Design and Analysis of Algorithms

Part I: Divide and Conquer

Lecture 4: Maximum Contiguous Subarray Problem

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Outline

- Introduction to Part I

- Maximum Contiguous Subarray Problem
  - Problem definition
  - A brute force algorithm
  - A data-reuse algorithm
  - A divide-and-conquer algorithm
  - Analysis of the divide-and-conquer algorithm

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  - **Divide**
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  - **Conquer**
    Solving each subproblem (directly if small enough or recursively)
  - **Combine**
    Combining the solutions of the subproblems into a global solution

Introduction to Part I

- In Part I, we will illustrate Divide-and-Conquer using several examples:
  - Maximum Contiguous Subarray (最大子数组)
  - Counting Inversions (逆序计数)
  - Polynomial Multiplication (多项式乘法)
  - QuickSort and Partition (快速排序与划分)
  - Randomized Selection (随机化选择)
  - Lower Bound for Sorting (基于比较的排序下界)

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Maximum Contiguous Subarray (MCS) Problem

ACME Corp[1] – PROFIT HISTORY

Year	1	2	3	4	5	6	7	8	9
Profit M$	-3	2	1	-4	5	2	-1	3	-1

[1] A Company that Makes Everything

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如果所有数组元素都是非负数，整个数组和肯定是最大

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Answer: Year 5-8, 9 M$

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Formal Definition

- Input: An array of reals *A*[1…n]

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Definition (Maximum Contiguous Subarray Problem)

Find $i \leq j$ such that $V(i,j)$ is maximized.

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```
VMAX ← A[1];
for i ← 1 to n do
    for j ← i to n do
        // calculate V(i,j)
        V ← 0;
        for x ← i to j do
            V ← V + A[x];
        end
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$O(n^3)$ arithmetic additions

