

Multiple sensor fusion for mobile robot localization and navigation using the Extended Kalman Filter

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Abstract — Navigation is an important topic in mobile robots. In this paper, an Extended Kalman Filter (EKF) is used to localize a mobile robot equipped with an encoder, compass, IMU and GPS utilizing three different approaches. Subsequently, an input output state feedback linearization (I-O SFL) method is used to control the robot along the desired robot trajectory. The presented algorithms are verified when the robot was steered along two different track shapes. Additionally, the performance of the method is demonstrated when a fault was simulated on the sensors.

Keywords — *Localization, Extended Kalman Filter (EKF), Navigation, Input Output State Feedback linearization (IO-SFL), Mobile Robot.*

I. INTRODUCTION

The problem of navigation can be summarized into answering three questions: “where am I?”, “where am I going?” and “how should I get there?” [1]. Localization answers the first question. Finding a robust solution to this problem is the first step for solving the other two questions.

In mobile robot navigation systems, onboard navigation sensors based on dead-reckoning are widely utilized. This type of sensors includes encoders and inertial measurement units (IMUs). These sensors measure the robot’s linear and angular velocity along with its acceleration. By integrating sensor measurements, the position and orientation of mobile robots are estimated. This is known as dead-reckoning [2]. Dead-reckoning method is not suitable for long-term navigation because of its drawbacks where the onboard sensors have unavoidable accumulated errors. Therefore, mobile robot needs an external reference for correction. External absolute position sensors can be installed on mobile robots. These sensors include cameras and Global Positioning System (GPS) receivers. These sensors measure the absolute position and direction of the robot. In [3] authors defined two groups for positioning systems for mobile robots. The first is relative position measurements which include odometry and inertial navigation. The second system uses absolute position measurements which include magnetic compasses, active beacons, global positioning systems (GPS), landmark navigation and model matching. The two groups have advantages and drawbacks. For example, the absolute position sensors have accurate but low rate measurement. On other hand, the relative position sensors have fast rate but with drifting bias [4, 15].

Sensor fusion is a useful technique to fuse both types of positioning sensors to provide high-accuracy estimates at high update rates. The most popular algorithms of sensor fusion are:

Kalman Filtering (KF) based approaches [5]-[12], Particle Filters (PF) [13], and intelligent-based filters [15]-[16].

In order to localize the robot, many sensors were used and fusion methods have been developed. In [9], authors used received signal strength indicator (RSSI) from wireless local area network (WLAN) and an IMU to obtain an estimate of the robots state based on an Extended Kalman Filter. In [10]-[11], an IMU, wheel encoders and GPS were integrated using a Kalman Filter (KF) to obtain an outdoor 3-D localization solution. In [12], authors used a low pass filter and a Kalman Filter (KF) to reduce the noise and combine the sensor data from IMU, an odometer and an active beacon system to obtain a precise navigation system. In [14], authors used Monte Carlo Localization (MCL) with vision and sonar sensor. On the other hand, authors in [15] fused IMU and GPS measurements using intelligent fusion based on an Adaptive Neuro-Fuzzy Inference System (ANFIS).

Finally, after the robot is localized and the states have been estimated, wheeled mobile robots (WMR) can be controlled to track a desired path. However, the difficulty in motion control of WMRs arises from the nature of these systems which has nonholonomic constraints due to perfect rolling (i.e. no lateral slippage) [16]. The trajectory tracking problem was solved in [17] by dynamic feedback linearization, and in [18] utilizing input-output state-feedback linearization (I-O SFL).

In this paper, we mainly discuss the localization and navigation algorithms using the Extended Kalman Filter (EKF) and utilizing the encoder, compass, IMU and GPS measurements. Three approaches are proposed to use these sensors in different combinations while guaranteeing to observe all states of the mobile robot. The first approach uses encoder, compass and GPS receiver. The second approach uses gyroscope, accelerometer and GPS. Finally, the third approach use all of the sensors in the filter’s measurement equation.

The organization of this paper is as follows: in Section II, the kinematic model of WMR is derived. Section III presents the Extended Kalman filter design. Section IV presents input output state feedback controller. Section V presents the simulation results followed by the conclusions in section VI.

