Recap RLS algorithm

Estimate x, given $y(k) = C(k)x + w(k)$

Recursive algorithm

Init: $\hat{x}(0) = \bar{x}$, $P(0) = P_0 = Var(x)$

Recursion: observe $y(k)$

Update: $K(k) = P(k-1)C(k)^T\left(C(k)P(k)C(k)^T + R(k)\right)^{-1}$

$\hat{x}(k) = \hat{x}(k-1) + K(k)\left(y(k) - C(k)\hat{x}(k-1)\right)$

$P(k) = (I - K(k)C(k))P(k-1)(\_\_\_)^T + K(k)R(k)K(k)^T$

$P(k) = Var(x - \hat{x}(k))$

Dynamic system

$x(k) = A(k-1)x(k-1) + v(k-1)$ process noise $\cancel{+x}$ ⟸ $x(0)$, $\{v(\cdot)\}$, $\{w(\cdot)\}$ independent, zero mean

state $y(k) = C(k)x(k) + w(k)$ measurement noise

$P_0 = Var(x(0))$, $Var(v(k)) = Q(k)$ $Var(w(k)) = R(k)$

$x(k) = A(k-1)x(k-1) + v(k-1)$

$= A(k-1)\ldots x(0) + A(k)\ldots v(k-2)\ldots + v(k-1)$

$y(k) = \_\_\ x(0) + \_\_\_\ v(k-1) \ldots + \_\ v(0) + \_\ w(k)$

$\mathcal{Y}(k) = (y(1), y(2) \ldots y(k))$

$\xi(k) = (x(0), v(1) \ldots, w(1) \ldots)$ ⟵

$Var(\xi)$

$\mathcal{Y}(k) = \mathcal{C}(k)\,\xi(k)$ ⟵ $\hat{\xi}^{WLS}(k) = \left(\mathcal{C}(k)^T P_\xi(k)\,\mathcal{C}(k)\right)^{-1}\mathcal{C}(k)^T P_\xi(k)\,\mathcal{Y}(k)$

Batch

Recursion

Assume: $\hat{x}_p(k)$ $\quad P_p(k)$ $\quad$ Estimate of x at the (k)

$\hat{x}_{k|k-1}$ $\quad \Sigma^y_{k|k-1}$

① Get $y(k) = C(k)\,x(k) + w(k)$

Easy!

$$K(k) = P_p(k)\,C^T(k)\Big(\underbrace{C(k)\,P_p(k)\,C(k)^T + R(k)}_{\Sigma^y_{k|k-1}}\Big)^{-1}$$

$$\hat{x}_m(k) = \hat{x}_p(k) + K(k)\big(y(k) - C(k)\,\hat{x}_p(k)\big)$$

$\hat{x}_{k|k}$

$$P_m(k) = (I - K(k)C(k)P_p(k)\,(\text{—})^T + K(k)\,R(k)\,K(k)^T$$

x

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② Need prediction $\hat{x}_p(k+1)$ from model, $\hat{x}_m(k)$ $P_m(k)$

$$x(k+1) = A(k)\,x(k) + v(k) + u(k)$$

$$\hat{x}_p(k+1) = A(k)\cdot\hat{x}_m(k) + \cancel{E[v(k)]} + u(k)$$

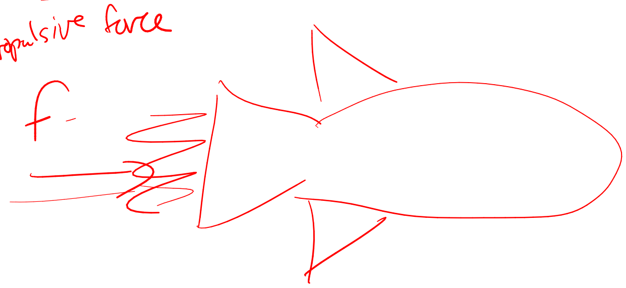
$$Var(e_p(k+1)) = P_p(k+1) = A(k)\,P_m(k)\,A(k)^T + Q(k) + \text{?}$$

$v(k)$ is independent

$\hat{x}_{k+2|k}$

Example Rocket ship

Propulsive force



Sensor: noisy meas. of position

Position p, Speed s

State $x(k) = \begin{bmatrix} p(k) \\ v(k) \end{bmatrix}$ $\quad$ timestep 1

desired force $\quad$ Noise on force

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} u_f(k) + \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} v_f(k)$$

$\ddot{p} = a$ $\qquad p(t+\Delta t) = p(t) + v(t)\cdot\Delta t + \frac{1}{2} a(t)\cdot \cancel{\Delta t^2}$

Define $v(k) = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} v_f(k)$ $\qquad Var(v(k)) = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} Var(v_f(k)) \begin{bmatrix} 1/2 & 1 \end{bmatrix}$

Meas. $y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + w(k)$

$E[v_f(k)] = 0$, $Var(v_f(k)) = 0.1$ $\qquad E[w(k)] = 0$, $Var(w(k)) = 1/2$

$E[x(0)] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\qquad Var(x(0)) = I$

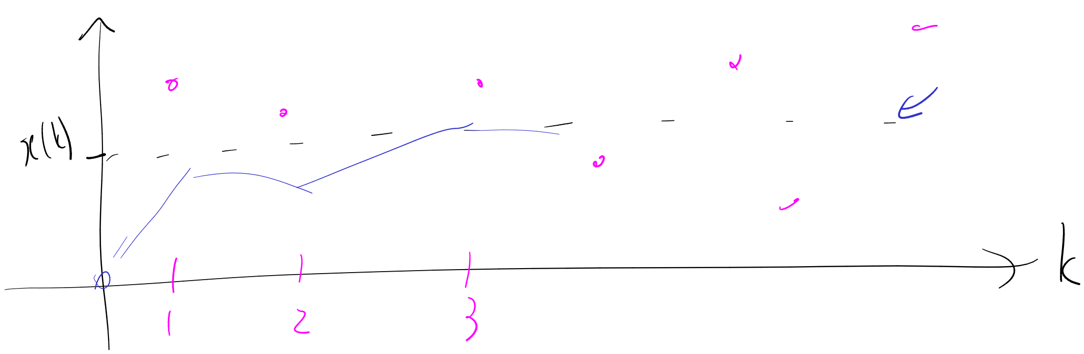
Linear?

System: $x(k) = x(k-1)$

$y(k) = x(k) + w(k)$

$w(k) \in [-1, 1]$ uniformly distributed

$x(0)$: Normally distributed $N(0,1)$



ML estimator

Init: $\ell(0) = +\infty \qquad u(0) = -\infty$

At time k, measurement $y(k)$: $\ell(k) = \min(\ell(k-1), y(1))$

$u(k) = \max(u(k-1), y(1))$