DEPARTMENT OF MECHANICAL ENGINEERING UNIVERSITY OF NIGERIA, NSUKKA

ME 573 — REFRIGERATION & AIR-CONDITIONING

Lecture notes on ME 573 — REFRIGERATION & AIR-CONDITIONING

by Prof. S. O. Enibe

2004 Updated 15/05/2006, 12/08/2010, 06/08/2016 Revised 02/07/2019 May be photocopied freely

Lecture notes on ME 573 — REFRIGERATION & AIR-CONDITIONING
©2004–2019 by S O Enibe
All rights reserved. A registered student of this course is allowed to make a photocopy of the text for academic purposes only. For other users, copies (whether mechanical, electronic, etc) may only be made with the express permision of the copyright owner.
Computer typeset by the author on 12 points roman using the LaTeX typesetting software engine.

Acknowledgements

The assistance of Mr. Peter Okafor in typing parts of the manuscript and preparing most of the illustrations is gratefully acknowledged. I also acknowledge the encouragement and support of my wife and children in enriching the text.

Preface

This text was prepared as a handy notebook and reference material for this introductory course in refrigeration and airconditioning. The basic concepts are first introduced, followed by a consideration of practical improvements to the basic cycle. Applications of refrigeration to air conditioning is then considered. The presenteation is then concluded with a discussion of cooling towers.

The text has been designed specifically to cover the basic requirements of ME 573 — Refrigeration & Air Conditioning in the University of Nigeria, but would be found useful by students in other Universities and institutions of higher learning.

SPECIAL INFORMATION

THERE ARE A TOTAL OF 13 EXERCISES IN THIS LECTURE NOTE. EVERY REGISTERED STUDENT IS REQUIRED TO ATTEMPT EACH OF THEM, ARRANGE THEM IN THE ORDER LISTED, AND SUBMIT THEM IN A FOLDER AT LEAST TWO WEEKS TO THE DATE OF THE EXAMINATION

Contents

1	Intro	oduction 1
	1.1	Reversed Heat Engine Cycle
	1.2	COP
	1.3	COP_{ref} and COP_{hp}
	1.4	Practical Refrigeration Cycles
	1.5	Refrigerants
2	VAP	OUR COMPRESSION 6
	2.1	Cycle Improvements
		2.1.1 Use of a Throttle Valve
		2.1.2 Vapour Superheating at Compressor inlet
		2.1.3 Condensate Undercooling
	2.2	Analysis
	2.3	Cooling Fluids
	2.4	Capacity
	2.5	p-h Diagram
	2.6	Compressors
	2.7	Flash Chamber
3	OTE	HER SYSTEMS 28
	3.1	Absorption Cycles
		3.1.1 Theoretical Analysis
	3.2	Gas Cycles
	3.3	Gas liquefaction
4	AIR	-CONDITIONING 44
	4.1	Psychrometry
		4.1.1 Specific Humidity
		4.1.2 Relative Humidity, ϕ
		4.1.3 Percentage Saturation, ψ
	4.2	ϕ Measurement
	4.3	Psychometric chart

	4.4	Moist	air Properties	49
		4.4.1	Specific Enthalpy of Moist Air	49
		4.4.2	Specific heat capacity	50
	4.5	Air- C	onditioning	50
		4.5.1	Heating Loads	50
		4.5.2	Air-Conditioning for Summer or Hot Season	50
		4.5.3	Air-Conditioning for Winter or Cold Season	51
	4.6	Cooli	ng Towers	63
5	Coo	ling Loa	ad Calculations	71
RI	REFERENCES			

