Design and Analysis of Algorithms Part II: Dynamic Programming Lecture 14: Minimum Edit Distance

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动态规划篇概述



- 在算法课程第二部分"动态规划"主题中,我们将主要聚焦于如下 经典问题:
 - 0-1 Knapsack (0-1背包问题)
 - Maximum Contiguous Subarray II (最大连续子数组 II)
 - Longest Common Subsequences (最长公共子序列)
 - Longest Common Substrings (最长公共子串)
 - Minimum Edit Distance (最小编辑距离)
 - Rod-Cutting (钢条切割)
 - Chain Matrix Multiplication (矩阵链乘法)

动态规划篇概述

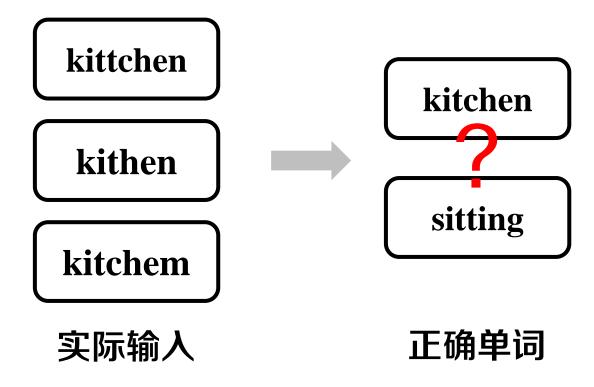


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问题背景



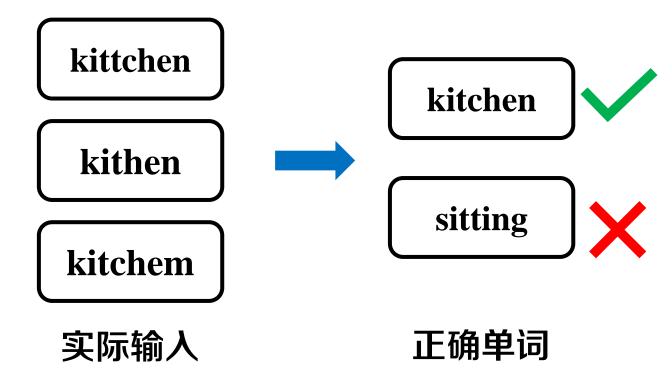
• 输入法自动更正



问题背景



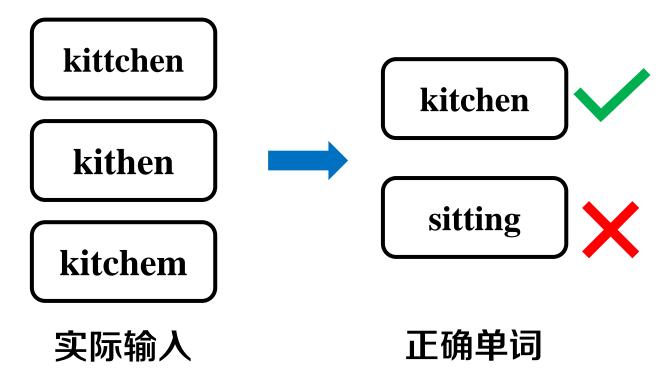
• 输入法自动更正



问题背景



• 输入法自动更正



问题: 如何衡量序列的相似程度?



