هوش جمعی : بهینه سازی ازدحام ذرات

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Outline

- Introduction
- Particle swarm optimization
- PSO algorithm
- PSO solution update in 2-D
- Example

Introduction

- Particle Swarm Optimization(PSO)
  - Proposed by James Kennedy & Russell Eberhart in 1995
  - Inspired by social behavior of birds and fishes
  - Combines self-experience with social experience
  - Population-based optimization

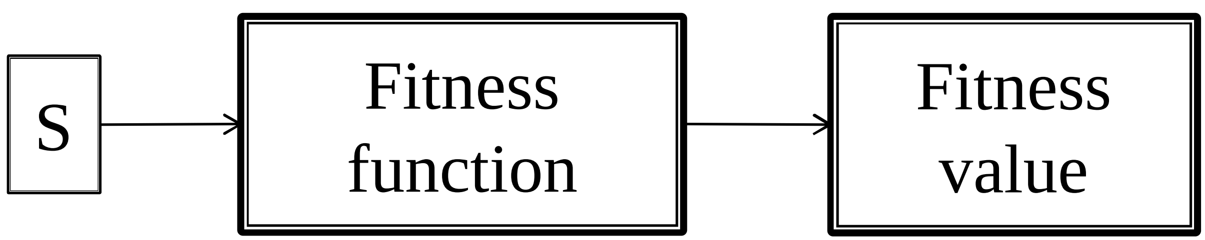
Concept

- Uses a number of particles that constitute a swarm moving around in the search space looking for the best solution.

- Each particle in search space adjusts its “flying” according to its own flying experience as well as the flying experience of other particles

Particle Swarm Optimization

- Swarm: a set of particles (S)
- Particle: a potential solution
  - Position: $\mathbf{x}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,n}) \in \Re^n$
  - Velocity: $\mathbf{v}_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,n}) \in \Re^n$
- Each particle maintains
  - Individual best position (PBest)
- Swarm maintains its global best (GBest)



PSO Algorithm

- Basic algorithm of PSO
  1. Initialize the swarm form the solution space
  2. Evaluate the fitness of each particle
  3. Update individual and global bests
  4. Update velocity and position of each particle
  5. Go to step2, and repeat until termination condition

PSO Algorithm

- Original velocity update equation

$$\mathbf{v}_i(k+1) = \text{Inertia} + \text{cognitive} + \text{social}$$

$$\begin{aligned}\mathbf{v}_i(k+1) = \omega \times \mathbf{v}_i(k) + c_1 \times random_1() \times (PBest_i - \mathbf{x}_i(k)) \\ + c_2 \times random_2() \times (GBest - \mathbf{x}_i(k))\end{aligned}$$