II. THE KINEMATIC MODEL OF DIFFERENTIAL MOBILE ROBOT

In order to determine the position of the robot on the plane, a relationship was established between the global reference frame

of the plane and the local reference frame of the robot, as shown in Fig. 1. The axes X_{Global} , Y_{Global} and X_{Robot} , Y_{Robot} define the global reference frame and robot reference frame respectively. The position of the robot in plane is described by point $S(x,y)$ on robot body, and by angular difference θ . The position of the robot is completely specified by the 3 variables x, y and θ . And a 3×1 -vector α and orthogonal rotation matrix $R(\theta)$ were defined to describe the robot position and orientation [20].

$$\alpha_{Global} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \quad (1)$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Where

$$\dot{\alpha}_{Robot} = R(\theta)\dot{\alpha}_{Global} \quad (3)$$

The differential drive robot has two wheels, the distance between the two wheels is l . The robot's motion is controlled by its linear velocity (V) and angular velocity (ω).

$$\dot{\alpha}_{Global} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(V, \omega) \quad (4)$$

$$\dot{\alpha}_{Global} = R(\theta)^{-1} \begin{bmatrix} V \\ 0 \\ \omega \end{bmatrix} \quad (5)$$

The motion of the robot can be described by the following kinematical model.

$$\dot{\alpha}_{Global} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} V \cos\theta \\ V \sin\theta \\ \omega \end{bmatrix} \quad (6)$$

As can be seen there are two inputs V, ω for the system. Where the linear velocity V of the robot is always heading in the X_{Robot} direction of the robot's reference frame, which means the robot cannot move sideways due to the nonholonomic

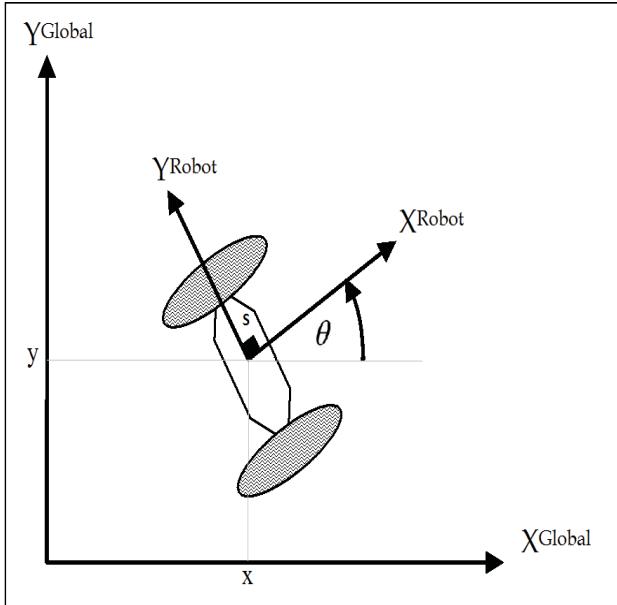


Fig. 1. The global reference frame and the robot local reference frame

constraint, and the angular velocity ω can be considered as the rotation rate of the local reference frame with respect to the global reference frame. If we redefine the states as $\beta = [x, y, \theta, V, \omega]$, the kinematical model will be as follows.

$$\dot{\beta} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{V} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} V \cos\theta \\ V \sin\theta \\ \omega \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

In order to perform the Kalman filter, this system needs to be discretized. So by writing it in discrete form with sampling time (Δt) the system model will be:

$$\beta_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \\ V_k \\ \omega_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + V_{k-1} \Delta t \cos\theta_{k-1} \\ y_{k-1} + V_{k-1} \Delta t \sin\theta_{k-1} \\ \theta_{k-1} + \omega_{k-1} \Delta t \\ V_{k-1} \\ \omega_{k-1} \end{bmatrix} \quad (8)$$

III. EXTENDED KALMAN FILTER

The Extended Kalman Filter was used to fuse both types of positioning sensors. The Kalman filter has the ability to make an optimal estimate of the state variable even with noisy measurements. The EKF was used instead of KF due to nonlinearity of a mobile robot kinematic model. EKF require system and measurement models, the system model and measurement models, respectively, are shown:

$$\beta_k = f(\beta_{k-1}) + w_{k-1} \quad (9)$$

$$z_k = h(\beta_k) + v_k \quad (10)$$

Where f and h represent the nonlinear system and measurement models, respectively. The dynamic system noise w_{k-1} , and measurement noise, v_k are both zero mean Gaussian noise with associated covariance matrices:

$$w_k \sim N(0, Q_k),$$

$$v_k \sim N(0, R_k)$$

In order to use nonlinear functions f and h , the functions must be linearized. By taking the Jacobians of f and h at the operating point, β_k during each time step. These matrices are:

$$F_{k-1} = \frac{\partial f(\beta)}{\partial \beta} \big|_{\beta_{k-1}^-} \quad (11)$$

$$F_k = \begin{bmatrix} 1 & 0 & -V_k \Delta t \sin\theta_k & \Delta t \cos\theta_k & 0 \\ 0 & 1 & V_k \Delta t \cos\theta_k & \Delta t \sin\theta_k & 0 \\ 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

