Medical Sampling Service System with Variable Technician Speed and Drone Energy Consumption Problem Model Formulation Section

Pham Cong Hoang

September 17, 2025

1 Model Formulation

In this section, we propose the mathematical formulation for the MSSVTDE problem. The model integrates two types of resources - technicians and drones - cooperating to serve a set of customers under operational constraints. The formulation captures the specific features of the system, such as time-dependent travel times for technicians, constant travel times for drones, service requirements at each customer, and limitations on payload and energy for drone operations. The objective is to jointly optimize routing and scheduling decisions to minimize service delays and operational costs. To that end, we first introduce the notation of sets, parameters, and decision variables used throughout the formulation.

1.1 Notation

Sets	
C	Set of customers
C_T	Customers that must be served by a technician
C_A	Customers that can be served by either a technician or a drone
N	All nodes, $N = \{0\} \cup C$; node 0 is the depot
$\mathcal{K}$	Set of technicians
$\mathcal{D}$	Set of drones
$\mathcal{R}$	Set of drone trips per drone
$\mathcal{R}_T$	Set of technician trips per technician
$\mathcal{L}$	Set of congestion intervals; $\mathcal{L}'$ are time marks
Parameters	
d_{ij}	Distance between nodes $i, j \in N$
σ_i	Technician service time at $i \in C$
σ'_i	Drone service time at $i \in C_A$
w_i	Mass of the sample collected at customer i
V	Technician base speed (no congestion)
θ_l	Congestion factor on interval l; technician speed is $\theta_l V$
T_l	Time mark delimiting interval, with $T_0 = 0$ and T_L horizon end
$W_{\max}$	Maximum drone payload capacity per trip
E	Maximum drone energy capacity per trip
h	Drone flight altitude
v_t, v_c, v_l	Drone velocities for take-off, cruise, and landing
β, γ	Energy coefficients: load-dependent (β) and baseline (γ)
T^D_{ij}	Drone flight time from node i to node j
$AT(i, j, l)$	Technician arrival time at j when leaving i at time mark T_l
$\overline{W}_i$	Maximum admissible waiting time for sample $i \in C$ (cold-chain limit)
Variables	
X^{kr}_{ij}	Binary variables; 1 if technician k (on trip r) travels arc $i \to j$; 0 otherwise
Y^{dr}_{ij}	Binary variables; $= 1$ if drone d takes arc (i, j) in trip r; 0 otherwise
S^{kr}_i	Service start time of technician k (trip r) at node i
S'^{kr}_i	Departure time of technician k (trip r) from node i
A^{kr}_{return}	Return time of technician k at the depot on trip r
S^{dr}_i	Service start time of drone d at node i on trip r
A^{dr}_{return}	Return time of drone d to the depot on trip r
L^{dr}_i	Drone d payload when leaving node i on trip r
E^{dr}_i	Cumulative energy consumed by the drone upon leaving node i
E^{dr}_{arr}	Total energy consumed by drone d during trip r
W_i	Waiting time of customer sample i until reaching the depot
$C_{\max}$	Makespan of the integrated system
α^{krl}_{ij}	Interpolation coefficient for technician (k, r) on arc (i, j) at time mark T_l
z^{krl}_{ij}	Binary variables; $= 1$ if S'^{kr}_i (arc $i \to j$) lies in $[T_l, T_{l+1})$

Table 1: Summary of notation

1.2 Objective

In our problem, we consider a single objective metric:

$$\min\ Z\ =\ C_{\max}. \tag{1}$$

The makespan $C_{\max}$ is the time when the last technician or drone returns to the depot. Cold-chain quality is enforced via explicit waiting-time limits in the constraints below.

