theory8

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1 Task

It will be error-free, but not necessarily will be linear.

2 Task

1. For linearly separable set we have:
   $a(x) = sign(\sum_{i=1}^{n} w_i x^i - \omega_0) = sign(< \omega, x > -\omega_0)$
   Where vector ω and scalar threshold ω_0 may be calculated in advance. Thus linear SVM has prediction complexity $O(n)$ where n is dimension.

2. For kernel SVM we have:
   $a(x) = sign(\sum_{i=1}^{n} \lambda_i y_i K(x_i, x) - w_0)$
   Say kernel calculations complexity equals n_{ker}. Then total complexity is $O(n_{SV} n_{ker})$ where n_{SV} is the number of support vectors.

3 Task

Both kernels are dependent on the same variables.$\phi_1(x)^T \phi_1(y), K_2(x, y) = \phi_2(x)^T \phi_2(y)$, where given set is separable in space ϕ_1. Then the sum of kernels is:

$$K_1(x, y) + K_2(x, y) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \end{bmatrix}^T \begin{bmatrix} \phi_1(y) \\ \phi_2(y) \end{bmatrix}$$

The space defined by sum of kernels is the product of the initial spaces and its dimension is the sum of dimensions of the given initial spaces. If the hyperplane that separates the dataset is defined by (ω, ω_0) then in resulting space the dataset is separable by hyperplane defined by $(\begin{bmatrix} \omega \\ 0 \end{bmatrix}, \omega_0)$.