Prediction:

$$\hat{\beta}_k^- = f(\hat{\beta}_{k-1}^+, u_k) \quad (13)$$

$$P_k^- = F_k P_{k-1}^- F_k^T + Q_k \quad (14)$$

Update:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad (15)$$

$$\hat{\beta}_k^+ = \hat{\beta}_k^- + K_k (z_k - h(\hat{\beta}_k^-)) \quad (16)$$

$$P_k^+ = P_k^- - K_k H_k P_k^- \quad (17)$$

The measurement model describes what the sensors measure. GPS measures the position of the robot, encoder measures the wheel's velocity, compass measures the heading, and the IMU

measures the angular velocity and acceleration rate in gyroscope and accelerometer, respectively.

Measurement model was derived for each sensor as follows.

a) GPS:

GPS measures the position of the robot., then the measurement matrix is:

$$h_{gps} = \begin{bmatrix} x_{gps} \\ y_{gps} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} \quad (18)$$

$$H_{gps} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

$$z_k = H_{gps} \beta_k + v_k \quad (20)$$

b) Odometry:

The two wheels of differential mobile robot are equipped with encoders. Encoders measure the wheel's velocity by counting the pulses per sampling period. Therefore, it will give the information about the displacement of the robot relative to its last known position. Then the measurement matrix is:

$$h_{encoder} = \begin{bmatrix} V_{r_{encoder}} \\ V_{l_{encoder}} \end{bmatrix} = \begin{bmatrix} V_r \\ V_l \end{bmatrix} \quad (21)$$

$$H_{encoder} = \begin{bmatrix} 0 & 0 & 0 & 1 & l/2 \\ 0 & 0 & 0 & 1 & -l/2 \end{bmatrix} \quad (22)$$

$$z_k = H_{encoder} \beta_k + v_k \quad (23)$$

c) Compass:

Compass is a navigational instrument for determining direction relative to the Earth's magnetic poles. It can be used to measure heading θ . Then the measurement matrix is:

$$h_{compass} = [\theta_{compass}] = \theta \quad (24)$$

$$H_{compass} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (25)$$

$$z_k = H_{compass} \beta_k + v_k \quad (26)$$

d) IMU:

In general IMU use a combination of accelerometers and gyroscopes.

1) Gyroscope :

Gyroscope measures the angular velocity ω . Then the measurement matrix is:

$$h_{gyroscope} = [\omega_{compass}] = \omega \quad (27)$$

$$H_{gyroscope} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (28)$$

$$z_k = H_{gyroscope} \beta_k + v_k \quad (29)$$

And by the integration, the heading θ can be obtained. Then the measurement matrix is:

$$h_{heading} = \int \omega_{gyroscope} . dt = \theta \quad (30)$$

$$H_{heading} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (31)$$

$$z_k = H_{heading} \beta_k + v_k \quad (32)$$

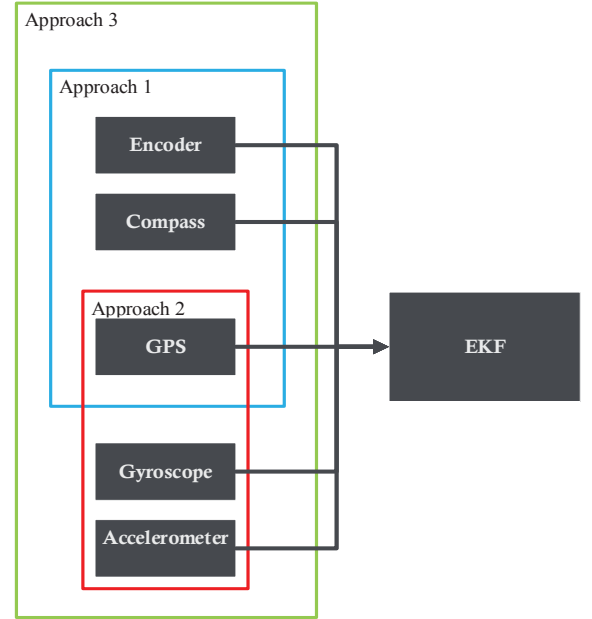


Fig. 2. The proposed approaches

2) Accelerometer

Accelerometer measures the linear acceleration, by integration, the linear velocity V can be obtained. Then the measurement matrix is:

