

Puzzle.c – Number Cross

0. Introduction

0-1. Number Cross consists of a MxN grid where every square, every row, and every column and is labeled with a positive integer

1. Constraints:

- 1-1. The color of the cell is black or white
- 1-2. The label of each column is the same as the sum of the numbers in the Black cell of the column.
- 1-3. The label of each row is the same as the sum of the numbers in the White cell of the row.

2. Definition:

- 2-1. I define $p(x, y)$ is the value that represents the black and white.(black is 1,white is 0).
- 2-2. The matrix range is from (0,0) to (n-1,m-1).
- 2-3. I define $N(x, y)$ is the value that represents the number range from 1 to 9 in a cell.
- 2-4. I define $B(y)$ is the value that represents the sum of the numbers in the Black cell of the column and $W(x)$ is the value that represents the numbers in the White cell of the row.

3. Logic formula and Constraints:

- 3-1. All $q(x, y)$ value has 0 or 1.

Q1.

$$\bigwedge_{x=0}^{n-1} \bigwedge_{y=0}^{m-1} (p(x, y) = 0) \vee (p(x, y) = 1)$$

- 3-2. Check the label of each row is the same as the sum of the numbers in the White cell of the row. Because black is 1 and white is 0, I subtract from $p(x, y)$ to 1. Then I multiply the result by $N(x, y)$. Repeat the previous process (m-1) times and add all result values. To make a positive number, multiply -1. Then compare results and $W(x)$.

Q2.

$$\bigwedge_{x=0}^{n-1} \{ (\sum_{y=0}^{m-1} (p(x, y) - 1) \times N(x, y)) \times (-1) = W(x) \}$$

- 3-3. Check the label of each column is the same as the sum of the numbers in the Black cell of the column. I multiply $p(x, y)$ by $N(x, y)$. Repeat the previous process (n-1) times and add all result values. Compare result and $B(y)$.

Q3.

$$\bigwedge_{y=0}^{m-1} \{ (\sum_{x=0}^{n-1} p(x, y) \times N(x, y)) = B(y) \}$$

4. Demonstration

4-1. input

Each of the following M lines has N+1 tokens; first N tokens are a sequence of the square labels in the row, and the last label for the row label.

4-2. output

1 is black and 0 is white. If there is no solution, print “No solution”

4-3. Test

4-3-1. Test 9x9 matrix

```
20 23 30 29 34 6 9 21 19
8 2 1 8 1 3 5 7 6 18
9 1 4 2 5 6 3 1 7 28
3 5 1 4 9 1 3 9 1 8
8 6 6 3 5 1 1 4 1 4
8 6 6 2 6 8 3 3 9 31
8 7 8 8 4 5 2 1 1 18
4 8 3 5 5 2 1 2 8 24
1 2 8 8 8 3 7 2 7 22
8 3 9 5 9 2 1 4 9 35
```

Input.txt file

```
1 1 0 1 0 0 1 0 0
0 1 1 0 1 0 0 0 0
1 1 1 0 1 0 0 1 1
1 1 1 1 1 1 1 0 1
0 0 0 1 1 0 0 1 1
0 1 1 1 0 0 1 1 0
0 0 1 0 0 0 1 1 1
1 1 1 1 0 1 0 1 0
0 0 0 0 1 1 0 1 0
```

output

4-3-2. Test There has no solution.

```
14 14 3 8 19
9 8 2 3 1 11
2 3 1 2 9 2
5 7 9 6 9 8
2 4 8 3 8 21
```

input.txt file

No solution

output

5. Discussion.

Found the solution of the puzzle by defining proposition using the various operation instead of simply comparing true or false. It is not only increasing understand of the proposition but also more and more learning about the z3 program. Through experience previous team projects and individual projects, I realized that output through problem and input is more efficient at being the proposition value to solve the problem than given numbers or clue like input.

6. How to build

Using the Makefile and argv Example) `~/Hongmath$ make` => `~/Hongmath$./puzzle input.txt`