• 基本思想

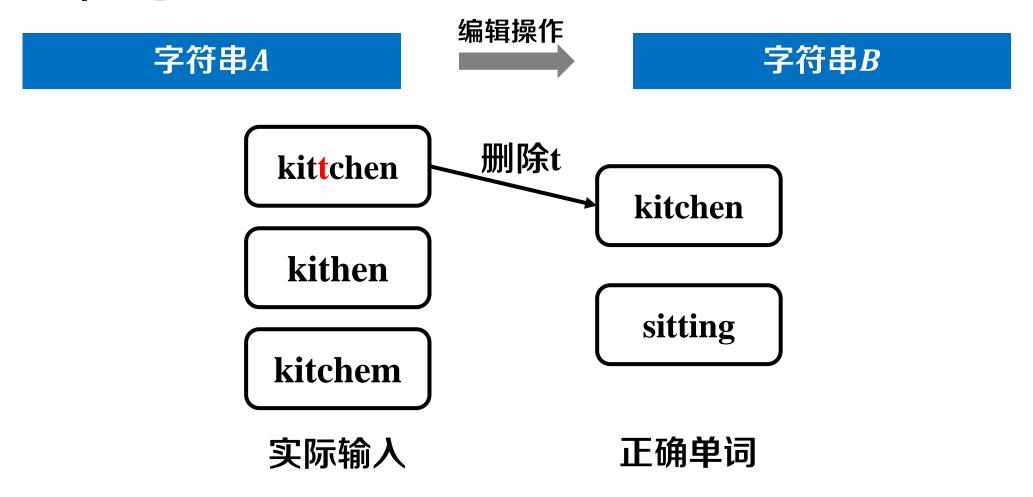
字符串A

编辑操作

字符串B

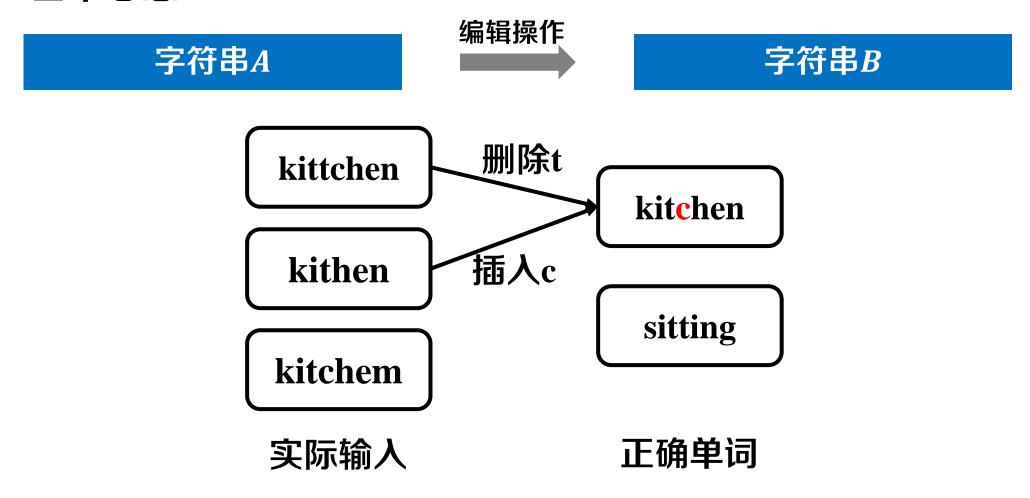


• 基本思想



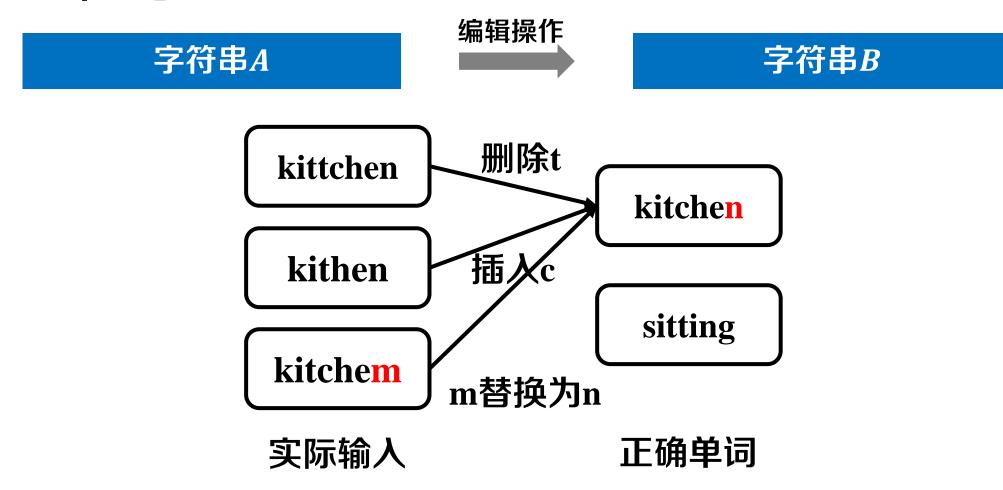


• 基本思想



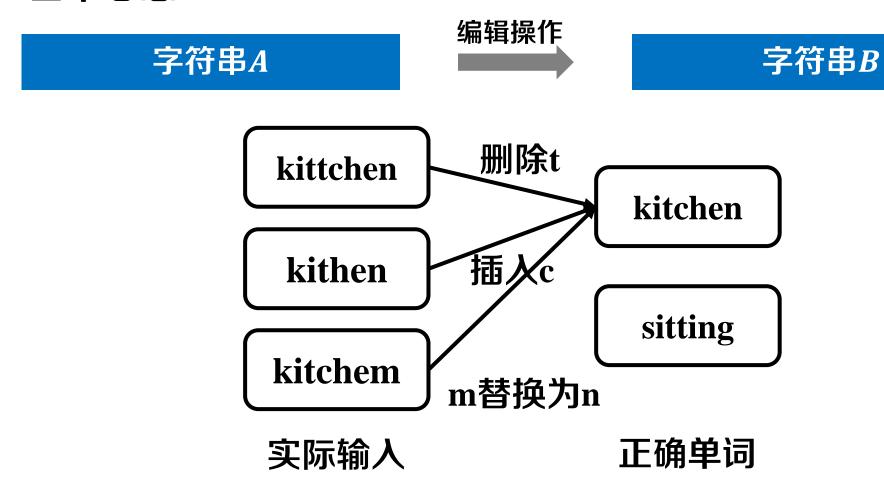


• 基本思想



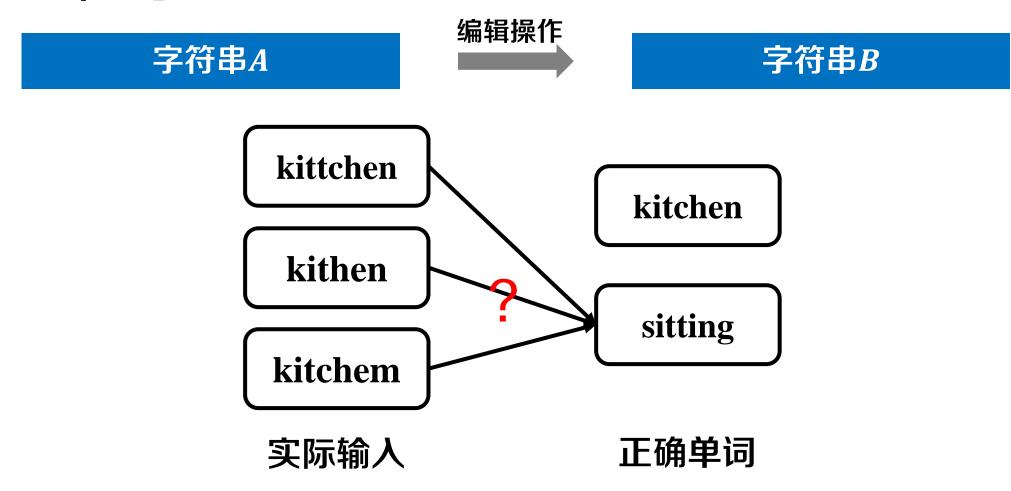


• 基本思想

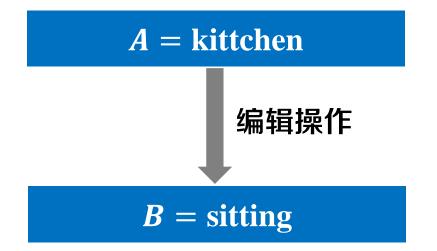




• 基本思想

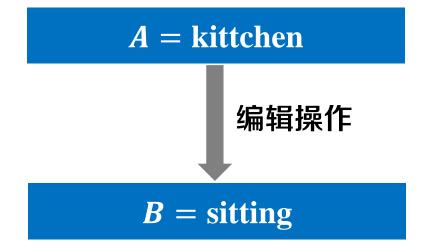






操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kitchen
替换	kitchem → kitchen

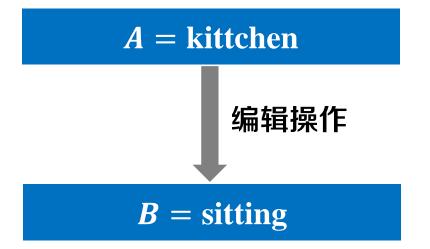




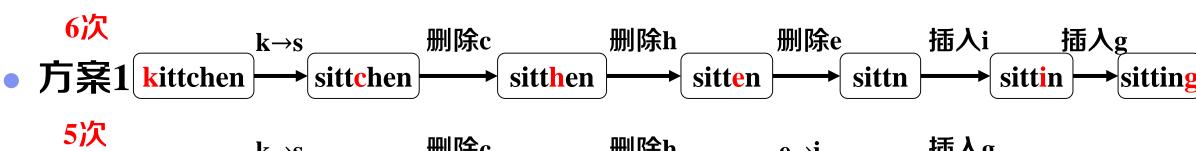
操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kitchen
替换	kitchem → kitche <mark>n</mark>





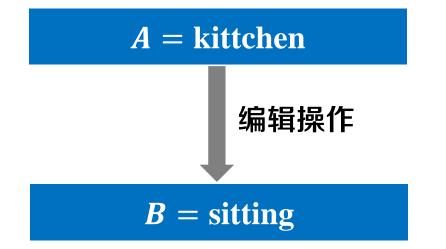


操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kitchen
替换	kitchem → kitche <mark>n</mark>

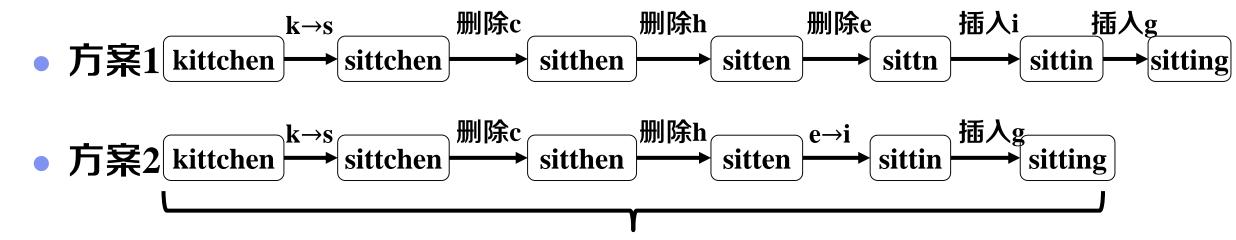


方案2 kittchen k→s sittchen 删除c sitthen 删除h sitten e→i sittin 插入g sitting



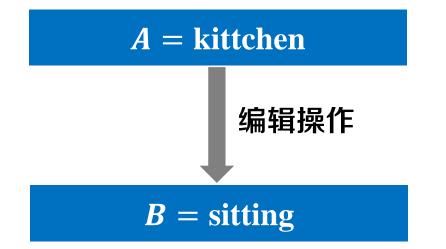


操作名称	操作示例
删除	kittchen → kitchen
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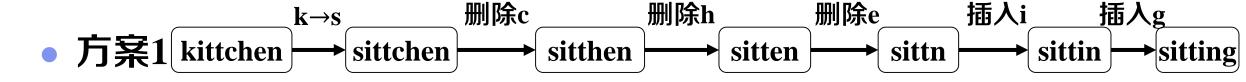


编辑距离:编辑操作次数





操作名称	操作示例
删除	kittchen → kitchen
插入	kithen → kit <mark>c</mark> hen
替换	kitchem → kitchen





问题: 如何求出最少的编辑操作数(最小编辑距离)?



