$$f(w,b,x) = \frac{1}{1+e^{-wx-b}}$$ Note that in logistic regression log is ln

$$J(w,b,x,y) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log f(w,b,x^{(i)}) + (1-y^{(i)}) \log(1-f(w,b,x)\right]$$

$$\frac{\partial J(w,b,x,y)}{\partial w} = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)}\left(\frac{\partial \log \cancel{f}(w,b,x^{(i)})}{\partial w}\right) + (1-y^{(i)}) \frac{\partial \log(1-f(w,b,x)]}{\partial w}\right.$$

1. $$\frac{\partial \log f(w,b,x^{(i)})}{\partial w} = (1+e^{-wx-b})\left(\frac{1}{1+e^{-wx-b}}\right)' = + \frac{x^{(i)} e^{-wx-b}}{1+e^{-wx-b}} =$$

$$(\log_a x)' = \frac{1}{x \ln a} \qquad \left(\frac{1}{x}\right)' = (x^{-1})' = -\frac{1}{x^2} \qquad = x^{(i)}(1-f(w,b,x)).$$

$$\frac{\partial \log[1-f(w,b,x)]}{\partial w} = -\left(\frac{\cancel{1+e^{-wx-b}}}{\cancel{e^{-wx-b}}}\right)\left(\frac{\cancel{e^{-wx-b}}}{(1+e^{-wx-b})^{\cancel{2}}}\right) = -\frac{x^{(i)}}{1+e^{-wx-b}} =$$

$$= -f(w,b,x)$$

$$\frac{\partial J(w,b,x,y)}{\partial w} = \frac{1}{m}\sum_{i=1}^{m}\left(-x^{(i)}\left(y^{(i)} - \cancel{f(w,b,x)\,y^{(i)}} - f(w,b,x) + \cancel{f(w,b,x)\,y^{(i)}}\right)\right) =$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left(f(w,b,x) - y^{(i)}\right)x^{(i)}$$

analogically

$$\frac{\partial J(w,b,x,y)}{\partial b} = \frac{1}{m}\sum_{i=1}^{m}\left(f(w,b,x) - y^{(i)}\right)$$