1.3 Constraints

Assignment Each customer in C_A is served exactly once by either a technician or a drone; a technician must serve each customer in C_T:

$$\sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}_T}\sum_{\substack{j\in N\\ j\neq i}} X_{ji}^{kr} + \sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}}\sum_{\substack{j\in N\setminus C_T\\ j\neq i}} Y_{ji}^{dr} = 1 \qquad \forall i\in C_A, \tag{2}$$

$$\sum_{k\in\mathcal{K}}\sum_{r\in\mathcal{R}_T}\sum_{\substack{j\in N\\ j\neq i}} X_{ji}^{kr} = 1 \qquad \forall i\in C_T. \tag{3}$$

Technician flow The flow of technicians must be consistent *per trip*: each trip departs from the depot at most once, returns to the depot if departed, and satisfies flow conservation at every visited customer:

$$\begin{aligned}
&\sum_{j\in C} X_{0j}^{kr} \le 1 && \forall k\in\mathcal{K},\ \forall r\in\mathcal{R}_T,\\
&\sum_{i\in C} X_{i0}^{kr} = \sum_{j\in C} X_{0j}^{kr} && \forall k\in\mathcal{K},\ \forall r\in\mathcal{R}_T,\\
&\sum_{\substack{j\in N\\ j\neq h}} X_{jh}^{kr} = \sum_{\substack{j\in N\\ j\neq h}} X_{hj}^{kr} && \forall h\in C,\ \forall k\in\mathcal{K},\ \forall r\in\mathcal{R}_T.
\end{aligned} \tag{4}$$

The start time of the next trip must be no earlier than the return time of the previous trip:

$$S_0^{\prime k,(r+1)} \ge A_{\text{return}}^{kr}, \qquad S_0^{\prime k,1} = 0. \tag{5}$$

Drone flow. Similarly, for each drone trip, departure and return must be consistent, and flow conservation holds at visited nodes:

$$\begin{aligned}
&\sum_{j\in C_A} Y_{0j}^{dr} \le 1 && \forall d\in\mathcal{D},\ \forall r\in\mathcal{R},\\
&\sum_{i\in C_A} Y_{i0}^{dr} = \sum_{j\in C_A} Y_{0j}^{dr} && \forall d\in\mathcal{D},\ \forall r\in\mathcal{R},\\
&\sum_{\substack{j\in N\setminus C_T\\ j\neq h}} Y_{jh}^{dr} = \sum_{\substack{j\in N\setminus C_T\\ j\neq h}} Y_{hj}^{dr} && \forall h\in C_A,\ \forall d\in\mathcal{D},\ \forall r\in\mathcal{R}.
\end{aligned} \tag{6}$$

Note that these constraints enforce flow conservation and depot return, but do not by themselves eliminate subtours. In this model, subtour elimination is naturally handled by the time-dependent constraints introduced in the next section and proved in the appendix.

Technician Timing with Congestion Discretization For each node i and technician (k,r), if a departure arc (i,j) is selected, then the departure time from i must follow its service start plus service duration. We set $S_0'^{k,1} = 0$ for all k and:

$$X_{ij}^{kr} = 1 \;\Rightarrow\; S_i'^{kr} \geq S_i^{kr} + \sigma_i, \qquad \forall i \in C, \forall j \in N,\ j \neq i,\ \forall k \in \mathcal{K},\ \forall r \in \mathcal{R}_T. \tag{7}$$

To handle congestion-dependent travel times, the time horizon is discretized into intervals $\mathcal{L}$ delimited by time marks $\mathcal{L}' = \{T_0, \ldots, T_L\}$. When technician (k,r) departs from node i to j, exactly one interval is activated by the binary variable z_{ij}^{krl}, while the continuous coefficients α_{ij}^{krl} interpolate the exact departure and arrival times within the chosen interval.

$$\sum_{l \in \mathcal{L}} z_{ij}^{krl} = X_{ij}^{kr}, \qquad \sum_{l \in \mathcal{L}'} \alpha_{ij}^{krl} = X_{ij}^{kr} \qquad \forall i \neq j,\ \forall k \in \mathcal{K},\ \forall r \in \mathcal{R}_T. \tag{8}$$

The coupling inequalities ensure that interpolation is only active around the selected interval:

$$\alpha_{ij}^{kr0} \leq z_{ij}^{kr0}, \quad \alpha_{ij}^{krl} \leq z_{ij}^{kr,l-1} + z_{ij}^{krl}, \quad \alpha_{ij}^{krL} \leq z_{ij}^{kr,L-1} \quad \forall i \neq j,\ \forall k \in \mathcal{K},\ \forall r \in \mathcal{R}_T,\ l = 1, \ldots, L-1. \tag{9}$$

