1. modifying limits in bicubic, we get -

bi-quadratic ⇒ $V(x,y) = \sum_{i=0}^{2}\sum_{j=0}^{2} a_{ij}x^i y^j$ (9 terms)

$= a_{00} + a_{01}y + a_{02}y^2 + a_{10}x + a_{11}xy + a_{12}xy^2 + a_{20}x^2 + a_{21}x^2y + a_{22}x^2y^2$

⇒ given 9 points, we can solve for all 9 coeffs.

in matrix form,

coeff matrix

$$V_{9\times1} = X_{9\times9}\, A_{9\times1}$$

(A has unknown coeff)

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix}_{9\times1} = \begin{bmatrix} 1 & y_1 & y_1^2 & x_1 & x_1y_1 & x_1y_1^2 & x_1^2 & x_1^2y_1 & x_1^2y_1^2 \\ 1 & y_2 & y_2^2 & x_2 & x_2y_2 & x_2y_2^2 & x_2^2 & x_2^2y_2 & x_2^2y_2^2 \\ 1 & y_3 & y_3^2 & x_3 & x_3y_3 & x_3y_3^2 & x_3^2 & x_3^2y_3 & x_3^2y_3^2 \\ 1 & y_4 & y_4^2 & x_4 & x_4y_4 & x_4y_4^2 & x_4^2 & x_4^2y_4 & x_4^2y_4^2 \\ 1 & y_5 & y_5^2 & x_5 & x_5y_5 & x_5y_5^2 & x_5^2 & x_5^2y_5 & x_5^2y_5^2 \\ 1 & y_6 & y_6^2 & x_6 & x_6y_6 & x_6y_6^2 & x_6^2 & x_6^2y_6 & x_6^2y_6^2 \\ 1 & y_7 & y_7^2 & x_7 & x_7y_7 & x_7y_7^2 & x_7^2 & x_7^2y_7 & x_7^2y_7^2 \\ 1 & y_8 & y_8^2 & x_8 & x_8y_8 & x_8y_8^2 & x_8^2 & x_8^2y_8 & x_8^2y_8^2 \\ 1 & y_9 & y_9^2 & x_9 & x_9y_9 & x_9y_9^2 & x_9^2 & x_9^2y_9 & x_9^2y_9^2 \end{bmatrix}_{9\times9} \begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{20} \\ a_{21} \\ a_{22} \end{bmatrix}_{9\times1}$$

$$A = X^{-1}V$$

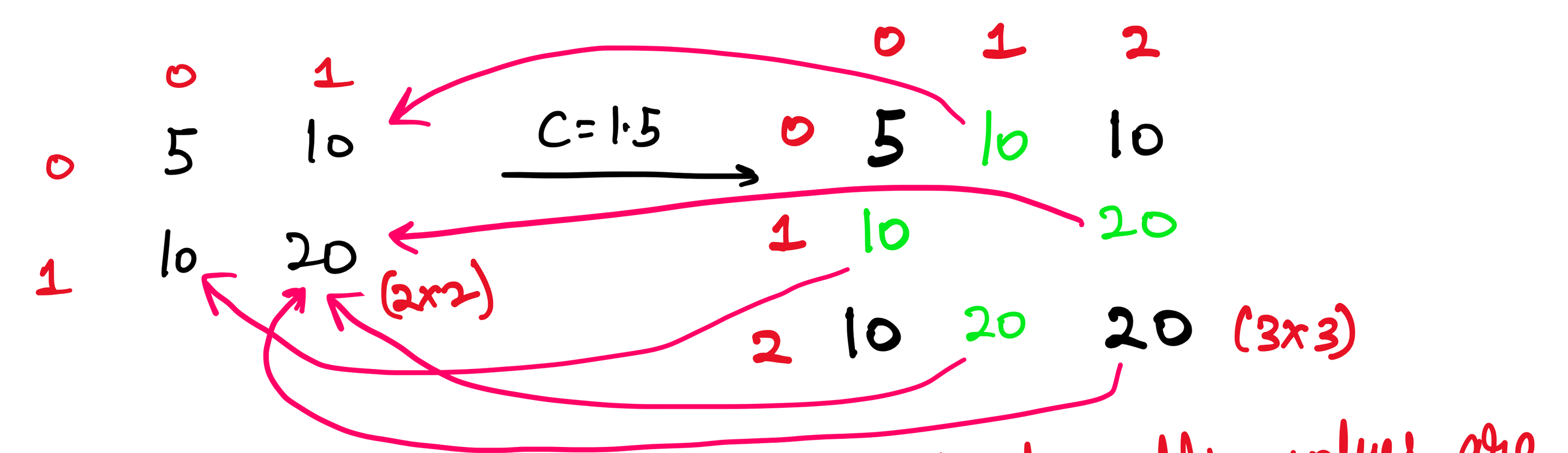
in case X^{-1} has small values, $A = (X+\lambda I)^{-1}V$

where λ is a small real number

2. given ⇒ image matrix $\begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}_{2\times2}$

★ using nearest neighbours to interpolate it by 1.5 along both x and y axis. Output image will be **3x3**

★ note: bilinear interpolation is only used for (1,1) coordinate



#note: for output → input coordinate mapping, values are being rounded off

output grid → input grid mapping.

(0,1) → $(0, \frac{1}{1.5})$ → (0, 0.67) ≈ (0,1)

(1,0) → (1/1.5, 0) → (0.67, 0) ≈ (1,0)

(2,1) → (2/1.5, 1/1.5) → (1.33, 0.67) ≈ (1,1)

(1,2) → (0.67, 1.33) ≈ (1,1)

(2,2) → (2/1.5, 2/1.5) → (1.33, 1.33) ≈ (1,1)

(2,0) → (1.33, 0) → (1,0)

(0,2) → (0, 1.33) → (0,1)

for (1,1) using bilinear interpolation

$x = \frac{1}{1.5} \approx 0.67$

$y = \frac{1}{1.5} \approx 0.67$

★ finding 4 nearest neighbors of (1,1) corresp. to input matrix,

$x_1 = \frac{1}{1.5} = 0.67 \approx 0$

$x_2 = \frac{1+1.5}{1.5} = 1.67 \approx 1$

$y_1 = \frac{1}{1.5} = 0.67 \approx 0$

$y_2 = \frac{1+1.5}{1.5} = 1.67 \approx 1$

Neighbours = [(0,0), (1,0), (0,1), (1,1)]

using $V = XA$

$$\begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

(columns: x, y, xy, 1)

on taking inverse and solving,

$$A = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

for (1,1),

x, y = 1/1.5, 1/1.5
= 0.67, 0.67

$$V = XA = \begin{bmatrix} 0.67 & 0.67 & 0.45 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

≈ 13.95 ≈ 14

∴, value for coordinate (1,1) in o/p matrix using bilinear interpolation is **14**