Contents

1 Init (Linux)

開場流程：

```
vim ~/.vimrc
mkdir contest && cd contest

vim template.cpp
for c in {A..P}; do
    cp template.cpp $c.cpp
done

vim run.sh && chmod 777 run.sh
```

1.1 vimrc

```
syn on
set nu rnu ru cul mouse=a
set cin et ts=4 sw=4 sts=4
set autochdir
set clipboard=unnamedplus

colo koehler

no <C-h> ^
no <C-l> $
no ; :

inoremap {<CR> {<CR>}<Esc>ko
```

1.2 template.cpp

```cpp
#include <bits/stdc++.h>
using namespace std;

void solve() {
}

int main() {
    ios_base::sync_with_stdio(false); cin.tie(0);
    int TEST = 1;
    //cin >> TEST;
    while (TEST--) solve();
    return 0;
}
```

1.3 run.sh

```bash
#!/bin/bash

g++ -std=c++17 -O2 -g -fsanitize=undefined,address $1
    && echo DONE COMPILE || exit 1
./a.out
```

2 Reminder

2.1 Observations and Tricks

- Contribution Technique
- 二分圖/Spanning Tree/DFS Tree
- 行、列操作互相獨立
- 奇偶性
- 當 s, t 遞增並且 $t = f(s)$，對 s 二分搜不好做，可以改成對 t 二分搜，再算 $f(t)$
- 啟發式合併
- Permutation Normalization（做一些平移對齊兩個 permutation）

- 枚舉 $a_1 \sim a_n$ 再枚舉 $a_n \sim a_1$ 可以包在一個迴圈
- 兩個凸型函數相加還是凸型函數，相減不一定

2.2 Bug List

- 沒開 long long
- 陣列戳出界／陣列開不夠大
- 寫好的函式忘記呼叫
- 0-base / 1-base
- 忘記初始化
- == 打成 =
- <= 打成 <+
- dp[i] 從 dp[i-1] 轉移時忘記特判 i > 0
- std::sort 比較運算子寫成 < 或是讓 = 的情況為 true
- 漏 case
- 線段樹改值懶標初始值不能設為 0
- DFS 的時候不小心覆寫到全域變數
- 浮點數誤差
- unsigned int128
- 多筆測資不能沒讀完直接 return
- 記得刪 cerr
- vector 超級肥，小 vector 請用 array，例如矩陣快速冪

3 Basic

3.1 template (optional)

```
#define F first
#define S second
#define ep emplace
#define eb emplace_back
#define endl '\n'

template<class T> using V=vector<T>;
typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
typedef pair<int, ll> pil;
typedef pair<ll, int> pli;

/* ========================================= */
// STL and I/O
// pair
template<typename T1, typename T2>
ostream& operator<<(ostream& os, pair<T1, T2> p) {
    return os << "(" << p.first << ", " << p.second <<
        ")";
}
template<typename T1, typename T2>
istream& operator>>(istream& is, pair<T1, T2>& p) {
    return is >> p.first >> p.second; }

// vector
template<typename T>
istream& operator>>(istream& is, vector<T>& v) {
    for (auto& x : v) is >> x;
    return is;
}
template<typename T>
ostream& operator<<(ostream& os, const vector<T>& v) {
    for (const auto& x : v) os << x << ' ';
    return os;
}
/* ========================================= */
// debug(), output()
#define RED      "\x1b[31m"
#define GREEN    "\x1b[32m"
#define YELLOW   "\x1b[33m"
#define GRAY     "\x1b[90m"
#define COLOREND "\x1b[0m"

void _debug() {}
template<typename A,typename... B> void _debug(A a,B...
    b) { cerr << a << ' ', _debug(b...); }
#define debug(...) cerr<<GRAY<<#__VA_ARGS__<<": "<<
    COLOREND,_debug(__VA_ARGS__),cerr<<endl
```

```
void _output() {}
template<typename A,typename... B> void _output(A a,B
    ... b) { cout << a << ' ', _output(b...); }
#define output(...) _output(__VA_ARGS__),cout<<endl
/* ========================================= */
// BASIC ALGORITHM
string binary(ll x, int b = -1) {
    if (b == -1) b = __lg(x) + 1;
    string s = "";
    for (int k = b - 1; k >= 0; k--) {
        s.push_back((x & (1LL<<k)) ? '1' : '0');
    }
    return s;
}
/* ========================================= */
// CONSTANT
const int INF = 1.05e9;
const ll LINF = 4e18;
const int MOD = 1e9 + 7;
//const int MOD = 998244353;
const int maxn = 2e5 + 3;
```

3.2 Stress

```
g++ gen.cpp -o gen.out
g++ ac.cpp -o ac.out
g++ wa.cpp -o wa.out
for ((i=0;;i++))
do
    echo "$i"
    ./gen.out > in.txt
    ./ac.out < in.txt > ac.txt
    ./wa.out < in.txt > wa.txt
    diff ac.txt wa.txt || break
done
```

3.3 PBDS

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

// map
tree<int, int, less<>, rb_tree_tag,
    tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// set
tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update> tr;
tr.order_of_key(element);
tr.find_by_order(rank);

// priority queue
__gnu_pbds::priority_queue<int, less<int> > big_q;  //
    Big First
__gnu_pbds::priority_queue<int, greater<int> > small_q;
    // Small First
q1.join(q2);  // join
```

3.4 Random

```
mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
#define RANDINT(a, b) uniform_int_distribution<int> (a,
    b)(rng) // inclusive
#define RANDLL(a, b) uniform_int_distribution<long long
    >(a, b)(rng)  // inclusive
#define RANDFLOAT(a, b) uniform_real_distribution<float
    >(a, b)(rng)  // exclusive
#define RANDDOUBLE(a, b) uniform_real_distribution<
    double>(a, b)(rng)  // exclusive
shuffle(v.begin(), v.end(), gen);
```

4 Python

4.1 I/O

```python
import sys
input = sys.stdin.readline

# Input
def readInt():
    return int(input())
def readList():
    return list(map(int,input().split()))
def readStr():
    s = input()
    return list(s[:len(s) - 1])
def readVars():
    return map(int,input().split())

# Output
sys.stdout.write(string)

# faster
def main():
    pass
main()
```

4.2 Decimal

```python
from decimal import *
getcontext().prec = 2500000
getcontext().Emax = 2500000
a,b = Decimal(input()),Decimal(input())
a*=b
print(a)
```

5 Data Structure

5.1 Segment Tree

```cpp
// Author: Gino
struct node {
    ll sum, add, mod; int ln;
    node(): sum(0), add(0), mod(0), ln(0) {}
};

struct segT {
    int n;
    vector<ll> ar;
    vector<node> st;

    void init(int _n) {
        n = _n;
        reset(ar, n, 0LL);
        reset(st, n*4);
    }
    void pull(int cl, int cr, int i) {
        st[i].sum = st[cl].sum + st[cr].sum;
    }
    void push(int cl, int cr, int i) {
        ll md = st[i].mod, ad = st[i].add;
        if (md) {
            st[cl].sum = md * st[cl].ln, st[cr].sum =
                md * st[cr].ln;
            st[cl].mod = md, st[cr].mod = md;
            st[i].mod = 0;
        }
        if (ad) {
            st[cl].sum += ad * st[cl].ln, st[cr].sum +=
                ad * st[cr].ln;
            st[cl].add += ad, st[cr].add += ad;
            st[i].add = 0;
        }
    }
    void build(int l, int r, int i) {
        if (l == r) {
            st[i].sum = ar[l];
            st[i].ln = 1;
            return;
        }
        int mid = (l+r)>>1, cl = i<<1, cr = i<<1|1;
        build(l, mid, cl);
        build(mid + 1, r, cr);
        pull(cl, cr, i);
        // DONT FORGET THIS
        st[i].ln = st[cl].ln + st[cr].ln;
    }
    void addval(int ql, int qr, ll val, int l, int r,
        int i) {
        if (qr < l || r < ql) return;
        if (ql <= l && r <= qr) {
            st[i].sum += val * st[i].ln;
            st[i].add += val;
            return;
        }
        int mid = (l+r)>>1, cl = i<<1, cr = i<<1|1;
        push(cl, cr, i);
        addval(ql, qr, val, l, mid, cl);
        addval(ql, qr, val, mid + 1, r, cr);
        pull(cl, cr, i);
    }
    void modify(int ql, int qr, ll val, int l, int r,
        int i) {
        if (qr < l || r < ql) return;
        if (ql <= l && r <= qr) {
            st[i].sum = val * st[i].ln;
            st[i].add = 0;
            st[i].mod = val;
            return;
        }
        int mid = (l+r)>>1, cl = i<<1, cr = i<<1|1;
        push(cl, cr, i);
        modify(ql, qr, val, l, mid, cl);
        modify(ql, qr, val, mid+1, r, cr);
        pull(cl, cr, i);
    }
    ll query(int ql, int qr, int l, int r, int i) {
        if (qr < l || r < ql) return 0;
        if (ql <= l && r <= qr) return st[i].sum;
        int mid = (l+r)>>1, cl = i<<1, cr = i<<1|1;
        push(cl, cr, i);
        return (query(ql, qr, l, mid, cl) +
                query(ql, qr, mid+1, r, cr));
    }
};
```

5.2 Heavy Light Decomposition

```cpp
// Author: Ian
void build(V<int>&v);
void modify(int p, int k);
int query(int ql, int qr);
// Insert [ql, qr) segment tree here
inline void solve(){
  int n, q; cin >> n >> q;
  V<int> v(n);
  for (auto& i: v) cin >> i;
  V<V<int>> e(n);
  for(int i = 1; i < n; i++){
    int a, b; cin >> a >> b, a--, b--;
    e[a].emplace_back(b);
    e[b].emplace_back(a);
  }
  V<int> d(n, 0), f(n, 0), sz(n, 1), son(n, -1);
  F<void(int, int)> dfs1 = [&](int x, int pre) {
    for (auto i: e[x]) if (i != pre) {
      d[i] = d[x]+1, f[i] = x;
      dfs1(i, x), sz[x] += sz[i];
      if (son[x] == -1 || sz[son[x]] < sz[i])
        son[x] = i;
    }
  }; dfs1(0,0);
  V<int> top(n, 0), dfn(n, -1);
  F<void(int,int)> dfs2 = [&](int x, int t) {
    static int cnt = 0;
    dfn[x] = cnt++, top[x] = t;
    if (son[x] == -1) return;
    dfs2(son[x], t);
    for (auto i: e[x]) if (!~dfn[i])
      dfs2(i,i);
  }; dfs2(0,0);
  V<int> dfnv(n);
  for (int i = 0; i < n; i++)
    dfnv[dfn[i]] = v[i];
  build(dfnv);
```

```cpp
38   while(q--){
39     int op, a, b, ans; cin >> op >> a >> b;
40     switch(op){
41       case 1:
42         modify(dfn[a-1], b);
43         break;
44       case 2:
45         a--, b--, ans = 0;
46         while (top[a] != top[b]) {
47           if (d[top[a]] > d[top[b]]) swap(a,b);
48           ans = max(ans, query(dfn[top[b]], dfn[b]+1));
49           b = f[top[b]];
50         }
51         if (dfn[a] > dfn[b]) swap(a,b);
52         ans = max(ans, query(dfn[a], dfn[b]+1));
53         cout << ans << endl;
54         break;
55     }
56   }
57 }
```

5.3　Skew Heap

```cpp
1  // Author: Ian
2  // Function: min-heap, with amortized O(lg n) merge
3  struct node {
4    node *l, *r; int v;
5    node(int x): v(x) { l = r = nullptr; }
6  };
7  node* merge(node* a,node* b) {
8    if (!a || !b) return a ?: b;
9    if (a->v > b->v) swap(a, b);
10   return a->r = merge(a->r, b), swap(a->l, a->r), a;
11 }
```

5.4　Leftist Heap

```cpp
1  // Author: Ian
2  // Function: min-heap, with worst-time O(lg n) merge
3  struct node {
4    node *l, *r; int d, v;
5    node(int x): d(1), v(x) { l = r = nullptr; }
6  };
7  static inline int d(node* x) { return x ? x->d : 0; }
8  node* merge(node* a, node* b) {
9    if (!a || !b) return a ?: b;
10   if (a->v>b->v) swap(a,b);
11   a->r = merge(a->r, b);
12   if (d(a->l) < d(a->r))
13     swap(a->l, a->r);
14   a->d = d(a->r) + 1;
15   return a;
16 }
```

5.5　Persistent Treap

```cpp
1  // Author: Ian
2  struct node {
3    node *l, *r;
4    char c; int v, sz;
5    node(char x = '$'): c(x), v(mt()), sz(1) {
6      l = r = nullptr;
7    }
8    node(node* p) {*this = *p;}
9    void pull() {
10     sz = 1;
11     for (auto i : {l, r})
12       if (i) sz += i->sz;
13   }
14 } arr[maxn], *ptr = arr;
15 inline int size(node* p) {return p ? p->sz : 0;}
16 node* merge(node* a, node* b) {
17   if (!a || !b) return a ? : b;
18   if (a->v < b->v) {
19     node* ret = new(ptr++) node(a);
20     ret->r = merge(ret->r, b), ret->pull();
21     return ret;
22   }
23   else {
```

```cpp
24     node* ret = new(ptr++) node(b);
25     ret->l = merge(a, ret->l), ret->pull();
26     return ret;
27   }
28 }
29 P<node*> split(node* p, int k) {
30   if (!p) return {nullptr, nullptr};
31   if (k >= size(p->l) + 1) {
32     auto [a, b] = split(p->r, k - size(p->l) - 1);
33     node* ret = new(ptr++) node(p);
34     ret->r = a, ret->pull();
35     return {ret, b};
36   }
37   else {
38     auto [a, b] = split(p->l, k);
39     node* ret = new(ptr++) node(p);
40     ret->l = b, ret->pull();
41     return {a, ret};
42   }
43 }
```

5.6　Li Chao Tree