编辑距离问题

Minimum Edit Distance, MED

输入

• 长度为n的字符串s,长度为m的字符串t

输出

• 求出一组编辑操作 $0 = \langle e_1, e_2, ... e_d \rangle$, 令

优化目标

min |0|

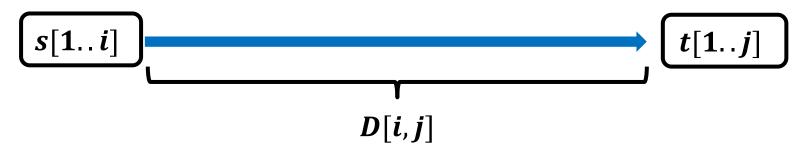
s.t. 字符串s经过o的操作后满足s=t

约束条件

问题结构分析



- 给出问题表示
 - D[i,j]: 字符串S[1..i]变为t[1..j]的最小编辑距离



- 明确原始问题
 - D[n,m]: 字符串s[1..n]变为t[1..m]的最小编辑距离

问题结构分析



递推关系建立



自底向上计算

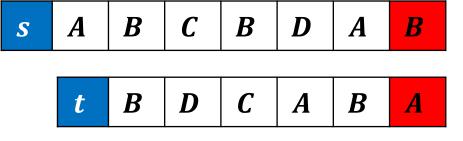


递推关系建立:回顾与启发



• 最长公共子序列

• 如果 $s_i \neq t_j$



S	A	B	С	B	D	A	В
t	В	D	С	A	В	A	

S	A	B	C	B	D	A	В
t	В	D	С	A	B	A	

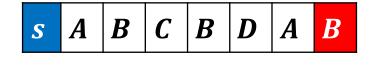
• 如果 $s_i = t_i$

S	A	B	<i>C</i>	B	D	A	B	C	A	B	Γ	B	D		R
		t	В	D	<i>C</i>	A	В	<i>3</i>			C		D]	<u> </u>
				•	•	•		$\boldsymbol{\mathcal{L}}$	B	$oldsymbol{D}$	L	A	В	<u> </u>	

递推关系建立



- 考察末尾元素
 - ●删除



 $t \mid B \mid D \mid C \mid A \mid B \mid A$

插入

s A B C B D A B ?

 $t \mid B \mid D \mid C \mid A \mid B \mid A$

● 替换

 $oldsymbol{s} oldsymbol{A} oldsymbol{B} oldsymbol{C} oldsymbol{B} oldsymbol{D} oldsymbol{A} oldsymbol{B}$

 $t \mid B \mid D \mid C \mid A \mid B \mid A$

问题结构分析

递推关系建立

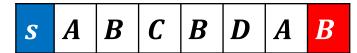
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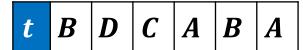
自底向上计算

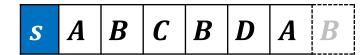
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• 考察末尾元素: 删除









问题结构分析



递推关系建立

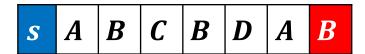


自底向上计算

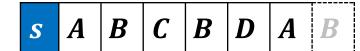




• 考察末尾元素: 删除



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



问题结构分析



递推关系建立



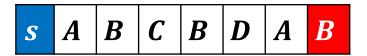
自底向上计算



删除s[i] s[1..i-1]

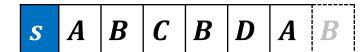


• 考察末尾元素: 删除



 $t \mid B \mid D \mid C \mid A \mid B \mid A$

s[1..i]



 $t \mid B \mid D \mid C \mid A \mid B \mid A$

t[1..*j*]





递推关系建立



自底向上计算



最优方案追踪

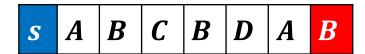


D[i,j]

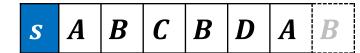
D[i-1,j]



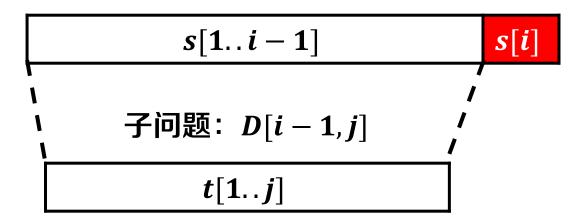
• 考察末尾元素: 删除



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



t B D C A B A



• D[i,j] = D[i-1,j] + 1

问题结构分析



递推关系建立

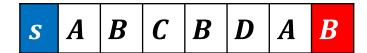


自底向上计算

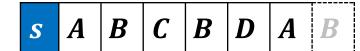




• 考察末尾元素: 删除



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



 $t \mid B \mid D \mid C \mid A \mid B \mid A$





递推关系建立

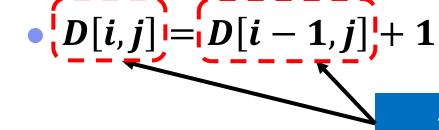


自底向上计算



最优方案追踪





最优子结构



• 考察末尾元素: 插入

s A B C B D A B ?