  - w, c_1, c_2: Constant
  - $random_1()$, $random_2()$: random variable

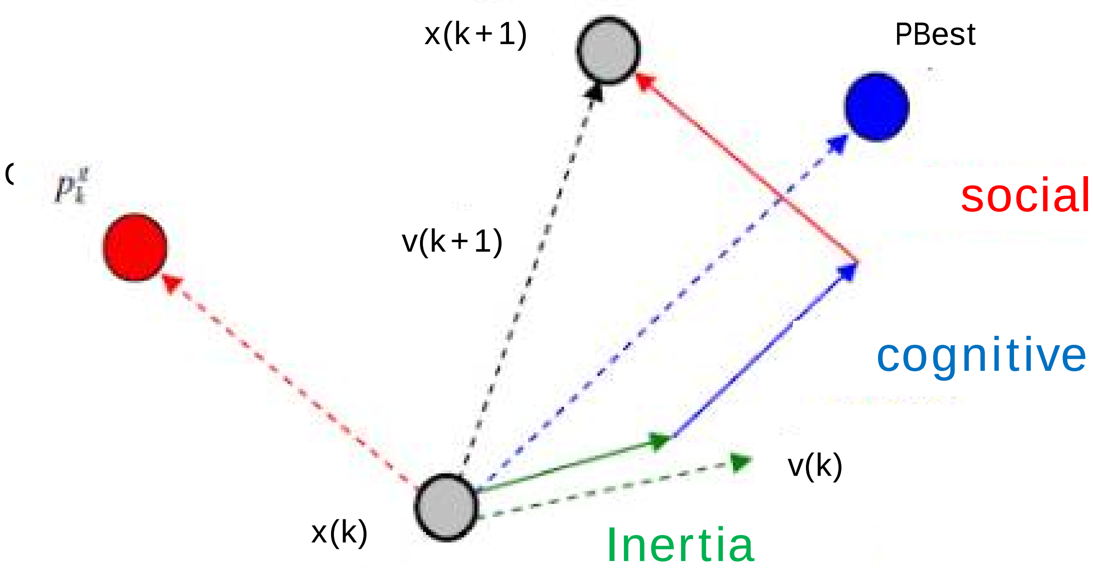
- Position update

$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + \mathbf{v}_i(k+1)$$

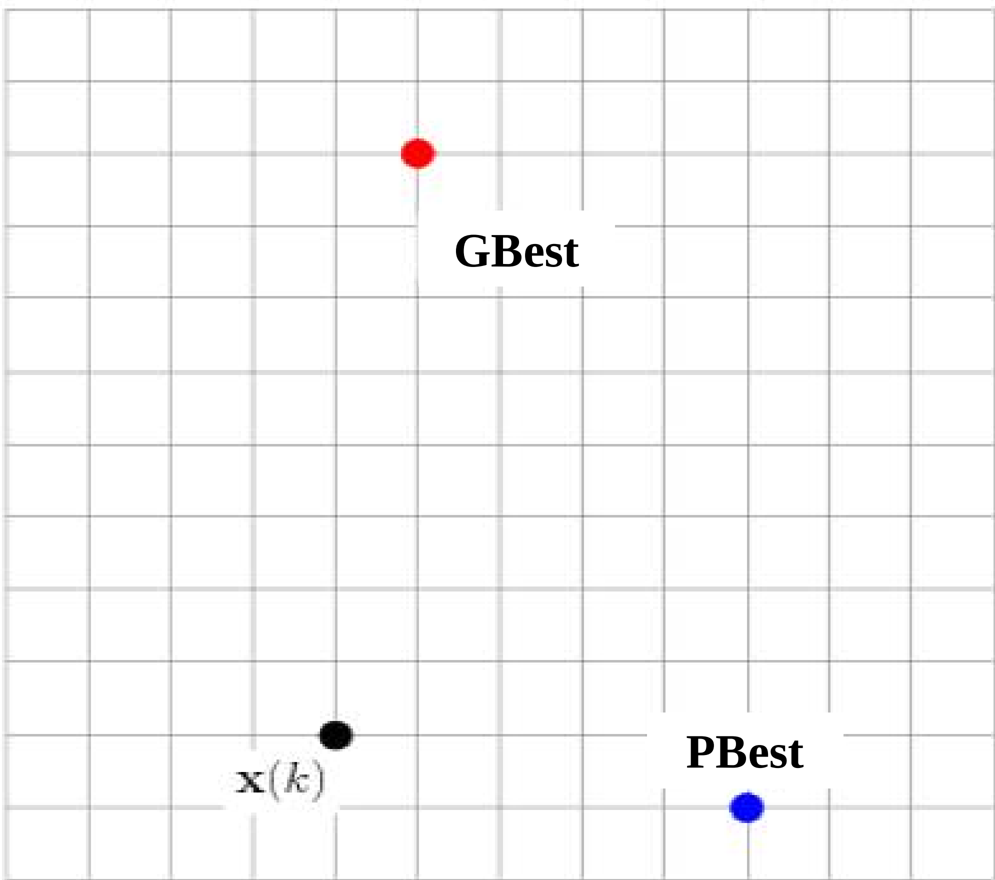
PSO Algorithm

- Particle's velocity

