remainder theorem:
  $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$
  Solve for $(p, q)$ using ExtGCD.
  $x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{lcm(m_1, m_2)}$
- Avoiding Overflow: $ca \mod cb = c(a \mod b)$
- Dirichlet Convolution: $(f * g)(n) = \sum_{d|n} f(n) g(n/d)$
- Important Multiplicative Functions + Proterties:
  1. $\epsilon(n) = [n = 1]$
  2. $1(n) = 1$
  3. $id(n) = n$
  4. $\mu(n) = 0$ if $n$ has squared prime factor
  5. $\mu(n) = (-1)^k$ if $n = p_1 p_2 \cdots p_k$
  6. $\epsilon = \mu * 1$
  7. $\phi = \mu * id$
  8. $[n = 1] = \sum_{d|n} \mu(d)$
  9. $[gcd = 1] = \sum_{d|gcd} \mu(d)$
- Möbius inversion: $f = g * 1 \Leftrightarrow g = f * \mu$

## 9.7 Polynomial

```cpp
const int maxk = 20;
const int maxn = 1<<maxk;
const ll LINF = 1e18;

/* P = r*2^k + 1
P                   r   k   g
998244353           119 23  3
1004535809          479 21  3

P                   r   k   g
3                   1   1   2
5                   1   2   2
17                  1   4   3
97                  3   5   5
193                 3   6   5
257                 1   8   3
7681                15  9   17
12289               3   12  11
40961               5   13  3
65537               1   16  3
786433              3   18  10
5767169             11  19  3
7340033             7   20  3
23068673            11  21  3
104857601           25  22  3
167772161           5   25  3
469762049           7   26  3
1004535809          479 21  3
2013265921          15  27  31
2281701377          17  27  3
3221225473          3   30  5
75161927681         35  31  3
77309411329         9   33  7
206158430209        3   36  22
2061584302081       15  37  7
2748779069441       5   39  3
6597069766657       3   41  5
39582418599937      9   42  5
79164837199873      9   43  5
263882790666241     15  44  7
1231453023109121    35  45  3
1337006139375617    19  46  3
3799912185593857    27  47  5
4222124650659841    15  48  19
7881299347898369    7   50  6
31525197391593473   7   52  3
180143985094819841  5   55  6
1945555039024054273 27  56  5
4179340454199820289 29  57  3
9097271247288401921 505 54  6 */

const int g = 3;
const ll MOD = 998244353;

ll pw(ll a, ll n) { /* fast pow */ }

#define siz(x) (int)x.size()

template<typename T>
vector<T>& operator+=(vector<T>& a, const vector<T>& b) {
    if (siz(a) < siz(b)) a.resize(siz(b));
    for (int i = 0; i < min(siz(a), siz(b)); i++) {
        a[i] += b[i];
        a[i] -= a[i] >= MOD ? MOD : 0;
    }
    return a;
}

template<typename T>
vector<T>& operator-=(vector<T>& a, const vector<T>& b) {
    if (siz(a) < siz(b)) a.resize(siz(b));
    for (int i = 0; i < min(siz(a), siz(b)); i++) {
        a[i] -= b[i];
        a[i] += a[i] < 0 ? MOD : 0;
    }
    return a;
}

template<typename T>
vector<T> operator-(const vector<T>& a) {
    vector<T> ret(siz(a));
    for (int i = 0; i < siz(a); i++) {
        ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
    }
    return ret;
}

vector<ll> X, iX;
vector<int> rev;

void init_ntt() {
    X.clear(); X.resize(maxn, 1);  // x1 = g^((p-1)/n)
    iX.clear(); iX.resize(maxn, 1);

    ll u = pw(g, (MOD-1)/maxn);
    ll iu = pw(u, MOD-2);

    for (int i = 1; i < maxn; i++) {
        X[i] = X[i-1] * u;
        iX[i] = iX[i-1] * iu;
        if (X[i] >= MOD) X[i] %= MOD;
        if (iX[i] >= MOD) iX[i] %= MOD;
    }

    rev.clear(); rev.resize(maxn, 0);
    for (int i = 1, hb = -1; i < maxn; i++) {
        if (!(i & (i-1))) hb++;
        rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
} }

template<typename T>
void NTT(vector<T>& a, bool inv=false) {

    int _n = (int)a.size();
    int k = __lg(_n) + ((1<<__lg(_n)) != _n);
    int n = 1<<k;
    a.resize(n, 0);

    short shift = maxk-k;
    for (int i = 0; i < n; i++)
        if (i > (rev[i]>>shift))
            swap(a[i], a[rev[i]>>shift]);

    for (int len = 2, half = 1, div = maxn>>1; len <= n; len<<=1, half<<=1, div>>=1) {
        for (int i = 0; i < n; i += len) {
            for (int j = 0; j < half; j++) {
                T u = a[i+j];
                T v = a[i+j+half] * (inv ? iX[j*div] : X[j*div]) % MOD;
                a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
                a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v);
    } } }

    if (inv) {
        T dn = pw(n, MOD-2);
        for (auto& x : a) {
            x *= dn;
            if (x >= MOD) x %= MOD;
} } }

template<typename T>
inline void resize(vector<T>& a) {
    int cnt = (int)a.size();
    for (; cnt > 0; cnt--) if (a[cnt-1]) break;
    a.resize(max(cnt, 1));
}

template<typename T>
vector<T>& operator*=(vector<T>& a, vector<T> b) {
    int na = (int)a.size();
    int nb = (int)b.size();
    a.resize(na + nb - 1, 0);
    b.resize(na + nb - 1, 0);

    NTT(a); NTT(b);
    for (int i = 0; i < (int)a.size(); i++) {
        a[i] *= b[i];
        if (a[i] >= MOD) a[i] %= MOD;
    }
    NTT(a, true);

    resize(a);
    return a;
}

template<typename T>
void inv(vector<T>& ia, int N) {
    vector<T> _a(move(ia));
    ia.resize(1, pw(_a[0], MOD-2));
    vector<T> a(1, -_a[0] + (-_a[0] < 0 ? MOD : 0));

    for (int n = 1; n < N; n<<=1) {
        // n -> 2*n
        // ia' = ia(2-a*ia);

        for (int i = n; i < min(siz(_a), (n<<1)); i++)
            a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD : 0));

        vector<T> tmp = ia;
        ia *= a;
        ia.resize(n<<1);
        ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia[0] + 2;
        ia *= tmp;
        ia.resize(n<<1);
    }
    ia.resize(N);
}

template<typename T>
void mod(vector<T>& a, vector<T>& b) {
    int n = (int)a.size()-1, m = (int)b.size()-1;
    if (n < m) return;

    vector<T> ra = a, rb = b;
    reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n-m+1));
    reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n-m+1));

    inv(rb, n-m+1);

    vector<T> q = move(ra);
    q *= rb;
    q.resize(n-m+1);
    reverse(q.begin(), q.end());

    q *= b;
    a -= q;
    resize(a);
}

/* Kitamasa Method (Fast Linear Recurrence):
Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j-1])
Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
Let R(x) = x^K mod B(x)   (get x^K using fast pow and use poly mod to get R(x))
Let r[i] = the coefficient of x^i in R(x)
=> a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */
```