 $\begin{array}{|c|c|c|c|c|c|} \hline t & B & D & C & A & B & A \end{array}$

问题结构分析



递推关系建立

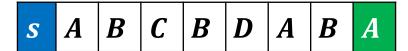


自底向上计算

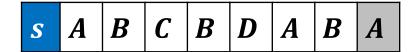




• 考察末尾元素: 插入



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



t B D C A B A

问题结构分析



递推关系建立

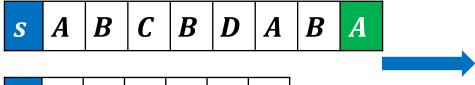


自底向上计算

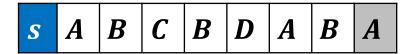




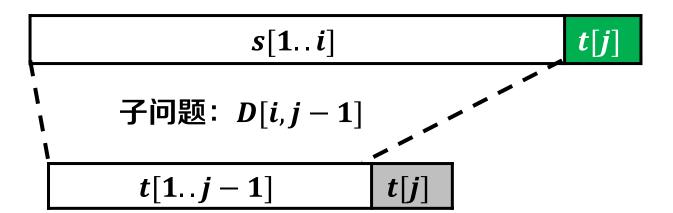
• 考察末尾元素: 插入



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



t B D C A B A



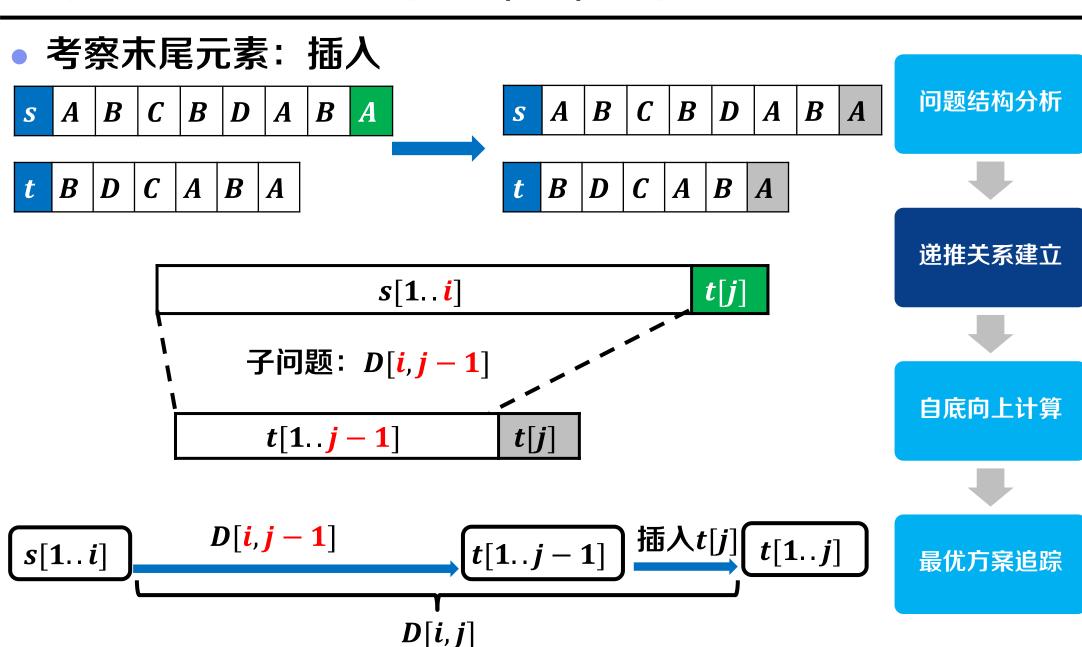
问题结构分析



自底向上计算

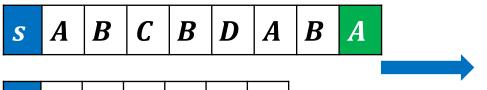




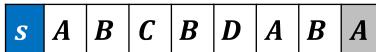




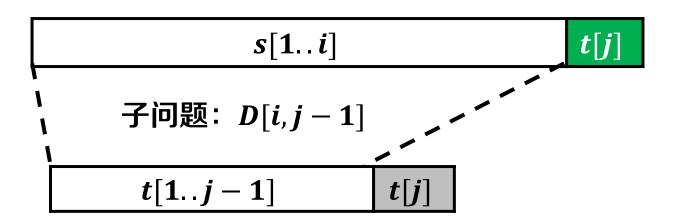
• 考察末尾元素: 插入



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



t B D C A B A



• D[i,j] = D[i,j-1] + 1

问题结构分析



递推关系建立

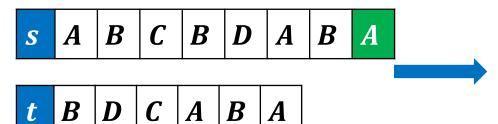


自底向上计算

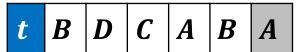


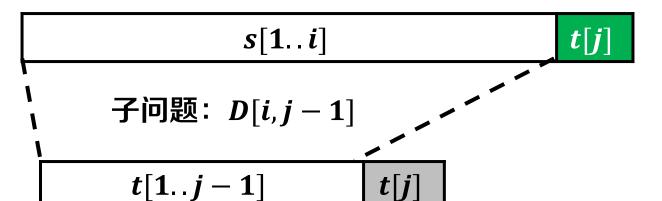


• 考察末尾元素: 插入



S A B C B D A B A





问题结构分析



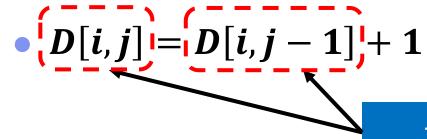
递推关系建立



自底向上计算



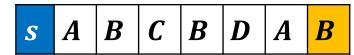
最优方案追踪

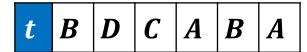


最优子结构



• 考察末尾元素: 替换





问题结构分析



递推关系建立



自底向上计算





• 考察末尾元素: 替换

s A B C B D A ?

t B D C A B A

问题结构分析



递推关系建立



自底向上计算



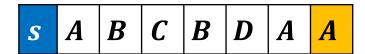
s[1..i-1]

子问题: D[i-1,j-1]

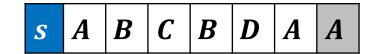
t[1..j-1]



• 考察末尾元素: 替换



t B D C A B A



t | B | D | C | A | B | A

s[i]

t[j]





递推关系建立



自底向上计算



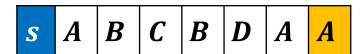
s[1..i-1]

子问题: D[i-1,j-1]

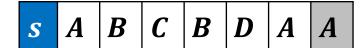
t[1..j-1]



• 考察末尾元素: 替换



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



t B D C A B A

s[i]

问题结构分析



递推关系建立



自底向上计算



 $\begin{array}{c|c}
s[1..i] \\
\hline
s[1..i-1] \\
\hline
s[i] \rightarrow t[j] \\
\hline
t[1..j-1] \\
\hline
t[1..j]
\end{array}$

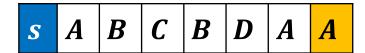
t[j]

D[i,j]

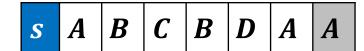
递推关系建立:分析最优(子)结构



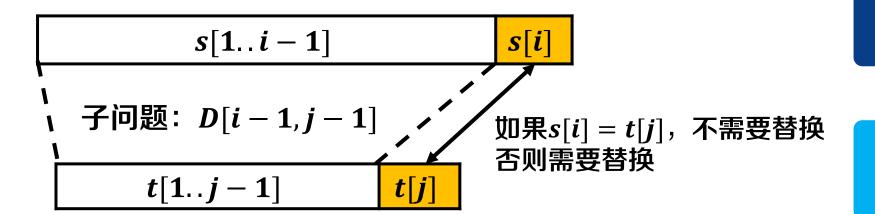
• 考察末尾元素: 替换



 $t \mid B \mid D \mid C \mid A \mid B \mid A$



t B D C A B A



•
$$D[i,j] = D[i-1,j-1] + \begin{cases} 0, & \text{if } s[i] = t[j] \\ 1, & \text{if } s[i] \neq t[j] \end{cases}$$

问题结构分析



递推关系建立



自底向上计算



递推关系建立:分析最优(子)结构



• 考察末尾元素: 替换

 $t \mid B \mid D \mid C \mid A \mid B \mid A$

t B D C A B A

s[i]





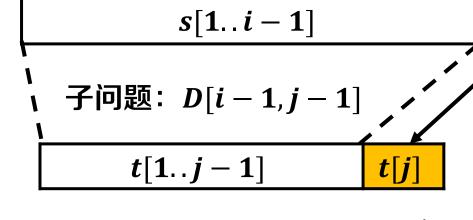
递推关系建立



自底向上计算



最优方案追踪



如果s[i] = t[j],不需要替换 否则需要替换

递推关系建立:构造递推公式



• 综合上面三种方式

•
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+iggl\{0,\ if\ s[i]=t[j]\ 1,\ if\ s[i]
otag by iggl\}$$

问题结构分析



递推关系建立



自底向上计算



递推关系建立: 构造递推公式



• 最小编辑距离 vs. 最长公共子序列

•
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+iggl\{0,\ if\ s[i]=t[j]\ 1,\ if\ s[i]
otag t[j] & ext{替换} \end{cases}$$

•
$$C[i,j] = \begin{cases} \max\{C[i-1,j], C[i,j-1]\}, x_i \neq y_j \\ C[i-1,j-1] + 1 \end{cases}$$
, $x_i = y_j$

问题结构分析



递推关系建立



自底向上计算

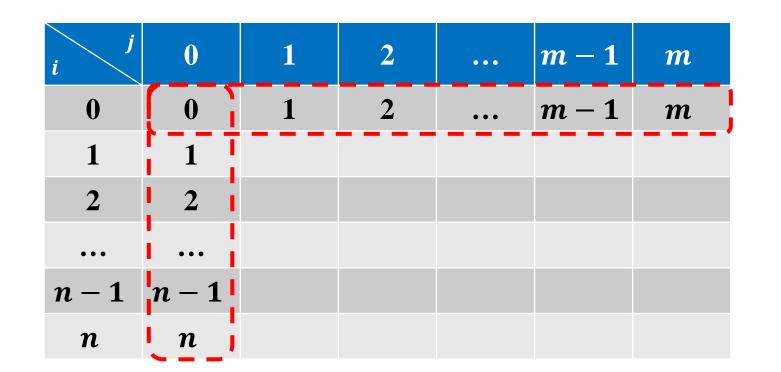


自底向上计算:确定计算顺序



• 初始化

- D[i,0]=i
 - 。 把长度为i的串变为空串至少需要i次操作(删除)
- D[0,j]=j
 - 把空串变为长度为j的串至少需要j次操作(插入)