$$\mathbf{v}_i(k+1) = \text{Inertia} + \text{cognitive} + \text{social}$$

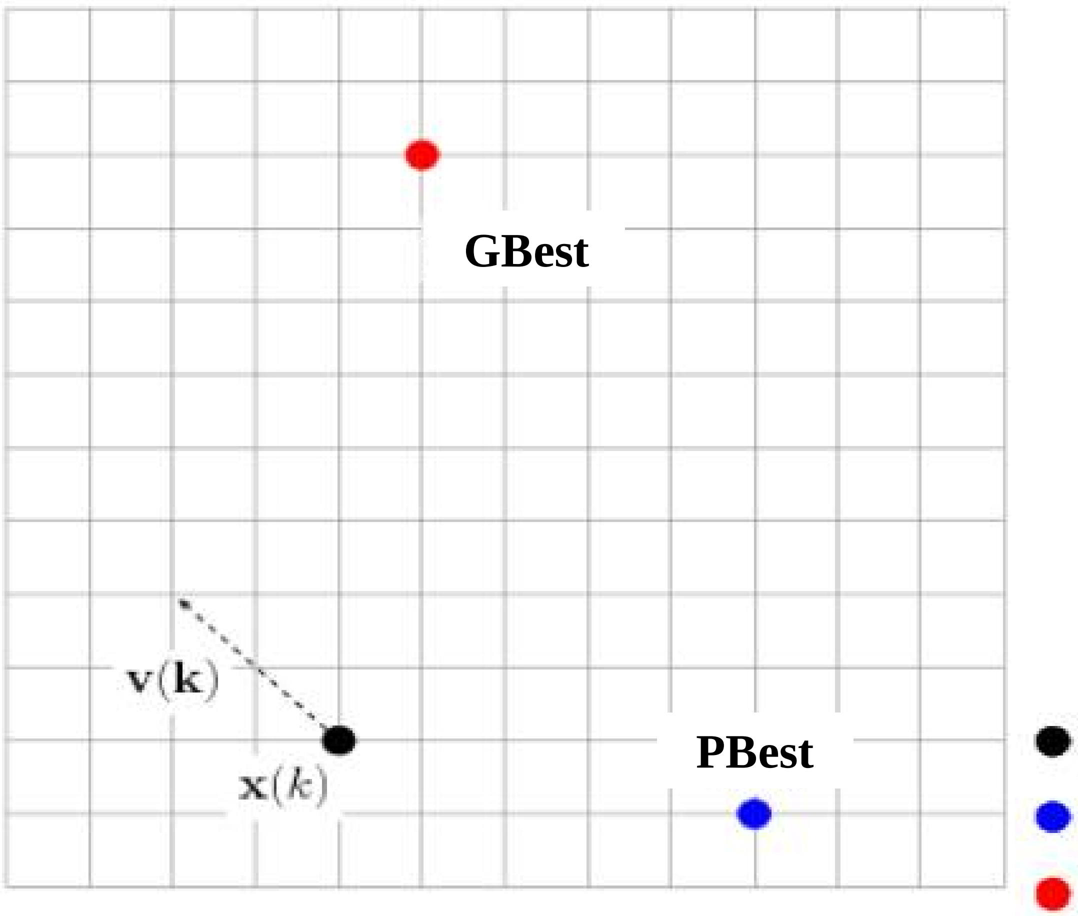


PSO solution update in 2D



- $\mathbf{x}(k)$ - Current solution (4, 2)
- PBest - Particle’s best solution (9, 1)
- GBest-Global best solution (5, 10)

PSO solution update in 2D



Inertia: **v**(k)=(-2, 2)

x(k) - Current solution (4, 2)