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        V ← V + A[j];
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            VMAX ← V;
        end
    end
end
return VMAX
```

$O(n^2)$ arithmetic additions

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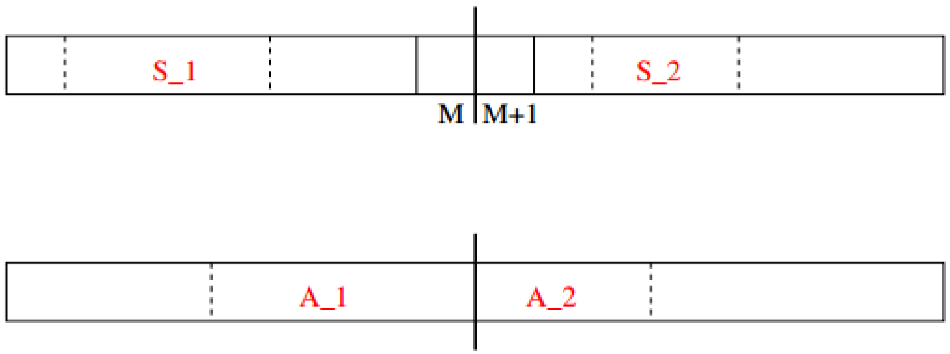
A Divide-and-Conquer Algorithm

Set $m = \lfloor (n+1)/2 \rfloor$



A_1 = MCS on left containing A[M]   A_2 = MCS on right containing A[M+1]

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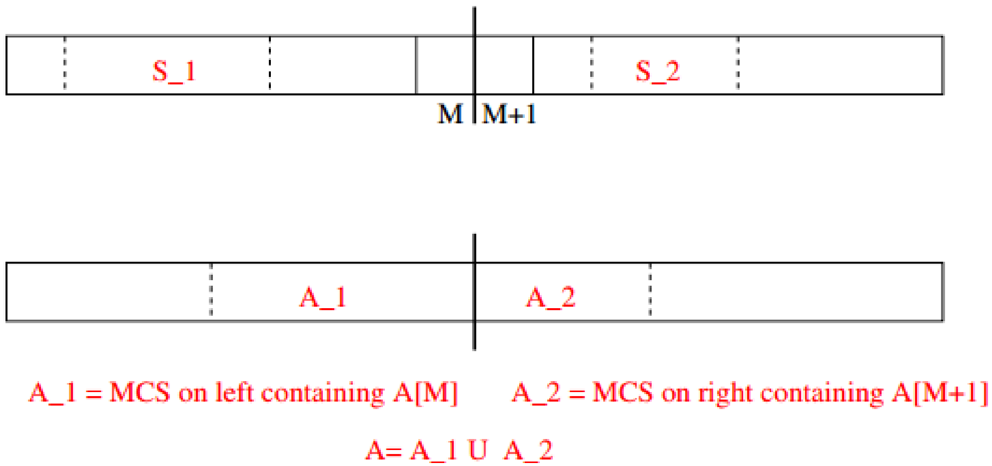

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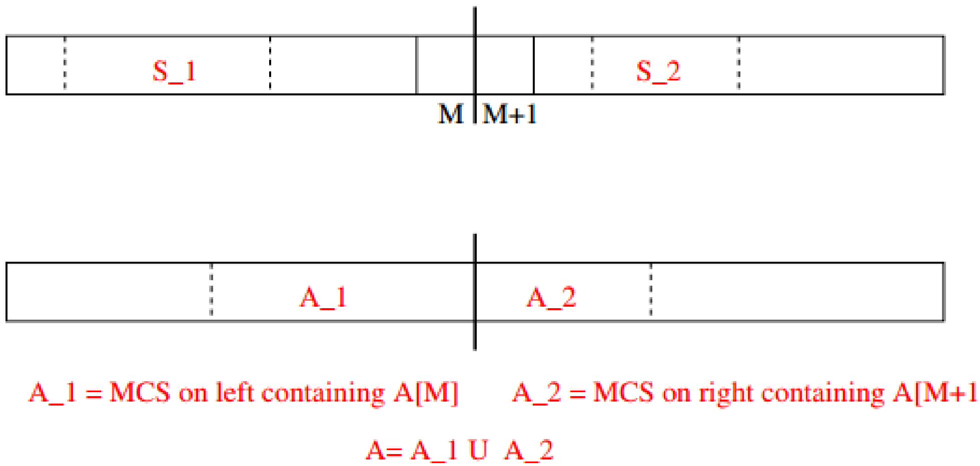

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1. S_1: the MCS in $A[1 \ldots m]$

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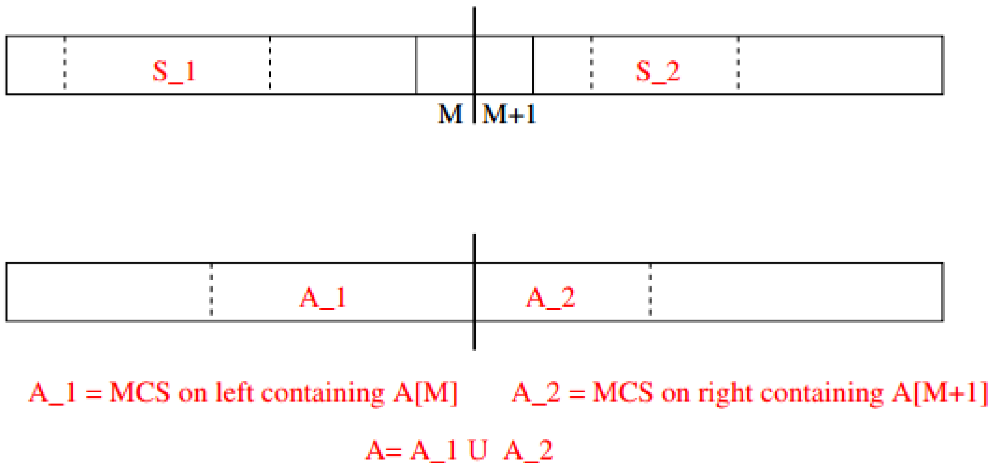

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1. S_1: the MCS in $A[1 \ldots m]$
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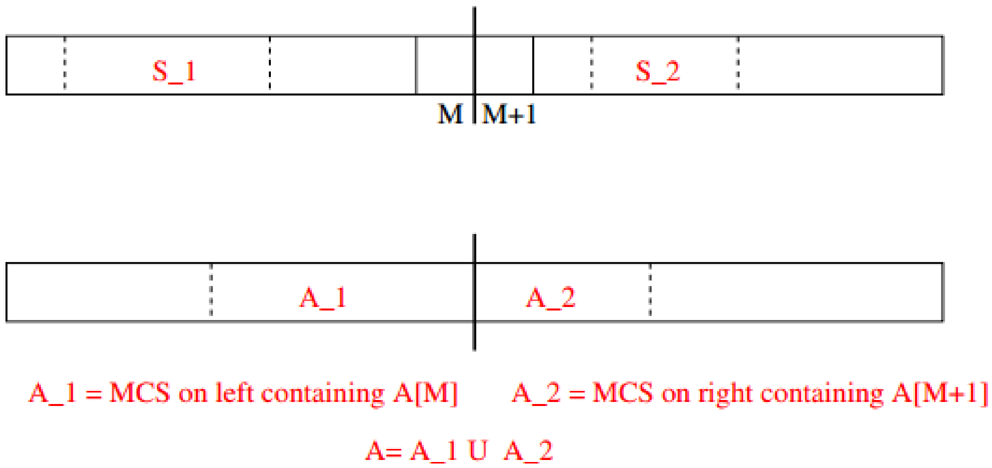

The MCS S must be one of

1. S_1: the MCS in $A[1 \ldots m]$
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3. A: the MCS across the cut.