List of Tables

3.1	Critical temperature of some common gases	42
4.1	Data for question 4.4	56
4.2	Computed data for question 4.4	57

List of Figures

1.1	Reversed heat engine cycle	2
1.2	Reversed heat engine system operating on the Carnot cycle	3
2.1	Refrigerator cycle using a throttle valve	7
2.2	Refrigeration Cycle with Vapour Superheat	8
2.3	Refrigeration cycle with vapour superheat and condensate undercooling	9
2.4	Schematic diagram for example 2.1	10
2.5	T.S diagram for case (i) in example 2.1	11
2.6	Sketch of a pressure-enthalpy chart for a refrigerant	15
2.7	Reversed Cycle on a ph-diagram	15
2.8	Cycle for example 2.3 on a ph-diagram	16
2.9	Schematic p-h diagram showing use of flash chamber	19
2.10	Two-stage refrigeration cycle using a flash chamber	20
2.11	Ammonia refrigeration plant for example 2	27
3.1	Schematic diagram of a vapour absorption refrigerator cycle with a liquid absorbent	28
3.2	TS diagram of vapour absorption refrigerator with liquid absorbent	30
3.3	Two-reversible engines equivalent to an absorption refrigerator	30
3.4	Enthalpy-concentration chart for lithium-bromide-water solution	32
3.5	Schematic diagram for example 3.1	33
3.6	Simple gas refrigeration cycle	38
3.7	T-s diagram showing irreversibility in the expansion and compression processes	39
3.8	Schematic diagram of a gas refrigeration plant	40
3.9	A simple gas liquifaction plant using the Linde process	43
4.1	T-s diagram for superheated water vapour in atmospheric air	45
4.2	Typical air-conditioning plant for heating season or summer	51
4.3	Air-conditioning system for winter or cold season	52
4.4	Schematic diagram for example 4.3	52
4.5	Schematic and T-s diagrams for an aircraft air-conditioning system	55
4.6	Plot compressor power requirement and turbine power output against pressure for example 4.4	57

4.7 4.8 4.9	Schematic diagram of the air-conditioning system in question 4.5	64
,		

Chapter 1

Introduction

In general, refrigeration is defined as any process of heat removal. More specifically, refrigeration is defined as that branch of science which deals with the process of reducing and maintaining the temperature of a space or material below the temperature of the surroundings.

To accomplish this, heat must be removed from the space being refrigerated and transferred to another space or reservoir whose temperature is above that of the refrigerated space. In view of this, it is evident that refrigeration and heating are actually opposite ends of the same process. Often, only the desired result distinguishes one from the other.

Dossat (1985) presents a detailed survey of many important applications of refrigeration, such as comfort air conditioning and the processing, storage, transportation and display of perisable goods. Indeed, mechanical refrigeration is used in the processing or manufacture of almost every article or commodity in the market today. The list of processes or products made possible or improved through the use of mechanical refrigeration is almost endless. For example, refrigeration has made possible the production of plastics, synthetic rubber and many other new and useful materials and products. Due to refrigeration, bakers can get more loaves of bread from a given quantity of flour, textile and paper manufactureres can speed up their machines and get more production, and better methods of hardening steels for machine tools are available.

For convenience of study, mechanical refrigeration is often grouped into six general categories, namely (1) domestic refrigeration, (2) commercial refrigeration, (3) industrial refrigeration, (4) marine and transportation refrigeration, (5) comfort air conditioning, and (6) industrial air conditioning.

1.1 Reversed Heat Engine Cycle

The basic cycle for a refrigerator is similar to a heat engine cycle except that the directions of heat and work input are reversed. For this purpose, a refrigeration cycle is often called a reversed heat engine cycle.

Figure 1.1 shows diagrammatically a refrigeration or reversed heat engine cycle.

In the system, a quantity of heat Q_1 is extracted from a low temperature reservoir at T_1 , while a larger quantity of heat Q_2 is supplied to a high temperature heat sink at T_2 . The work input to the cycle is W.

To analyse the performance of the reversed heat engine, recall the first law of thermodynamics which states;

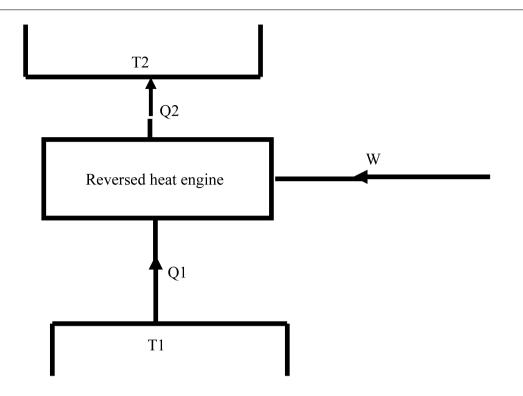


Figure 1.1: Reversed heat engine cycle

When a system undergoes a thermodynamic cycle, then the net heat supplied to the system from its surroundings plus the net work input to the system from its surroundings is equal to zero.