```cpp
1  // Author: Ian
2  // Function: For a set of lines L, find the maximum L_i
         (x) in L in O(lg n).
3  typedef long double ld;
4  constexpr int maxn = 5e4 + 5;
5  struct line {
6    ld a, b;
7    ld operator()(ld x) {return a * x + b;}
8  } arr[(maxn + 1) << 2];
9  bool operator<(line a, line b) {return a.a < b.a;}
10 #define m ((l+r)>>1)
11 void insert(line x, int i = 1, int l = 0, int r = maxn)
        {
12   if (r - l == 1) {
13     if (x(l) > arr[i](l))
14       arr[i] = x;
15     return;
16   }
17   line a = max(arr[i], x), b = min(arr[i], x);
18   if (a(m) > b(m))
19     arr[i] = a, insert(b, i << 1, l, m);
20   else
21     arr[i] = b, insert(a, i << 1 | 1, m, r);
22 }
23 ld query(int x, int i = 1, int l = 0, int r = maxn) {
24   if (x < l || r <= x) return -numeric_limits<ld>::max
        ();
25   if (r - l == 1) return arr[i](x);
26   return max({arr[i](x), query(x, i << 1, l, m), query(
        x, i << 1 | 1, m, r)});
27 }
28 #undef m
```

5.7　Time Segment Tree

```cpp
1  // Author: Ian
2  constexpr int maxn = 1e5 + 5;
3  V<P<int>> arr[(maxn + 1) << 2];
4  V<int> dsu, sz;
5  V<tuple<int, int, int>> his;
6  int cnt, q;
7  int find(int x) {
8    return x == dsu[x] ? x : find(dsu[x]);
9  };
10 inline bool merge(int x, int y) {
11   int a = find(x), b = find(y);
12   if (a == b) return false;
13   if (sz[a] > sz[b]) swap(a, b);
14   his.emplace_back(a, b, sz[b]), dsu[a] = b, sz[b] +=
        sz[a];
15   return true;
16 };
17 inline void undo() {
18   auto [a, b, s] = his.back(); his.pop_back();
19   dsu[a] = a, sz[b] = s;
20 }
21 #define m ((l + r) >> 1)
```

```
22  void insert(int ql, int qr, P<int> x, int i = 1, int l
        = 0, int r = q) {
23      // debug(ql, qr, x); return;
24      if (qr <= l || r <= ql) return;
25      if (ql <= l && r <= qr) {arr[i].push_back(x);
            return;}
26      if (qr <= m)
27          insert(ql, qr, x, i << 1, l, m);
28      else if (m <= ql)
29          insert(ql, qr, x, i << 1 | 1, m, r);
30      else {
31          insert(ql, qr, x, i << 1, l, m);
32          insert(ql, qr, x, i << 1 | 1, m, r);
33      }
34  }
35  void traversal(V<int>& ans, int i = 1, int l = 0, int r
        = q) {
36      int opcnt = 0;
37      // debug(i, l, r);
38      for (auto [a, b] : arr[i])
39          if (merge(a, b))
40              opcnt++, cnt--;
41      if (r - l == 1) ans[l] = cnt;
42      else {
43          traversal(ans, i << 1, l, m);
44          traversal(ans, i << 1 | 1, m, r);
45      }
46      while (opcnt--)
47          undo(), cnt++;
48      arr[i].clear();
49  }
50  #undef m
51  inline void solve() {
52      int n, m; cin>>n>>m>>q,q++;
53      dsu.resize(cnt = n), sz.assign(n, 1);
54      iota(dsu.begin(), dsu.end(), 0);
55      // a, b, time, operation
56      unordered_map<ll, V<int>> s;
57      for (int i = 0; i < m; i++) {
58          int a, b; cin>>a>>b;
59          if (a > b) swap(a, b);
60          s[((ll)a << 32) | b].emplace_back(0);
61      }
62      for (int i = 1; i < q; i++) {
63          int op,a, b;
64          cin>>op>>a>>b;
65          if (a > b) swap(a, b);
66          switch (op) {
67          case 1:
68              s[((ll)a << 32) | b].push_back(i);
69              break;
70          case 2:
71              auto tmp = s[((ll)a << 32) | b].back();
72              s[((ll)a << 32) | b].pop_back();
73              insert(tmp, i, P<int> {a, b});
74          }
75      }
76      for (auto [p, v] : s) {
77          int a = p >> 32, b = p & -1;
78          while (v.size()) {
79              insert(v.back(), q, P<int> {a, b});
80              v.pop_back();
81          }
82      }
83      V<int> ans(q);
84      traversal(ans);
85      for (auto i : ans)
86          cout<<i<<' ';
87      cout<<endl;
88  }
```

6 DP

6.1 Aliens

```
1  // Author: Gino
2  // Function: TODO
3  int n; ll k;
4  vector<ll> a;
5  vector<pll> dp[2];
```

```
6  void init() {
7      cin >> n >> k;
8      for (auto& d : dp) d.clear(), d.resize(n);
9      a.clear(); a.resize(n);
10     for (auto& i : a) cin >> i;
11 }
12 pll calc(ll p) {
13     dp[0][0] = make_pair(0, 0);
14     dp[1][0] = make_pair(-a[0], 0);
15     for (int i = 1; i < n; i++) {
16         if (dp[0][i-1].first > dp[1][i-1].first + a[i] - p)
               {
17             dp[0][i] = dp[0][i-1];
18         } else if (dp[0][i-1].first < dp[1][i-1].first + a[
               i] - p) {
19             dp[0][i] = make_pair(dp[1][i-1].first + a[i] - p,
                   dp[1][i-1].second+1);
20         } else {
21             dp[0][i] = make_pair(dp[0][i-1].first, min(dp[0][
                   i-1].second, dp[1][i-1].second+1));
22         }
23         if (dp[0][i-1].first - a[i] > dp[1][i-1].first) {
24             dp[1][i] = make_pair(dp[0][i-1].first - a[i], dp
                   [0][i-1].second);
25         } else if (dp[0][i-1].first - a[i] < dp[1][i-1].
               first) {
26             dp[1][i] = dp[1][i-1];
27         } else {
28             dp[1][i] = make_pair(dp[1][i-1].first, min(dp[0][
                   i-1].second, dp[1][i-1].second));
29         }
30     }
31     return dp[0][n-1];
32 }
33 void solve() {
34     ll l = 0, r = 1e7;
35     pll res = calc(0);
36     if (res.second <= k) return cout << res.first << endl
           , void();
37     while (l < r) {
38         ll mid = (l+r)>>1;
39         res = calc(mid);
40         if (res.second <= k) r = mid;
41         else l = mid+1;
42     }
43     res = calc(l);
44     cout << res.first + k*l << endl;
45 }
```

6.2 SOS DP

```
1  // Author: Gino
2  // Function: Solve problems that enumerates subsets of
       subsets (3^n => n*2^n)
3  for (int msk = 0; msk < (1<<n); msk++) {
4      for (int i = 1; i <= n; i++) {
5          if (msk & (1<<(i - 1))) {
6              // dp[msk][i] = dp[msk][i - 1] + dp[msk ^
                   (1<<(i - 1))][i - 1];
7          } else {
8              // dp[msk][i] = dp[msk][i - 1];
9          }
10     }
11 }
```

7 Graph

7.1 Tree Centroid

```
1  int n;
2  vector<vector<int>> G;
3
4  pii centroid;
5  vector<int> sz, mxcc;  // mxcc[u]: max component size
       after removing u
6
7  void dfs(int u, int p) {
8      sz[u] = 1;
9      for (auto& v : G[u]) {
```

```
10        if (v == p) continue;
11        dfs(v, u);
12        sz[u] += sz[v];
13        mxcc[u] = max(mxcc[u], sz[v]);
14    }
15    mxcc[u] = max(mxcc[u], n - sz[u]);
16 }
17
18 void find_centroid() {
19     centroid = pii{-1, -1};
20     reset(sz, n + 1, 0);
21     reset(mxcc, n + 1, 0);
22     dfs(1, 1);
23     for (int u = 1; u <= n; u++) {
24         if (mxcc[u] <= n / 2) {
25             if (centroid.first != -1) centroid.second =
                     u;
26             else centroid.first = u;
27         }
28     }
29 }
```

7.2 Bellman-Ford + SPFA

```
 1 int n, m;
 2
 3 // Graph
 4 vector<vector<pair<int, ll> > > g;
 5 vector<ll> dis;
 6 vector<bool> negCycle;
 7
 8 // SPFA
 9 vector<int> rlx;
10 queue<int> q;
11 vector<bool> inq;
12 vector<int> pa;
13 void SPFA(vector<int>& src) {
14     dis.assign(n+1, LINF);
15     negCycle.assign(n+1, false);
16     rlx.assign(n+1, 0);
17     while (!q.empty()) q.pop();
18     inq.assign(n+1, false);
19     pa.assign(n+1, -1);
20
21     for (auto& s : src) {
22         dis[s] = 0;
23         q.push(s); inq[s] = true;
24     }
25
26     while (!q.empty()) {
27         int u = q.front();
28         q.pop(); inq[u] = false;
29         if (rlx[u] >= n) {
30             negCycle[u] = true;
31         }
32         else for (auto& e : g[u]) {
33             int v = e.first;
34             ll w = e.second;
35             if (dis[v] > dis[u] + w) {
36                 dis[v] = dis[u] + w;
37                 rlx[v] = rlx[u] + 1;
38                 pa[v] = u;
39                 if (!inq[v]) {
40                     q.push(v);
41                     inq[v] = true;
42 } } } } }
43
44
45 // Bellman-Ford
46 queue<int> q;
47 vector<int> pa;
48 void BellmanFord(vector<int>& src) {
49     dis.assign(n+1, LINF);
50     negCycle.assign(n+1, false);
51     pa.assign(n+1, -1);
52
53     for (auto& s : src) dis[s] = 0;
54
55     for (int rlx = 1; rlx <= n; rlx++) {
56         for (int u = 1; u <= n; u++) {
```

```
57            if (dis[u] == LINF) continue; // Important
                   !!
58            for (auto& e : g[u]) {
59                int v = e.first; ll w = e.second;
60                if (dis[v] > dis[u] + w) {
61                    dis[v] = dis[u] + w;
62                    pa[v] = u;
63                    if (rlx == n) negCycle[v] = true;
64 } } } } }
65
66
67 // Negative Cycle Detection
68 void NegCycleDetect() {
69 /* No Neg Cycle: NO
70 Exist Any Neg Cycle:
71 YES
72 v0 v1 v2 ... vk v0 */
73
74     vector<int> src;
75     for (int i = 1; i <= n; i++)
76         src.emplace_back(i);
77
78     SPFA(src);
79     // BellmanFord(src);
80
81     int ptr = -1;
82     for (int i = 1; i <= n; i++) if (negCycle[i])
83         { ptr = i; break; }
84
85     if (ptr == -1) { return cout << "NO" << endl, void
               (); }
86
87     cout << "YES\n";
88     vector<int> ans;
89     vector<bool> vis(n+1, false);
90
91     while (true) {
92         ans.emplace_back(ptr);
93         if (vis[ptr]) break;
94         vis[ptr] = true;
95         ptr = pa[ptr];
96     }
97     reverse(ans.begin(), ans.end());
98
99     vis.assign(n+1, false);
100    for (auto& x : ans) {
101        cout << x << ' ';
102        if (vis[x]) break;
103        vis[x] = true;
104    }
105    cout << endl;
106 }
107
108 // Distance Calculation
109 void calcDis(int s) {
110    vector<int> src;
111    src.emplace_back(s);
112    SPFA(src);
113    // BellmanFord(src);
114
115    while (!q.empty()) q.pop();
116    for (int i = 1; i <= n; i++)
117        if (negCycle[i]) q.push(i);
118
119    while (!q.empty()) {
120        int u = q.front(); q.pop();
121        for (auto& e : g[u]) {
122            int v = e.first;
123            if (!negCycle[v]) {
124                q.push(v);
125                negCycle[v] = true;
126 } } } }
```

7.3 BCC - AP

```
 1 int n, m;
 2 int low[maxn], dfn[maxn], instp;
 3 vector<int> E, g[maxn];
 4 bitset<maxn> isap;
 5 bitset<maxm> vis;
 6 stack<int> stk;
```

```cpp
7    int bccnt;
8    vector<int> bcc[maxn];
9    inline void popout(int u) {
10     bccnt++;
11     bcc[bccnt].emplace_back(u);
12     while (!stk.empty()) {
13       int v = stk.top();
14       if (u == v) break;
15       stk.pop();
16       bcc[bccnt].emplace_back(v);
17     }
18   }
19   void dfs(int u, bool rt = 0) {
20     stk.push(u);
21     low[u] = dfn[u] = ++instp;
22     int kid = 0;
23     Each(e, g[u]) {
24       if (vis[e]) continue;
25       vis[e] = true;
26       int v = E[e]^u;
27       if (!dfn[v]) {
28         // tree edge
29         kid++; dfs(v);
30         low[u] = min(low[u], low[v]);
31         if (!rt && low[v] >= dfn[u]) {
32           // bcc found: u is ap
33           isap[u] = true;
34           popout(u);
35         }
36       } else {
37         // back edge
38         low[u] = min(low[u], dfn[v]);
39       }
40     }
41     // special case: root
42     if (rt) {
43       if (kid > 1) isap[u] = true;
44       popout(u);
45     }
46   }
47   void init() {
48     cin >> n >> m;
49     fill(low, low+maxn, INF);
50     REP(i, m) {
51       int u, v;
52       cin >> u >> v;
53       g[u].emplace_back(i);
54       g[v].emplace_back(i);
55       E.emplace_back(u^v);
56     }
57   }
58   void solve() {
59     FOR(i, 1, n+1, 1) {
60       if (!dfn[i]) dfs(i, true);
61     }
62     vector<int> ans;
63     int cnt = 0;
64     FOR(i, 1, n+1, 1) {
65       if (isap[i]) cnt++, ans.emplace_back(i);
66     }
67     cout << cnt << endl;
68     Each(i, ans) cout << i << ' ';
69     cout << endl;
70   }
```

7.4 BCC - Bridge