问题结构分析



递推关系建立



自底向上计算



自底向上计算:确定计算顺序



• 递推公式

•
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+egin{cases} 0, \ if \ s[i]=t[j] \ 1, \ if \ s[i]
eq t[j] \end{cases}$$
 替换

i	0	1	2	•••	m-1	m
0	0	1	2	•••	m-1	m
1	1	0[i-1,i]	- 1] + {	0	- 1, <i>j</i>] +	
2	2	[-,]	-, ($\begin{matrix} 1 & D[i] \end{matrix}$	-1, j] +	1
•••	•••			D[i,j]		
n-1	n-1		D[i,j-1]	1]+1		
\boldsymbol{n}	\boldsymbol{n}					

问题结构分析



递推关系建立



自底向上计算



自底向上计算: 依次计算问题



• 递推公式

•
$$D[i,j] = \min egin{cases} D[i-1,j]+1 & ext{删除} \ D[i,j-1]+1 & ext{插入} \ D[i-1,j-1]+egin{cases} 0, \ if \ s[i]=t[j] \ 1, \ if \ s[i]
eq t[j] \end{cases}$$
 替换

i j	0	1	2	•••	m-1	m
0	0	1	2	• • •	m-1	m
1	1					
2	2	<u> </u>				.= >
•••	• • •	<u> </u>				. =>
n-1	n-1	4				.=>
\boldsymbol{n}	\boldsymbol{n}	<u> </u>				*

问题结构分析



递推关系建立



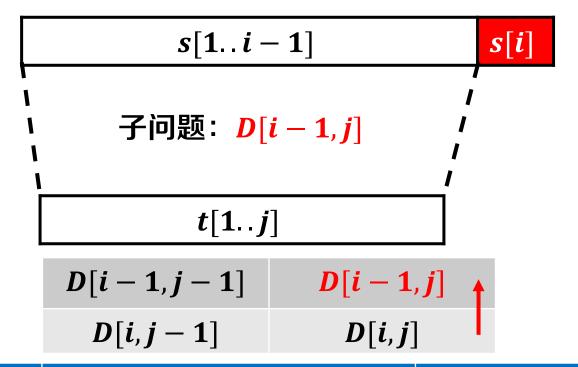
自底向上计算



最优方案追踪: 记录决策过程



• 追踪数组Rec,记录子问题来源



Rec[i,j]	子问题来源	操作
U	上侧,即 <i>D</i> [<i>i</i> — 1, <i>j</i>]	删除s[i]

问题结构分析



递推关系建立



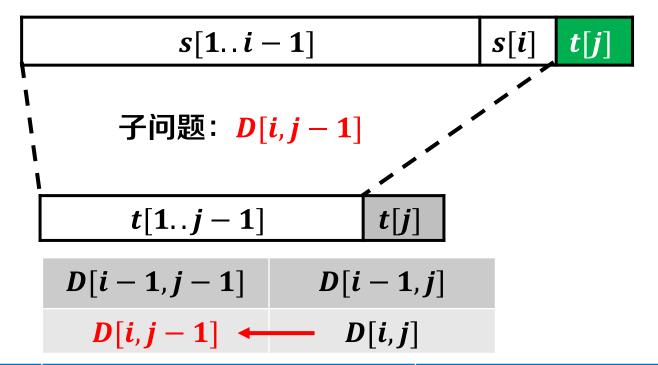
自底向上计算



最优方案追踪:记录决策过程



• 追踪数组Rec,记录子问题来源



Rec[i,j]	子问题来源	操作
U	上侧,即 <i>D</i> [<i>i</i> — 1, <i>j</i>]	删除s[i]
L	左侧,即 $D[i,j-1]$	插入t[j]

问题结构分析



递推关系建立



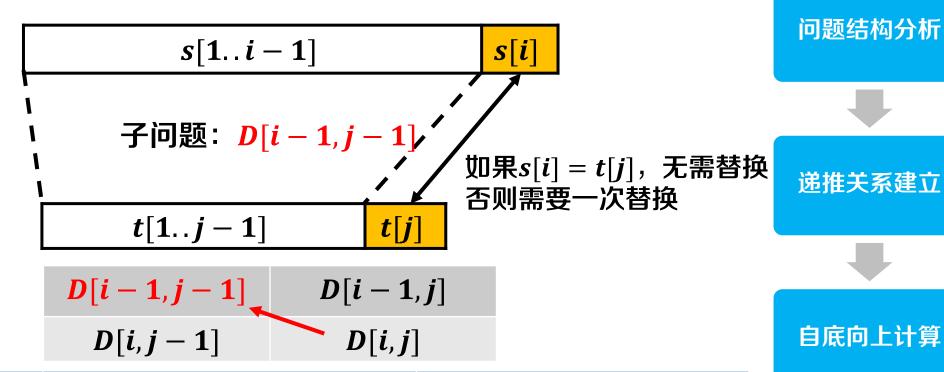
自底向上计算



最优方案追踪:记录决策过程



• 追踪数组Rec,记录子问题来源

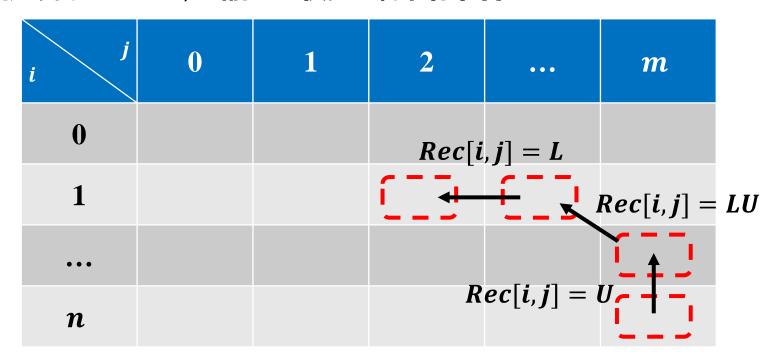


Rec[i,j]	子问题来源	操作
U	上侧,即 <i>D</i> [<i>i</i> — 1, <i>j</i>]	删除s[i]
L	左侧,即 <i>D</i> [<i>i,j</i> — 1]	插入t[j]
LU	左上,即 $D[i-1,j-1]$	用 $t[j]$ 替换 $s[i]$ /无操作

最优方案追踪:输出最优方案



• 根据数组Rec,输出最少编辑操作



Rec[i,j]	子问题来源	操作
U	上侧,即 <i>D</i> [<i>i</i> — 1, <i>j</i>]	删除s[i]
L	左侧,即 <i>D</i> [<i>i,j</i> — 1]	插入 <i>t</i> [<i>j</i>]
LU	左上,即 <i>D</i> [<i>i</i> – 1, <i>j</i> – 1]	用 $t[j]$ 替换 $s[i]$ /无操作

问题结构分析



递推关系建立



自底向上计算





	1	2	3	4	5	6	7
S	A	В	C	В	D	A	В
t	В	D	C	A	В	A	

D								Rec							
i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1							1	U						
2	2		初始	化.				2	U						
3	3			7.0				3	U						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	B	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1							1	U						
2	2							2	U						
3	3							3	U						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \left\{$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D[i-1,j]Rec D L U U U U U U U