Thus, we have

$$\sum dQ + \sum dW = 0$$

$$\Rightarrow Q_1 - Q_2 + W = 0$$

Hence

$$W + Q_1 = Q_2 (1.1)$$

If the main objective of the reversed heat engine is to extract heat from a low temperature source, the system is usually called a *refrigerator*. In that case, the quantity of heat Q_1 extracted from the source is of interest. On the other hand, if the primary objective of the heat engine is to reject heat from the system and supply it to the surroundings, the system is called a *heat pump*. In this case, the quantity of heat Q_2 is of interest.

The power input to the reversed heat engine, W, is also very important because it contributes significantly to the running cost.

1.2 Coefficient of Performance

The performance of a refrigerator or heat pump is measured in terms of the coefficient of performance, (COP), and this is defined as follows:

$$COP_{ref} = \frac{Q_1}{\sum W} \tag{1.2}$$

$$COP_{hp} = \frac{Q_2}{\sum W} \tag{1.3}$$

To illustrate further, consider the performance of a reversed heat engine operating on the Carnot cycle, as shown in figure 1.2(a). The cycle processes are illustrated in the T.S diagram shown in figure 1.2(b) and may be described as follows:

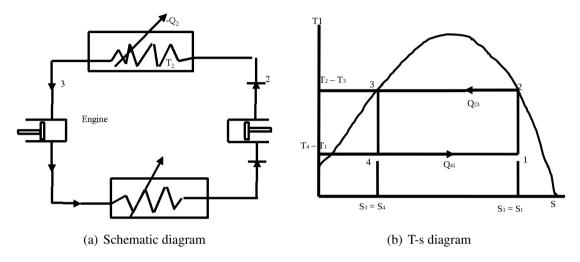


Figure 1.2: Reversed heat engine system operating on the Carnot cycle

- **1–2 Compression** Wet vapour at state 1 enters the compressor and is compressed isentropically to state 2. The work input is W_{12} .
- **2–3 Condensation** The vapour enters the condenser at state 2 and is condensed at constant pressure to state 3 where it is completely liquid. Heat Q_{23} is rejected by the refrigerant.
- **3..4 Expansion** The liquid expands isentropically behind the piston of the engine doing work of amount W_{34}
- **4–1 Evaporation** The refrigerant enters the evaporator where the heat necessary for evaporation, Q_{41} , is extracted from the cold source.

It is clear that

net work input
$$\sum W = W_{12} - W_{34}$$
 (1.4)

Net heat input
$$\sum Q = (Q_{41} - Q_{23})$$
 (1.5)

From first law,

$$\sum Q + \sum W = 0 \tag{1.6}$$

Hence

$$W_{12} - W_{34} = Q_{23} - Q_{41}$$

Thus, from equations 1.2 and 1.3, we have

$$COP_{ref} = \frac{Q_{41}}{\sum W} = \frac{Q_{41}}{Q_{23} - Q_{41}} \tag{1.7}$$

$$COP_{hp} = \frac{Q_{23}}{\sum W} = \frac{Q_{23}}{Q_{23} - Q_{41}} \tag{1.8}$$

Now, from the T-S diagram of Figure 1.2(b), it is clear that

$$Q_{41} = T_1(s_1 - s_4)$$

$$Q_{23} = T_2(s_2 - s_3) = T_2(s_1 - s_4)$$

Thus,

$$\sum W = Q_{23} - Q_{41}$$

$$= T_2(s_1 - s_4) - T_1(s_1 - s_4)$$

$$= (T_2 - T_1)(s_1 - s_4)$$

Hence

$$COP_{ref} = \frac{T_1(s_1 - s_4)}{(T_2 - T_1)(s_1 - s_4)} = \frac{T_1}{T_2 - T_1}$$
(1.9)

$$COP_{hp} = \frac{T_2(s_1 - s_4)}{(T_2 - T_1)(s_1 - s_4)} = \frac{T_2}{T_2 - T_1}$$
(1.10)

For any given values of T_1 and T_2 , equations (2.4) and (2.5) give the maximum values of COP_{ref} and COP_{hp} respectively. Actual refrigerators or heat pumps will usually perform below these maximum values.