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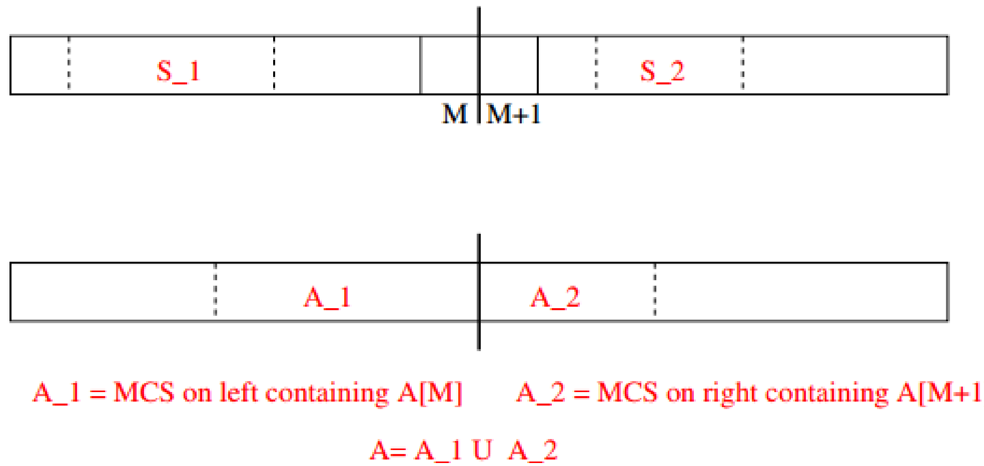

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So,

最终，在S_1, S_2和A(跨越中点的最大子数组)这三种情况中选取和最大者

$$S = \text{the best among } \{S_1, S_2, A\}$$

An Example of Divide-and-Conquer Algorithm

1	-5	4	2	-7	3	6	-1	2	-4	7	-10	2	6	1	-3

- $S_1 =$

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- $A_1 = [3, 6, -1]$ and $A_2 = [2, -4, 7]$
- $A = A_1 \cup A_2 =$

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- $Value(S_1) = 9;\ Value(S_2) = 9;\ Value(A) = 13$
- solution:

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- solution: A

Divide: MCS across The Cut

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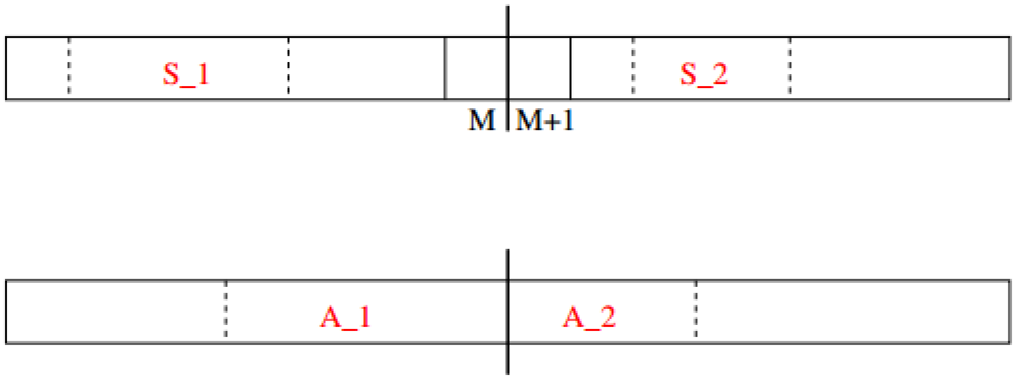

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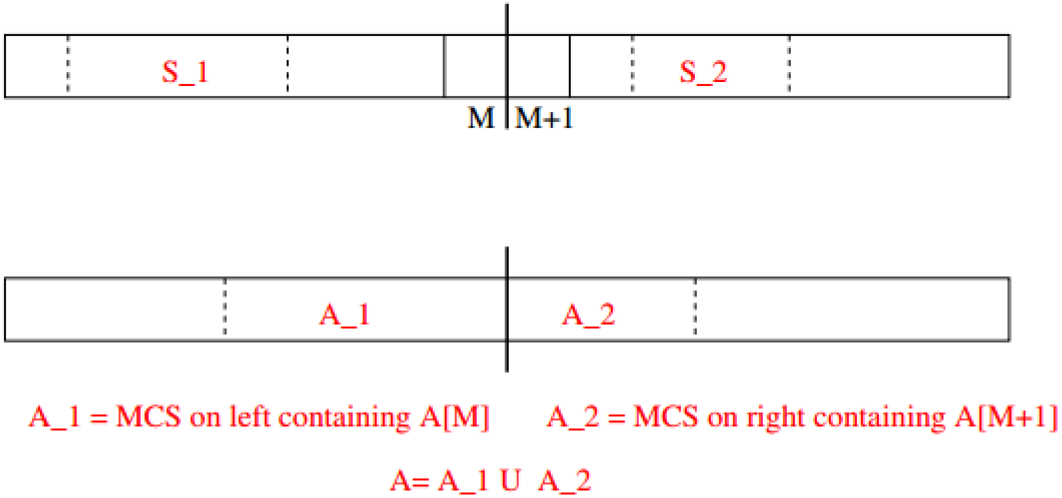

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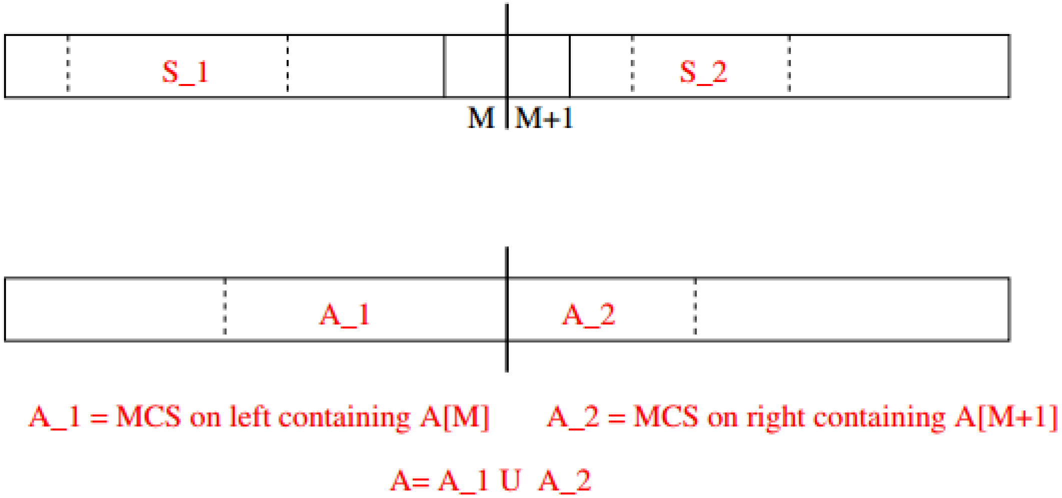

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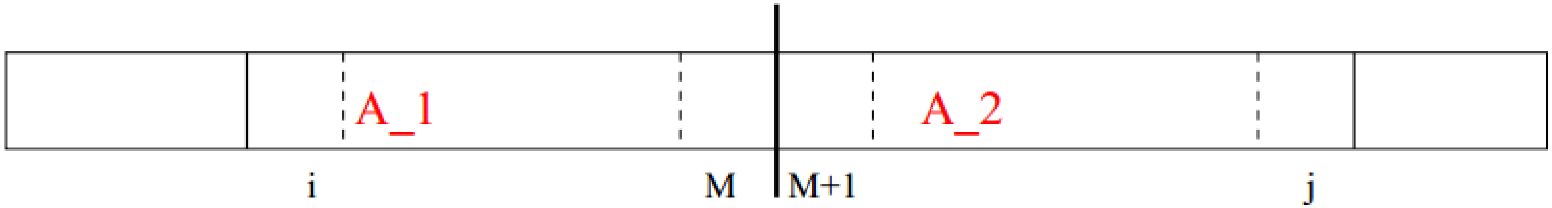
Conquer: Finding the "A_1" Subarrays