	1	2	3	4	5	6	7	$ \begin{pmatrix} D[i-1,j]+1 & 删除A \\ D[i-i-1]+1 & tf & D \end{pmatrix} $
S	<u>A</u>	В	C	В	D	A	В	$D[i,j] = \min $ $ D[i,j-1] + 1 $ 插入B $ (0,if s[i] = t[i] $
t	B	D	C	A	В	A		$D[i,j] = \min \left\{ D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases} \right\}$

D			D[i	-1,j				Rec							
i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1							1	\mathbf{U}						
2	2							2	U						
3	3		D[i,j]	[-1]				3	\mathbf{U}						
4	4							4	U						
5	5							5	U						
6	6							6	U						
7	7							7	U						



	1	2	,	3	4	5	6	7					i – 1, j]	+ 1 + 1	删除A 插入B
S	A	B		C	В	D	A	В	D[i,j]	= min			_		
t	В	, D		$\mathbf{C}^{S[i]}$	$[t] \neq t[$	<i>j</i>]	A				D[i -	- 1 , j –	1] + $\{$ 1	, if $s[i]$	$\neq t[j]$
D			D[i]	-1,j	7]			Rec					A칱	替换为I	3
i j	0	1	2	3	4	5	6	i	0	1	2	3	4	5	6
0	0	$\begin{bmatrix} 1 \end{bmatrix}$	2	3	4	5	6	0		L	L	L	L	L	L
1	1			D	[i-1]	j-1		1	U						
2	2							2	U						
3	3		D[i,]	[j-1]				3	U						
4	4							4	U						
5	5							5	\mathbf{U}						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C S	$[i] \neq t$	j	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

\boldsymbol{D}			D[i]	-1,j				Rec							
i	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0_	1	2	3	4	5	6	0		L	L	L	L	L	L
1	(1)	1		D[i-1,	$\overline{j-1}$		1	\mathbf{U}	LU					
2	2							2	\mathbf{U}						
3	3		D[i,j]	[-1]				3	\mathbf{U}						
4	4							4	U						
5	5							5	\mathbf{U}						
6	6							6	U						
7	7							7	U						



		1		2	3	4	5	6	7					[i-1,j]		
	S	A		B	C	В	D	A	В	D[i,j]	= min	{ _		[i,j-1]		= t[i]
	t	В		D	C	AS	$[t] \neq t[$	j]				D[i -	\cdot 1, j $-$	$1] + \{1$), if s[i] ., if s[i]	$\neq t[j]$
D)					D[i-1]	., j]		Rec							
i	j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
	0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
	1	1	1	2			o[i-1]	[,j-1]	1	U	LU	LU				
	2	2							2	\mathbf{U}						
	3	3			D	[i,j-1]	.]		3	U						
	4	4							4	U						
	5	5							5	U						
	6	6							6	U						
	7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3				1	\mathbf{U}	LU	LU	LU			
2	2							2	U						
3	3							3	\mathbf{U}						
4	4							4	U						
5	5							5	\mathbf{U}						
6	6							6	U						
7	7							7	U						



	1	2	2	3	4	5	6	7					[i-1,j]		
S	A		3	C	В	D	A	В	D[i,j]	= min	}		[i,j-1]		=t[i]
t	В	I)	C	A	B	s[i] = t	j]			D[i -	-1,j-	$oldsymbol{1}$] + $igg\{egin{matrix} oldsymbol{0} \ oldsymbol{1} \ \end{matrix}$, if $s[i]$	$\neq t[j]$
D						D	[i-1,j]	i] c					无	需替换	
i j	0	1	2	3	4	5	6	i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3		D[i-1,j	-1]	LU	LU	LU	LU		
2	2							2	U						
3	3					D[i,	j - 1]	3	\mathbf{U}						
4	4							4	\mathbf{U}						
5	5							5	\mathbf{U}						
6	6							6	\mathbf{U}						
7	7							7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	i j	0
0	0	1	2	3	4	5	6	0	
1	1	1	2	3	3	4		1	U
2	2							2	\mathbf{U}
3	3							3	U
4	4							4	U
5	5							5	U
6	6							6	U
7	7							7	U

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	
2	\mathbf{U}						
3	\mathbf{U}						
4	U						
5	\mathbf{U}						
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6_
1	1	1	2	3	3	4	5
2	2						
3	3						
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	U						
3	\mathbf{U}						
4	U						
5	\mathbf{U}						
6	U						
7	\mathbf{U}						



L LU

	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	B	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	i j	0	1	2	3	4
0	0	1	2	3	4	5	6	0		L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU
2	2	1						2	U	LU			
3	3							3	\mathbf{U}				
4	4							4	U				
5	5							5	\mathbf{U}				
6	6							6	\mathbf{U}				
7	7							7	U				



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	2	[1	2	,				2	U	LU	LU				
3	3							3	\mathbf{U}						
4	4							4	\mathbf{U}						
5	5							5	\mathbf{U}						
6	6							6	\mathbf{U}						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	$\begin{bmatrix} \mathbf{B} \end{bmatrix}$			D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	\mathbf{C}	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3			
3	3						
4	4						
5	5						
6	6						
7	7						

j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU			
3	\mathbf{U}						
4	\mathbf{U}						
5	\mathbf{U}						
6	\mathbf{U}						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	$oldsymbol{A}$	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	
0	0	1	2	3	4	5	6	
1	1	1	2	3	3	4	5	
2	2	1	2	3	4			
3	3							
4	4							
5	5							
6	6							
7	7							

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU		
3	\mathbf{U}						
4	\mathbf{U}						
5	\mathbf{U}						
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C		D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	[3	•
3	3						
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	
3	\mathbf{U}						
4	\mathbf{U}						
5	\mathbf{U}						
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	$oxed{L}$
3	3							3	\mathbf{U}						
4	4							4	U						
5	5							5	\mathbf{U}						
6	6							6	U						
7	7							7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	$\begin{bmatrix} \mathbf{B} \end{bmatrix}$	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

) Rec

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	(1)	2	3	4	3	4
3	3	2					
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	U					
4	\mathbf{U}						
5	\mathbf{U}						
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	i
0	0	1	2	3	4	5	6	0
1	1	1	2	3	3	4	5	1
2	2	1	2	3	4	3	4	2
3	3	2	2					3
4	4							4
5	5							5
6	6							6
7	7							7

i	0	1	2	3	4	5	6
0		L	\mathbf{L}	${f L}$	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU	LU	L
3	\mathbf{U}	U	LU				
4	\mathbf{U}						
5	\mathbf{U}						
6	\mathbf{U}						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2			
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU			
4	\mathbf{U}						
5	\mathbf{U}						
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1		3			4
3	3	2	2	2	3		
4	4						
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L		
4	U						
5	\mathbf{U}						
6	U						
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	\mathbf{A}			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	
4	4						
5	5						
6	6						
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	
4	\mathbf{U}						
5	\mathbf{U}						
6	\mathbf{U}						
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4						
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	U						
5	\mathbf{U}						
6	U						
7	U						



	1	2	3	4	5	6	7		$ \begin{pmatrix} D[i-1,j]+1 \\ D[i-1]+1 \end{pmatrix} $
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[j]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	\mathbf{U}	U	LU	LU	L	L	LU
4	4	3						4	U	LU					
5	5							5	\mathbf{U}						
6	6							6	U						
7	7							7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	\mathbf{U}	U	LU	LU	L	L	LU
4	4	3	3					4	U	LU	LU				
5	5							5	\mathbf{U}						
6	6							6	U						
7	7							7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	\mathbf{U}	U	LU	LU	L	L	LU
4	4	3	3	3				4	U	LU	LU	LU			
5	5							5	\mathbf{U}						
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C		В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3		
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	${f L}$	LU
4	U	LU	LU	LU	LU		
5	\mathbf{U}						
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	$\left(\begin{array}{c} \mathbf{B} \end{array} \right)$	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	B	\mathbf{A}			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	${f L}$	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	
5	\mathbf{U}						
6	U						
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4_
4	4	3	3	3	3	3	4
5	5						
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}						
6	\mathbf{U}						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	\mathbf{A}			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	U	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	U	U	LU	LU	L	L	LU
4	4	3	3	3	3	3	4	4	U	LU	LU	LU	LU	LU	L
5	5	4	3					5	U	U	LU				
6	6							6	U						
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	\mathbb{C}	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	j		
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU			
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		$ \begin{pmatrix} D[i-1,j]+1 \\ D[i-1]+1 \end{pmatrix} $
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4		
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU		
6	\mathbf{U}						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	B	\mathbf{A}			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	${f L}$	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	
6	U						
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6						
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	U						
7	U						