Finally, departure and arrival times are linked to the time marks through interpolation. If arc (i,j) is chosen for technician (k,r), then the departure time from i and the arrival time at j are forced to match the interpolated values from the time-mark grid:

$$X_{ij}^{kr} = 1 \;\Rightarrow\; S_i'^{kr} = \sum_{l \in \mathcal{L}'} \alpha_{ij}^{krl} T_l, \qquad \forall i \neq j,\ \forall k \in \mathcal{K},\ \forall r \in \mathcal{R}_T, \tag{10}$$

$$X_{ij}^{kr} = 1 \;\Rightarrow\; S_j^{kr} = \sum_{l \in \mathcal{L}'} \alpha_{ij}^{krl}\, AT(i,j,l), \qquad \forall i \neq j,\ \forall k \in \mathcal{K},\ \forall r \in \mathcal{R}_T. \tag{11}$$

If the return arc $(i,0)$ is selected for technician (k,r), the return time equals the interpolated arrival time at the depot:

$$X_{i0}^{kr} = 1 \;\Rightarrow\; A_{\text{return}}^{kr} = \sum_{l \in \mathcal{L}'} \alpha_{i0}^{krl}\, AT(i,0,l), \qquad \forall i \in C,\ \forall k \in \mathcal{K},\ \forall r \in \mathcal{R}_T. \tag{12}$$

Drone timing. If an arc (i,j) is selected for drone d in trip r, then the service start time at j is fixed by the departure time at i plus service and flight time. The precomputed T_{ij}^D already includes take-off, cruise, and landing durations:

$$Y_{0j}^{dr} = 1 \;\Rightarrow\; S_j^{dr} = S_0^{dr} + T_{0j}^D, \qquad \forall j \in C_A,\ \forall d \in \mathcal{D},\ \forall r \in \mathcal{R}, \tag{13}$$

$$Y_{ij}^{dr} = 1 \;\Rightarrow\; S_j^{dr} = S_i^{dr} + \sigma_i' + T_{ij}^D, \qquad \forall i \in C_A,\ j \in N \setminus C_T,\ i \neq j,\ \forall d \in \mathcal{D},\ \forall r \in \mathcal{R}, \tag{14}$$

$$Y_{i0}^{dr} = 1 \;\Rightarrow\; A_{\text{return}}^{dr} = S_i^{dr} + \sigma_i' + T_{i0}^D, \qquad \forall i \in C_A,\ \forall d \in \mathcal{D},\ \forall r \in \mathcal{R}. \tag{15}$$

The start time of the next trip must be no earlier than the return time of the previous trip:

$$S_0^{d,(r+1)} \geq A_{\text{return}}^{dr}, \qquad S_0^{d,1} = 0. \tag{16}$$

Drone payload. Payload evolves additively along the selected route. If arc (i,j) is chosen, the payload at j equals the payload at i plus the collected mass w_j. At departure from the depot, the payload is initialized to zero:

$$Y_{ij}^{dr} = 1 \;\Rightarrow\; L_j^{dr} = L_i^{dr} + w_j, \qquad \forall i \in N \setminus C_T,\ j \in C_A,\ i \neq j,\ \forall d \in \mathcal{D},\ \forall r \in \mathcal{R}, \tag{17}$$

$$Y_{0j}^{dr} = 1 \;\Rightarrow\; L_j^{dr} = w_j, \qquad L_0^{dr} = 0. \tag{18}$$

Payload must not exceed the maximum capacity:

$$L_i^{dr} \leq W_{\max}, \qquad \forall i \in N \setminus C_T,\ \forall d \in \mathcal{D},\ \forall r \in \mathcal{R}. \tag{19}$$

Drone energy. Energy consumption is load-dependent. If arc (i, j) is selected, the energy at j equals the energy at i plus the term $(\beta L_i^{dr} + \gamma)T_{ij}^D$. If arc $(i, 0)$ is selected, the total return energy E_{arr}^{dr} is computed analogously:

$$Y_{ij}^{dr} = 1 \;\Rightarrow\; E_j^{dr} = E_i^{dr} + (\beta L_i^{dr} + \gamma)\, T_{ij}^D, \qquad \forall i \neq j,\; i, j \in N \setminus C_T,\; \forall d \in \mathcal{D},\; \forall r \in \mathcal{R}, \tag{20}$$

$$Y_{i0}^{dr} = 1 \;\Rightarrow\; E_{\text{arr}}^{dr} = E_i^{dr} + (\beta L_i^{dr} + \gamma)\, T_{i0}^D. \tag{21}$$

