TTK4135 – Optimalization and Control

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Contents

1 Equations

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LP Remember to read the task closely, and see whether it asks for the given resources to be $fullyutilized$. If not, then introduce slack variables in your constraints.

Elements of Analysis

Lipschitz continuous

$$||f(x_1) - f(x_0)|| \leq L||x_1 - x_0||, \quad \forall\, x_0, x_1 \in \mathcal{N} \tag{1}$$

Mean value theorem

$$f(x+p) = f(x) + \nabla f(x + \alpha p)^\top p \tag{2}$$

Matrix calculus

Derivative

$$\begin{aligned} \nabla(c^\top \mathbf{x}) &= c \\ \nabla(\mathbf{x}^\top c) &= c \end{aligned} \tag{3}$$

$$\nabla\left(\frac{1}{2}\mathbf{x}^\top G\mathbf{x}\right) = \frac{1}{2}G\mathbf{x} + \frac{1}{2}G^\top \mathbf{x} \tag{4}$$

Gradient

$$\nabla f(\mathbf{x}) = \left[\frac{\partial f}{\partial \mathbf{x}}^\top\right] = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} \tag{5}$$

Hessian

$$\nabla_{xx} f(\mathbf{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1^2} & \frac{\partial f}{\partial x_1 \partial x_2} & \cdots \\ \frac{\partial f}{\partial x_2 \partial x_1} & \frac{\partial f}{\partial x_2^2} & \\ \vdots & & \ddots \end{bmatrix} \tag{6}$$

Jacobian

$$Jf(\mathbf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \\ \vdots & & \ddots \end{bmatrix} \tag{7}$$

2 Algorithms

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Simplex

One Step of Simplex()

Given $\mathcal{B}, \mathcal{N}, x_B = B^{-1}b \geq, x_N = 0$;
Solve $B^\top \lambda = C_B for \lambda$,
Compute $s_N = c_N - N^\top \lambda$; (* pricing *)
if $s_N \geq 0$
    stop; (* optimal point found *)
Select $q \in \mathcal{N}$ with $s_q < 0$ as the entering index;
Solve $Bd = A_q$ for d;
if $d \leq 0$
    stop; (* problem is unbounded *)
Calculate $x_q^+ = \min_{i|d_i>0}(x_B)_i/d_i$, and use p to denote the minimizing i;
Update $x_B^+ = x_B - dx_q^+, x_N^+ = (0, \ldots, 0, x_q^+, 0, \ldots, 0)^\top$;
Change $\mathcal{B}$ by adding q and removing the basic variable corresponding to column p of B