	1	2	3	4	5	6	7		$ \begin{pmatrix} D[i-1,j]+1 \\ D[i-1,j]+1 \end{pmatrix} $
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle a_i \rangle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	B	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	<u>5</u>	4	3	4	4	4	4
6	6	5					
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	U	U					
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	
0	0	1	2	3	4	5	6	
1	1	1	2	3	3	4	5	
2	2	1	2	3	4	3	4	
3	3	2	2	2	3	4	4	
4	4	3	3	3	3	3	4	
5	5	4	3	4	4	4	4	
6	6	5	4					
7	7							

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	${f L}$
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	U	U	U				
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3		4	4	4
6	6	5	4	4			
7	7						

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	${f L}$	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	U	U	U	LU			
7	\mathbf{U}						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4		
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU	LU	L
3	\mathbf{U}	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	\mathbf{U}	\mathbf{U}	\mathbf{U}	LU	LU		
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	B	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

D

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	
7	7						

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	U	LU	LU	L	${f L}$	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	
7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle a_i \rangle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	U	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	U	U	LU	LU	L	L	LU
4	4	3	3	3	3	3	4	4	U	LU	LU	LU	LU	LU	L
5	5	4	3	4	4	4	4	5	U	U	LU	LU	LU	LU	LU
6	6	5	4	4	4	5	4	6	U	U	U	LU	LU	LU	LU
7	7							7	U						



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	B	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	U	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	\mathbf{U}	U	LU	LU	L	L	LU
4	4	3	3	3	3	3	4	4	U	LU	LU	LU	LU	LU	L
5	5	4	3	4	4	4	4	5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	6	5	4	4	4	5	4	6	U	U	U	LU	LU	LU	LU
7	7	6						7	U	LU					



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6	i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6	0		L	L	L	L	L	L
1	1	1	2	3	3	4	5	1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	2	1	2	3	4	3	4	2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	3	2	2	2	3	4	4	3	\mathbf{U}	U	LU	LU	L	L	LU
4	4	3	3	3	3	3	4	4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	5	4	3	4	4	4	4	5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	6	5	4	4	4	5	4	6	U	U	U	LU	LU	LU	LU
7	7	6	5					7	U	LU	U				



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	\mathbb{C}	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5			

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	${f L}$	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU			



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C		В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5	5		

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	${f L}$	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU		



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \left\langle \right.$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	B	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5	5	4	

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	U	LU	U	LU	LU	LU	



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	B	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3	4	4	4	4
6	6	5	4	4	4	5	4
7	7	6	5	5	5	4	5

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	U	LU	LU	LU	LU	LU	L
3	U	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	U	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $O[i,j-1]=t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

i j	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	2	3	3	4	5
2	2	1	2	3	4	3	4
3	3	2	2	2	3	4	4
4	4	3	3	3	3	3	4
5	5	4	3		最优角	2	4
6	6	5	4	4	4	3	4
7	7	6	5	5	5	4	5

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	U	LU	LU	L	L	LU
4	U	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	L



	1	2	3	4	5	6	7		$ \begin{pmatrix} D[i-1,j]+1 \\ D[i-1]+1 \end{pmatrix} $
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0,if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

插人A

Rec							
i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	U	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	U	U	\mathbf{U}	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	(L)



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \left\langle \right.$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	B	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

无需操作 插人A

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	${f L}$	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	${f L}$	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	\mathbf{U}	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	${f L}$



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i, j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

无需操作 无需操作 插人A

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i, j-1]+1 $(0. if s[i] = t[i]$
t	В	D	\mathbb{C}	A	В	\mathbf{A}			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

用C替换D 无需操作 无需操作 插人A

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	U	LU	LU	LU	LU	LU
6	U	U	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	B	D	A	В	$D[i,j] = \min \langle$	D[i, j-1]+1 $(0. if s[i] = t[i]$
t	В	$\left(\begin{array}{c} \mathbf{D} \end{array} \right)$	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

用D替换B 用C替换D 无需操作 无需操作 插人A

i	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	U	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	${f L}$
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	U	U	\mathbf{U}	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	${f L}$



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

删除C 用D替换B 用C替换D 无需操作 无需操作 插人A

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	U	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	${f L}$
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	\mathbf{U}	\mathbf{U}	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	${f L}$



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	B	C	В	D	A	\mathbf{B} $D[i, j]$	j] = $\min \left\{ \right.$	D[i,j-1]+1
t	$\begin{bmatrix} \mathbf{B} \end{bmatrix}$	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作:

无需操作 删除C 用D替换D 用C替换D 无需操作 插入A

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	\mathbf{U}	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	${f L}$	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	L
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	\mathbf{U}	U	U	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	L



	1	2	3	4	5	6	7		D[i-1,j]+1
S	A	В	C	В	D	A	В	$D[i,j] = \min \langle$	D[i,j-1]+1 $(0.if s[i] = t[i]$
t	В	D	C	A	В	A			$D[i-1,j-1] + \begin{cases} 0, if \ s[i] = t[j] \\ 1, if \ s[i] \neq t[j] \end{cases}$

操作: 删除A

无需操作

删除C

用D替换B

用C替换D

无需操作

无需操作

插人A

i j	0	1	2	3	4	5	6
0		L	L	L	L	L	L
1	U	LU	LU	LU	LU	L	LU
2	\mathbf{U}	LU	LU	LU	LU	LU	L
3	\mathbf{U}	\mathbf{U}	LU	LU	L	L	LU
4	\mathbf{U}	LU	LU	LU	LU	LU	${f L}$
5	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU	LU
6	\mathbf{U}	\mathbf{U}	\mathbf{U}	LU	LU	LU	LU
7	\mathbf{U}	LU	\mathbf{U}	LU	LU	LU	L



```
输入: 字符串s和t
输出: s和t的最小编辑距离
n \leftarrow \text{length}(s)
m \leftarrow \text{length}(t)
新建D[0..n,0..m],Rec[0..n,0..m]两个二维数组
//初始化
for i \leftarrow 0 to n do
                                           初始化
    D[i,0] \leftarrow i
    Rec[i,0] \leftarrow "U"
end
for j \leftarrow 0 to m do
    D[0,j] \leftarrow j
    Rec[0,j] \leftarrow ``L"
```



```
<u>//动态</u>规划
for i \leftarrow 1 to n do
                                                    依次计算子问题
   for j \leftarrow 1 to m do
        if s_i \neq t_i then
        c \leftarrow 1
        end
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        if replace = min\{delete, insert, replace\} then
           D[i,j] \leftarrow D[i-1,j-1] + c
           Rec[i,j] \leftarrow "LU"
        end
        else if insert = min\{delete, insert, replace\} then
           D[i,j] \leftarrow D[i,j-1] + 1
           Rec[i,j] \leftarrow "L"
        end
        else
           D[i,j] \leftarrow D[i-1,j] + 1
           Rec[i,j] \leftarrow "U"
        end
    end
end
```



```
//动态规划
for i \leftarrow 1 to n do
    \underline{\mathbf{for}} \ \underline{j} \leftarrow 1 \ \underline{to} \ \underline{m} \ \underline{\mathbf{do}}
                                                               替换/无需操作
         if s_i \neq t_j then
         end
        replace \leftarrow D[i-1,j-1] + c
         delete \leftarrow D[i-1,j]+1
         insert \leftarrow D[i, j-1] + 1
         if replace = min\{delete, insert, replace\} then
             D[i,j] \leftarrow D[i-1,j-1] + c
             Rec[i,j] \leftarrow "LU"
         end
         else if insert = min\{delete, insert, replace\} then
             D[i,j] \leftarrow D[i,j-1] + 1
             Rec[i,j] \leftarrow ``L"
         end
         else
             D[i,j] \leftarrow D[i-1,j] + 1
             Rec[i,j] \leftarrow "U"
         end
    end
end
```