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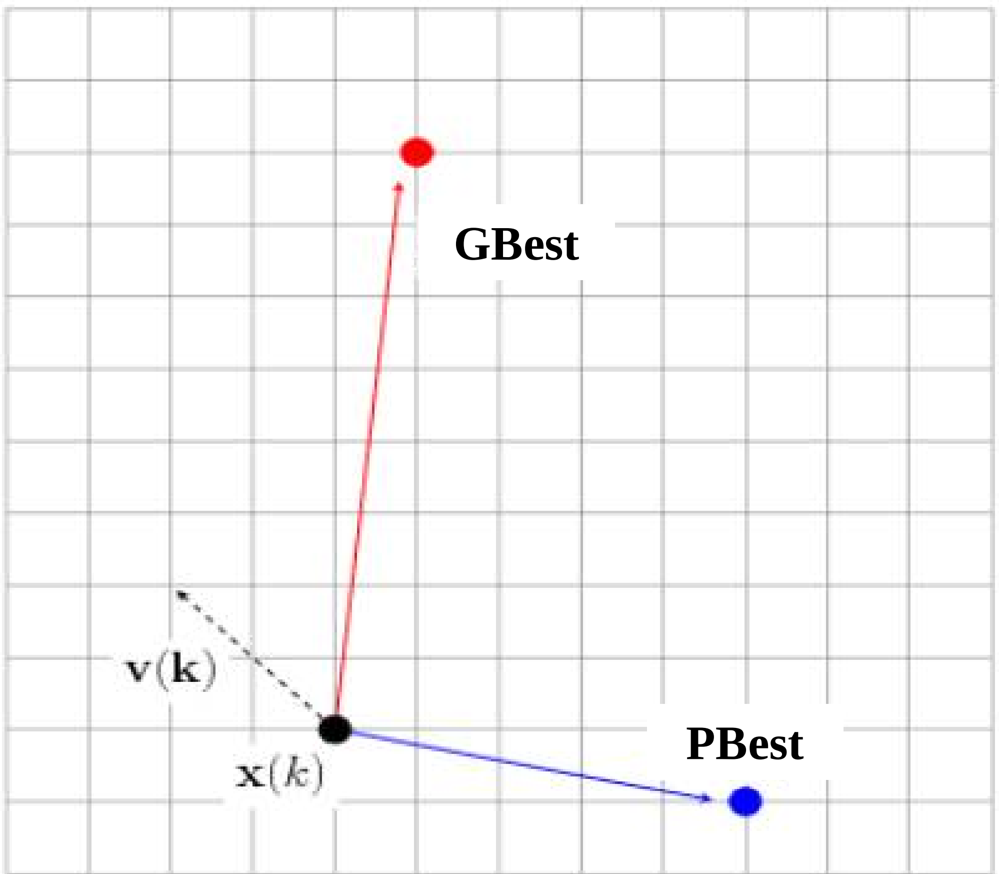
PSO solution update in 2D



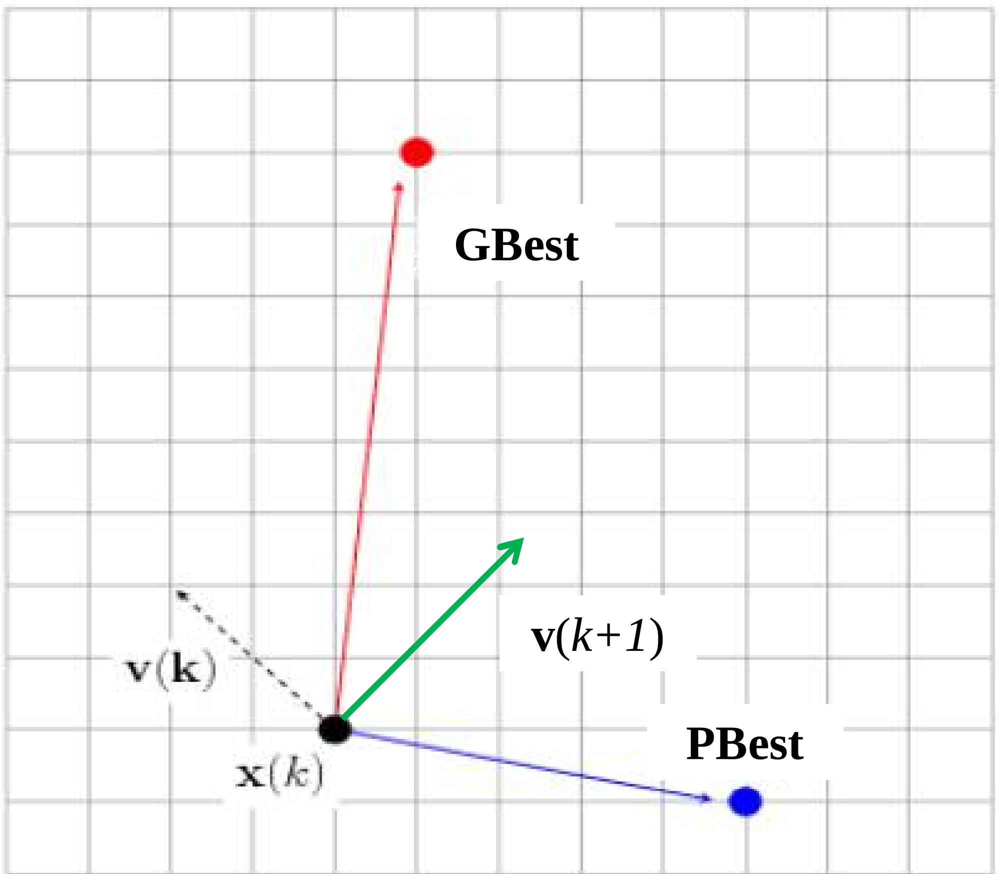
- Inertia: **v**(k)=(-2,2)
- Cognitive:
  PBest-**x**(k)=(9,1)-(4,2)=(5,-1)
- Social:
  GBest-**x**(k)=(5,10)-(4,2)=(1,8)