Energy is bounded by the capacity:

$$E_{\text{arr}}^{dr} \leq E, \qquad E_0^{dr} = 0. \tag{22}$$

Makespan The makespan is at least the return time of every technician and drone trip:

$$C_{\max} \geq A_{\text{return}}^{kr},\; \forall k \in \mathcal{K},\; \forall r \in \mathcal{R}_T, \qquad C_{\max} \geq A_{\text{return}}^{dr},\; \forall d \in \mathcal{D},\; \forall r \in \mathcal{R}. \tag{23}$$

Cold-chain waiting If a technician serves customer i, the waiting time W_i must not be less than the difference between depot return and service completion; similarly if a drone serves i. Additionally, waiting time is capped by the admissible limit $\overline{W}$:

$$X_{ji}^{kr} = 1 \;\Rightarrow\; W_i \geq A_{\text{return}}^{kr} - (S_i^{kr} + \sigma_i), \qquad \forall i \in C,\; \forall k \in \mathcal{K},\; \forall r \in \mathcal{R}_T,\; \forall j \neq i, \tag{24}$$

$$Y_{ji}^{dr} = 1 \;\Rightarrow\; W_i \geq A_{\text{return}}^{dr} - (S_i^{dr} + \sigma_i'), \qquad \forall i \in C_A,\; \forall d \in \mathcal{D},\; \forall r \in \mathcal{R},\; \forall j \neq i, \tag{25}$$

$$W_i \leq \overline{W}_i, \qquad \forall i \in C. \tag{26}$$

Appendices

A Precomputation of Travel-Time Quantities

This appendix describes the offline computation of drone flight times T^D_{ij} and the technician arrival map $AT(i,j,l)$ used by the formulation.

Drone flight times T^D_{ij}

Drone motion is decomposed into three phases: take-off, cruise, and landing. Let

$$t_{\text{takeoff}} \;=\; \frac{h}{v_t}, \qquad t_{\text{landing}} \;=\; \frac{h}{v_l}, \qquad t_{\text{cruise},ij} \;=\; \frac{d_{ij}}{v_c},$$

where h is the flight altitude, v_t and v_l are vertical velocities for take-off and landing, v_c is the horizontal cruise velocity, and d_{ij} is the Euclidean/metric distance between nodes i and j.

The precomputed drone travel time T^D_{ij} for an admissible arc (i,j) is the sum of these phase durations. Consistent with the domain restrictions in the model (drones serve only customers in C_A and may not serve C_T), we set:

$$\begin{aligned} T^D_{0j} &\;=\; t_{\text{takeoff}} + t_{\text{cruise},0j} + t_{\text{landing}}, \quad \forall\, j \in C_A, \\ T^D_{ij} &\;=\; t_{\text{takeoff}} + t_{\text{cruise},ij} + t_{\text{landing}}, \quad \forall\, i \in C_A,\ \forall\, j \in N \setminus C_T, \\ T^D_{i0} &\;=\; t_{\text{takeoff}} + t_{\text{cruise},i0} + t_{\text{landing}}, \quad \forall\, i \in C_A. \end{aligned} \tag{27}$$

By convention, T^D_{ii} is undefined, and $T^D_{0,0} = 0$. Since $h > 0$ and $v_t, v_l, v_c > 0$, we have $T^D_{ij} > 0$ for $i \neq j$. These values are independent of road congestion and are reused throughout optimization.

Technician arrival map $AT(i,j,l)$

For $(i,j) \in N^2$, $i \neq j$, and time mark $T_l \in \mathcal{L}'$, $AT(i,j,l)$ is the arrival time at j if a technician departs i at T_l. Travel is time–dependent: on each interval $[T_p, T_{p+1})$ ($p \in \mathcal{L}$) the speed is $v_p = \theta_p V$.