Note: The quantity of heat extracted from the cold reservoir is often called the *refrigerating effect*

Example 1.1 The temperature in the evaporator coil of a certain refrigerator is -10° C, while that in the condenser is 35°C. Assuming that the machine operates on the reversed Carnot cycle, calculate the COP_{ref} , the refrigerating effect per kilowatt of work input, and the heat rejected to the condenser.

Solution

We have $T_1 = -10^{\circ}C = 263$ K and $T_2 = 35^{\circ}C = 308$ K.

1. The COP_{ref} is given by

$$COP_{ref} = \frac{T_1}{T_2 - T_1} = \frac{263}{308 - 263} = \frac{263}{45} = 5.8444$$

- 2. The refrigerating effect per kilowatt of work input = $COP_{ref} \times 1kW = 5.8444$
- 3. Heat rejected to the condenser, Q_{23} is given by $Q_{23} = W + Q_{41} = 1 + (COP_{ref} \times 1)$ kW. Thus, $Q_{23} = 1 + (5.8444 \times 1) = 6.8444$.

1.3 Relationship between COP_{ref} and COP_{hp}

Now, from equation 1.1, we have $Q_2 = Q_1 + W$. Dividing through by W gives

$$\frac{Q_2}{W} = \frac{Q_1}{W} + 1$$

From equations 1.2 and 1.3, we have

$$COP_{hv} = COP_{ref} + 1 (1.11)$$

Both coefficients decrease as the temperature range of the cycle is increased.

Exercise 1.1 *1. What is the essential difference between a refrigerator and a heat pump.*

- 2. What is the relationship between the COP_{ref} and COP_{hp}
- 3. Mention a disadvantage of using water as a refrigerant.

Exercise 1.2 *In the following questions, answer* true *or* false:

- 1. The COP_{ref} of a heat pump is always higher than that of a refrigerator operating within the same temperature limits.
- 2. For the same refrigerating effect and same cold space temperature, a refrigerator when operating in a hot environment comsumes more work than when in a colder environment.
- 3. The COP of a refrigerator improves when

the refrigerant is undercooled before expansion.

- 4. The expansion engine in a reversed Carnot cycle is replaced with a throttle valve in a refrigerator because the valve is more efficient.
- 5. A vapour absorption refrigerator has a lower work input compared with its vapour compression counterpart.

1.4 Practical Refrigeration Cycles

A number of practical considerations have necessitated the introduction of certain modifications in aspects of the ideal reversed Carnot cycle in order to build refrigerating machines that are technically effective and commercially viable. Many practical refrigeration systems have therefore been developed, the most important of which are as follows:

- 1. Vapour-compression refrigeration systems
- 2. Vapour-absorption refrigeration system
- 3. Gas refrigeration systems
- 4. Steam jet refrigeration systems.

We shall consider each of these systems in turn in the following sections.

1.5 Refrigerants

Chapter 2

VAPOUR COMPRESSION SYSTEMS

In a vapour-compression refrigeration or heat pump cycle, a condensable vapour is used as the refrigerant. The bulk of the heat transfer processes to and from the fluid take place at the evaporation and condensing temperatures of the refrigerant. Since heat transfer coefficient during change of phase (evaporation or condensation) are higher than for the liquid or gasous states, the physical size of the refrigerating machines is reduced since smaller areas are required for heat transfer.

2.1 Cycle Improvements

In order to achieve a practical yet technically efficient cycle, at least three modifications are made to the basic reversed Carnot cycle, namely

- 1. replacement of the expansion engine with a throttle valve
- 2. vapour superheating in the compressor
- 3. undercooling of the condensed refrigerant

These modifications shall now be considered in detail.