A_1 is in the form $A[i \ldots m]$, $V(i, m) = V(i+1, m) + A[i]$

MAX $\leftarrow A[m]$;
SUM $\leftarrow A[m]$;

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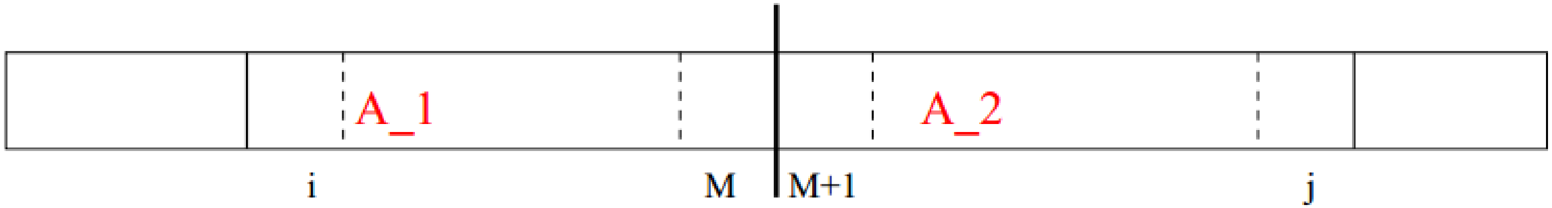

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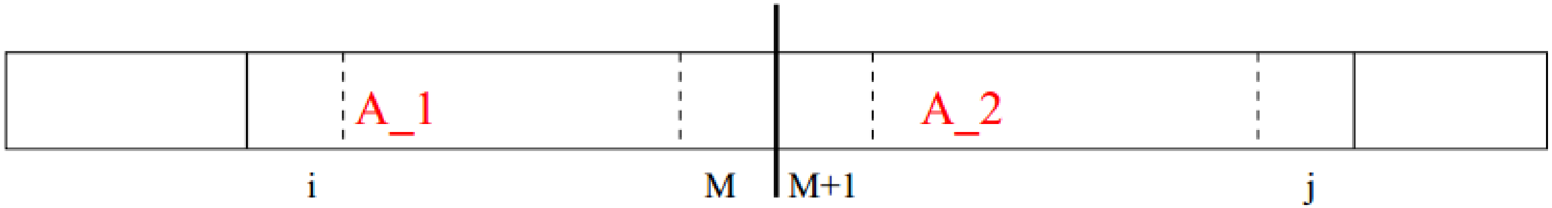

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for $i \leftarrow m-1$ **downto** *1* **do**
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  if $SUM > MAX$ **then**
    MAX $\leftarrow$ SUM;
  end
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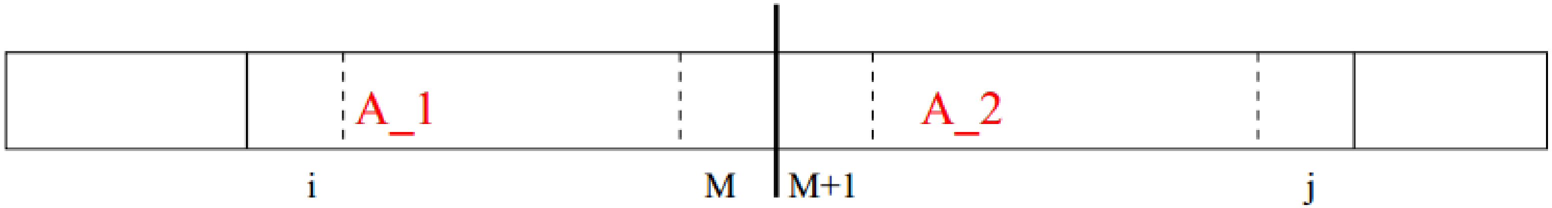

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  if $SUM > MAX$ **then**
    MAX $\leftarrow$ SUM;
  end
end
A_1=MAX;

Conquer: Finding *"A"* with A Linear Time

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- $A = A_1 \cup A_2$ can be found in $O(n)$ time

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- $A = A_1 \cup A_2$ can be found in $O(n)$ time
  - linear to the input size

The Complete Divide-and-Conquer Algorithm

$MCS(A, s, t)$

Input: $A[s \dots t]$ with $s \leq t$
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begin
