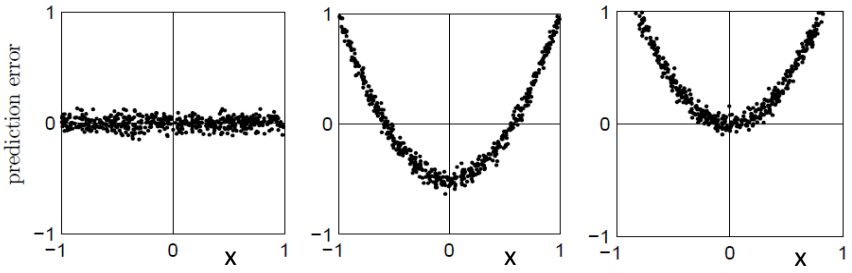
# Theoretical task 4.

*Recommendations: all solutions should be short, mathematically strict (unless qualitative explanation is needed), precise with respect to the stated question and clearly written.*

1. Consider linear regression task in one-dimensional space. $\{(x_i, y_i)\}_{i=1}^N$ is a training dataset and for object $i$: $x_i \in [-1, 1]$ is a feature, $y_i$ is an answer we want to predict, $\hat{y}_i = kx_i + b$ is our prediction.

   At the picture below you can see three different plots of the prediction error $(y - \hat{y})$ against $x$. Which of these plots cannot be obtained if least squares method is used to train a regression model?




2. Consider linear regression task in one-dimensional space $y = kx+b$ and two datasets: $\{(x_1^1, y_1^1), \ldots, (x_n^1, y_n^1)\}$ and $\{(x_1^2, y_1^2), \ldots, (x_m^2, y_m^2)\}$. Assume that the least squares method is used to train a regression models in this task.

   It turns out, that if we train a regression model on the first dataset we obtain a coefficient $k_1 > 0$. Similarly, if we train a regression model on the second dataset we obtain a coefficient $k_2 > 0$.

   Is it true that if we train the regression model on both datasets together then the obtained coefficient $k$ will also be positive?

   Answer to the previous question if additionally we know that $\sum_{i=1}^n x_i^1 = \sum_{i=1}^m x_i^2 = 0$.

3. Consider the dataset with the following features: height, age(from 10 to 45), sex(=1 for females, =0 for males). You would like to build a regression model to estimate person's height based on other features.

   Write down the regression model that would consider all the following facts:

   - on average the height of males and females is different,
   - at the age of 25 human rate of growth dramatically drops down, but still assumed to be linear.

   Explain your answer.