# 10 Linear Algebra

## 10.1 Gaussian-Jordan Elimination

```cpp
int n; vector<vector<ll> > v;
void gauss(vector<vector<ll>>& v) {
int r = 0;
for (int i = 0; i < n; i++) {
    bool ok = false;
    for (int j = r; j < n; j++) {
        if (v[j][i] == 0) continue;
        swap(v[j], v[r]);
        ok = true; break;
    }
    if (!ok) continue;
    ll div = inv(v[r][i]);
    for (int j = 0; j < n+1; j++) {
        v[r][j] *= div;
        if (v[r][j] >= MOD) v[r][j] %= MOD;
    }
    for (int j = 0; j < n; j++) {
        if (j == r) continue;
        ll t = v[j][i];
        for (int k = 0; k < n+1; k++) {
            v[j][k] -= v[r][k] * t % MOD;
            if (v[j][k] < 0) v[j][k] += MOD;
    } }
    r++;
} }
```

## 10.2 Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then det = 0, otherwise det = product of diagonal elements.

2. Properties of det:

   - Transpose: Unchanged
   - Row Operation 1 - Swap 2 rows: $-det$
   - Row Operation 2 - $k\vec{r_i}$: $k \times det$
   - Row Operation 3 - $k\vec{r_i}$ add to $\vec{r_j}$: Unchaged