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Output: MCS of $A[s \ldots t]$
begin
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  else
    $m \leftarrow \lfloor \frac{s+t}{2} \rfloor$;
    Find $MCS(A, s, m)$;

The Complete Divide-and-Conquer Algorithm

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    Find $MCS(A, m+1, t)$;
    Find MCS that contains both $A[m]$ and $A[m+1]$;

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    Find MCS that contains both $A[m]$ and $A[m+1]$;
    return maximum of the three sequences found
  end

The Complete Divide-and-Conquer Algorithm

$MCS(A, s, t)$

Input: $A[s \dots t]$ with $s \le t$
Output: MCS of $A[s \dots t]$
begin
    if $s = t$ **then return** $A[s]$;
    else
        $m \leftarrow \lfloor \frac{s+t}{2} \rfloor$;
        Find $MCS(A, s, m)$;
        Find $MCS(A, m+1, t)$;
        Find MCS that contains both $A[m]$ and $A[m+1]$;
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First Call: *MCS*(*A, 1, n*)

A Full Illustration of the D&C Algorithm

6 -4 7 -4 0 1

A Full Illustration of the D&C Algorithm

6 -4 7 ¦ -4 0 1

A Full Illustration of the D&C Algorithm

6 -4 7 | -4 0 1

6 -4 7 **-4 0 1**

A Full Illustration of the D&C Algorithm

6 -4 7 | -4 0 1

6 -4 | 7

-4 0 | 1

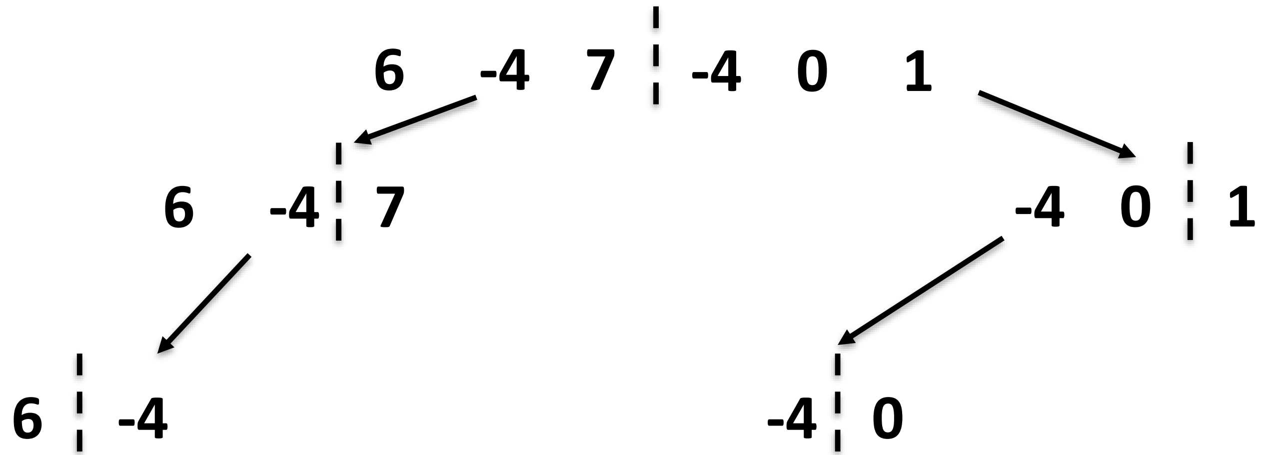
A Full Illustration of the D&C Algorithm

6 -4 7 | -4 0 1

6 -4 | 7 **-4 0 | 1**

6 -4 **-4 0**

A Full Illustration of the D&C Algorithm



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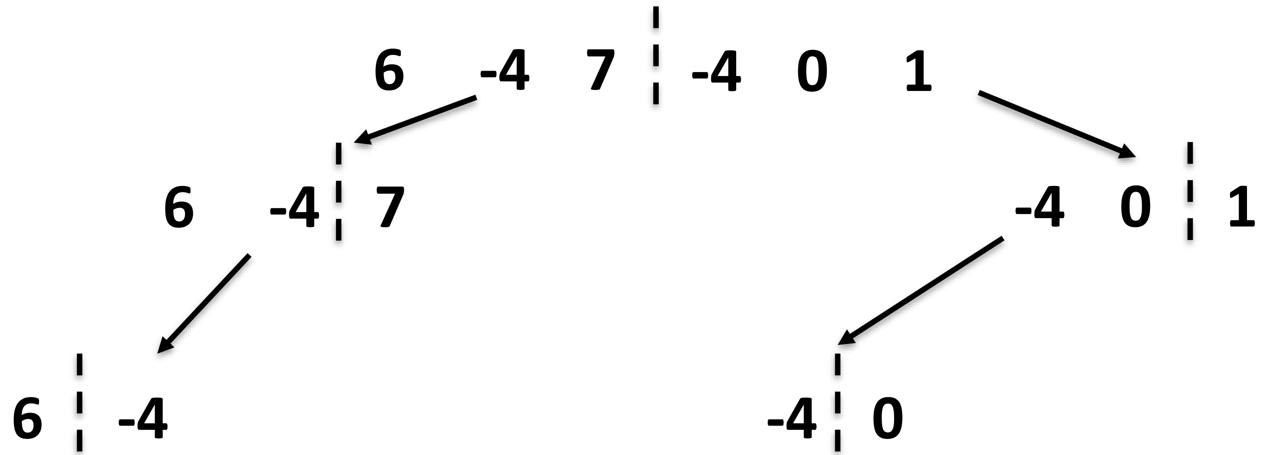

Divide

A Full Illustration of the D&C Algorithm

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6 -4 ¦ 7 **-4 0 ¦ 1**

6 ¦ -4 **-4 ¦ 0**

Divide

Conquer

A Full Illustration of the D&C Algorithm

6 -4 7 | -4 0 1

6 -4 | 7   **-4 0 | 1**

6 | -4   **-4 | 0**

Divide

MCS={6} **MCS={-4}**

Conquer

A Full Illustration of the D&C Algorithm

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Divide

Conquer

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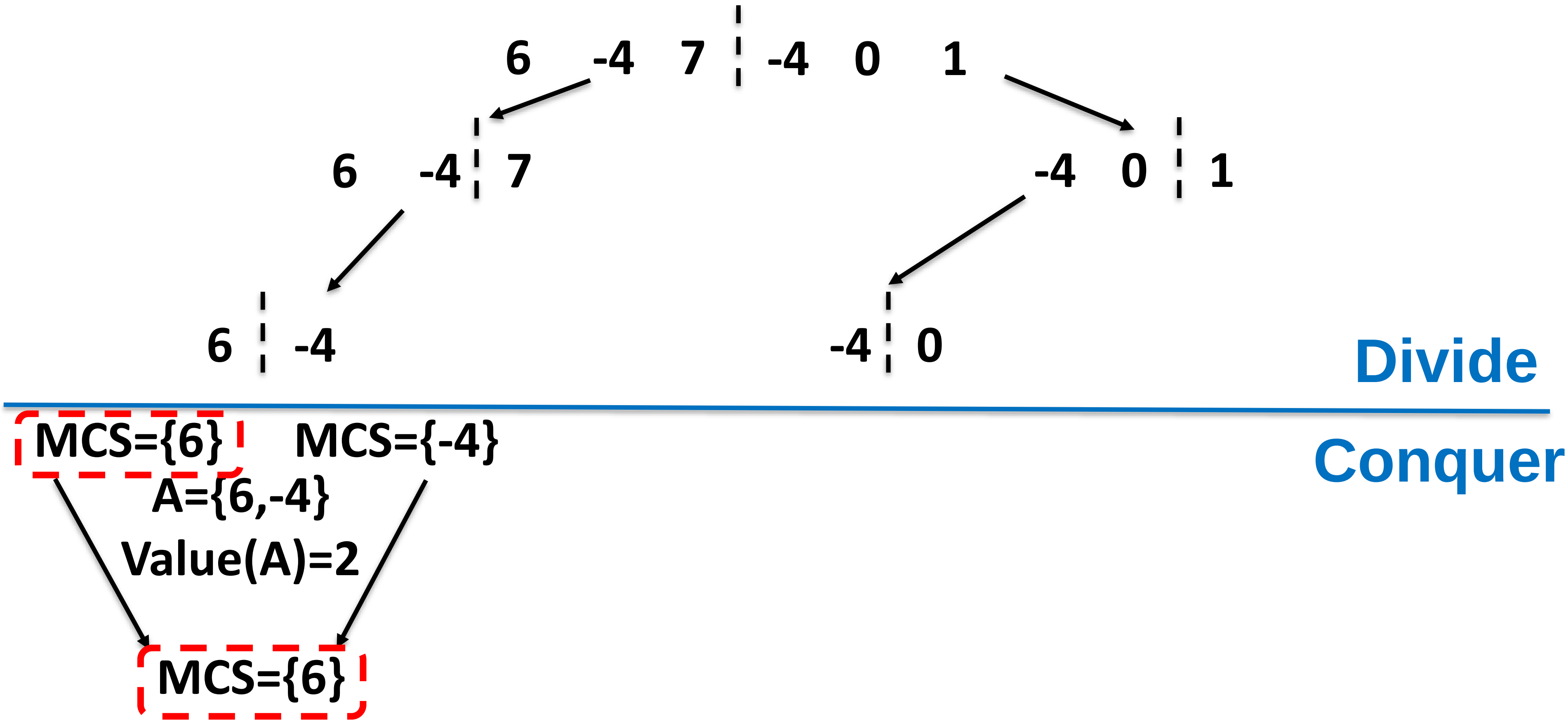
A={6,-4}

Value(A)=2

A Full Illustration of the D&C Algorithm



A Full Illustration of the D&C Algorithm

6 -4 7 ¦ -4 0 1

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6 ¦ -4

-4 ¦ 0