● **x**(k) - Current solution (4, 2)

● PBest - Particle's best solution (9, 1)

● GBest-Global best solution (5, 10)

PSO solution update in 2D



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- Social:
  GBest-$\mathbf{x}(k)=(5,10)-(4,2)=(1,8)$

$\mathbf{v}(k+1)=(-2,2)+0.8*(5,-1)$
$+0.2*(1,8) = (2.2,2.8)$

- $\mathbf{x}(k)$ - Current solution (4, 2)
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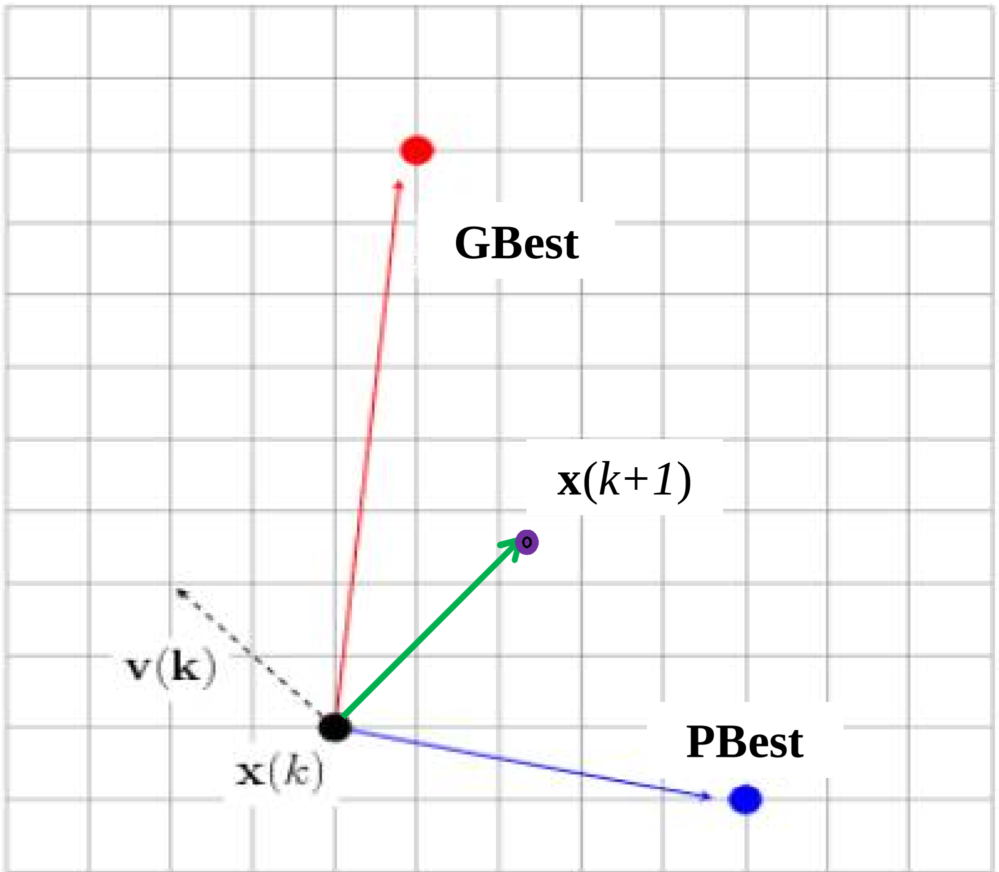
PSO solution update in 2D