Set remaining distance $d \leftarrow d_{ij}$ and clock $t \leftarrow T_l$; let p be such that $t \in [T_p, T_{p+1})$ (if $t \geq T_L$, use the last interval). Repeat: (i) $\Delta t \leftarrow T_{p+1} - t$, $\Delta x \leftarrow v_p\, \Delta t$; (ii) if $d \leq \Delta x$, return $AT(i,j,l) = t + d/v_p$; (iii) else set $d \leftarrow d - \Delta x$, $t \leftarrow T_{p+1}$, $p \leftarrow p+1$ and continue (clamping p to the last interval if needed).

B Subtour Elimination Proof

We justify that the formulation precludes disconnected subtours for both technicians and drones. The argument relies on the assignment and flow constraints, the timing constraints, and the positivity of travel and service times.

Technicians. Suppose technician $k \in \mathcal{K}$ uses arc (i,j) with $X^k_{ij} = 1$. From the timing constraints we have

$$S^k_j \;\geq\; S'^k_i + \Delta^{\text{tech}}_{ij}, \qquad S'^k_i \;\geq\; S^k_i + \sigma_i,$$

where $\Delta^{\text{tech}}_{ij} > 0$ is the effective travel time derived from $AT(i,j,l)$. Thus

$$S^k_j \;\geq\; S^k_i + \sigma_i + \Delta^{\text{tech}}_{ij}.$$

Time strictly increases along every used arc. Summing around any cycle $C \subseteq C$ yields a contradiction.

$$S_u^k \;\geq\; S_u^k + \sum_{(i,j)\in C} \big(\sigma_i + \Delta_{ij}^{\text{tech}}\big),$$

Hence, subtours without depot are impossible.

Drones. If drone $d \in \mathcal{D}$ on trip $r \in \mathcal{R}$ uses arc (i,j) with $Y_{ij}^{dr} = 1$, then

$$S_j^{dr} \;\geq\; S_i^{dr} + \sigma_i' + T_{ij}^D,$$

with $T_{ij}^D > 0$ for $i \neq j$. Time strictly increases along every used drone arc. Summing along a cycle not including the depot yields a contradiction. Thus, each trip (d,r) is depot-anchored and free of subtours.

C Linearization of Conditional Constraints

For completeness we list the linear reformulations of the logical implications used in the model. If the solver supports indicator constraints, those can replace the Big-M encodings.

Template. For binary $b \in \{0,1\}$ and condition $y \geq f(x)$ to hold if $b = 1$, use

$$y \;\geq\; f(x) - M(1-b).$$

Applications in the model.

- **Technician service-departure link:**

$$S_i'^{kr} \;\geq\; S_i^{kr} + \sigma_i - M\Big(1 - \sum_{j\in N\setminus\{i\}} X_{ij}^{kr}\Big), \quad \forall i \in C, k \in \mathcal{K}, r \in \mathcal{R}_T.$$

- **Interpolation coupling (technicians):**

$$S_i'^{kr} \;\geq\; \sum_{l\in\mathcal{L}'} \alpha_{ij}^{krl} T_l - M(1 - X_{ij}^{kr}),$$

$$S_j^{kr} \;\geq\; \sum_{l\in\mathcal{L}'} \alpha_{ij}^{krl} AT(i,j,l) - M(1 - X_{ij}^{kr}).$$

- **Technician return:**

$$A_{\text{return}}^{kr} \;\geq\; \sum_{l\in\mathcal{L}'} \alpha_{i0}^{krl} AT(i,0,l) - M(1 - X_{i0}^{kr}).$$

- **Drone timing:**

$$S_j^{dr} \;\geq\; S_i^{dr} + \sigma_i' + T_{ij}^D - M(1 - Y_{ij}^{dr}),$$
$$A_{\text{return}}^{dr} \;\geq\; S_i^{dr} + \sigma_i' + T_{i0}^D - M(1 - Y_{i0}^{dr}).$$

- **Drone payload:**

$$L_j^{dr} \;\leq\; L_i^{dr} - w_j + M(1 - Y_{ij}^{dr}).$$

- **Drone energy:**

$$E_j^{dr} \;\geq\; E_i^{dr} + (\beta L_i^{dr} + \gamma) T_{ij}^D - M(1 - Y_{ij}^{dr}).$$