$$h_{accelerometer} = \int a_{accelerometer} . dt = V \quad (33)$$

$$H_{accelerometer} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (34)$$

$$z_k = H_{accelerometer} \beta_k + v_k \quad (35)$$

In this paper, three approaches were proposed, the first one uses encoder, compass and GPS, the second one uses gyroscope, accelerometer and GPS, and third one uses all of them as measurements. Each approach consist of a group of sensors have the ability to observe all states of the mobile robot. Fig.2 summarizes the proposed approaches.

IV. THE KINEMATIC CONTROLLER

In order to navigate WMR on a path, the kinematic model is used to design feedback laws to achieve the trajectory tracking of WMR. The input output state feedback linearization was implemented. By taking a point B outside the wheel axle of the mobile robot, as the output (reference point) of the system as shown in Fig.3, where the position of B is :

$$x_B = x + b \cos \theta \quad (36)$$

$$y_B = y + b \sin \theta \quad (37)$$

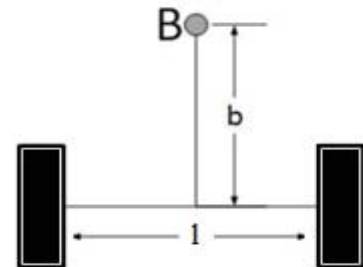


Fig. 3. Robot top view where the point B outside the wheel axle with distance b.

The kinematic model of the robot with respect to the coordinate transformation :

$$\dot{x}_B = v \cos \theta - \omega b \sin \theta \quad (38)$$

$$\dot{y}_B = v \sin \theta - \omega b \cos \theta \quad (39)$$

$$\dot{\theta} = \omega \quad (40)$$

By rearranging eq. (37) – eq. (39) and putting them in matrix form.

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\frac{1}{b} \sin \theta & \frac{1}{b} \cos \theta \end{bmatrix} \begin{bmatrix} \dot{x}_B \\ \dot{y}_B \end{bmatrix} \quad (41)$$

Then the control law with given a trajectory (x_{des}, y_{des}) :

$$\dot{x}_B = \dot{x}_{des} + k_1 e_x \quad (42)$$

$$\dot{y}_B = \dot{y}_{des} + k_2 e_y \quad (43)$$

Where:

$$e_x = x_{des} - x_B \quad (44)$$

$$e_y = y_{des} - y_B \quad (45)$$

The whole system is summarized in Fig. 4.

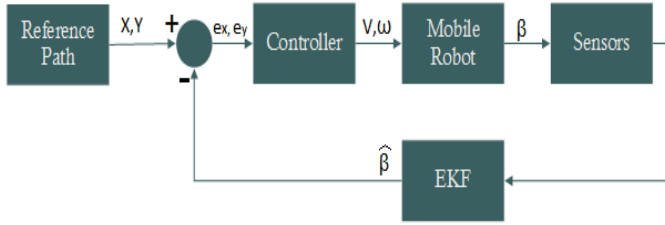


Fig. 4. The Whole System Structure.

V. RESULTS

The performance of the presented approaches were tested in different paths including circular and 8-shaped paths. Also, they were tested when a fault occurred in some sensors. Additionally, the measurement rates have been taken into account in this simulation i.e. GPS readings are updated on every fifteenth iteration and the other sensors are updated every iteration.

Fig. 5 shows the performance of the three approaches in 8-shaped path. Approach 3 has the best performance, followed by the approach 2 and finally approach 1. The results agree with the fact that more measurements will give a higher accuracy estimate. On the other hand, it can be noticed there is no guarantee for the robot to track the trajectory based only on odometry without sensor fusion due to the bias that will accumulate on the estimate.

Fig. 6 shows the performance of the three approaches in a circular path when a fault was occurred in the encoder after half of the simulation time. Approach 1 has the worst performance because it depends on the encoder, and no effect on approach 2. Finally, approach 3 has accurate performance even with bad encoder's measurement because it is updated the states using other sensors.