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  PBest-$\mathbf{x}(k)$=(9,1)-(4,2)=(5,-1)
- Social:
  GBest-$\mathbf{x}(k)$=(5,10)-(4,2)=(1,8)
- $\mathbf{v}(k+1)$=(2.2,2.8)

$\mathbf{x}(k+1)=\mathbf{x}(k)+\mathbf{v}(k+1)=$
(4,2)+(2.2,2.8)=(6.2,4.8)

- $\mathbf{x}(k)$ - Current solution (4, 2)
- PBest - Particle's best solution (9, 1)
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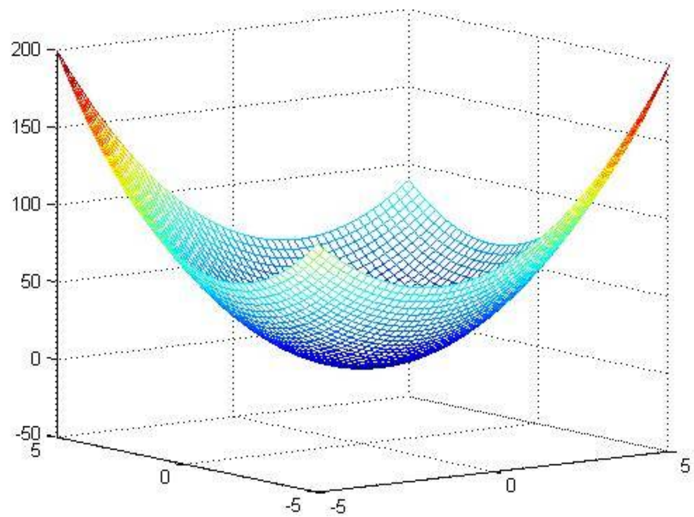
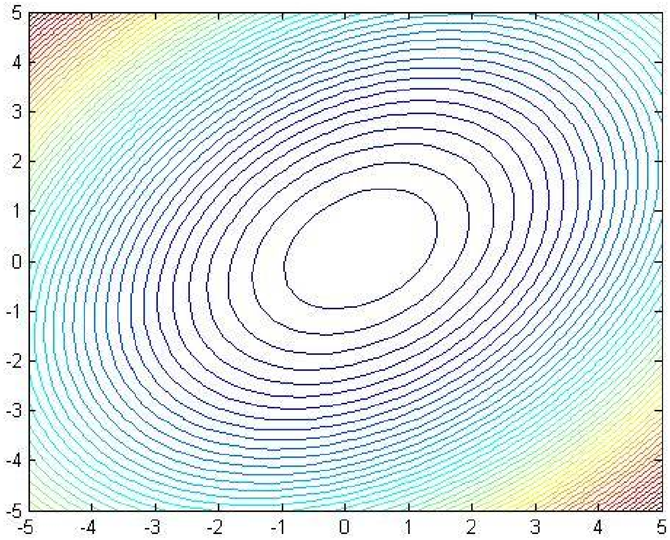
Example