# 11 Flow / Matching

## 11.1 Dinic

```cpp
struct Dinic {
    struct Edge {
        int t, c, r;
        Edge() {}
        Edge(int _t, int _c, int _r):
            t(_t), c(_c), r(_r) {}
    };
    vector<vector<Edge>> G;
    vector<int> dis, iter;
    int s, t;
    void init(int n) {
        G.resize(n), dis.resize(n), iter.resize(n);
        for(int i = 0; i < n; ++i)
            G[i].clear();
    }
    void add(int a, int b, int c) {
        G[a].eb(b, c, G[b].size());
        G[b].eb(a, 0, G[a].size() - 1);
    }
    bool bfs() {
        fill(ALL(dis), -1);
        dis[s] = 0;
        queue<int> que;
        que.push(s);
        while(!que.empty()) {
            int u = que.front(); que.pop();
            for(auto& e : G[u]) {
                if(e.c > 0 && dis[e.t] == -1) {
                    dis[e.t] = dis[u] + 1;
                    que.push(e.t);
                }
            }
        }
        return dis[t] != -1;
    }
    int dfs(int u, int cur) {
        if(u == t) return cur;
        for(int &i = iter[u]; i < (int)G[u].size(); ++i ) {
            auto& e = G[u][i];
            if(e.c > 0 && dis[u] + 1 == dis[e.t]) {
                int ans = dfs(e.t, min(cur, e.c));
                if(ans > 0) {
                    G[e.t][e.r].c += ans;
                    e.c -= ans;
                    return ans;
                }
            }
        }
        return 0;
    }

    int flow(int a, int b) {
        s = a, t = b;
        int ans = 0;
        while(bfs()) {
            fill(ALL(iter), 0);
            int tmp;
            while((tmp = dfs(s, INF)) > 0)
                ans += tmp;
        }
        return ans;
    }
};
```

## 11.2 ISAP

```cpp
#define SZ(c) ((int)(c).size())
struct Maxflow{
  static const int MAXV=50010;
  static const int INF =1000000;
  struct Edge{
    int v,c,r;
    Edge(int _v,int _c,int _r):v(_v),c(_c),r(_r){}
  };
  int s,t; vector<Edge> G[MAXV];
  int iter[MAXV],d[MAXV],gap[MAXV],tot;
  void init(int n,int _s,int _t){
    tot=n,s=_s,t=_t;
    for(int i=0;i<=tot;i++){
      G[i].clear(); iter[i]=d[i]=gap[i]=0;
    }
  }
  void addEdge(int u,int v,int c){
    G[u].push_back(Edge(v,c,SZ(G[v])));
    G[v].push_back(Edge(u,0,SZ(G[u])-1));
  }
  int DFS(int p,int flow){
    if(p==t) return flow;
    for(int &i=iter[p];i<SZ(G[p]);i++){
      Edge &e=G[p][i];
      if(e.c>0&&d[p]==d[e.v]+1){
        int f=DFS(e.v,min(flow,e.c));
        if(f){ e.c-=f; G[e.v][e.r].c+=f; return f; }
      }
    }
    if((--gap[d[p]])==0) d[s]=tot;
    else{ d[p]++; iter[p]=0; ++gap[d[p]]; }
    return 0;
  }
  int flow(){
    int res=0;
    for(res=0,gap[0]=tot;d[s]<tot;res+=DFS(s,INF));
    return res;
  } // reset: set iter,d,gap to 0
} flow;
```

## 11.3 MCMF

```cpp
struct MCMF {
    struct Edge {
        int to, cap, rev;
        ll cost;
        Edge() {}
        Edge(int _to, int _cap, int _rev, ll _cost) :
            to(_to), cap(_cap), rev(_rev), cost(_cost) {}
    };
    static const int N = 2000;
    vector<Edge> G[N];
    int n, s, t;
    void init(int _n, int _s, int _t) {
        n = _n, s = _s, t = _t;
        for(int i = 0; i <= n; ++i)
            G[i].clear();
    }
    void add_edge(int from, int to, int cap, ll cost) {
        G[from].eb(to, cap, (int)G[to].size(), cost);
        G[to].eb(from, 0, (int)G[from].size() - 1, -cost);
    }

    bool vis[N];
    int iter[N];
    ll dis[N];
    bool SPFA() {
        for(int i = 0; i <= n; ++i)
            vis[i] = 0, dis[i] = LINF;

        dis[s] = 0; vis[s] = 1;
        queue<int> que; que.push(s);
        while(!que.empty()) {
            int u = que.front(); que.pop();
            vis[u] = 0;
            for(auto& e : G[u]) if(e.cap > 0 && dis[e.to] > dis[u] + e.cost) {
                dis[e.to] = dis[u] + e.cost;
                if(!vis[e.to]) {
                    que.push(e.to);
                    vis[e.to] = 1;
                }
            }
        }
        return dis[t] != LINF;
    }

    int dfs(int u, int cur) {
        if(u == t) return cur;
        int ret = 0; vis[u] = 1;
        for(int &i = iter[u]; i < (int)G[u].size(); ++i) {
            auto &e = G[u][i];
            if(e.cap > 0 && dis[e.to] == dis[u] + e.cost && !vis[e.to]) {
                int tmp = dfs(e.to, min(cur, e.cap));
                e.cap -= tmp;
                G[e.to][e.rev].cap += tmp;
                cur -= tmp;
                ret += tmp;
                if(cur == 0) {
                    vis[u] = 0;
                    return ret;
                }
            }
        }
        vis[u] = 0;
        return ret;
    }
    pair<int, ll> flow() {
        int flow = 0; ll cost = 0;
        while(SPFA()) {
            memset(iter, 0, sizeof(iter));
            int tmp = dfs(s, INF);
            flow += tmp, cost += tmp * dis[t];
        }
        return {flow, cost};
    }
};
```