For further evaluation, the Mean Error (ME) and the Mean Square Error (MSE) of the system are shown in Table 1 and 2, respectively, they are obtained as follows:

$$ME = \frac{1}{n} \sum_{k=1}^n NE_k \quad (46)$$

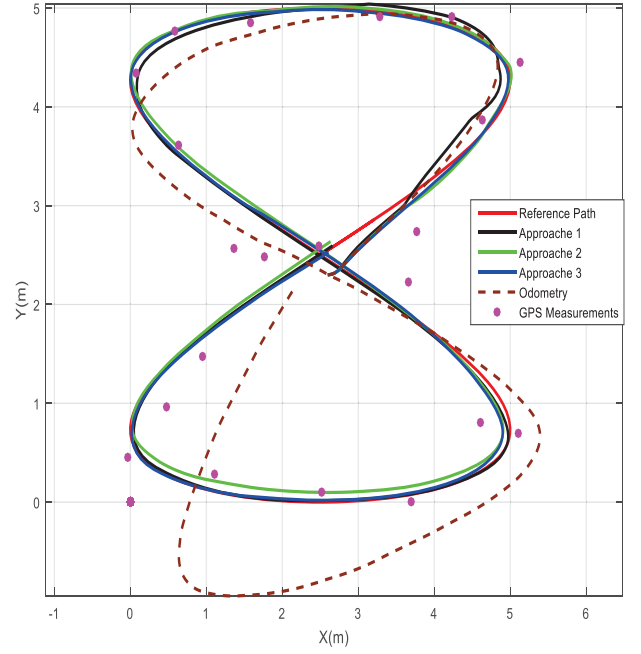


Fig. 5. Test 1 with 8-shaped path.

$$MSE = \frac{1}{n} \sum_{k=1}^n (NE_k - ME)^2 \quad (47)$$

where the norm of the error (NE) is:

$$NE_k = \sqrt{e_{x_k}^2 + e_{y_k}^2} \quad (48)$$

The results show that approach 3 outperformed the other approaches in all paths and tests.

Table 1. The mean error.

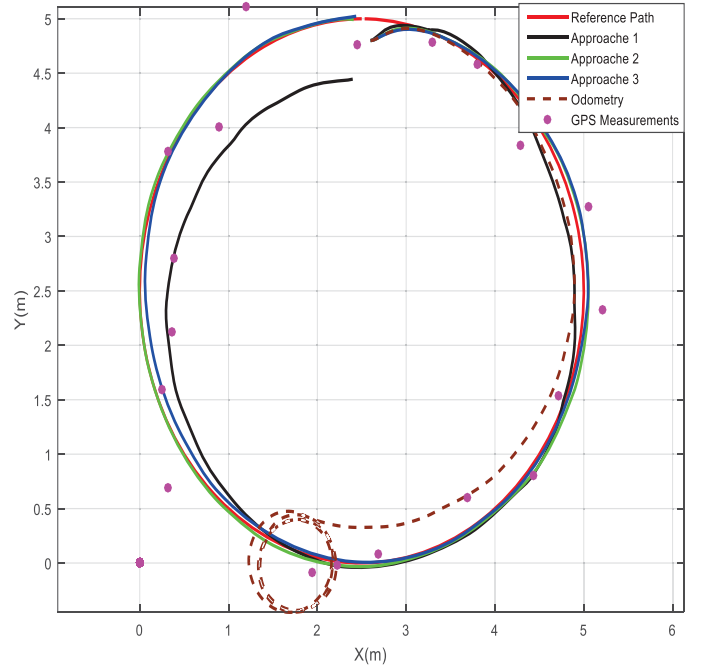


Fig. 6. Test 2 with Circular path

	8-shape	Circular
Approach 1	0.29688	0.27739
Approach 2	0.2936	0.11565
Approach 3	0.28235	0.11215

Table 2. Mean Square Error (MSE).

	8-shape	Circular
Approach 1	0.13635	0.17077
Approach 2	0.1491	0.076845
Approach 3	0.12569	0.01398

VI. CONCLUSION

In this paper, localization, control and navigation approaches for differential mobile robot were proposed. The method combines the robot controller with a navigation component using sensor fusion, which combines the measurements of onboard sensor and GPS measurements. Three approaches were verified in a simulation environment. The performance of the proposed algorithms was demonstrated when a fault on the encoder was simulated. In approaches 1 and 2, a group of sensors were chosen to observe all the states of the mobile robot. The encoder, compass and GPS were utilized in approach 1 where as the IMU and GPS were utilized in approach 2. On the other hand, all the sensors were utilized in approach 3. The results demonstrated that approach 3 outperformed the two other approaches. This agrees with the fact that more measurements will give a better estimation accuracy. Also, if one sensor failed, the algorithm still gives an accurate result when using approach 3.

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