2.1.1 Use of a Throttle Valve

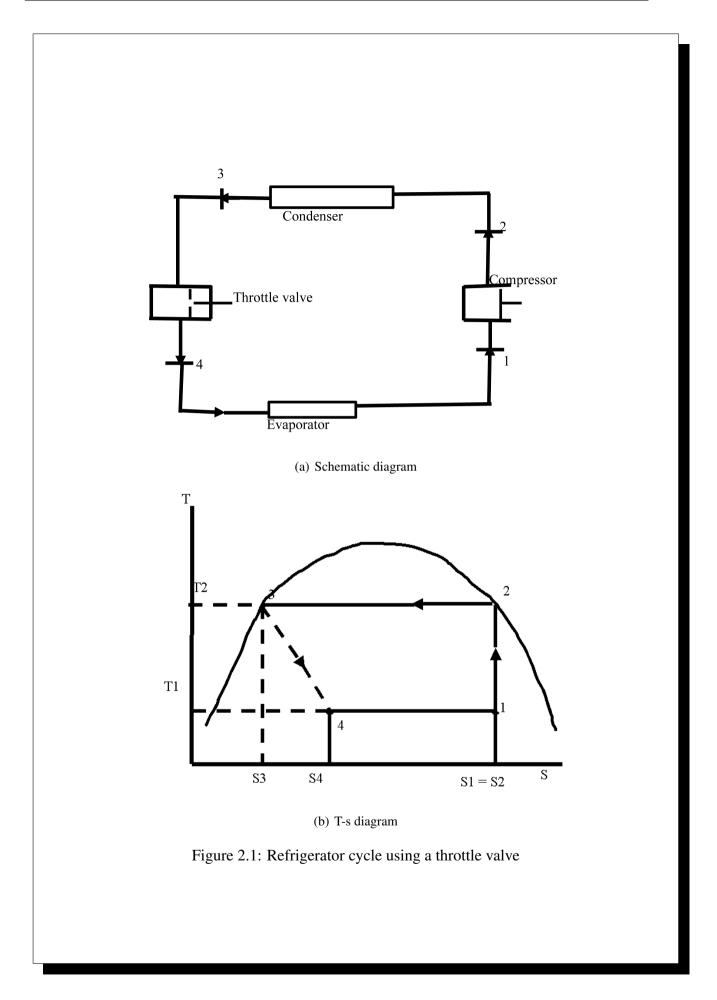
In order to reduce the size and hence cost of the plant, the expansion engine in the basic reversed Carnot cycle is replaced by a simple throttle valve, as shown in figure 2.1.

Because the throttling process is irreversible, the entire cycle becomes irreversible. From figure 2.1(b), it is clear that the refrigerating effect,

 $Q_1 = T_1$ (S₁-S₄), is less than $Q_1 = T_1(S_1 - S_3)$, namely its value using an isentropic expansion engine.

2.1.2 Vapour Superheating at Compressor inlet

The carry-over of liquid refrigerant into the compressor usually interferes with its lubrication. Further, it is desirable to make complete use of the enthalpy of evaporation of the refrigerant. For this reason, the refrigerant leaving the evaporator is allowed to become dry-saturated or even attain a small degree of superheat, as shown in figure 2.2



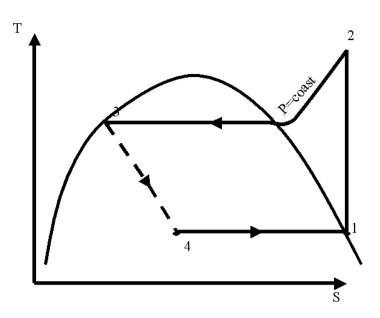


Figure 2.2: Refrigeration Cycle with Vapour Superheat

2.1.3 Condensate Undercooling

The refrigerating effect in process 4-1 can be increased by cooling the condensed vapour at constant pressure just below the corresponding saturation temperature, as shown in figure 2.3.

The effect of undercooling is to move the line 3-4 which represents the throttling process to the left of the diagram. The available cooling water temperature limits the amount of undercooling possible.

