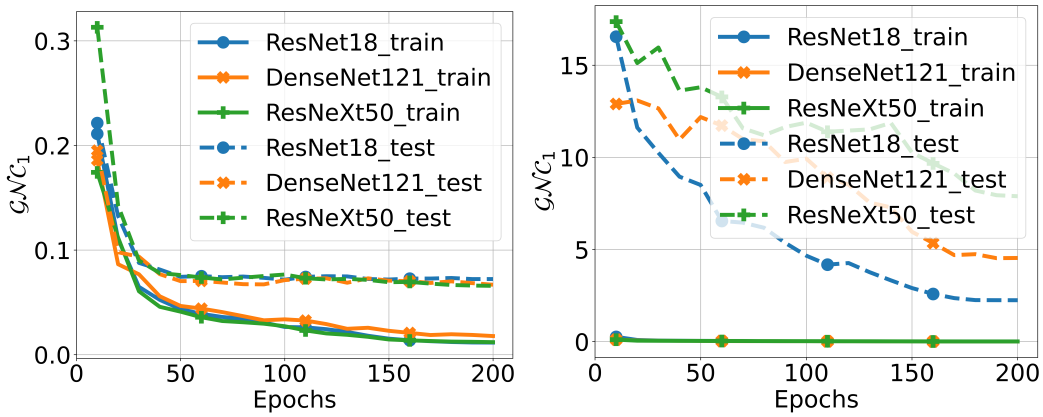
Appendix

A. Experiment

A.1. Test Collapse for $\mathcal{GNC}$

Previous studies (Hui et al., 2022; Xue et al., 2023) have demonstrated that even when collapse occurs on training data, it may not necessarily happen on unseen test data. However, these studies primarily focus on regular Neural Collapse (NC) and NC in contrastive learning. This same observation could apply to generalized NC. Specifically, differences might exist between the geometric arrangement of training and test data in the case of generalized neural collapse.

To verify that, we first train the ResNet18, DenseNet121 and RestNeXt50 networks on CIFAR10 and Tiny-ImageNet training set following the setting in additional results on prevalence of $\mathcal{GNC}$ in B.3. Then we evaluate the $\mathcal{GNC}_1$ on both the training set and validation set of CIFAR10 and Tiny-ImageNet across these network architectures. The result can be found in figure 6.



(a) $\mathcal{GNC}_1$ (CIFAR100) (b) $\mathcal{GNC}_1$ (Tiny)

Figure 6. **Illustration of Train vs. Test collapse of $\mathcal{GNC}$ across different network architectures on CIFAR100(left) and Tiny-ImageNet(right) dataset.** We train the networks on CIFAR100 with $d = 10,\ K = 100$, Tiny-ImageNet with $d = 10,\ K = 200$. Then $\mathcal{GNC}_1$ is evaluated on training and validation set across different network architectures.

The findings reveal a "generalization gap" when comparing train vs. test collapse in $\mathcal{GNC}_1$: as t approaches infinity, Train $\mathcal{GNC}_1$ appears to converge to zero while Test $\mathcal{GNC}_1$ does not. This suggests that $\mathcal{GNC}_1$ is more of an optimization issue rather than a generalization one since the test set does not diminish to an insignificant value under any circumstances.