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Quasilinear utility

In economics and consumer theory, **quasilinear utility** functions are linear in one argument, generally the numeraire. Quasilinear preferences can be represented by the utility function $u(x_1, x_2, \ldots, x_n) = x_1 + \theta(x_2, \ldots, x_n)$ where θ is strictly concave.[1]:164 A nice property of the quasilinear utility function is that, the Marshallian/Walrasian demand for $x_2, \ldots, x_n$ does not depend on wealth and therefore is not subject to a wealth effect.[1]:165-166 The absence of a wealth effect simplifies analysis[1]:222 and makes quasilinear utility functions a common choice for modelling. Furthermore, when utility is quasilinear, compensating variation (CV), equivalent variation (EV), and consumer's surplus are algebraically equivalent.[1]:163 In mechanism design, quasilinear utility ensures that agents can compensate each other with side payments.

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Definition in terms of preferences

A preference relation $\succsim$ is quasilinear with respect to commodity 1 (called, in this case, the *numeraire* commodity) if:

- All the indifference sets are parallel displacements of each other along the axis of commodity 1. That is, if a bundle "x" is indifferent to a bundle "y" (x~y), then $(x + \alpha e_1) \sim (y + \alpha e_1), \forall \alpha \in \mathbb{R}, e_1 = (1, 0, \ldots, 0)$[2]
- Good 1 is desirable; that is, $(x + \alpha e_1) \succ (x), \forall \alpha > 0$

In other words: a preference relation is quasilinear if there is one commodity, called the numeraire, which shifts the indifference curves outward as consumption of it increases, without changing their slope.

In two dimensional case, the indifference curves are parallel; which is useful because the entire utility function can be determined from a single indifference curve.

Definition in terms of utility functions

A utility function is quasilinear in commodity 1 if it is in the form

$$u(x) = x_1 + \theta(x_2, \ldots, x_L)$$

where θ is a function.[3] In the case of two goods, this function could be, for example $u(x) = x_1 + \sqrt{x_2}$.

The quasilinear form is special in that the demand function for the consumption goods depends only on the prices and *not* on the income. E.g, with two commodities, if

$$u(x, y) = x + \theta(y)$$

then the demand for y is derived from the equation

$$\theta'(y) = p_y$$

so

$$y(p, I) = (\theta')^{-1}(p_y),$$

which is independent of the income I.

The indirect utility function in this case is

$$v(p, I) = v(p) + I,$$

which is a special case of the Gorman polar form.[1]:154, 169

Equivalence of definitions

The cardinal and ordinal definitions are equivalent in the case of a convex consumption set with continuous preferences that are locally non-satiated in the first argument.

See also

- Quasiconvex function
- Linear utility function - a special type of a quasilinear utility function.

References

1. Varian, Hal (1992). *Microeconomic Analysis* (Third ed.). New York: Norton. ISBN 0-393-95735-7.
2. Mas-Colell, Andreu; Whinston, Michael; Green, Jerry (1995). "3". *Microeconomic Theory*. New York: Oxford University Press. p. 45.
3. "Topics in Consumer Theory" (https://web.archive.org/web/20111215230240/http://www.hks.harvard.edu/nhm/notes2006/notes4.pdf) (PDF). *hks.harvard.edu*. August 2006. pp. 87–88. Archived from the original (http://www.hks.harvard.edu/nhm/notes2006/notes4.pdf) (PDF) on 15 December 2011.