```cpp
1    int n, m;
2    vector<int> g[maxn], E;
3    int low[maxn], dfn[maxn], instp;
4    int bccnt, bccid[maxn];
5    stack<int> stk;
6    bitset<maxm> vis, isbrg;
7    void init() {
8      cin >> n >> m;
9      REP(i, m) {
10       int u, v;
11       cin >> u >> v;
12       E.emplace_back(u^v);
13       g[u].emplace_back(i);
14       g[v].emplace_back(i);
```

```cpp
15       }
16     fill(low, low+maxn, INF);
17   }
18   void popout(int u) {
19     bccnt++;
20     while (!stk.empty()) {
21       int v = stk.top();
22       if (v == u) break;
23       stk.pop();
24       bccid[v] = bccnt;
25     }
26   }
27   void dfs(int u) {
28     stk.push(u);
29     low[u] = dfn[u] = ++instp;
30
31     Each(e, g[u]) {
32       if (vis[e]) continue;
33       vis[e] = true;
34
35       int v = E[e]^u;
36       if (dfn[v]) {
37         // back edge
38         low[u] = min(low[u], dfn[v]);
39       } else {
40         // tree edge
41         dfs(v);
42         low[u] = min(low[u], low[v]);
43         if (low[v] == dfn[v]) {
44           isbrg[e] = true;
45           popout(u);
46         }
47       }
48     }
49   }
50   void solve() {
51     FOR(i, 1, n+1, 1) {
52       if (!dfn[i]) dfs(i);
53     }
54     vector<pii> ans;
55     vis.reset();
56     FOR(u, 1, n+1, 1) {
57       Each(e, g[u]) {
58         if (!isbrg[e] || vis[e]) continue;
59         vis[e] = true;
60         int v = E[e]^u;
61         ans.emplace_back(mp(u, v));
62       }
63     }
64     cout << (int)ans.size() << endl;
65     Each(e, ans) cout << e.F << ' ' << e.S << endl;
66   }
```

7.5 SCC - Tarjan

```cpp
1    // 2-SAT
2    vector<int> E, g[maxn];  // 1~n, n+1~2n
3    int low[maxn], in[maxn], instp;
4    int sccnt, sccid[maxn];
5
6    stack<int> stk;
7    bitset<maxn> ins, vis;
8
9    int n, m;
10
11   void init() {
12     cin >> m >> n;
13     E.clear();
14     fill(g, g+maxn, vector<int>());
15     fill(low, low+maxn, INF);
16     memset(in, 0, sizeof(in));
17     instp = 1;
18     sccnt = 0;
19     memset(sccid, 0, sizeof(sccid));
20     ins.reset();
21     vis.reset();
22   }
23
24   inline int no(int u) {
25     return (u > n ? u-n : u+n);
26   }
```

```
27
28  int ecnt = 0;
29  inline void clause(int u, int v) {
30      E.eb(no(u)^v);
31      g[no(u)].eb(ecnt++);
32      E.eb(no(v)^u);
33      g[no(v)].eb(ecnt++);
34  }
35
36  void dfs(int u) {
37      in[u] = instp++;
38      low[u] = in[u];
39      stk.push(u);
40      ins[u] = true;
41
42      Each(e, g[u]) {
43          if (vis[e]) continue;
44          vis[e] = true;
45
46          int v = E[e]^u;
47          if (ins[v]) low[u] = min(low[u], in[v]);
48          else if (!in[v]) {
49              dfs(v);
50              low[u] = min(low[u], low[v]);
51          }
52      }
53
54      if (low[u] == in[u]) {
55          sccnt++;
56          while (!stk.empty()) {
57              int v = stk.top();
58              stk.pop();
59              ins[v] = false;
60              sccid[v] = sccnt;
61              if (u == v) break;
62          }
63      }
64  }
65
66
67  int main() {
68      WiwiHorz
69      init();
70
71      REP(i, m) {
72          char su, sv;
73          int u, v;
74          cin >> su >> u >> sv >> v;
75          if (su == '-') u = no(u);
76          if (sv == '-') v = no(v);
77          clause(u, v);
78      }
79
80      FOR(i, 1, 2*n+1, 1) {
81          if (!in[i]) dfs(i);
82      }
83
84      FOR(u, 1, n+1, 1) {
85          int du = no(u);
86          if (sccid[u] == sccid[du]) {
87              return cout << "IMPOSSIBLE\n", 0;
88          }
89      }
90
91      FOR(u, 1, n+1, 1) {
92          int du = no(u);
93          cout << (sccid[u] < sccid[du] ? '+' : '-') << ' ';
94      }
95      cout << endl;
96
97      return 0;
98  }
```

7.6 Eulerian Path - Undir

```
1  // from 1 to n
2  #define gg return cout << "IMPOSSIBLE\n", void();
3
4  int n, m;
5  vector<int> g[maxn];
6  bitset<maxn> inodd;
7
8  void init() {
9  cin >> n >> m;
10 inodd.reset();
11 for (int i = 0; i < m; i++) {
12   int u, v; cin >> u >> v;
13   inodd[u] = inodd[u] ^ true;
14   inodd[v] = inodd[v] ^ true;
15   g[u].emplace_back(v);
16   g[v].emplace_back(u);
17 } }
18 stack<int> stk;
19 void dfs(int u) {
20     while (!g[u].empty()) {
21         int v = g[u].back();
22         g[u].pop_back();
23         dfs(v);
24     }
25 stk.push(u);}}
```

7.7 Eulerian Path - Dir

```
1  // from node 1 to node n
2  #define gg return cout << "IMPOSSIBLE\n", 0
3
4  int n, m;
5  vector<int> g[maxn];
6  stack<int> stk;
7  int in[maxn], out[maxn];
8
9  void init() {
10 cin >> n >> m;
11 for (int i = 0; i < m; i++) {
12   int u, v; cin >> u >> v;
13   g[u].emplace_back(v);
14   out[u]++, in[v]++;
15 }
16 for (int i = 1; i <= n; i++) {
17   if (i == 1 && out[i]-in[i] != 1) gg;
18   if (i == n && in[i]-out[i] != 1) gg;
19   if (i != 1 && i != n && in[i] != out[i]) gg;
20 } }
21 void dfs(int u) {
22     while (!g[u].empty()) {
23         int v = g[u].back();
24         g[u].pop_back();
25         dfs(v);
26     }
27     stk.push(u);
28 }
29 void solve() {
30   dfs(1)
31     for (int i = 1; i <= n; i++)
32         if ((int)g[i].size()) gg;
33     while (!stk.empty()) {
34         int u = stk.top();
35         stk.pop();
36         cout << u << ' ';
37 } }
```

7.8 Hamilton Path

```
1  // top down DP
2  // Be Aware Of Multiple Edges
3  int n, m;
4  ll dp[maxn][1<<maxn];
5  int adj[maxn][maxn];
6
7  void init() {
8      cin >> n >> m;
9      fill(dp[0], dp[maxn-1]+(1<<maxn), -1);
10 }
11
12 void DP(int i, int msk) {
13     if (dp[i][msk] != -1) return;
14     dp[i][msk] = 0;
15     REP(j, n) if (j != i && (msk & (1<<j)) && adj[j][i]) {
16         int sub = msk ^ (1<<i);
```

```
17        if (dp[j][sub] == -1) DP(j, sub);
18        dp[i][msk] += dp[j][sub] * adj[j][i];
19        if (dp[i][msk] >= MOD) dp[i][msk] %= MOD;
20      }
21 }
22
23
24 int main() {
25     WiwiHorz
26     init();
27
28     REP(i, m) {
29         int u, v;
30         cin >> u >> v;
31         if (u == v) continue;
32         adj[--u][--v]++;
33     }
34
35     dp[0][1] = 1;
36     FOR(i, 1, n, 1) {
37         dp[i][1] = 0;
38         dp[i][1|(1<<i)] = adj[0][i];
39     }
40     FOR(msk, 1, (1<<n), 1) {
41         if (msk == 1) continue;
42         dp[0][msk] = 0;
43     }
44
45
46     DP(n-1, (1<<n)-1);
47     cout << dp[n-1][(1<<n)-1] << endl;
48
49     return 0;
50 }
```

7.9 Kth Shortest Path

```
1  // time: O(|E| \lg |E|+|V| \lg |V|+K)
2  // memory: O(|E| \lg |E|+|V|)
3  struct KSP{ // 1-base
4    struct nd{
5      int u,v; ll d;
6      nd(int ui=0,int vi=0,ll di=INF){ u=ui; v=vi; d=di;
           }
7    };
8    struct heap{ nd* edge; int dep; heap* chd[4]; };
9    static int cmp(heap* a,heap* b)
10   { return a->edge->d > b->edge->d; }
11   struct node{
12     int v; ll d; heap* H; nd* E;
13     node(){}
14     node(ll _d,int _v,nd* _E){ d =_d; v=_v; E=_E; }
15     node(heap* _H,ll _d){ H=_H; d=_d; }
16     friend bool operator<(node a,node b)
17     { return a.d>b.d; }
18   };
19   int n,k,s,t,dst[N]; nd *nxt[N];
20   vector<nd*> g[N],rg[N]; heap *nullNd,*head[N];
21   void init(int _n,int _k,int _s,int _t){
22     n=_n; k=_k; s=_s; t=_t;
23     for(int i=1;i<=n;i++){
24       g[i].clear(); rg[i].clear();
25       nxt[i]=NULL; head[i]=NULL; dst[i]=-1;
26     }
27   }
28   void addEdge(int ui,int vi,ll di){
29     nd* e=new nd(ui,vi,di);
30     g[ui].push_back(e); rg[vi].push_back(e);
31   }
32   queue<int> dfsQ;
33   void dijkstra(){
34     while(dfsQ.size()) dfsQ.pop();
35     priority_queue<node> Q; Q.push(node(0,t,NULL));
36     while (!Q.empty()){
37       node p=Q.top(); Q.pop(); if(dst[p.v]!=-1)continue
             ;
38       dst[p.v]=p.d; nxt[p.v]=p.E; dfsQ.push(p.v);
39       for(auto e:rg[p.v]) Q.push(node(p.d+e->d,e->u,e))
             ;
40     }
41   }
```

```
42   heap* merge(heap* curNd,heap* newNd){
43     if(curNd==nullNd) return newNd;
44     heap* root=new heap;memcpy(root,curNd,sizeof(heap))
           ;
45     if(newNd->edge->d<curNd->edge->d){
46       root->edge=newNd->edge;
47       root->chd[2]=newNd->chd[2];
48       root->chd[3]=newNd->chd[3];
49       newNd->edge=curNd->edge;
50       newNd->chd[2]=curNd->chd[2];
51       newNd->chd[3]=curNd->chd[3];
52     }
53     if(root->chd[0]->dep<root->chd[1]->dep)
54       root->chd[0]=merge(root->chd[0],newNd);
55     else root->chd[1]=merge(root->chd[1],newNd);
56     root->dep=max(root->chd[0]->dep,
57             root->chd[1]->dep)+1;
58     return root;
59   }
60   vector<heap*> V;
61   void build(){
62     nullNd=new heap; nullNd->dep=0; nullNd->edge=new nd
           ;
63     fill(nullNd->chd,nullNd->chd+4,nullNd);
64     while(not dfsQ.empty()){
65       int u=dfsQ.front(); dfsQ.pop();
66       if(!nxt[u]) head[u]=nullNd;
67       else head[u]=head[nxt[u]->v];
68       V.clear();
69       for(auto&& e:g[u]){
70         int v=e->v;
71         if(dst[v]==-1) continue;
72         e->d+=dst[v]-dst[u];
73         if(nxt[u]!=e){
74           heap* p=new heap;fill(p->chd,p->chd+4,nullNd)
                 ;
75           p->dep=1; p->edge=e; V.push_back(p);
76         }
77       }
78       if(V.empty()) continue;
79       make_heap(V.begin(),V.end(),cmp);
80 #define L(X) ((X<<1)+1)
81 #define R(X) ((X<<1)+2)
82       for(size_t i=0;i<V.size();i++){
83         if(L(i)<V.size()) V[i]->chd[2]=V[L(i)];
84         else V[i]->chd[2]=nullNd;
85         if(R(i)<V.size()) V[i]->chd[3]=V[R(i)];
86         else V[i]->chd[3]=nullNd;
87       }
88       head[u]=merge(head[u],V.front());
89     }
90   }
91   vector<ll> ans;
92   void first_K(){
93     ans.clear(); priority_queue<node> Q;
94     if(dst[s]==-1) return;
95     ans.push_back(dst[s]);
96     if(head[s]!=nullNd)
97       Q.push(node(head[s],dst[s]+head[s]->edge->d));
98     for(int _=1;_<k and not Q.empty();_++){
99       node p=Q.top(),q; Q.pop(); ans.push_back(p.d);
100      if(head[p.H->edge->v]!=nullNd){
101        q.H=head[p.H->edge->v]; q.d=p.d+q.H->edge->d;
102        Q.push(q);
103      }
104      for(int i=0;i<4;i++)
105        if(p.H->chd[i]!=nullNd){
106          q.H=p.H->chd[i];
107          q.d=p.d-p.H->edge->d+p.H->chd[i]->edge->d;
108          Q.push(q);
109 } } }
110  void solve(){ // ans[i] stores the i-th shortest path
111    dijkstra(); build();
112    first_K(); // ans.size() might less than k
113  }
114 } solver;
```

7.10 System of Difference Constraints