Divide

Conquer

MCS={6} MCS={-4}

A={6,-4}

Value(A)=2

MCS={6} MCS={7}

A Full Illustration of the D&C Algorithm

6 -4 7 ¦ -4 0 1

6 -4 ¦ 7

-4 0 ¦ 1

6 ¦ -4

-4 ¦ 0

Divide

Conquer

MCS={6} **MCS={-4}**

A={6,-4}

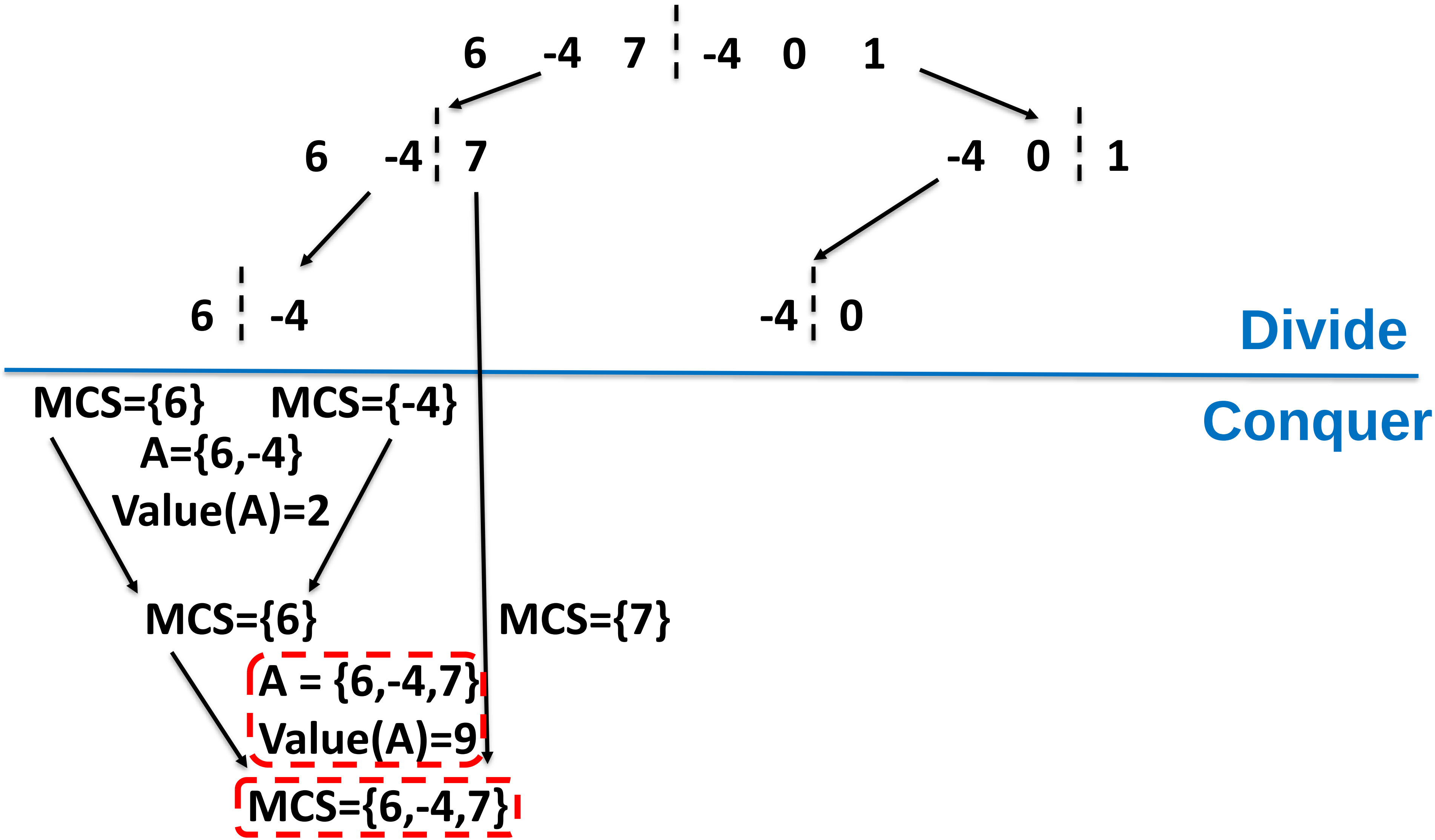
Value(A)=2

MCS={6} **MCS={7}**

A = {6,-4,7}

Value(A)=9

A Full Illustration of the D&C Algorithm



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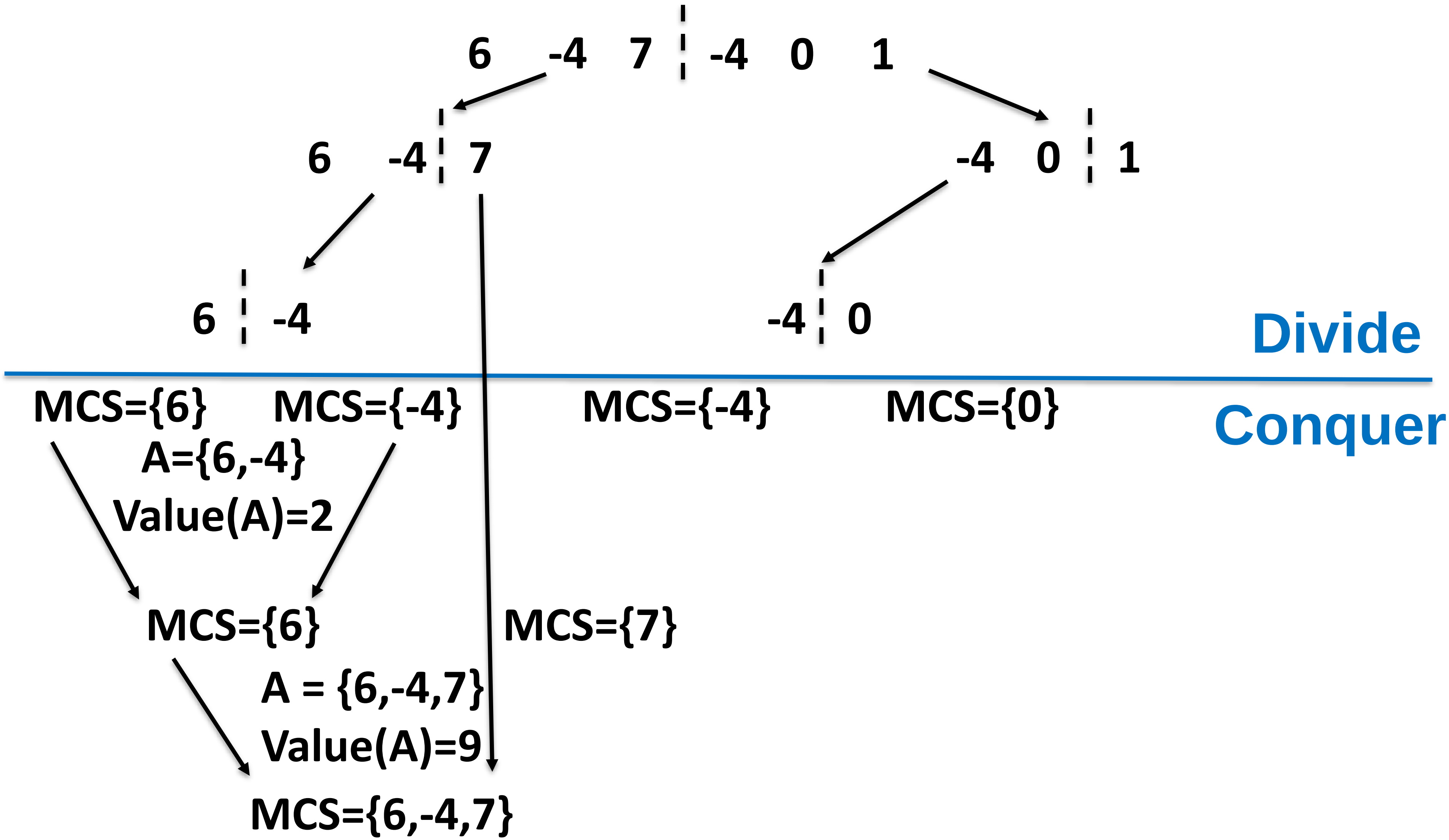

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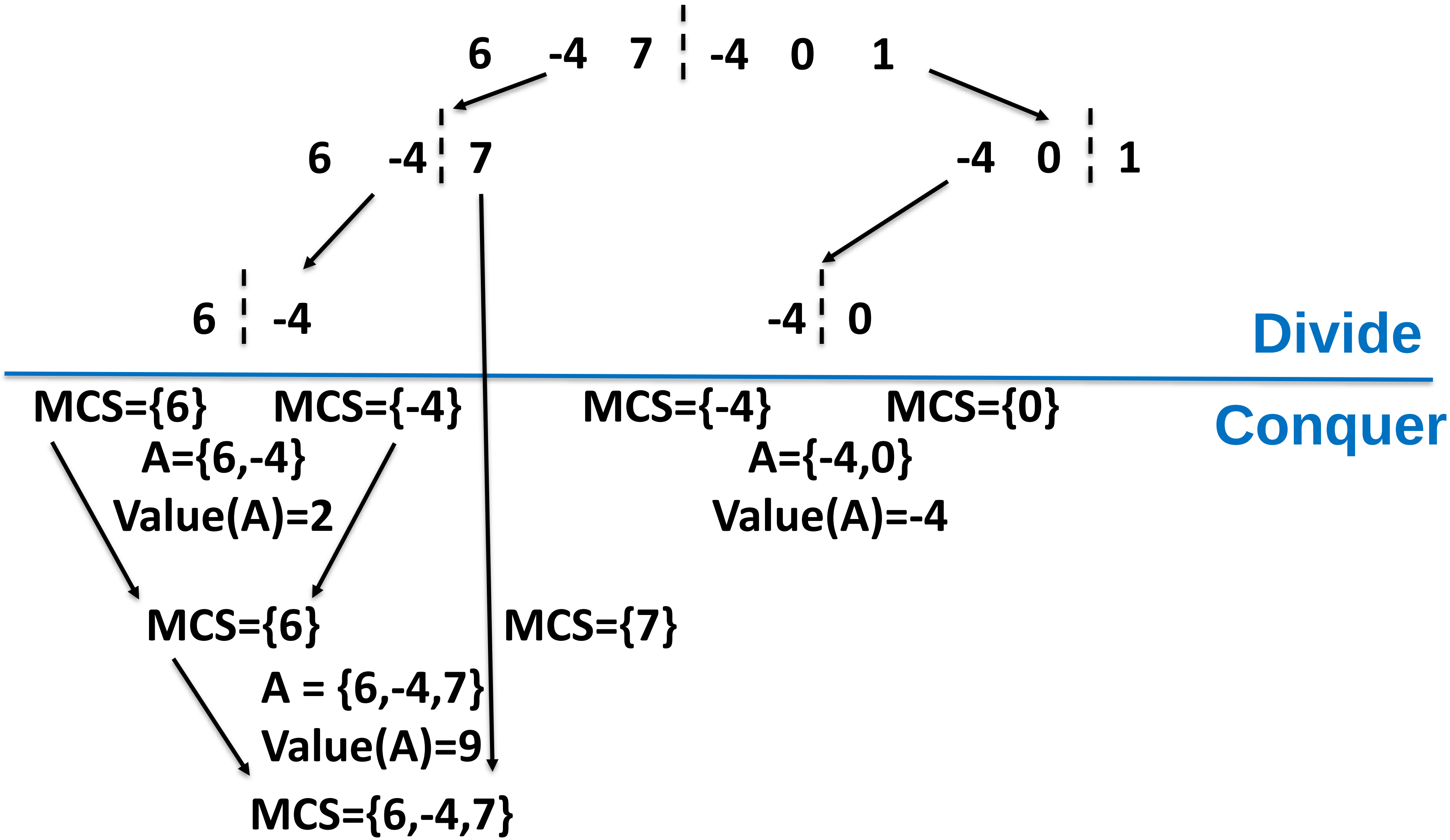

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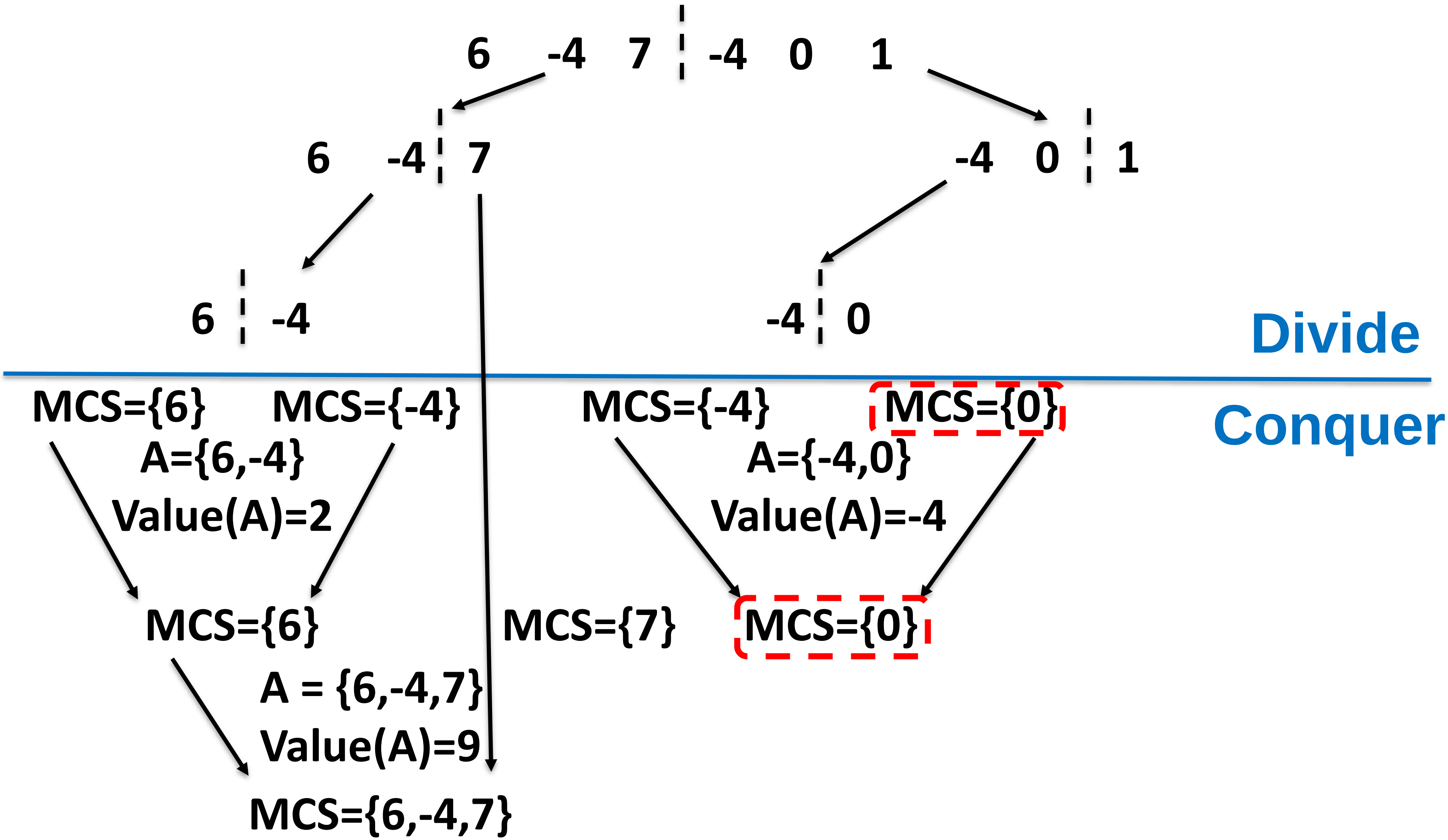

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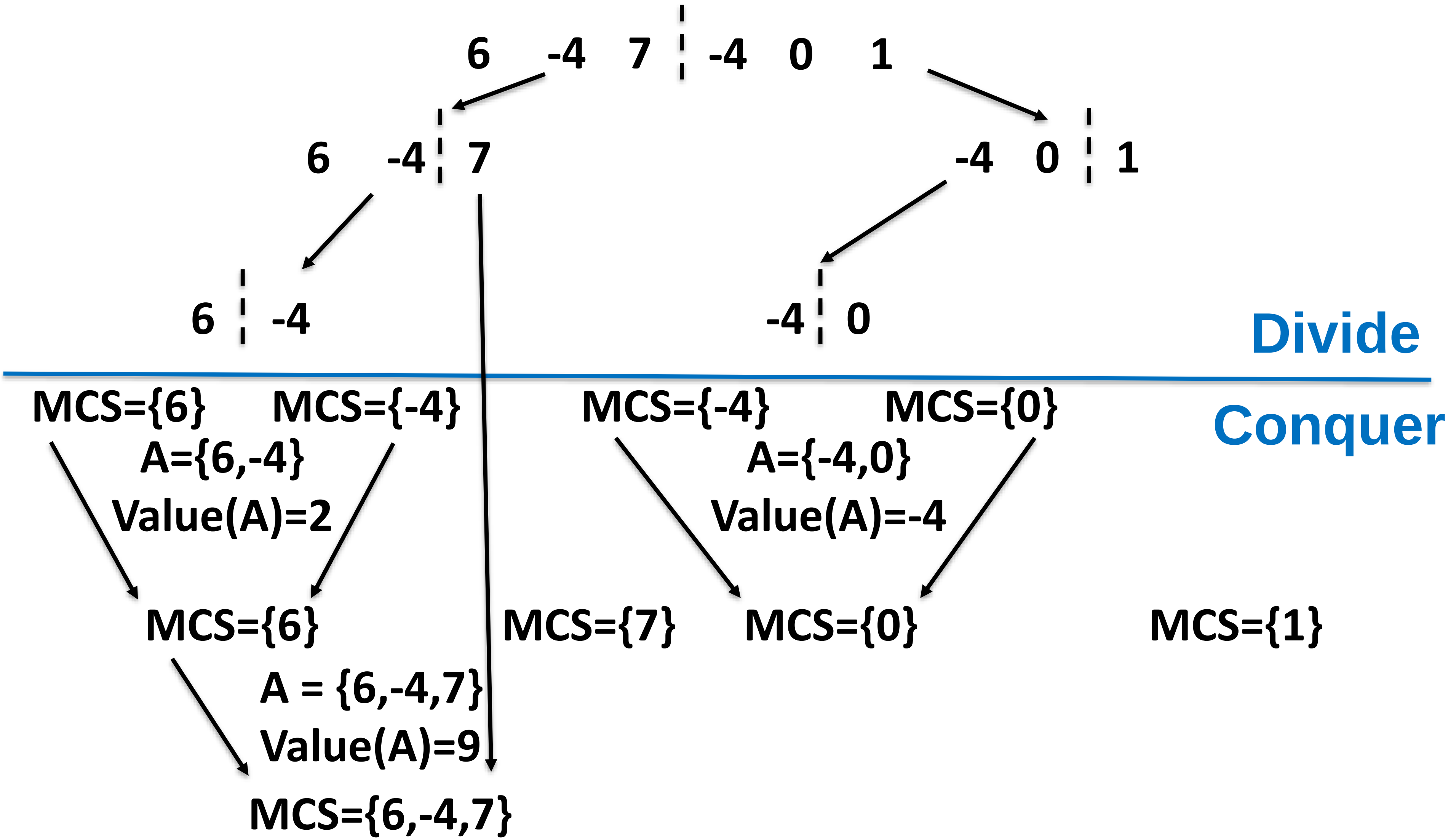

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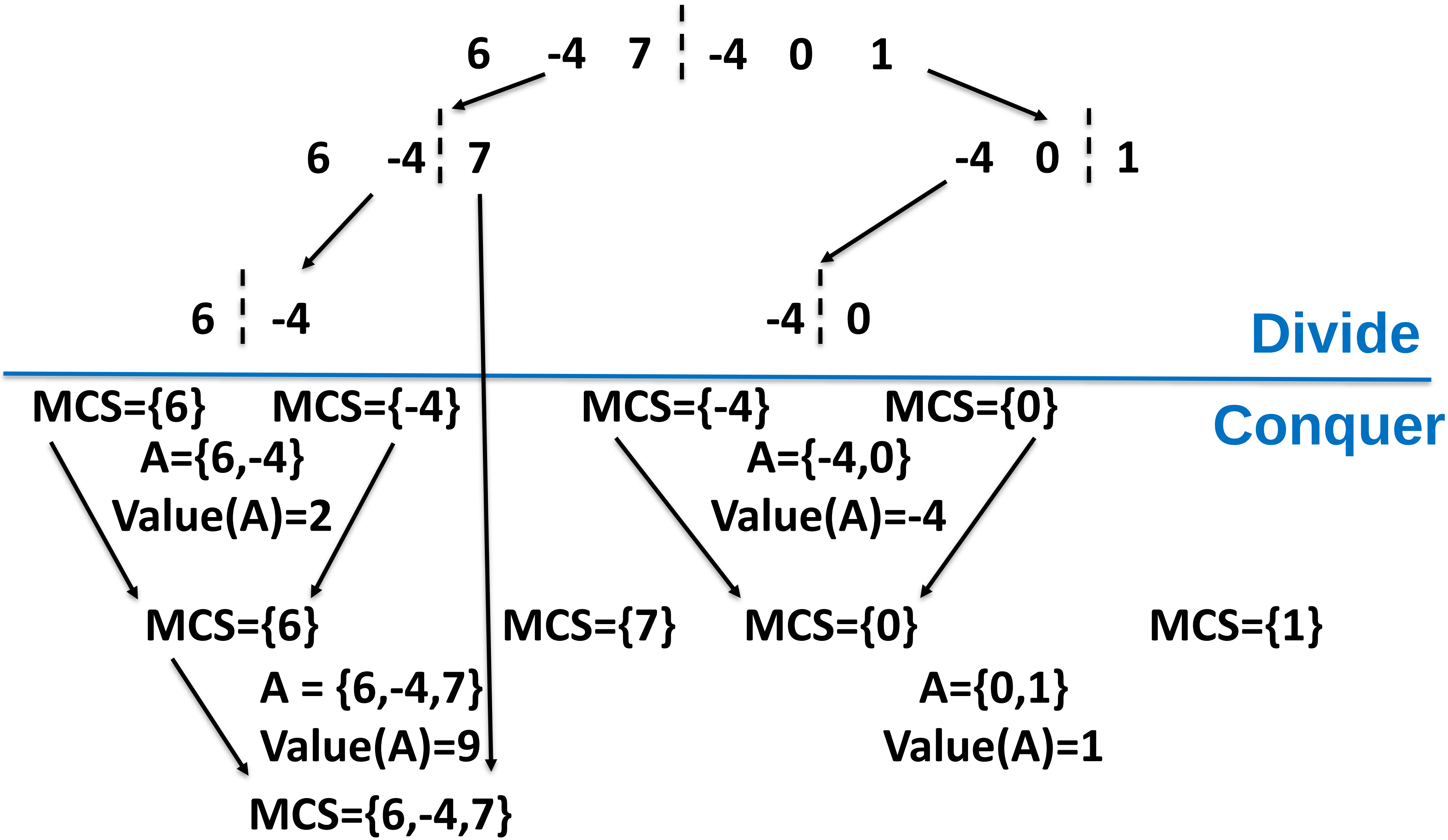

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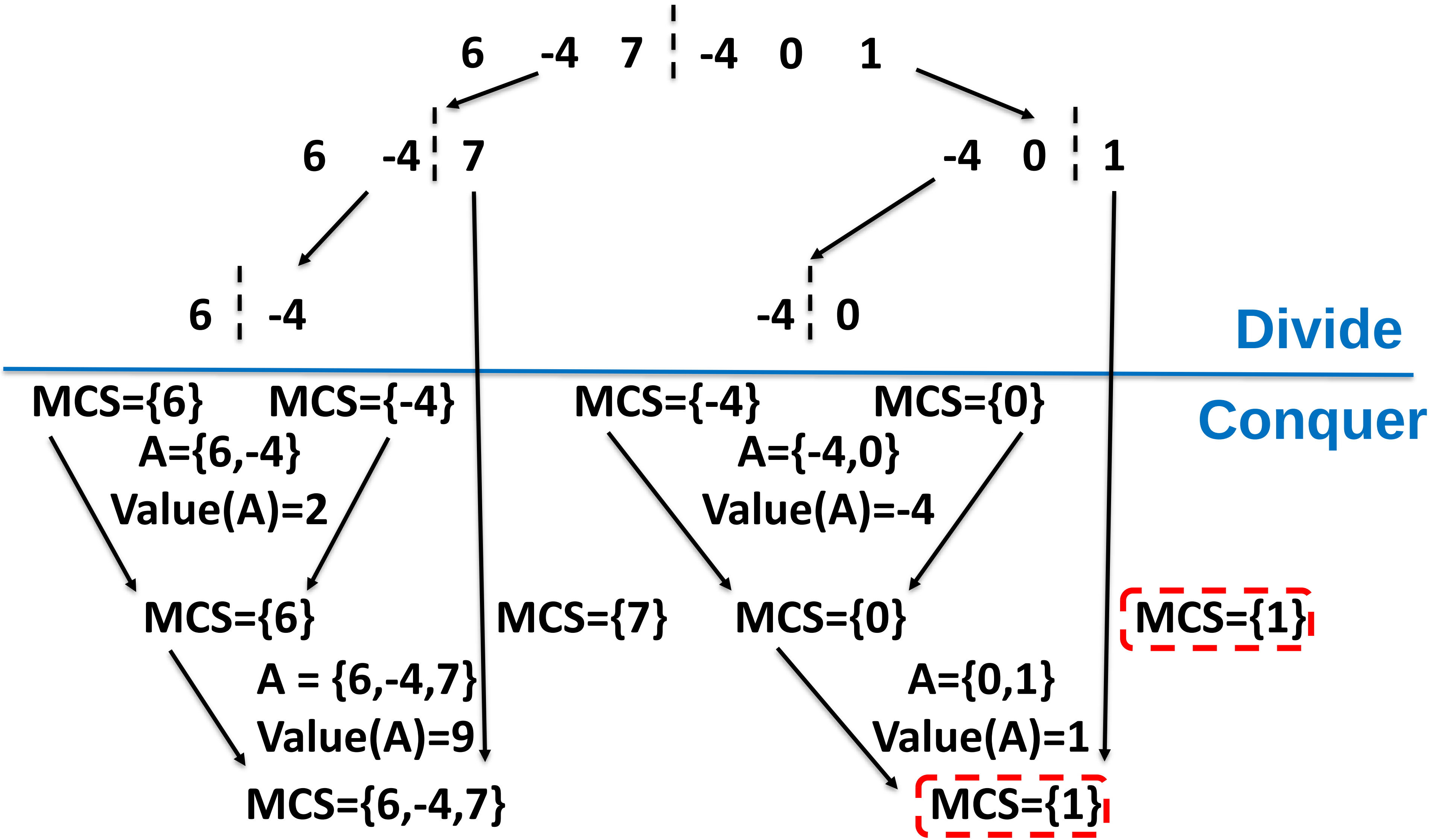

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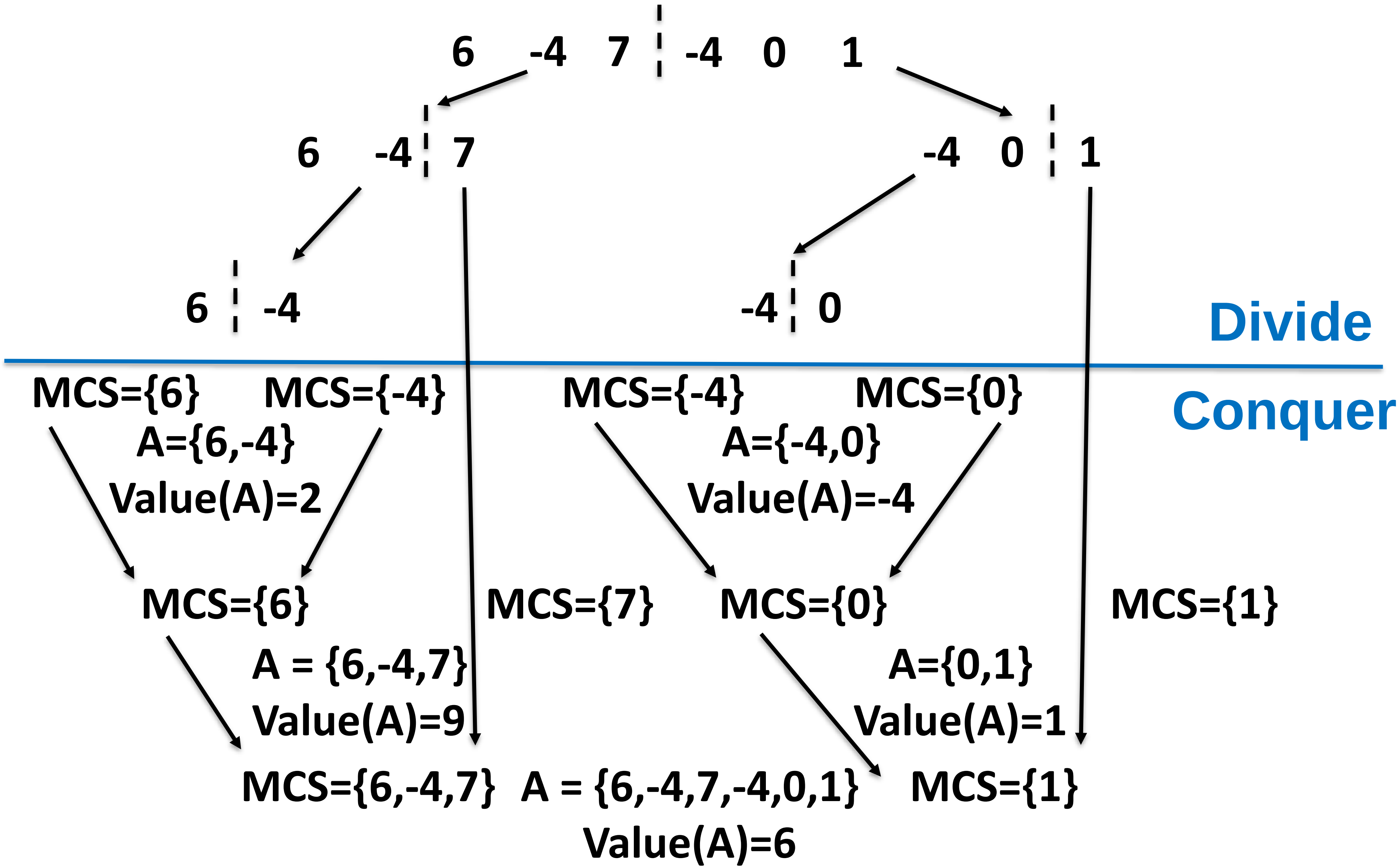

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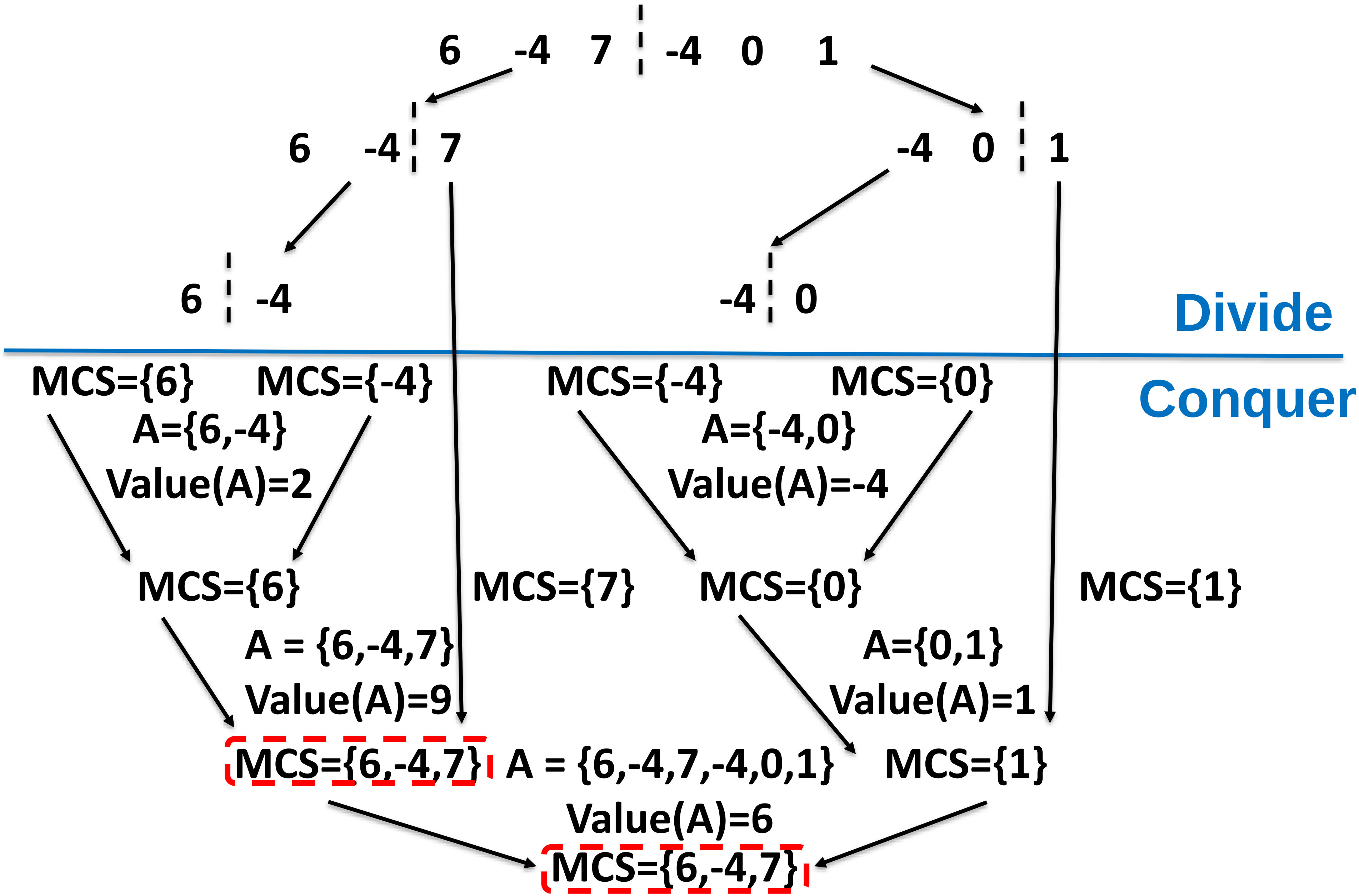

Outline