## 11.4 Hopcroft-Karp

```cpp
struct HopcroftKarp {
    // id: X = [1, nx], Y = [nx+1, nx+ny]
    int n, nx, ny, m, MXCNT;
    vector<vector<int> > g;
    vector<int> mx, my, dis, vis;
    void init(int nnx, int nny, int mm) {
        nx = nnx, ny = nny, m = mm;
        n = nx + ny + 1;
        g.clear(); g.resize(n);
    }
    void add(int x, int y) {
        g[x].emplace_back(y);
        g[y].emplace_back(x);
    }
    bool dfs(int x) {
        vis[x] = true;
        Each(y, g[x]) {
            int px = my[y];
            if (px == -1 ||
                (dis[px] == dis[x]+1 &&
                !vis[px] && dfs(px))) {
                mx[x] = y;
                my[y] = x;
                return true;
            }
        }
        return false;
    }
    void get() {
        mx.clear(); mx.resize(n, -1);
        my.clear(); my.resize(n, -1);

        while (true) {
            queue<int> q;
            dis.clear(); dis.resize(n, -1);
            for (int x = 1; x <= nx; x++){
                if (mx[x] == -1) {
                    dis[x] = 0;
                    q.push(x);
                }
            }
            while (!q.empty()) {
                int x = q.front(); q.pop();
                Each(y, g[x]) {
                    if (my[y] != -1 && dis[my[y]] == -1) {
                        dis[my[y]] = dis[x] + 1;
                        q.push(my[y]);
                    }
                }
            }

            bool brk = true;
            vis.clear(); vis.resize(n, 0);
            for (int x = 1; x <= nx; x++)
                if (mx[x] == -1 && dfs(x))
                    brk = false;

            if (brk) break;
        }
        MXCNT = 0;
        for (int x = 1; x <= nx; x++) if (mx[x] != -1) MXCNT++;
    }
} hk;
```

## 11.5 Cover / Independent Set

```
V(E) Cover: choose some V(E) to cover all E(V)
V(E) Independ: set of V(E) not adj to each other

M = Max Matching
Cv = Min V Cover
Ce = Min E Cover
Iv = Max V Ind
Ie = Max E Ind (equiv to M)

M = Cv (Konig Theorem)
Iv = V \ Cv
Ce = V - M

Construct Cv:
1. Run Dinic
2. Find s-t min cut
3. Cv = {X in T} + {Y in S}
```