2.2 Analysis of the Modified Cycle

All the modifications to the Basic reversed Carnot cycle are shown in figure 2.3. Since the cycle consists of a number of flow processes, the steady flow energy equation can be applied to each process between states 1 and 2, namely,

$$h_1 + \frac{c_1^2}{2} + Q + W = h_2 + \frac{c_2^2}{2}$$
 (2.1)

Neglecting the changes in the kinetic energy, the equation reduces to

$$h_1 + Q + W = h_2 (2.2)$$

Applying equation (2.2) to each process in figure 2.3, we have the following expressions

Process 4–1: (Evaporation) For the evaporation process, W=0, hence equation (2.2) reduces to

$$h_4 + Q_{41} + 0 = h_1$$

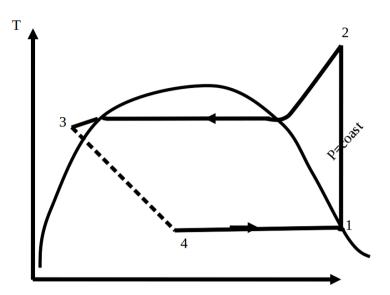


Figure 2.3: Refrigeration cycle with vapour superheat and condensate undercooling

$$\Rightarrow Q_{41} = (h_1 - h_4) \tag{2.3}$$

Note that Q_{41} is the refrigerating effect.

Process 1-2 (compression) Assuming that the compression process is adiabatic, Q = 0, thus we have $h_1 + 0 + W = h_2$, hence

$$W = (h_2 - h_1) (2.4)$$

We recall that W is the work done on the refrigerant, or net work in the cycle.

If process 1-2 is isentropic (i.e. reversible and adiabatic), then $s_1 = s_2$.

Process 2–3 (Condensation) In this case, the work transfer is zero, and we have $h_2 + Q_{23} + 0 = h_3$, hence

$$-Q_{23} = (h_2 - h_3) (2.5)$$

Recall that $-Q_{23}$ is the heat rejected by the refrigerant in the condenser.

Process 3–4: (Throttling) For this process, we have zero heat and work transfers, thus $h_3 + 0 + 0 = h_4$, hence

$$h_3 = h_4 \tag{2.6}$$

This is to be expected for a throttling process

Example 2.1 An ammonia vapour-compression refrigerator operates between an evaporator pressure of 2.077 bar and a condenser pressure of 12.37 bar. The following cycles are to be compared; in each case there is no undercooling in the condenser, and isentropic compression may be assumed;

1. The vapour has a dryness fraction of 0.9 at entry to the compressor,

- 2. The vapour is dry saturated at entry to the compressor.
- 3. The vapour has 5K of superheat at entry to the compressor.

In each case calculate COP_{ref} and the refrigerating effect per unit mass. What would be the COP_{ref} of a reversed Carnot cycle operating between the same saturation temperatures?

Solution

The schematic diagram of the plant is shown in figure 2.4.

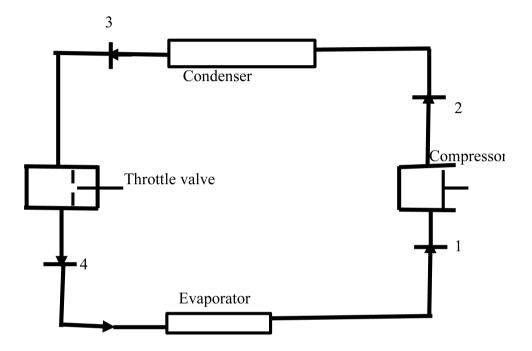


Figure 2.4: Schematic diagram for example 2.1

Properties of ammonia refrigerant are taken from the tables of Thermodynamic properties by Mayhew & Rogers (1987).

1. Vapour has a Dryness Fraction of 0.9 at entry to the compressor

The T-S diagram for this case is shown in figure 2.5.