- Find the minimum of this function

$$f(\mathbf{x}) = 3x_1^2 - 2x_1x_2 + 3x_2^2 - x_1 - x_2$$





Example

$$\mathbf{x}_1 = \begin{bmatrix} 2.2824 & 0.6238 & 4.0005 & 3.1717 & -4.0058 \\ -0.4894 & -2.7580 & -2.7043 & -3.3118 & 1.5771 \end{bmatrix}$$

$$\mathbf{v}_1 = \begin{bmatrix} -0.6321 & 0.1712 & 0.6942 & 0.0264 & 0.2207 \\ 0.2133 & -0.5598 & -0.2500 & 0.6079 & 0.3122 \end{bmatrix}$$

$$\Downarrow$$

$$\mathbf{x}_2 = \begin{bmatrix} 1.7767 & 1.4300 & 2.5656 & 2.2018 & 3.3541 \\ -0.3187 & -2.2903 & -0.3385 & 0.3199 & -0.5338 \end{bmatrix}$$

$$\mathbf{v}_2 = \begin{bmatrix} -0.5057 & 0.8063 & -1.4349 & -0.9700 & 7.3599 \\ 0.1706 & 0.4677 & 2.3657 & 3.6317 & -2.1109 \end{bmatrix}$$

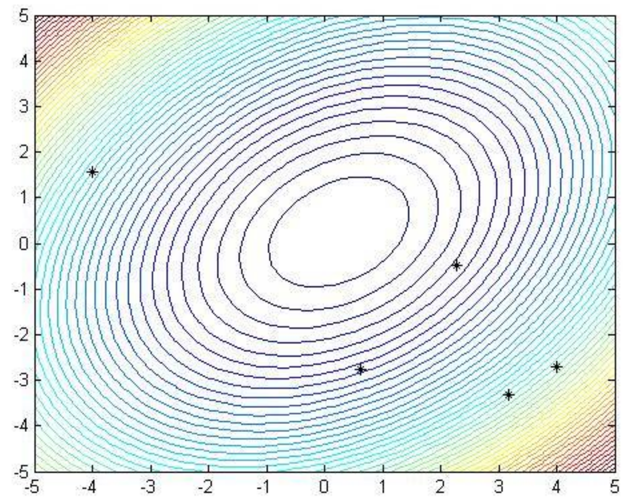
$$\Downarrow$$

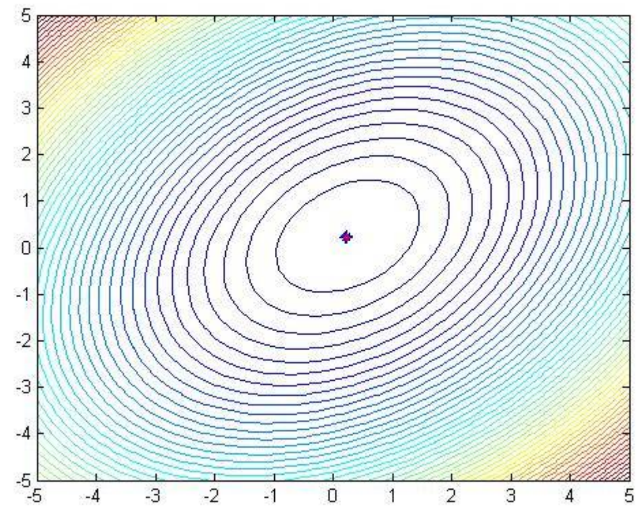
$$\mathbf{x}_3 = \begin{bmatrix} 1.3721 & 2.4464 & 1.0728 & 1.1350 & 7.9656 \\ -0.1822 & 0.1959 & 1.5627 & 2.7884 & -2.0485 \end{bmatrix}$$

$$\mathbf{v}_3 = \begin{bmatrix} -0.4046 & 1.0163 & -1.4928 & -1.0667 & 4.6114 \\ 0.1365 & 2.4862 & 1.9012 & 2.4685 & -1.5146 \end{bmatrix}$$

$$\vdots$$

$$\mathbf{x}_t = \begin{bmatrix} 0.2230 & 0.2197 & 0.2400 & 0.2293 & 0.2167 \\ 0.2056 & 0.2436 & 0.2378 & 0.2156 & 0.2106 \end{bmatrix}$$





$$GBest = \begin{bmatrix} 0.2227 \\ 0.2057 \end{bmatrix} \quad fitness = -0.25$$