## 11.6 KM

```cpp
#include <bits/stdc++.h>

using namespace std;
const int inf = 1e9;

struct KuhnMunkres {
    int n;
    vector<vector<int>> g;
    vector<int> lx, ly, slack;
    vector<int> match, visx, visy;
    KuhnMunkres(int n) : n(n), g(n, vector<int>(n)),
        lx(n), ly(n), slack(n), match(n), visx(n), visy(n) {}
    vector<int> & operator[](int i) { return g[i]; }
    bool dfs(int i, bool aug) { // aug = true 表示要更新 match
        if(visx[i]) return false;
        visx[i] = true;
        for(int j = 0; j < n; j++) {
            if(visy[j]) continue;
            // 一邊擴增交錯樹、尋找增廣路徑
            // 一邊更新slack：樹上的點跟樹外的點所造成的最小權重
            int d = lx[i] + ly[j] - g[i][j];
            if(d == 0) {
                visy[j] = true;
                if(match[j] == -1 || dfs(match[j], aug)) {
                    if(aug)
                        match[j] = i;
                    return true;
                }
            } else {
                slack[j] = min(slack[j], d);
            }
        }
        return false;
    }
    bool augment() { // 回傳是否有增廣路
        for(int j = 0; j < n; j++) if(!visy[j] && slack[j] == 0) {
            visy[j] = true;
            if(match[j] == -1 || dfs(match[j], false)) {
                return true;
            }
        }
        return false;
    }
    void relabel() {
        int delta = inf;
        for(int j = 0; j < n; j++) if(!visy[j]) delta = min(delta, slack[j]);
        for(int i = 0; i < n; i++) if(visx[i]) lx[i] -= delta;
        for(int j = 0; j < n; j++) {
            if(visy[j]) ly[j] += delta;
            else slack[j] -= delta;
        }
    }
    int solve() {
        for(int i = 0; i < n; i++) {
            lx[i] = 0;
            for(int j = 0; j < n; j++) lx[i] = max(lx[i], g[i][j]);
        }
        fill(ly.begin(), ly.end(), 0);
        fill(match.begin(), match.end(), -1);
        for(int i = 0; i < n; i++) {
            // slack 在每一輪要初始化
            fill(slack.begin(), slack.end(), inf);
            fill(visx.begin(), visx.end(), false);
            fill(visy.begin(), visy.end(), false);
            if(dfs(i, true)) continue;
            // 重複調整頂標直到找到增廣路徑
            while(!augment()) relabel();
            fill(visx.begin(), visx.end(), false);
            fill(visy.begin(), visy.end(), false);
            dfs(i, true);
        }
        int ans = 0;
        for(int j = 0; j < n; j++) if(match[j] != -1) ans += g[match[j]][j];
        return ans;
    }
};
signed main() {
    ios_base::sync_with_stdio(0), cin.tie(0);
    int n;
    while(cin >> n && n) {
        KuhnMunkres KM(n);
        for(int i = 0; i < n; i++) {
            for(int j = 0; j < n; j++) {
                int c;
                cin >> c;
                if(c > 0)
                    KM[i][j] = c;
            }
        }
        cout << KM.solve() << '\n';
    }
}
```

# 12 Combinatorics

## 12.1 Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 1 | 2 | 5 |
| 4 | 14 | 42 | 132 | 429 |
| 8 | 1430 | 4862 | 16796 | 58786 |
| 12 | 208012 | 742900 | 2674440 | 9694845 |

## 12.2 Burnside's Lemma

Let $X$ be the original set.
Let $G$ be the group of operations acting on $X$.
Let $X^g$ be the set of $x$ not affected by $g$.
Let $X/G$ be the set of orbits.
Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

# 13 Special Numbers

## 13.1 Fibonacci Series

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 3 |
| 5 | 5 | 8 | 13 | 21 |
| 9 | 34 | 55 | 89 | 144 |
| 13 | 233 | 377 | 610 | 987 |
| 17 | 1597 | 2584 | 4181 | 6765 |
| 21 | 10946 | 17711 | 28657 | 46368 |
| 25 | 75025 | 121393 | 196418 | 317811 |
| 29 | 514229 | 832040 | 1346269 | 2178309 |
| 33 | 3524578 | 5702887 | 9227465 | 14930352 |

$f(45) \approx 10^9, f(88) \approx 10^{18}$

## 13.2 Prime Numbers

- First 50 prime numbers:

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 5 | 7 | 11 |
| 6 | 13 | 17 | 19 | 23 | 29 |
| 11 | 31 | 37 | 41 | 43 | 47 |
| 16 | 53 | 59 | 61 | 67 | 71 |
| 21 | 73 | 79 | 83 | 89 | 97 |
| 26 | 101 | 103 | 107 | 109 | 113 |
| 31 | 127 | 131 | 137 | 139 | 149 |
| 36 | 151 | 157 | 163 | 167 | 173 |
| 41 | 179 | 181 | 191 | 193 | 197 |
| 46 | 199 | 211 | 223 | 227 | 229 |

- Very large prime numbers:
  1000001333 1000500889 2500001909
  2000000659 900004151 850001359

- $\pi(n) \equiv$ Number of primes $\leq n \approx n/((\ln n) - 1)$
  $\pi(100) = 25, \pi(200) = 46$
  $\pi(500) = 95, \pi(1000) = 168$
  $\pi(2000) = 303, \pi(4000) = 550$
  $\pi(10^4) = 1229, \pi(10^5) = 9592$
  $\pi(10^6) = 78498, \pi(10^7) = 664579$