Process 4-1 in the evaporator *Here, we have*

$$Q_{41} = h_1 - h_4$$

$$h_1 = h(2.077bar, 0.9)$$

$$= h_f + x(h_g - h_f)$$

$$= 98.8 + 0.9(1422.7 - 98.8)$$

$$\therefore h_1 = 1290.31kJ/kg$$

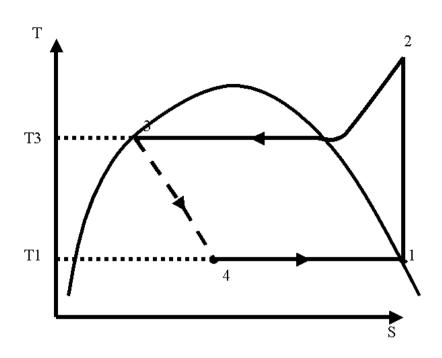


Figure 2.5: T.S diagram for case (i) in example 2.1

Similarly,

$$s_1 = s_f + x(s_g - s_f)$$

= 0.404 + 0.9(5.593 - 0.404)
\therefore\tau_1 = 5.0741

Process 1–2 (Compressor) Since flow in the compressor is isentropic,

$$s_2 = s_1 = 5.0741$$

 $P_2 = 12.37bar$

Since $s_2 > s_g$ at 12.37bar, the vapour is superheated. Now, at P_2 , $h_{g2} = 1469.9$, hence

$$\begin{split} h_2 = &h_{g2} + (h_{2(50)} - h_{g2}) \left[\frac{s_2 - s_{g2}}{s_{2(50)} - s_{g2}} \right] \\ = &1469.9 + (1613 - 1469.9) \left[\frac{5.0741 - 4.962}{5.397 - 4.962} \right] \\ = &1469.9 + 143.1 \times 0.2577011 \\ or \ h_2 = &1506.777 \ kJ/kg \end{split}$$

Process 2-3 in the condensers Since there is no undercooling of the refrigerant, it is saturated liquid at 3, hence $h_3 = h_f$ at 12.37bar = 332.8kJ/kg.

Process 3–4 in throttling valve The process is irreversible but adiabatic, hence $h_4 = h_3 = 332.8kJ/kg$

Hence, the refrigerating effect, Q_{41} is given by

$$Q_{41} = h_1 - h_4 = 1290.31 - 332.8$$
$$= 957.51kJ/kq$$

Further, the heat rejected in the condenser, Q_{23} is given by

$$-Q_{23} = h_2 - h_3 = 1506.777 - 332.8$$

or $-Q_{23} = 1173.977kJ/kg$

Also,

$$W_{12} = h_2 - h_1 = 1506.777 - 1290.31$$

or $W_{12} = 216.467kJ/kg$

Hence, the coefficient of performance becomes

$$COP_{ref} = \frac{Q_{41}}{-Q_{23} - Q_{41}} = \frac{Q_{41}}{W}$$

or $COP_{ref} = \frac{957.51}{216.467} = 4.42335$

Thus, the refrigerating effect $Q_{41} = 957.51$ kJ/kg while the $COP_{ref} = 4.42335$. For a Carnot cycle operating between the two pressure ranges,

$$COP_{ref(Carnot)} = \frac{T_1}{T_2' - T_1}$$

$$T_1 = -18^{\circ}C$$
 = 255K, $T_2 = 32^{\circ}C$ = 305K. Hence,

$$COP_{ref(Carnot)} = \frac{255}{305 - 255} = \frac{255}{50}$$
$$\therefore COP_{ref(Carnot)} = 5.1$$

2. Case (ii) Vapour is dry saturated at entry to the compressor. In this case, $h_1 = h_g$ at 2.077 bar = 1422.7kJ/kg, hence $Q_{41} = h_1 - h_4 = 1422.7 - 332.8 = 1089.9kJ/kg$. The coefficient of performance the becomes $COP_{ref} = Q_{41}/W_{12} = 1089.9/216.467 = 5.0349$. Determine s_1, s_2, h_2 , and then compute W_{12} We have

$$h_1 = 1422.7 kJ/kg$$

 $s_1 = s_g \text{ at } 2.077 \text{ bar } = 5.593 \text{ kJ/kg K}$
 $s_2 = s_1 = 5.593 \text{ kJ/kg K}$