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
         c \leftarrow 0
         if s_i \neq t_j then
           c \leftarrow 1
          end
         replace \leftarrow D[i-1,j-1]+c
         delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
         \begin{array}{c} \textbf{if} \ replace = min\{delete, insert, replace\} \ \textbf{then} \\ + \ \mathcal{D}[i,j] \leftarrow \mathcal{D}[i-1,j-1] + c \end{array} 
                                                                                        采用替换操作
             Rec[i,j] \leftarrow "LU"
          end
          else if insert = min\{delete, insert, replace\} then
              D[i,j] \leftarrow D[i,j-1] + 1
              Rec[i,j] \leftarrow ``L"
          end
          else
              D[i,j] \leftarrow D[i-1,j] + 1
              Rec[i,j] \leftarrow "U"
          end
    \mathbf{end}
end
```



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
         c \leftarrow 0
          if s_i \neq t_j then
           c \leftarrow 1
          end
          replace \leftarrow D[i-1,j-1]+c
          delete \leftarrow D[i-1,j]+1
          insert \leftarrow D[i, j-1] + 1
         \begin{array}{l} \textbf{if } replace = min\{delete, insert, replace\} \textbf{ then} \\ | D[i,j] \leftarrow D[i-1,j-1] + c \end{array}
                                                                                            记录编辑距离和操作
             Rec[i,j] \leftarrow ``LU"
          \mathbf{else} \ \mathbf{if} \ insert = min\{delete, insert, replace\} \ \mathbf{then}
               D[i,j] \leftarrow D[i,j-1] + 1
              Rec[i,j] \leftarrow "L"
          end
          else
              D[i,j] \leftarrow D[i-1,j] + 1
              Rec[i,j] \leftarrow "U"
          \mathbf{end}
     \mathbf{end}
end
```



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
        c \leftarrow 0
        if s_i \neq t_i then
         c \leftarrow 1
        end
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        if replace = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i-1,j-1] + c
            Rec[i,j] \leftarrow ``LU"
        \mathbf{end}
      \mathbf{f} else if insert = min\{delete, insert, replace\} then
                                                                                   采取插入操作
            D[i,j] \leftarrow D[i,j-1] + 1
           Rec[i,j] \leftarrow ``L"
        end
        else
            D[i,j] \leftarrow D[i-1,j] + 1
           Rec[i,j] \leftarrow "U"
        \mathbf{end}
    \mathbf{end}
end
```



```
//动态规划
for i \leftarrow 1 to n do
   for j \leftarrow 1 to m do
        c \leftarrow 0
        if s_i \neq t_j then
         c \leftarrow 1
        end
        replace \leftarrow D[i-1,j-1]+c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        if replace = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i-1,j-1] + c
           Rec[i,j] \leftarrow "LU"
        end
        else if insert = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i,j-1] + 1
            Rec[i,j] \leftarrow "L"
        end
      /else
                                                                                采取删除操作
          D[i,j] \leftarrow D[i-1,j] + 1 Rec[i,j] \leftarrow ``U"
       end
    end
end
```



Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i,j] = "LU" then
                                         采取替换操作
   Print-MED(Rec, s, t, i-1, j-1)
   if s_i = t_i then
      print "无需操作"
   \mathbf{end}
   else
      \operatorname{print} "用t_j替换s_i"
   end
end
else if Rec[i, j] = "U" then
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i, j] = "LU" then
  Print-MED(Rec, s, t, i - 1, j - 1)
                                         递归输出子问题方案
   if s_i = t_i then
      print "无需操作"
   \mathbf{end}
   else
      \operatorname{print} "用t_j替换s_i"
   end
end
else if Rec[i, j] = "U" then
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
  return NULL
end
if Rec[i,j] = "LU" then
   Print-MED(Rec, s, t, i-1, j-1)
  if s_i = t_j then
                                替换/无操作
      print "无需操作"
  end
  else
  ■ print "用t_j替换s_i"
  end
end
else if Rec[i, j] = "U" then
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



• Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i,j] = "LU" then
   Print-MED(Rec, s, t, i - 1, j - 1)
   if s_i = t_i then
      print "无需操作"
   \mathbf{end}
   else
      \operatorname{print} "用t_j替换s_i"
   end
else if Rec[i,j] = "U" then
                                         采取删除操作
   Print-MED(Rec, s, t, i - 1, j)
   print "删除s_i"
end
else
   Print-MED(Rec, s, t, i, j - 1)
   print "插入t_j"
end
```



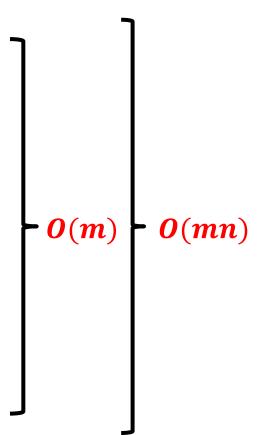
• Print-MED(Rec, s, t, i, j)

```
输入: 矩阵Rec, 字符串s,t, 位置索引i和j
输出: 操作序列
if i = 0 and j = 0 then
   return NULL
end
if Rec[i,j] = "LU" then
    Print-MED(Rec, s, t, i - 1, j - 1)
    if s_i = t_i then
       print "无需操作"
   \mathbf{end}
    else
      \operatorname{print} "用t_j替换s_i"
    end
end
else if Rec[i, j] = "U" then
    Print-MED(Rec, s, t, i - 1, j)
   \operatorname{print} "删除s_i"
end
/else
                                           采取插入操作
    Print-MED(Rec, s, t, i, j - 1)
   print "插入t_i"
end
```

时间复杂度分析



```
//动态规划
for i \leftarrow 1 to n do
    for j \leftarrow 1 to m do
        c \leftarrow 0
        if s_i \neq t_j then
        end
        replace \leftarrow D[i-1, j-1] + c
        delete \leftarrow D[i-1,j]+1
        insert \leftarrow D[i, j-1] + 1
        \mathbf{if}\ replace = min\{delete, insert, replace\}\ \mathbf{then}
            D[i,j] \leftarrow D[i-1,j-1] + c
           Rec[i, j] \leftarrow "LU"
        \mathbf{end}
        else if insert = min\{delete, insert, replace\} then
            D[i,j] \leftarrow D[i,j-1] + 1
            Rec[i,j] \leftarrow "L"
        end
        else
            D[i,j] \leftarrow D[i-1,j] + 1
            Rec[i,j] \leftarrow "U"
        end
    end
                                     时间复杂度: O(mn)
end
```





删除

插入

替换

最长公共子序列

 $如果 s_i \neq t_j$

如果 $s_i = t_j$

$$C[i,j] = \begin{cases} \max\{C[i-1,j], C[i,j-1]\}, x_i \neq y_j \\ C[i-1,j-1] + 1, x_i = y_j \end{cases}$$

最小编辑距离

$$t \mid B \mid D \mid C \mid A \mid B \mid A$$

$$t \mid B \mid D \mid C \mid A \mid B \mid A$$

$$D[i,j] = \min egin{cases} D[i-1,j] + 1 \ D[i,j-1] + 1 \ D[i-1,j-1] + egin{cases} 0, if \ s[i] = t[j] \ 1, if \ s[i]
eq t[j] \end{cases}$$