```
1 vector<vector<pair<int, ll>>> G;
2 void add(int u, int v, ll w) {
```

```
3      G[u].emplace_back(make_pair(v, w));
4  }
```

- $x_u - x_v \le c \Rightarrow$ add(v, u, c)

- $x_u - x_v \ge c \Rightarrow$ add(u, v, -c)

- $x_u - x_v = c \Rightarrow$ add(v, u, c), add(u, v -c)

- $x_u \ge c \Rightarrow$ add super vertex $x_0 = 0$, then $x_u - x_0 \ge c \Rightarrow$ add(u, 0, -c)

- Don't for get non-negative constraints for every variable if specified implicitly.

- Interval sum $\Rightarrow$ Use prefix sum to transform into differential constraints. Don't for get $S_{i+1} - S_i \ge 0$ if x_i needs to be non-negative.

- $\frac{x_u}{x_v} \le c \Rightarrow \log x_u - \log x_v \le \log c$

8 String

8.1 Rolling Hash

```cpp
1  const ll C = 27;
2  inline int id(char c) {return c-'a'+1;}
3  struct RollingHash {
4      string s; int n; ll mod;
5      vector<ll> Cexp, hs;
6      RollingHash(string& _s, ll _mod):
7          s(_s), n((int)_s.size()), mod(_mod)
8      {
9          Cexp.assign(n, 0);
10         hs.assign(n, 0);
11         Cexp[0] = 1;
12         for (int i = 1; i < n; i++) {
13             Cexp[i] = Cexp[i-1] * C;
14             if (Cexp[i] >= mod) Cexp[i] %= mod;
15         }
16         hs[0] = id(s[0]);
17         for (int i = 1; i < n; i++) {
18             hs[i] = hs[i-1] * C + id(s[i]);
19             if (hs[i] >= mod) hs[i] %= mod;
20     } }
21     inline ll query(int l, int r) {
22         ll res = hs[r] - (l ? hs[l-1] * Cexp[r-l+1] :
             0);
23         res = (res % mod + mod) % mod;
24         return res; }
25 };
```

8.2 Trie

```cpp
1  struct node {
2      int c[26]; ll cnt;
3      node(): cnt(0) {memset(c, 0, sizeof(c));}
4      node(ll x): cnt(x) {memset(c, 0, sizeof(c));}
5  };
6  struct Trie {
7      vector<node> t;
8      void init() {
9          t.clear();
10         t.emplace_back(node());
11     }
12     void insert(string s) { int ptr = 0;
13         for (auto& i : s) {
14             if (!t[ptr].c[i-'a']) {
15                 t.emplace_back(node());
16                 t[ptr].c[i-'a'] = (int)t.size()-1; }
17             ptr = t[ptr].c[i-'a']; }
18         t[ptr].cnt++; }
19 } trie;
```

8.3 KMP

```cpp
1  int n, m;
2  string s, p;
3  vector<int> f;
4  void build() {
5    f.clear(); f.resize(m, 0);
6    int ptr = 0; for (int i = 1; i < m; i++) {
7      while (ptr && p[i] != p[ptr]) ptr = f[ptr-1];
8      if (p[i] == p[ptr]) ptr++;
9      f[i] = ptr;
10 }}
11 void init() {
12   cin >> s >> p;
13   n = (int)s.size();
14   m = (int)p.size();
15   build(); }
16 void solve() {
17   int ans = 0, pi = 0;
18   for (int si = 0; si < n; si++) {
19     while (pi && s[si] != p[pi]) pi = f[pi-1];
20     if (s[si] == p[pi]) pi++;
21     if (pi == m) ans++, pi = f[pi-1];
22   }
23 cout << ans << endl; }
```

8.4 Z Value

```cpp
1  string is, it, s;
2  int n; vector<int> z;
3  void init() {
4      cin >> is >> it;
5      s = it+'0'+is;
6      n = (int)s.size();
7      z.resize(n, 0); }
8  void solve() {
9      int ans = 0; z[0] = n;
10     for (int i = 1, l = 0, r = 0; i < n; i++) {
11         if (i <= r) z[i] = min(z[i-l], r-i+1);
12         while (i+z[i] < n && s[z[i]] == s[i+z[i]]) z[i
             ]++;
13         if (i+z[i]-1 > r) l = i, r = i+z[i]-1;
14         if (z[i] == (int)it.size()) ans++;
15     }
16     cout << ans << endl; }
```

8.5 Manacher

```cpp
1  int n; string S, s;
2  vector<int> m;
3  void manacher() {
4  s.clear(); s.resize(2*n+1, '.');
5  for (int i = 0, j = 1; i < n; i++, j += 2) s[j] = S[i];
6  m.clear(); m.resize(2*n+1, 0);
7  // m[i] := max k such that s[i-k, i+k] is palindrome
8  int mx = 0, mxk = 0;
9  for (int i = 1; i < 2*n+1; i++) {
10   if (mx-(i-mx) >= 0) m[i] = min(m[mx-(i-mx)], mx+mxk-i
        );
11   while (0 <= i-m[i]-1 && i+m[i]+1 < 2*n+1 &&
12       s[i-m[i]-1] == s[i+m[i]+1]) m[i]++;
13   if (i+m[i] > mx+mxk) mx = i, mxk = m[i];
14 } }
15 void init() { cin >> S; n = (int)S.size(); }
16 void solve() {
17   manacher();
18   int mx = 0, ptr = 0;
19   for (int i = 0; i < 2*n+1; i++) if (mx < m[i])
20   { mx = m[i]; ptr = i; }
21   for (int i = ptr-mx; i <= ptr+mx; i++)
22     if (s[i] != '.') cout << s[i];
23 cout << endl; }
```

8.6 Suffix Array

```cpp
1  #define F first
2  #define S second
3  struct SuffixArray {  // don't forget s += "$";
4      int n; string s;
```

```
 5     vector<int> suf, lcp, rk;
 6     vector<int> cnt, pos;
 7     vector<pair<pii, int> > buc[2];
 8     void init(string _s) {
 9         s = _s; n = (int)s.size();
10 // resize(n): suf, rk, cnt, pos, lcp, buc[0~1]
11     }
12     void radix_sort() {
13         for (int t : {0, 1}) {
14             fill(cnt.begin(), cnt.end(), 0);
15             for (auto& i : buc[t]) cnt[ (t ? i.F.F : i.
                  F.S) ]++;
16             for (int i = 0; i < n; i++)
17                 pos[i] = (!i ? 0 : pos[i-1] + cnt[i-1])
                      ;
18             for (auto& i : buc[t])
19                 buc[t^1][pos[ (t ? i.F.F : i.F.S) ]++]
                      = i;
20     }}
21     bool fill_suf() {
22         bool end = true;
23         for (int i = 0; i < n; i++) suf[i] = buc[0][i].
              S;
24         rk[suf[0]] = 0;
25         for (int i = 1; i < n; i++) {
26             int dif = (buc[0][i].F != buc[0][i-1].F);
27             end &= dif;
28             rk[suf[i]] = rk[suf[i-1]] + dif;
29         } return end;
30     }
31     void sa() {
32         for (int i = 0; i < n; i++)
33             buc[0][i] = make_pair(make_pair(s[i], s[i])
                  , i);
34         sort(buc[0].begin(), buc[0].end());
35         if (fill_suf()) return;
36         for (int k = 0; (1<<k) < n; k++) {
37             for (int i = 0; i < n; i++)
38                 buc[0][i] = make_pair(make_pair(rk[i],
                      rk[(i + (1<<k)) % n]), i);
39             radix_sort();
40             if (fill_suf()) return;
41     }}
42     void LCP() {  int k = 0;
43         for (int i = 0; i < n-1; i++) {
44             if (rk[i] == 0) continue;
45             int pi = rk[i];
46             int j = suf[pi-1];
47             while (i+k < n && j+k < n && s[i+k] == s[j+
                  k]) k++;
48             lcp[pi] = k;
49             k = max(k-1, 0);
50     }}
51 };
52 SuffixArray suffixarray;
```

8.7 SA-IS

```
 1 const int N=300010;
 2 struct SA{
 3 #define REP(i,n) for(int i=0;i<int(n);i++)
 4 #define REP1(i,a,b) for(int i=(a);i<=int(b);i++)
 5   bool _t[N*2]; int _s[N*2],_sa[N*2];
 6   int _c[N*2],x[N],_p[N],_q[N*2],hei[N],r[N];
 7   int operator [](int i){ return _sa[i]; }
 8   void build(int *s,int n,int m){
 9     memcpy(_s,s,sizeof(int)*n);
10     sais(_s,_sa,_p,_q,_t,_c,n,m); mkhei(n);
11   }
12   void mkhei(int n){
13     REP(i,n) r[_sa[i]]=i;
14     hei[0]=0;
15     REP(i,n) if(r[i]) {
16       int ans=i>0?max(hei[r[i-1]]-1,0):0;
17       while(_s[i+ans]==_s[_sa[r[i]-1]+ans]) ans++;
18       hei[r[i]]=ans;
19     }
20   }
21   void sais(int *s,int *sa,int *p,int *q,bool *t,int *c
        ,int n,int z){
22     bool uniq=t[n-1]=true,neq;
```

```
23     int nn=0,nmxz=-1,*nsa=sa+n,*ns=s+n,lst=-1;
24 #define MS0(x,n) memset((x),0,n*sizeof(*(x)))
25 #define MAGIC(XD) MS0(sa,n);\
26 memcpy(x,c,sizeof(int)*z); XD;\
27 memcpy(x+1,c,sizeof(int)*(z-1));\
28 REP(i,n) if(sa[i]&&!t[sa[i]-1]) sa[x[s[sa[i]-1]]++]=sa[
      i]-1;\
29 memcpy(x,c,sizeof(int)*z);\
30 for(int i=n-1;i>=0;i--) if(sa[i]&&t[sa[i]-1]) sa[--x[s[
      sa[i]-1]]]=sa[i]-1;
31     MS0(c,z); REP(i,n) uniq&=++c[s[i]]<2;
32     REP(i,z-1) c[i+1]+=c[i];
33     if(uniq) { REP(i,n) sa[--c[s[i]]]=i; return; }
34     for(int i=n-2;i>=0;i--)
35       t[i]=(s[i]==s[i+1]?t[i+1]:s[i]<s[i+1]);
36     MAGIC(REP1(i,1,n-1) if(t[i]&&!t[i-1]) sa[--x[s[i
        ]]]=p[q[i]=nn++]=i);
37     REP(i,n) if(sa[i]&&t[sa[i]]&&!t[sa[i]-1]){
38       neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
          [i])*sizeof(int));
39       ns[q[lst=sa[i]]]=nmxz+=neq;
40     }
41     sais(ns,nsa,p+nn,q+n,t+n,c+z,nn,nmxz+1);
42     MAGIC(for(int i=nn-1;i>=0;i--) sa[--x[s[p[nsa[i
        ]]]]]=p[nsa[i]]);
43   }
44 }sa;
45 int H[N],SA[N],RA[N];
46 void suffix_array(int* ip,int len){
47   // should padding a zero in the back
48   // ip is int array, len is array length
49   // ip[0..n-1] != 0, and ip[len]=0
50   ip[len++]=0; sa.build(ip,len,128);
51   memcpy(H,sa.hei+1,len<<2); memcpy(SA,sa._sa+1,len<<2)
        ;
52   for(int i=0;i<len;i++) RA[i]=sa.r[i]-1;
53   // resulting height, sa array \in [0,len)
54 }
```

8.8 Minimum Rotation

```
 1 //rotate(begin(s), begin(s)+minRotation(s), end(s))
 2 int minRotation(string s) {
 3 int a = 0, n = s.size(); s += s;
 4 for(int b = 0; b < n; b++) for(int k = 0; k < n; k++) {
 5     if(a + k == b ||| s[a + k] < s[b + k]) {
 6         b += max(0, k - 1);
 7         break; }
 8     if(s[a + k] > s[b + k]) {
 9         a = b;
10         break;
11     } }
12 return a; }
```

8.9 Aho Corasick

```
 1 struct ACautomata{
 2   struct Node{
 3     int cnt;
 4     Node *go[26], *fail, *dic;
 5     Node (){
 6       cnt = 0; fail = 0; dic=0;
 7       memset(go,0,sizeof(go));
 8     }
 9   }pool[1048576],*root;
10   int nMem;
11   Node* new_Node(){
12     pool[nMem] = Node();
13     return &pool[nMem++];
14   }
15   void init() { nMem = 0; root = new_Node(); }
16   void add(const string &str) { insert(root,str,0); }
17   void insert(Node *cur, const string &str, int pos){
18     for(int i=pos;i<str.size();i++){
19       if(!cur->go[str[i]-'a'])
20         cur->go[str[i]-'a'] = new_Node();
21       cur=cur->go[str[i]-'a'];
22     }
23     cur->cnt++;
24   }
```

```cpp
25    void make_fail(){
26      queue<Node*> que;
27      que.push(root);
28      while (!que.empty()){
29        Node* fr=que.front(); que.pop();
30        for (int i=0; i<26; i++){
31          if (fr->go[i]){
32            Node *ptr = fr->fail;
33            while (ptr && !ptr->go[i]) ptr = ptr->fail;
34            fr->go[i]->fail=ptr=(ptr?ptr->go[i]:root);
35            fr->go[i]->dic=(ptr->cnt?ptr:ptr->dic);
36            que.push(fr->go[i]);
37    } } } }
38 }AC;
```

9 Geometry

9.1 Basic Operations