- Introduction to Part I

- Maximum Contiguous Subarray Problem
  - Problem definition
  - A brute force algorithm
  - A data-reuse algorithm
  - A divide-and-conquer algorithm
  - Analysis of the divide-and-conquer algorithm

Analysis of the D&C Algorithm

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- $T(n)$: time needed to run $MCS(A, s, t)$

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```
begin
    if s = t then return A[s] // O(1)
    else
        m ← ⌊(s+t)/2⌋;
        Find MCS(A, s, m); // T(⌈n/2⌉)
        Find MCS(A, m+1, t); // T(⌊n/2⌋)
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$$T(1) = O(1)$$

$$T(n) = T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + O(n) \quad \text{for } n > 1$$

Analysis of the D&C Algorithm

To simplify the analysis, we assume that n is a power of 2

$$T(n) = \begin{cases} 2T(n/2) + n, & \text{if } n > 1, \\ 1, & \text{if } n = 1. \end{cases}$$

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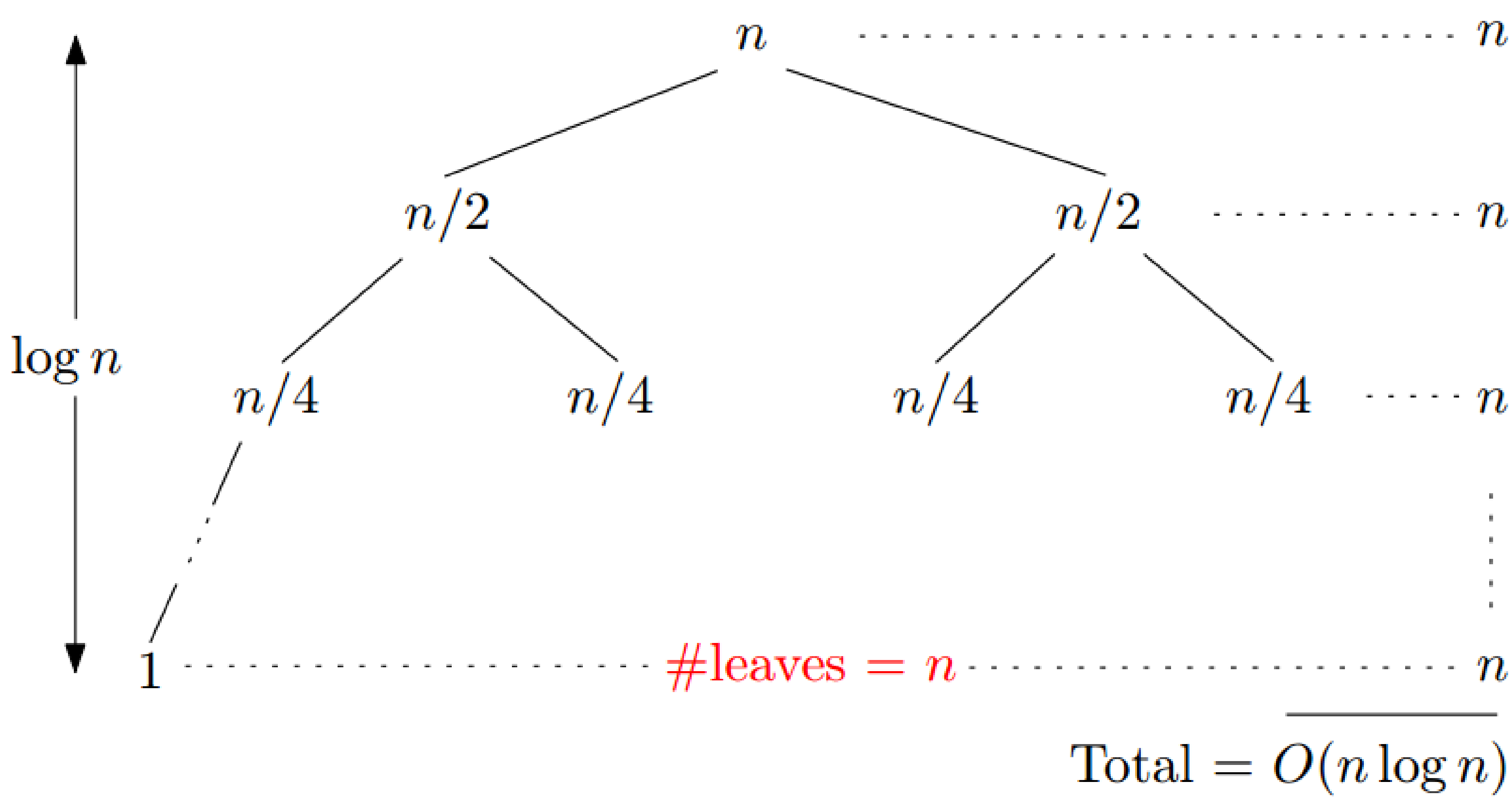

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Can you solve the problem in $O(n)$ time?

谢谢

BDA
Beihang University