Theoretical task 9.

Recommendations: all solutions should be short, mathematically strict (unless qualitative explanation is needed), precise with respect to the stated question and clearly written.

1. Does the principal component analysis (PCA) transformation require the preliminary feature normalization procedure? Explain your answer.
2. There is a dataset X containing 10^6 100-dimensional objects. It has been compressed with PCA (transformation matrix $P \in \mathbf{R}^{100\times 100}$) and the compressed dataset $\hat{X}$ contains 10^6 20-dimensional objects. How can one reconstruct the original uncompressed dataset given only P and $\hat{X}$?
3. Provide an example of the two-dimensional ($d = 2$) dataset in the binary classification problem for which the preliminary application of PCA compression to dimensionality $d = 1$ would hurt the classification accuracy seriously. Explain why PCA can hurt the classification accuracy.