```cpp
1  // Author: Gino
2  typedef long long T;
3  // typedef long double T;
4  const long double eps = 1e-8;
5
6  short sgn(T x) {
7      if (abs(x) < eps) return 0;
8      return x < 0 ? -1 : 1;
9  }
10
11 struct Pt {
12 T x, y;
13 Pt(T _x=0, T _y=0):x(_x), y(_y) {}
14 Pt operator+(Pt a) { return Pt(x+a.x, y+a.y); }
15 Pt operator-(Pt a) { return Pt(x-a.x, y-a.y); }
16 Pt operator*(T a)  { return Pt(x*a, y*a); }
17 Pt operator/(T a)  { return Pt(x/a, y/a); }
18 T operator*(Pt a)  { return x*a.x + y*a.y; }
19 T operator^(Pt a)  { return x*a.y - y*a.x; }  // 不要打
   反
20 bool operator<(Pt a)
21     { return x < a.x || (x == a.x && y < a.y); }
22 //return sgn(x-a.x) < 0 || (sgn(x-a.x) == 0 && sgn(y-a.
   y) < 0); }
23 bool operator==(Pt a)
24     { return sgn(x-a.x) == 0 && sgn(y-a.y) == 0; }
25 };
26
27 Pt mv(Pt a, Pt b) { return b-a; }
28 T len2(Pt a) { return a*a; }
29 T dis2(Pt a, Pt b) { return len2(b-a); }
30
31 short ori(Pt a, Pt b) { return ((a^b)>0) - ((a^b)<0); }
32 bool onseg(Pt p, Pt l1, Pt l2) {
33     Pt a = mv(p, l1), b = mv(p, l2);
34     return ((a^b) == 0) && ((a*b) <= 0);
35 }
```

9.2 InPoly

```cpp
1  // Author: Gino
2  // Function: Check if a point P sits in a polygon (
      doesn't have to be convex hull)
3  // 0 = Bound, 1 = In, -1 = Out
4  short inPoly(Pt p) {
5  for (int i = 0; i < n; i++)
6      if (onseg(p, E[i], E[(i+1)%n])) return 0;
7  int cnt = 0;
8  for (int i = 0; i < n; i++)
9      if (banana(p, Pt(p.x+1, p.y+2e9), E[i], E[(i+1)%n])
          )
10         cnt ^= 1;
11 return (cnt ? 1 : -1);
12 }
```

9.3 Sort by Angle

```cpp
1  // Author: Gino
2  int ud(Pt a) {  // up or down half plane
```

```cpp
3      if (a.y > 0) return 0;
4      if (a.y < 0) return 1;
5      return (a.x >= 0 ? 0 : 1);
6  }
7  sort(ALL(E), [&](const Pt& a, const Pt& b){
8      if (ud(a) != ud(b)) return ud(a) < ud(b);
9      return (a^b) > 0;
10 });
```

9.4 Line Intersect Check

```cpp
1  // Author: Gino
2  // Function: check if (p1---p2) (q1---q2) banana
3  inline bool banana(Pt p1, Pt p2, Pt q1, Pt q2) {
4  if (onseg(p1, q1, q2) || onseg(p2, q1, q2) ||
5      onseg(q1, p1, p2) || onseg(q2, p1, p2)) {
6      return true;
7  }
8  Pt p = mv(p1, p2), q = mv(q1, q2);
9  return (ori(p, mv(p1, q1)) * ori(p, mv(p1, q2)) < 0 &&
10         ori(q, mv(q1, p1)) * ori(q, mv(q1, p2)) < 0);
11 }
```

9.5 Line Intersection

```cpp
1  // Author: Gino
2  // T: long double
3  Pt bananaPoint(Pt p1, Pt p2, Pt q1, Pt q2) {
4  if (onseg(q1, p1, p2)) return q1;
5  if (onseg(q2, p1, p2)) return q2;
6  if (onseg(p1, q1, q2)) return p1;
7  if (onseg(p2, q1, q2)) return p2;
8  double s = abs(mv(p1, p2) ^ mv(p1, q1));
9  double t = abs(mv(p1, p2) ^ mv(p1, q2));
10 return q2 * (s/(s+t)) + q1 * (t/(s+t));
11 }
```

9.6 Convex Hull

```cpp
1  // Author: Gino
2  vector<Pt> hull;
3  void convexHull() {
4  hull.clear(); sort(E.begin(), E.end());
5  for (int t : {0, 1}) {
6      int b = (int)hull.size();
7      for (auto& ei : E) {
8          while ((int)hull.size() - b >= 2 &&
9                  ori(mv(hull[(int)hull.size()-2], hull.
                    back()),
10                     mv(hull[(int)hull.size()-2], ei)) ==
                       -1) {
11             hull.pop_back();
12         }
13         hull.emplace_back(ei);
14     }
15     hull.pop_back();
16     reverse(E.begin(), E.end());
17 } }
```

9.7 Lower Concave Hull

```cpp
1  // Author: Unknown
2  struct Line {
3    mutable ll m, b, p;
4    bool operator<(const Line& o) const { return m < o.m;
         }
5    bool operator<(ll x) const { return p < x; }
6  };
7
8  struct LineContainer : multiset<Line, less<>> {
9    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
10   const ll inf = LLONG_MAX;
11   ll div(ll a, ll b) { // floored division
12     return a / b - ((a ^ b) < 0 && a % b); }
13   bool isect(iterator x, iterator y) {
14     if (y == end()) { x->p = inf; return false; }
15     if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
16     else x->p = div(y->b - x->b, x->m - y->m);
```

```
17      return x->p >= y->p;
18    }
19    void add(ll m, ll b) {
20      auto z = insert({m, b, 0}), y = z++, x = y;
21      while (isect(y, z)) z = erase(z);
22      if (x != begin() && isect(--x, y)) isect(x, y =
          erase(y));
23      while ((y = x) != begin() && (--x)->p >= y->p)
24        isect(x, erase(y));
25    }
26    ll query(ll x) {
27      assert(!empty());
28      auto l = *lower_bound(x);
29      return l.m * x + l.b;
30    }
31 };
```

9.8 Polygon Area

```
1  // Author: Gino
2  // Function: Return doubled area of a polygon
3  T dbarea(vector<Pt>& e) {
4  ll res = 0;
5  for (int i = 0; i < (int)e.size(); i++)
6      res += e[i]^e[(i+1)%SZ(e)];
7  return abs(res);
8  }
```

9.9 Pick's Theorem

Consider a polygon which vertices are all lattice points.
Let i = number of points inside the polygon.
Let b = number of points on the boundary of the polygon.
Then we have the following formula:

$$Area = i + \frac{b}{2} - 1$$

9.10 Minimum Enclosing Circle

```
1  // Author: Gino
2  // Function: Find Min Enclosing Circle using Randomized
       O(n) Algorithm
3  Pt circumcenter(Pt A, Pt B, Pt C) {
4  // a1(x-A.x) + b1(y-A.y) = c1
5  // a2(x-A.x) + b2(y-A.y) = c2
6  // solve using Cramer's rule
7  T a1 = B.x-A.x, b1 = B.y-A.y, c1 = dis2(A, B)/2.0;
8  T a2 = C.x-A.x, b2 = C.y-A.y, c2 = dis2(A, C)/2.0;
9  T D  = Pt(a1, b1) ^ Pt(a2, b2);
10 T Dx = Pt(c1, b1) ^ Pt(c2, b2);
11 T Dy = Pt(a1, c1) ^ Pt(a2, c2);
12 if (D == 0) return Pt(-INF, -INF);
13 return A + Pt(Dx/D, Dy/D);
14 }
15 Pt center; T r2;
16
17 void minEncloseCircle() {
18 mt19937 gen(chrono::steady_clock::now().
     time_since_epoch().count());
19 shuffle(ALL(E), gen);
20 center = E[0], r2 = 0;
21
22 for (int i = 0; i < n; i++) {
23     if (dis2(center, E[i]) <= r2) continue;
24     center = E[i], r2 = 0;
25     for (int j = 0; j < i; j++) {
26         if (dis2(center, E[j]) <= r2) continue;
27         center = (E[i] + E[j]) / 2.0;
28         r2 = dis2(center, E[i]);
29         for (int k = 0; k < j; k++) {
30             if (dis2(center, E[k]) <= r2) continue;
31             center = circumcenter(E[i], E[j], E[k]);
32             r2 = dis2(center, E[i]);
33         }
34     }
35 } }
```

9.11 PolyUnion

```
1  // Author: Unknown
2  struct PY{
3    int n; Pt pt[5]; double area;
4    Pt& operator[](const int x){ return pt[x]; }
5    void init(){ //n,pt[0~n-1] must be filled
6      area=pt[n-1]^pt[0];
7      for(int i=0;i<n-1;i++) area+=pt[i]^pt[i+1];
8      if((area/=2)<0)reverse(pt,pt+n),area=-area;
9    }
10 };
11 PY py[500]; pair<double,int> c[5000];
12 inline double segP(Pt &p,Pt &p1,Pt &p2){
13   if(dcmp(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
14   return (p.x-p1.x)/(p2.x-p1.x);
15 }
16 double polyUnion(int n){ //py[0~n-1] must be filled
17   int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
18   for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
19   for(i=0;i<n;i++){
20     for(ii=0;ii<py[i].n;ii++){
21       r=0;
22       c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
23       for(j=0;j<n;j++){
24         if(i==j) continue;
25         for(jj=0;jj<py[j].n;jj++){
26           ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]))
              ;
27           tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj
              +1]));
28           if(ta==0 && tb==0){
29             if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[
                i][ii])>0&&j<i){
30               c[r++]=make_pair(segP(py[j][jj],py[i][ii
                  ],py[i][ii+1]),1);
31               c[r++]=make_pair(segP(py[j][jj+1],py[i][
                  ii],py[i][ii+1]),-1);
32             }
33           }else if(ta>=0 && tb<0){
34             tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
35             td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
36             c[r++]=make_pair(tc/(tc-td),1);
37           }else if(ta<0 && tb>=0){
38             tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
39             td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
40             c[r++]=make_pair(tc/(tc-td),-1);
41     } } }
42       sort(c,c+r);
43       z=min(max(c[0].first,0.0),1.0); d=c[0].second; s
          =0;
44       for(j=1;j<r;j++){
45         w=min(max(c[j].first,0.0),1.0);
46         if(!d) s+=w-z;
47         d+=c[j].second; z=w;
48       }
49       sum+=(py[i][ii]^py[i][ii+1])*s;
50     }
51   }
52   return sum/2;
53 }
```

9.12 Minkowski Sum

```
1  // Author: Unknown
2  /* convex hull Minkowski Sum*/
3  #define INF 100000000000000LL
4  int pos( const Pt& tp ){
5    if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
6    return tp.Y > 0 ? 0 : 1;
7  }
8  #define N 300030
9  Pt pt[ N ], qt[ N ], rt[ N ];
10 LL Lx,Rx;
11 int dn,un;
12 inline bool cmp( Pt a, Pt b ){
13   int pa=pos( a ),pb=pos( b );
14   if(pa==pb) return (a^b)>0;
15   return pa<pb;
16 }
17 int minkowskiSum(int n,int m){
```

```
18   int i,j,r,p,q,fi,fj;
19   for(i=1,p=0;i<n;i++){
20     if( pt[i].Y<pt[p].Y ||
21         (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
22   for(i=1,q=0;i<m;i++){
23     if( qt[i].Y<qt[q].Y ||
24         (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
25   rt[0]=pt[p]+qt[q];
26   r=1; i=p; j=q; fi=fj=0;
27   while(1){
28     if((fj&&j==q) ||
29        ( (!fi||i!=p) &&
30          cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
31       rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
32       p=(p+1)%n;
33       fi=1;
34     }else{
35       rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
36       q=(q+1)%m;
37       fj=1;
38     }
39     if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
       r++;
40     else rt[r-1]=rt[r];
41     if(i==p && j==q) break;
42   }
43   return r-1;
44 }
45 void initInConvex(int n){
46   int i,p,q;
47   LL Ly,Ry;
48   Lx=INF; Rx=-INF;
49   for(i=0;i<n;i++){
50     if(pt[i].X<Lx) Lx=pt[i].X;
51     if(pt[i].X>Rx) Rx=pt[i].X;
52   }
53   Ly=Ry=INF;
54   for(i=0;i<n;i++){
55     if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i; }
56     if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }
57   }
58   for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
59   qt[dn]=pt[q]; Ly=Ry=-INF;
60   for(i=0;i<n;i++){
61     if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i; }
62     if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
63   }
64   for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
65   rt[un]=pt[q];
66 }
67 inline int inConvex(Pt p){
68   int L,R,M;
69   if(p.X<Lx || p.X>Rx) return 0;
70   L=0;R=dn;
71   while(L<R-1){ M=(L+R)/2;
72     if(p.X<qt[M].X) R=M; else L=M; }
73     if(tri(qt[L],qt[R],p)<0) return 0;
74     L=0;R=un;
75   while(L<R-1){ M=(L+R)/2;
76     if(p.X<rt[M].X) R=M; else L=M; }
77     if(tri(rt[L],rt[R],p)>0) return 0;
78     return 1;
79 }
80 int main(){
81   int n,m,i;
82   Pt p;
83   scanf("%d",&n);
84   for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y);
85   scanf("%d",&m);
86   for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
87   n=minkowskiSum(n,m);
88   for(i=0;i<n;i++) pt[i]=rt[i];
89   scanf("%d",&m);
90   for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
91   n=minkowskiSum(n,m);
92   for(i=0;i<n;i++) pt[i]=rt[i];
93   initInConvex(n);
94   scanf("%d",&m);
95   for(i=0;i<m;i++){
96     scanf("%lld %lld",&p.X,&p.Y);
97     p.X*=3; p.Y*=3;
98     puts(inConvex(p)?"YES":"NO");
99   }
100 }
```

10 Number Theory

10.1 Basic

```
1  // Author: Gino
2  const int maxc = 5e5;
3  ll pw(ll a, ll n) {
4      ll res = 1;
5      while (n) {
6          if (n & 1) res = res * a % MOD;
7          a = a * a % MOD;
8          n >>= 1;
9      }
10     return res;
11 }
12
13 vector<ll> fac, ifac;
14 void build_fac() {
15     reset(fac, maxc + 1, 1LL);
16     reset(ifac, maxc + 1, 1LL);
17     for (int x = 2; x <= maxc; x++) {
18         fac[x] = x * fac[x - 1] % MOD;
19         ifac[x] = pw(fac[x], MOD - 2);
20     }
21 }
22
23 ll C(ll n, ll k) {
24     if (n < k) return 0LL;
25     return fac[n] * ifac[n - k] % MOD * ifac[k] % MOD;
26 }
```

10.2 Prime Seive and Defactor

```
1  // Author: Gino
2  const int maxc = 1e6 + 1;
3  vector<int> lpf;
4  vector<int> prime;
5
6  void seive() {
7      prime.clear();
8      lpf.resize(maxc, 1);
9      for (int i = 2; i < maxc; i++) {
10         if (lpf[i] == 1) {
11             lpf[i] = i;
12             prime.emplace_back(i);
13         }
14         for (auto& j : prime) {
15             if (i * j >= maxc) break;
16             lpf[i * j] = j;
17             if (j == lpf[i]) break;
18 } } }
19 vector<pii> fac;
20 void defactor(int u) {
21     fac.clear();
22     while (u > 1) {
23         int d = lpf[u];
24         fac.emplace_back(make_pair(d, 0));
25         while (u % d == 0) {
26             u /= d;
27             fac.back().second++;
28 } } }
```

10.3 Harmonic Series

```
1  // Author: Gino
2  // O(n log n)
3  for (int i = 1; i <= n; i++) {
4      for (int j = i; j <= n; j += i) {
5          // O(1) code
6      }
7  }
8
9  // PIE
10 // given array a[0], a[1], ..., a[n - 1]
```

```
11  // calculate dp[x] = number of pairs (a[i], a[j]) such
        that
12  //                    gcd(a[i], a[j]) = x    // (i < j)
13  //
14  // idea: let mc(x) = # of y s.t. x|y
15  //           f(x) = # of pairs s.t. gcd(a[i], a[j]) >=
        x
16  //           f(x) = C(mc(x), 2)
17  //           dp[x] = f(x) - sum(dp[y], x < y and x|y)
18  const int maxc = 1e6;
19  vector<int> cnt(maxc + 1, 0), dp(maxc + 1, 0);
20  for (int i = 0; i < n; i++)
21      cnt[a[i]]++;
22
23  for (int x = maxc; x >= 1; x--) {
24      ll cnt_mul = 0;   // number of multiples of x
25      for (int y = x; y <= maxc; y += x)
26          cnt_mul += cnt[y];
27
28      dp[x] = cnt_mul * (cnt_mul - 1) / 2;   // number of
            pairs that are divisible by x
29      for (int y = x + x; y <= maxc; y += x)
30          dp[x] -= dp[y];   // PIE: subtract all dp[y] for
                y > x and x|y
31  }
```

10.4 Count Number of Divisors

```
1  // Author: Gino
2  // Function: Count the number of divisors for all x <=
       10^6 using harmonic series
3  const int maxc = 1e6;
4  vector<int> facs;
5
6  void find_all_divisors() {
7      facs.clear(); facs.resize(maxc + 1, 0);
8      for (int x = 1; x <= maxc; x++) {
9          for (int y = x; y <= maxc; y += x) {
10             facs[y]++;
11         }
12     }
13 }
```

10.5 數論分塊

```
1  // Author: Gino
2  /*
3  n = 17
4    i:  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17
5  n/i: 17  8  5  4  3  2  2  2  1  1  1  1  1  1  1  1  1
6                        ^        ^
7                      L(2)     R(2)
8
9  L(x) :=  left bound for n/i = x
10 R(x) := right bound for n/i = x
11
12 ====== FORMULA ======
13 >>> R = n / (n/L) <<<
14 ====================
15
16 Example: L(2) = 6
17          R(2) = 17 / (17 / 6)
18               = 17 / 2
19               = 8
20 */
21 // ======= CODE ========
22
23 for (ll l = 1, r = 1, q = n; l <= n; l = r + 1) {
24     q = n/l;
25     r = n/q;
26     // Process your code here
27 }
28 // q, l, r: 17 1 1
29 // q, l, r: 8 2 2
30 // q, l, r: 5 3 3
31 // q, l, r: 4 4 4
32 // q, l, r: 3 5 5
33 // q, l, r: 2 6 8
34 // q, l, r: 1 9 17
```

10.6 Pollard's rho

```
1  // Author: Unknown
2  // Function: Find a non-trivial factor of a big number
       in O(n^(1/4) log^2(n))
3
4  ll find_factor(ll number) {
5      __int128 x = 2;
6      for (__int128 cycle = 1; ; cycle++) {
7          __int128 y = x;
8          for (int i = 0; i < (1<<cycle); i++) {
9              x = (x * x + 1) % number;
10             __int128 factor = __gcd(x - y, number);
11             if (factor > 1)
12                 return factor;
13         }
14     }
15 }
```

```
1  # Author: Unknown
2  # Function: Find a non-trivial factor of a big number
       in O(n^(1/4) log^2(n))
3  from itertools import count
4  from math import gcd
5  from sys import stdin
6
7  for s in stdin:
8      number, x = int(s), 2
9      brk = False
10     for cycle in count(1):
11         y = x
12         if brk:
13             break
14         for i in range(1 << cycle):
15             x = (x * x + 1) % number
16             factor = gcd(x - y, number)
17             if factor > 1:
18                 print(factor)
19                 brk = True
20                 break
```

10.7 Miller Rabin

```
1  // Author: Unknown
2  // Function: Check if a number is a prime in O(100 *
       log^2(n))
3  //      miller_rabin(): return 1 if prime, 0 otherwise
4
5  // n < 4,759,123,141         3 :  2, 7, 61
6  // n < 1,122,004,669,633     4 :  2, 13, 23, 1662803
7  // n < 3,474,749,660,383     6 :  pirmes <= 13
8  // n < 2^64                  7 :
9  // 2, 325, 9375, 28178, 450775, 9780504, 1795265022
10 bool witness(ll a,ll n,ll u,int t){
11   if(!(a%=n)) return 0;
12   ll x=mypow(a,u,n);
13   for(int i=0;i<t;i++) {
14     ll nx=mul(x,x,n);
15     if(nx==1&&x!=1&&x!=n-1) return 1;
16     x=nx;
17   }
18   return x!=1;
19 }
20 bool miller_rabin(ll n,int s=100) {
21   // iterate s times of witness on n
22   if(n<2) return 0;
23   if(!(n&1)) return n == 2;
24   ll u=n-1; int t=0;
25   while(!(u&1)) u>>=1, t++;
26   while(s--){
27     ll a=randll()%(n-1)+1;
28     if(witness(a,n,u,t)) return 0;
29   }
30   return 1;
31 }
```

10.8 Fast Power

Note: $a^n \equiv a^{(n \bmod (p-1))} \pmod p$

10.9 Extend GCD

```
// Author: Gino
// [Usage]
// bezout(a, b, c):
//     find solution to ax + by = c
//     return {-LINF, -LINF} if no solution
// inv(a, p):
//     find modulo inverse of a under p
//     return -1 if not exist
// CRT(vector<ll>& a, vector<ll>& m)
//     find a solution pair (x, mod) satisfies all x =
    a[i] (mod m[i])
//     return {-LINF, -LINF} if no solution

const ll LINF = 4e18;
typedef pair<ll, ll> pll;
template<typename T1, typename T2>
T1 chmod(T1 a, T2 m) {
    return (a % m + m) % m;
}

ll GCD;
pll extgcd(ll a, ll b) {
    if (b == 0) {
        GCD = a;
        return pll{1, 0};
    }
    pll ans = extgcd(b, a % b);
    return pll{ans.second, ans.first - a/b * ans.second
        };
}
pll bezout(ll a, ll b, ll c) {
    bool negx = (a < 0), negy = (b < 0);
    pll ans = extgcd(abs(a), abs(b));
    if (c % GCD != 0) return pll{-LINF, -LINF};
    return pll{ans.first * c/GCD * (negx ? -1 : 1),
               ans.second * c/GCD * (negy ? -1 : 1)};
}
ll inv(ll a, ll p) {
    if (p == 1) return -1;
    pll ans = bezout(a % p, -p, 1);
    if (ans == pll{-LINF, -LINF}) return -1;
    return chmod(ans.first, p);
}
pll CRT(vector<ll>& a, vector<ll>& m) {
    for (int i = 0; i < (int)a.size(); i++)
        a[i] = chmod(a[i], m[i]);

    ll x = a[0], mod = m[0];
    for (int i = 1; i < (int)a.size(); i++) {
        pll sol = bezout(mod, m[i], a[i] - x);
        if (sol.first == -LINF) return pll{-LINF, -LINF
            };

        // prevent long long overflow
        ll p = chmod(sol.first, m[i] / GCD);
        ll lcm = mod / GCD * m[i];
        x = chmod((__int128)p * mod + x, lcm);
        mod = lcm;
    }
    return pll{x, mod};
}
```

10.10 Mu + Phi

```
// Author: Gino
const int maxn = 1e6 + 5;
ll f[maxn];
vector<int> lpf, prime;
void build() {
lpf.clear(); lpf.resize(maxn, 1);
prime.clear();
f[1] = ...;  /* mu[1] = 1, phi[1] = 1 */
for (int i = 2; i < maxn; i++) {
    if (lpf[i] == 1) {
        lpf[i] = i; prime.emplace_back(i);
        f[i] = ...;  /* mu[i] = 1, phi[i] = i-1 */
    }
    for (auto& j : prime) {
        if (i*j >= maxn) break;
```

```
        lpf[i*j] = j;
        if (i % j == 0) f[i*j] = ...;  /* 0, phi[i]*j
            */
        else f[i*j] = ...;  /* -mu[i], phi[i]*phi[j] */
        if (j >= lpf[i]) break;
} } }
```

10.11 Other Formulas

- Inversion:
  $aa^{-1} \equiv 1 \pmod m$. a^{-1} exists iff $\gcd(a,m)=1$.

- Linear inversion:
  $a^{-1} \equiv (m - \lfloor \frac{m}{a} \rfloor) \times (m \bmod a)^{-1} \pmod m$

- Fermat's little theorem:
  $a^p \equiv a \pmod p$ if p is prime.

- Euler function:
  $\phi(n) = n \prod_{p|n} \frac{p-1}{p}$

- Euler theorem:
  $a^{\phi(n)} \equiv 1 \pmod n$ if $\gcd(a,n)=1$.

- Extended Euclidean algorithm:
  $ax + by = \gcd(a,b) = \gcd(b, a \bmod b) = \gcd(b, a - \lfloor \frac{a}{b} \rfloor b) = bx_1 + (a - \lfloor \frac{a}{b} \rfloor b)y_1 = ay_1 + b(x_1 - \lfloor \frac{a}{b} \rfloor y_1)$

- Divisor function:
  $\sigma_x(n) = \sum_{d|n} d^x$. $n = \prod_{i=1}^r p_i^{a_i}$.
  $\sigma_x(n) = \prod_{i=1}^r \frac{p_i^{(a_i+1)x}-1}{p_i^x-1}$ if $x \neq 0$. $\sigma_0(n) = \prod_{i=1}^r (a_i + 1)$.

- Chinese remainder theorem (Coprime Moduli):
  $x \equiv a_i \pmod{m_i}$.
  $M = \prod m_i$. $M_i = M/m_i$. $t_i = M_i^{-1}$.
  $x = kM + \sum a_i t_i M_i, k \in \mathbb{Z}$.

- Chinese remainder theorem:
  $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2} \Rightarrow x = m_1 p + a_1 = m_2 q + a_2 \Rightarrow m_1 p - m_2 q = a_2 - a_1$
  Solve for (p,q) using ExtGCD.
  $x \equiv m_1 p + a_1 \equiv m_2 q + a_2 \pmod{lcm(m_1, m_2)}$

- Avoiding Overflow: $ca \bmod cb = c(a \bmod b)$

- Dirichlet Convolution: $(f * g)(n) = \sum_{d|n} f(n)g(n/d)$

- Important Multiplicative Functions + Proterties:
  1. $\epsilon(n) = [n = 1]$
  2. $1(n) = 1$
  3. $id(n) = n$
  4. $\mu(n) = 0$ if n has squared prime factor
  5. $\mu(n) = (-1)^k$ if $n = p_1 p_2 \cdots p_k$
  6. $\epsilon = \mu * 1$
  7. $\phi = \mu * id$
  8. $[n = 1] = \sum_{d|n} \mu(d)$
  9. $[gcd = 1] = \sum_{d|gcd} \mu(d)$

- Möbius inversion: $f = g * 1 \Leftrightarrow g = f * \mu$

10.12 Polynomial

```
// Author: Gino
// Preparation: first implement pw(a, n), then call
    set_mod(), set g to corresponding primitive root
    and init_ntt() in order

// [Usage]
// polynomial: vector<ll> a, b
// negation: -a
// add/subtract: a += b, a -= b
// convolution: a *= b
```

```
9   // in-place modulo: mod(a, b)
10  // in-place inversion under mod x^N: inv(ia, N)
11
12  /* P = r*2^k + 1
13  P                    r    k    g
14  998244353            119  23   3
15  1004535809           479  21   3
16
17  P                    r    k    g
18  3                    1    1    2
19  5                    1    2    2
20  17                   1    4    3
21  97                   3    5    5
22  193                  3    6    5
23  257                  1    8    3
24  7681                 15   9    17
25  12289                3    12   11
26  40961                5    13   3
27  65537                1    16   3
28  786433               3    18   10
29  5767169              11   19   3
30  7340033              7    20   3
31  23068673             11   21   3
32  104857601            25   22   3
33  167772161            5    25   3
34  469762049            7    26   3
35  1004535809           479  21   3
36  2013265921           15   27   31
37  2281701377           17   27   3
38  3221225473           3    30   5
39  75161927681          35   31   3
40  77309411329          9    33   7
41  206158430209         3    36   22
42  2061584302081        15   37   7
43  2748779069441        5    39   3
44  6597069766657        3    41   5
45  39582418599937       9    42   5
46  79164837199873       9    43   5
47  263882790666241      15   44   7
48  1231453023109121     35   45   3
49  1337006139375617     19   46   3
50  3799912185593857     27   47   5
51  4222124650659841     15   48   19
52  7881299347898369     7    50   6
53  31525197391593473    7    52   3
54  180143985094819841   5    55   6
55  1945555039024054273  27   56   5
56  4179340454199820289  29   57   3
57  9097271247288401921  505  54   6  */
58
59  const int maxk = 19;
60  const int maxn = 1<<maxk;
61
62  using u64 = unsigned long long;
63  using u128 = __uint128_t;
64
65  int g = 3;
66  u64 MOD;
67  u64 BARRETT_IM; // ⌊ 2^64 / MOD ⌋
68
69  inline void set_mod(u64 m) {
70      MOD = m;
71      BARRETT_IM = (u128(1) << 64) / m;
72  }
73  inline u64 chmod(u128 x) {
74      u64 q = (u64)((x * BARRETT_IM) >> 64);
75      u64 r = (u64)(x - (u128)q * MOD);
76      if (r >= MOD) r -= MOD;
77      return r;
78  }
79  inline u64 mmul(u64 a, u64 b) {
80      return chmod((u128)a * b);
81  }
82  ll pw(ll a, ll n) {
83      ll ret = 1;
84      while (n > 0) {
85          if (n & 1) ret = mmul(ret, a);
86          a = mmul(a, a);
87          n >>= 1;
88      }
89      return ret;
90  }
91  template<typename T>
92  vector<T>& operator+=(vector<T>& a, const vector<T>& b)
        {
93      if (siz(a) < siz(b)) a.resize(siz(b));
94      for (int i = 0; i < min(siz(a), siz(b)); i++) {
95          a[i] += b[i];
96          a[i] -= a[i] >= MOD ? MOD : 0;
97      }
98      return a;
99  }
100 template<typename T>
101 vector<T>& operator-=(vector<T>& a, const vector<T>& b)
        {
102     if (siz(a) < siz(b)) a.resize(siz(b));
103     for (int i = 0; i < min(siz(a), siz(b)); i++) {
104         a[i] -= b[i];
105         a[i] += a[i] < 0 ? MOD : 0;
106     }
107     return a;
108 }
109 template<typename T>
110 vector<T> operator-(const vector<T>& a) {
111     vector<T> ret(siz(a));
112     for (int i = 0; i < siz(a); i++) {
113         ret[i] = -a[i] < 0 ? -a[i] + MOD : -a[i];
114     }
115     return ret;
116 }
117 vector<ll> X, iX;
118 vector<int> rev;
119 void init_ntt() {
120     X.assign(maxn, 1);   // x1 = g^((p-1)/n)
121     iX.assign(maxn, 1);
122
123     ll u = pw(g, (MOD-1)/maxn);
124     ll iu = pw(u, MOD-2);
125     for (int i = 1; i < maxn; i++) {
126         X[i] = mmul(X[i - 1], u);
127         iX[i] = mmul(iX[i - 1], iu);
128     }
129
130     if ((int)rev.size() == maxn) return;
131     rev.assign(maxn, 0);
132     for (int i = 1, hb = -1; i < maxn; i++) {
133         if (!(i & (i-1))) hb++;
134         rev[i] = rev[i ^ (1<<hb)] | (1<<(maxk-hb-1));
135 } }
136 template<typename T>
137 void NTT(vector<T>& a, bool inv=false) {
138     int _n = (int)a.size();
139     int k = __lg(_n) + ((1<<__lg(_n)) != _n);
140     int n = 1<<k;
141     a.resize(n, 0);
142
143     short shift = maxk-k;
144     for (int i = 0; i < n; i++)
145         if (i > (rev[i]>>shift))
146             swap(a[i], a[rev[i]>>shift]);
147     for (int len = 2, half = 1, div = maxn>>1; len <= n
            ; len<<=1, half<<=1, div>>=1) {
148         for (int i = 0; i < n; i += len) {
149             for (int j = 0; j < half; j++) {
150                 T u = a[i+j];
151                 T v = mmul(a[i+j+half], (inv ? iX[j*div
                        ] : X[j*div]));
152                 a[i+j] = (u+v >= MOD ? u+v-MOD : u+v);
153                 a[i+j+half] = (u-v < 0 ? u-v+MOD : u-v)
                        ;
154     } } }
155     if (inv) {
156         T dn = pw(n, MOD-2);
157         for (auto& x : a) {
158             x = mmul(x, dn);
159 } } }
160 template<typename T>
161 inline void shrink(vector<T>& a) {
162     int cnt = (int)a.size();
163     for (; cnt > 0; cnt--) if (a[cnt-1]) break;
164     a.resize(max(cnt, 1));
165 }
166 template<typename T>
167 vector<T>& operator*=(vector<T>& a, vector<T> b) {
```

```
168    int na = (int)a.size();
169    int nb = (int)b.size();
170    a.resize(na + nb - 1, 0);
171    b.resize(na + nb - 1, 0);
172
173    NTT(a); NTT(b);
174    for (int i = 0; i < (int)a.size(); i++)
175        a[i] = mmul(a[i], b[i]);
176    NTT(a, true);
177
178    shrink(a);
179    return a;
180 }
181 inline ll crt(ll a0, ll a1, ll m1, ll m2, ll
        inv_m1_mod_m2){
182    // x ≡ a0 (mod m1), x ≡ a1 (mod m2)
183    // t = (a1 - a0) * inv(m1) mod m2
184    // x = a0 + t * m1  (mod m1*m2)
185    ll t = chmod(a1 - a0);
186    if (t < 0) t += m2;
187    t = (ll)((__int128)t * inv_m1_mod_m2 % m2);
188    return a0 + (ll)((__int128)t * m1);
189 }
190 void mul_crt() {
191    // a copy to a1, a2 | b copy to b1, b2
192    ll M1 = 998244353, M2 = 1004535809;
193    g = 3; set_mod(M1); init_ntt(); a1 *= b1;
194    g = 3, set_mod(M2); init_ntt(); a2 *= b2;
195
196    ll inv_m1_mod_m2 = pw(M1, M2 - 2);
197    for (int i = 2; i <= 2 * k; i++)
198        cout << crt(a1[i], a2[i], M1, M2, inv_m1_mod_m2
                ) << ' ';
199    cout << endl;
200 }
201 template<typename T>
202 void inv(vector<T>& ia, int N) {
203    vector<T> _a(move(ia));
204    ia.resize(1, pw(_a[0], MOD-2));
205    vector<T> a(1, -_a[0] + (-_a[0] < 0 ? MOD : 0));
206
207    for (int n = 1; n < N; n<<=1) {
208        // n -> 2*n
209        // ia' = ia(2-a*ia);
210        for (int i = n; i < min(siz(_a), (n<<1)); i++)
211            a.emplace_back(-_a[i] + (-_a[i] < 0 ? MOD :
                    0));
212
213        vector<T> tmp = ia;
214        ia *= a;
215        ia.resize(n<<1);
216        ia[0] = ia[0] + 2 >= MOD ? ia[0] + 2 - MOD : ia
                [0] + 2;
217        ia *= tmp;
218        ia.resize(n<<1);
219    }
220    ia.resize(N);
221 }
222 template<typename T>
223 void mod(vector<T>& a, vector<T>& b) {
224    int n = (int)a.size()-1, m = (int)b.size()-1;
225    if (n < m) return;
226
227    vector<T> ra = a, rb = b;
228    reverse(ra.begin(), ra.end()); ra.resize(min(n+1, n
            -m+1));
229    reverse(rb.begin(), rb.end()); rb.resize(min(m+1, n
            -m+1));
230    inv(rb, n-m+1);
231
232    vector<T> q = move(ra);
233    q *= rb;
234    q.resize(n-m+1);
235    reverse(q.begin(), q.end());
236
237    q *= b;
238    a -= q;
239    resize(a);
240 }
241 /* Kitamasa Method (Fast Linear Recurrence):
242 Find a[K] (Given a[j] = c[0]a[j-N] + ... + c[N-1]a[j
        -1])
243 Let B(x) = x^N - c[N-1]x^(N-1) - ... - c[1]x^1 - c[0]
244 Let R(x) = x^K mod B(x)   (get x^K using fast pow and
        use poly mod to get R(x))
245 Let r[i] = the coefficient of x^i in R(x)
246 => a[K] = a[0]r[0] + a[1]r[1] + ... + a[N-1]r[N-1] */
```

11   Linear Algebra

11.1   Gaussian-Jordan Elimination

```
1 int n; vector<vector<ll> > v;
2 void gauss(vector<vector<ll>>& v) {
3    int r = 0;
4    for (int i = 0; i < n; i++) {
5        bool ok = false;
6        for (int j = r; j < n; j++) {
7            if (v[j][i] == 0) continue;
8            swap(v[j], v[r]);
9            ok = true; break;
10       }
11       if (!ok) continue;
12       ll div = inv(v[r][i]);
13       for (int j = 0; j < n+1; j++) {
14           v[r][j] *= div;
15           if (v[r][j] >= MOD) v[r][j] %= MOD;
16       }
17       for (int j = 0; j < n; j++) {
18           if (j == r) continue;
19           ll t = v[j][i];
20           for (int k = 0; k < n+1; k++) {
21               v[j][k] -= v[r][k] * t % MOD;
22               if (v[j][k] < 0) v[j][k] += MOD;
23       } }
24       r++;
25 } }
```

11.2   Determinant

1. Use GJ Elimination, if there's any row consists of only 0, then det = 0, otherwise det = product of diagonal elements.

2. Properties of det:

   - Transpose: Unchanged
   - Row Operation 1 - Swap 2 rows: $-det$
   - Row Operation 2 - $k\overrightarrow{r_i}$: $k \times det$
   - Row Operation 3 - $k\overrightarrow{r_i}$ add to $\overrightarrow{r_j}$: Unchaged

12   Flow / Matching

12.1   Dinic

```
1 // Author: Benson
2 // Function: Max Flow, O(V^2 E)
3 struct Dinic {
4   struct Edge {
5     int t, c, r;
6     Edge() {}
7     Edge(int _t, int _c, int _r):
8       t(_t), c(_c), r(_r) {}
9   };
10  vector<vector<Edge>> G;
11  vector<int> dis, iter;
12  int s, t;
13  void init(int n) {
14    G.resize(n), dis.resize(n), iter.resize(n);
15    for(int i = 0; i < n; ++i)
16      G[i].clear();
17  }
18  void add(int a, int b, int c) {
19    G[a].eb(b, c, G[b].size());
20    G[b].eb(a, 0, G[a].size() - 1);
21  }
```

```cpp
22    bool bfs() {
23      fill(ALL(dis), -1);
24      dis[s] = 0;
25      queue<int> que;
26      que.push(s);
27      while(!que.empty()) {
28        int u = que.front(); que.pop();
29        for(auto& e : G[u]) {
30          if(e.c > 0 && dis[e.t] == -1) {
31            dis[e.t] = dis[u] + 1;
32            que.push(e.t);
33          }
34        }
35      }
36      return dis[t] != -1;
37    }
38    int dfs(int u, int cur) {
39      if(u == t) return cur;
40      for(int &i = iter[u]; i < (int)G[u].size(); ++i) {
41        auto& e = G[u][i];
42        if(e.c > 0 && dis[u] + 1 == dis[e.t]) {
43          int ans = dfs(e.t, min(cur, e.c));
44          if(ans > 0) {
45            G[e.t][e.r].c += ans;
46            e.c -= ans;
47            return ans;
48          }
49        }
50      }
51      return 0;
52    }
53
54    int flow(int a, int b) {
55      s = a, t = b;
56      int ans = 0;
57      while(bfs()) {
58        fill(ALL(iter), 0);
59        int tmp;
60        while((tmp = dfs(s, INF)) > 0)
61          ans += tmp;
62      }
63      return ans;
64    }
65 };
```

12.2 ISAP

```cpp
1  // Author: CRyptoGRapheR
2  #define SZ(c) ((int)(c).size())
3  struct Maxflow{
4    static const int MAXV=50010;
5    static const int INF =1000000;
6    struct Edge{
7      int v,c,r;
8      Edge(int _v,int _c,int _r):v(_v),c(_c),r(_r){}
9    };
10   int s,t; vector<Edge> G[MAXV];
11   int iter[MAXV],d[MAXV],gap[MAXV],tot;
12   void init(int n,int _s,int _t){
13     tot=n,s=_s,t=_t;
14     for(int i=0;i<=tot;i++){
15       G[i].clear(); iter[i]=d[i]=gap[i]=0;
16     }
17   }
18   void addEdge(int u,int v,int c){
19     G[u].push_back(Edge(v,c,SZ(G[v])));
20     G[v].push_back(Edge(u,0,SZ(G[u])-1));
21   }
22   int DFS(int p,int flow){
23     if(p==t) return flow;
24     for(int &i=iter[p];i<SZ(G[p]);i++){
25       Edge &e=G[p][i];
26       if(e.c>0&&d[p]==d[e.v]+1){
27         int f=DFS(e.v,min(flow,e.c));
28         if(f){ e.c-=f; G[e.v][e.r].c+=f; return f; }
29       }
30     }
31     if((--gap[d[p]])==0) d[s]=tot;
32     else{ d[p]++; iter[p]=0; ++gap[d[p]]; }
33     return 0;
34   }
```

```cpp
35   int flow(){
36     int res=0;
37     for(res=0,gap[0]=tot;d[s]<tot;res+=DFS(s,INF));
38     return res;
39   } // reset: set iter,d,gap to 0
40 } flow;
```

12.3 Bounded Max Flow

```cpp
1  // Author: CRyptoGRapheR
2  // Max flow with lower/upper bound on edges
3  // use with ISAP, l,r,a,b must be filled
4  int in[N],out[N],l[M],r[M],a[M],b[M];
5  int solve(int n, int m, int s, int t){
6    flow.init(n+2,n,n+1);
7    for(int i=0;i<m;i ++){
8      in[r[i]]+=a[i]; out[l[i]]+=a[i];
9      flow.addEdge(l[i],r[i],b[i]-a[i]);
10     // flow from l[i] to r[i] must in [a[i], b[i]]
11   }
12   int nd=0;
13   for(int i=0;i <= n;i ++){
14     if(in[i]<out[i]){
15       flow.addEdge(i,flow.t,out[i]-in[i]);
16       nd+=out[i]-in[i];
17     }
18     if(out[i]<in[i])
19       flow.addEdge(flow.s,i,in[i]-out[i]);
20   }
21   // original sink to source
22   flow.addEdge(t,s,INF);
23   if(flow.flow()!=nd) return -1; // no solution
24   int ans=flow.G[s].back().c; // source to sink
25   flow.G[s].back().c=flow.G[t].back().c=0;
26   // take out super source and super sink
27   for(size_t i=0;i<flow.G[flow.s].size();i++){
28     Maxflow::Edge &e=flow.G[flow.s][i];
29     flow.G[flow.s][i].c=0; flow.G[e.v][e.r].c=0;
30   }
31   for(size_t i=0;i<flow.G[flow.t].size();i++){
32     Maxflow::Edge &e=flow.G[flow.t][i];
33     flow.G[flow.t][i].c=0; flow.G[e.v][e.r].c=0;
34   }
35   flow.addEdge(flow.s,s,INF);flow.addEdge(t,flow.t,INF)
         ;
36   flow.reset(); return ans+flow.flow();
37 }
```

12.4 MCMF

```cpp
1  // Author: Unknown
2  // Function: First Maximize flow, then minimize flow
       cost
3  struct MCMF {
4    struct Edge {
5      int to, cap, rev;
6      ll cost;
7      Edge() {}
8      Edge(int _to, int _cap, int _rev, ll _cost) :
9        to(_to), cap(_cap), rev(_rev), cost(_cost) {}
10   };
11   static const int N = 2000;
12   vector<Edge> G[N];
13   int n, s, t;
14   void init(int _n, int _s, int _t) {
15     n = _n, s = _s, t = _t;
16     for(int i = 0; i <= n; ++i)
17       G[i].clear();
18   }
19   void add_edge(int from, int to, int cap, ll cost) {
20     G[from].eb(to, cap, (int)G[to].size(), cost);
21     G[to].eb(from, 0, (int)G[from].size() - 1, -cost);
22   }
23
24   bool vis[N];
25   int iter[N];
26   ll dis[N];
27   bool SPFA() {
28     for(int i = 0; i <= n; ++i)
29       vis[i] = 0, dis[i] = LINF;
```

```
30     dis[s] = 0; vis[s] = 1;
31     queue<int> que; que.push(s);
32     while(!que.empty()) {
33       int u = que.front(); que.pop();
34       vis[u] = 0;
35       for(auto& e : G[u]) if(e.cap > 0 && dis[e.to] >
           dis[u] + e.cost) {
36         dis[e.to] = dis[u] + e.cost;
37         if(!vis[e.to]) {
38           que.push(e.to);
39           vis[e.to] = 1;
40         }
41       }
42     }
43     return dis[t] != LINF;
44   }
45
46   int dfs(int u, int cur) {
47     if(u == t) return cur;
48     int ret = 0; vis[u] = 1;
49     for(int &i = iter[u]; i < (int)G[u].size(); ++i) {
50       auto &e = G[u][i];
51       if(e.cap > 0 && dis[e.to] == dis[u] + e.cost && !
           vis[e.to]) {
52         int tmp = dfs(e.to, min(cur, e.cap));
53         e.cap -= tmp;
54         G[e.to][e.rev].cap += tmp;
55         cur -= tmp;
56         ret += tmp;
57         if(cur == 0) {
58           vis[u] = 0;
59           return ret;
60         }
61       }
62     }
63     vis[u] = 0;
64     return ret;
65   }
66   pair<int, ll> flow() {
67     int flow = 0; ll cost = 0;
68     while(SPFA()) {
69       memset(iter, 0, sizeof(iter));
70       int tmp = dfs(s, INF);
71       flow += tmp, cost += tmp * dis[t];
72     }
73     return {flow, cost};
74   }
75 };
```

12.5   Hopcroft-Karp

```
1  // Author: Gino
2  // Function: Max Bipartite Matching in O(V sqrt(E))
3  // init() -> get() -> Ans = hk.MXCNT
4  struct HopcroftKarp {
5    // id: X = [1, nx], Y = [nx+1, nx+ny]
6    int n, nx, ny, m, MXCNT;
7    vector<vector<int> > g;
8    vector<int> mx, my, dis, vis;
9    void init(int nnx, int nny, int mm) {
10     nx = nnx, ny = nny, m = mm;
11     n = nx + ny + 1;
12     g.clear(); g.resize(n);
13   }
14   void add(int x, int y) {
15     g[x].emplace_back(y);
16     g[y].emplace_back(x);
17   }
18   bool dfs(int x) {
19     vis[x] = true;
20     for (auto& y : g[x]) {
21       int px = my[y];
22       if (px == -1 ||
23           (dis[px] == dis[x]+1 &&
24            !vis[px] && dfs(px))) {
25         mx[x] = y;
26         my[y] = x;
27         return true;
28       }
29     }
```

```
30     return false;
31   }
32   void get() {
33     mx.clear(); mx.resize(n, -1);
34     my.clear(); my.resize(n, -1);
35
36     while (true) {
37       queue<int> q;
38       dis.clear(); dis.resize(n, -1);
39       for (int x = 1; x <= nx; x++){
40         if (mx[x] == -1) {
41           dis[x] = 0;
42           q.push(x);
43         }
44       }
45       while (!q.empty()) {
46         int x = q.front(); q.pop();
47         for (auto& y : g[x]) {
48           if (my[y] != -1 && dis[my[y]] == -1) {
49             dis[my[y]] = dis[x] + 1;
50             q.push(my[y]);
51           }
52         }
53       }
54
55       bool brk = true;
56       vis.clear(); vis.resize(n, 0);
57       for (int x = 1; x <= nx; x++)
58         if (mx[x] == -1 && dfs(x))
59           brk = false;
60
61       if (brk) break;
62     }
63     MXCNT = 0;
64     for (int x = 1; x <= nx; x++) if (mx[x] != -1)
           MXCNT++;
65   }
66 } hk;
```

12.6   Cover / Independent Set

```
1  V(E) Cover: choose some V(E) to cover all E(V)
2  V(E) Independ: set of V(E) not adj to each other
3
4  M = Max Matching
5  Cv = Min V Cover
6  Ce = Min E Cover
7  Iv = Max V Ind
8  Ie = Max E Ind (equiv to M)
9
10 M = Cv (Konig Theorem)
11 Iv = V \ Cv
12 Ce = V - M
13
14 Construct Cv:
15 1. Run Dinic
16 2. Find s-t min cut
17 3. Cv = {X in T} + {Y in S}
```

12.7   KM

```
1  // Author: Unknown
2  // Function: Weighted Max Bipartite Matching in O(V^3)
3  #include <bits/stdc++.h>
4
5  using namespace std;
6  const int inf = 1e9;
7
8  struct KuhnMunkres {
9    int n;
10   vector<vector<int>> g;
11   vector<int> lx, ly, slack;
12   vector<int> match, visx, visy;
13   KuhnMunkres(int n) : n(n), g(n, vector<int>(n)),
14   lx(n), ly(n), slack(n), match(n), visx(n), visy(n) {}
15   vector<int> & operator[](int i) { return g[i]; }
16   bool dfs(int i, bool aug) { // aug = true 表示要更新
         match
17     if(visx[i]) return false;
18     visx[i] = true;
```

```cpp
19      for(int j = 0; j < n; j++) {
20        if(visy[j]) continue;
21        // 一邊擴增交錯樹、尋找增廣路徑
22        // 一邊更新slack：樹上的點跟樹外的點所造成的最小
            權重
23        int d = lx[i] + ly[j] - g[i][j];
24        if(d == 0) {
25          visy[j] = true;
26          if(match[j] == -1 || dfs(match[j], aug)) {
27            if(aug)
28              match[j] = i;
29            return true;
30          }
31        } else {
32          slack[j] = min(slack[j], d);
33        }
34      }
35      return false;
36    }
37    bool augment() { // 回傳是否有增廣路
38      for(int j = 0; j < n; j++) if(!visy[j] && slack[j]
            == 0) {
39        visy[j] = true;
40        if(match[j] == -1 || dfs(match[j], false)) {
41          return true;
42        }
43      }
44      return false;
45    }
46    void relabel() {
47      int delta = inf;
48      for(int j = 0; j < n; j++) if(!visy[j]) delta = min
            (delta, slack[j]);
49      for(int i = 0; i < n; i++) if(visx[i]) lx[i] -=
            delta;
50      for(int j = 0; j < n; j++) {
51        if(visy[j]) ly[j] += delta;
52        else slack[j] -= delta;
53      }
54    }
55    int solve() {
56      for(int i = 0; i < n; i++) {
57        lx[i] = 0;
58        for(int j = 0; j < n; j++) lx[i] = max(lx[i], g[i
            ][j]);
59      }
60      fill(ly.begin(), ly.end(), 0);
61      fill(match.begin(), match.end(), -1);
62      for(int i = 0; i < n; i++) {
63        // slack 在每一輪都要初始化
64        fill(slack.begin(), slack.end(), inf);
65        fill(visx.begin(), visx.end(), false);
66        fill(visy.begin(), visy.end(), false);
67        if(dfs(i, true)) continue;
68        // 重複調整頂標直到找到增廣路徑
69        while(!augment()) relabel();
70        fill(visx.begin(), visx.end(), false);
71        fill(visy.begin(), visy.end(), false);
72        dfs(i, true);
73      }
74      int ans = 0;
75      for(int j = 0; j < n; j++) if(match[j] != -1) ans
            += g[match[j]][j];
76      return ans;
77    }
78 };
79 signed main() {
80   ios_base::sync_with_stdio(0), cin.tie(0);
81   int n;
82   while(cin >> n && n) {
83     KuhnMunkres KM(n);
84     for(int i = 0; i < n; i++) {
85       for(int j = 0; j < n; j++) {
86         int c;
87         cin >> c;
88         if(c > 0)
89           KM[i][j] = c;
90       }
91     }
92     cout << KM.solve() << '\n';
93   }
94 }
```

13   Combinatorics

13.1   Catalan Number

$$C_0 = 1, C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}, C_n = C_n^{2n} - C_{n-1}^{2n}$$

0	1	1	2	5
4	14	42	132	429
8	1430	4862	16796	58786
12	208012	742900	2674440	9694845

13.2   Bertrand's Ballot Theorem

- A always $> B$: $C(p+q, p) - 2C(p+q-1, p)$

- A always $\geq B$: $C(p+q, p) \times \frac{p+1-q}{p+1}$

13.3   Burnside's Lemma

Let X be the original set.
Let G be the group of operations acting on X.
Let X^g be the set of x not affected by g.
Let X/G be the set of orbits.
Then the following equation holds:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

14   Special Numbers

14.1   Fibonacci Series

1	1	1	2	3
5	5	8	13	21
9	34	55	89	144
13	233	377	610	987
17	1597	2584	4181	6765
21	10946	17711	28657	46368
25	75025	121393	196418	317811
29	514229	832040	1346269	2178309
33	3524578	5702887	9227465	14930352

$f(45) \approx 10^9, f(88) \approx 10^{18}$

14.2   Prime Numbers

- First 50 prime numbers:

1	2	3	5	7	11
6	13	17	19	23	29
11	31	37	41	43	47
16	53	59	61	67	71
21	73	79	83	89	97
26	101	103	107	109	113
31	127	131	137	139	149
36	151	157	163	167	173
41	179	181	191	193	197
46	199	211	223	227	229

- Very large prime numbers:

1000001333	1000500889	2500001909
2000000659	900004151	850001359

- $\pi(n) \equiv$ Number of primes $\le n \approx n/((\ln n) - 1)$
  $\pi(100) = 25, \pi(200) = 46$
  $\pi(500) = 95, \pi(1000) = 168$
  $\pi(2000) = 303, \pi(4000) = 550$
  $\pi(10^4) = 1229, \pi(10^5) = 9592$
  $\pi(10^6) = 78498, \pi(10^7) = 664579$

- $\pi(n) \equiv$ Number of primes $\le n \approx n/((\ln n) - 1)$
  $\pi(100) = 25, \pi(200) = 46$
  $\pi(500) = 95, \pi(1000) = 168$
  $\pi(2000) = 303, \pi(4000) = 550$
  $\pi(10^4) = 1229, \pi(10^5) = 9592$
  $\pi(10^6) = 78498, \pi(10^7) = 664579$