Alternatively, since $s_2 > s_{50}$, we interpolate between s_{50K} and s_{100K} using the interpolation formular

$$h_2 = h_{50K} + (h_{100K} - h_{50K}) \times \frac{s_2 - s_{50K}}{s_{100K} - s_{50K}}$$

$$= 1613 + (1739.3 - 1613) \times \frac{5.593 - 5.397}{5.731 - 5.397}$$

$$= 1613 + 126.3 \times \frac{0.196}{0.334}$$

$$= 1613 + 74.116 = 1687.1162$$

The heat transfer Q_{41} and work transfer W_{12} are given as

$$Q_{41} = h_1 - h_4 = 1422.7 - 332.8 = 1089.9$$

 $W_{12} = h_2 - h_1 = 1687.1162 - 1422.7 = 264.4161 = 264.4162$

Thus, the COP becomes

$$COP_{ref} = \frac{Q_{41}}{W_{12}} = \frac{1089.9}{264.4162} = 4.1219$$

3. Case (iii) Vapour has 5K of superheat at entry to the compressor. In this case, the vapour is superheated, and h_1 may be determined by interpolation. Solution is obtained in this way: Determine s_1 , calculate h_2 , and then compute W_{12} , and proceed as before. Please complete the solution as an exercise.

Exercise 2.1 In a refrigerating plant, the vapour leaves the evaporator dry saturated at 1.826 bar and is compressed to 7.449 bar. The temperature of the vapour leaving the compressor is 45°C. The liquid leaves the condenser at 25°C and is throttled to the evaporating pressure.

- 1. If the refrigerant is R12, calculate
 - (a) the refrigerating effect
 - (b) the work input per unit mass of refrigerant
 - (c) the COP_{ref}
- 2. Compare the results found in 1 with a plant using R134a between the same saturation temperatures, with the same maximum cycle temperature and the same degree of undercooling of the condensate

Answers: 121.27 kJ/kg; 29.66 kJ/kg; 4.09; 152.05 kJ/kg; 43.65 kJ/kg; 3.48 Adapted from Eastop & McConkey (2002)

2.3 Secondary Cooling Fluids

Many of the refrigerants in common use may cause damages to goods if brought into direct contact with them. Further, in many applications, a cold fluid resulting from a refrigeration application

may have to be transfered over large distances to where it is applied. For these reasons, many refrigeration applications often use another fluid other than the refrigerant as the heat transfer fluid between the heat source and the heat sink. In such a case, the space to be cooled is surrounded by a pipe network carrying the secondary or heat transfer fluid. The secondary fluid is cooled in the evaporator section of the refrigerator before being pumped through the pipe network. This approach reduces the chances of damage to goods in case of a refrigerant leakage.

The most common secondary fluids for refrigeration applications are solutions of sodium chloride or calcium chloride in water. They are often called brines.

Sodium chloride brine begins to freeze at about -18° C, hence applications requiring lower temperatures may have to use the calcium chloride brine.

In air-conditioning, the secondary fluids most commonly used are water and air.

2.4 Refrigeration Capacity

As observed previously, the primary quantity of interest in a refrigeration application is the rate of heat extraction per unit time from the cold chamber. This is often called the <u>refrigeration capacity</u>. The refrigeration capacity determines the required mass flow rate of the refrigerant for a given application. Thus, the mass flow rate of the refrigerant, \dot{m} , is given by

$$\dot{m} = \frac{\text{refrigeration capacity}}{\text{Refrigerating effect per unit Mass of refrigerant}}$$
 (2.7)

Example 2.2 Determine the mass flow rate of the refrigerant for each case of example 2.1 for a plant with the refrigeration capacity of 1 kW

Solution

For case (i), refrigerating effect, $Q_{41} = 957.51 k$ J/kg. Hence, mass flow rate of refrigerant is given by

$$\dot{m} = \frac{1}{957.51} = 0.00104 \, \text{kg/s}$$

For case (ii), the refrigerating effect $Q_{41}=1089.9$ kJ/kg. Hence the refrigerant mass flow rate becomes $\dot{m}=1/1089.9=0.0009175$ kg/s.

For case (iii), $Q_{41} =$

2.5 Use of Pressure Enthalpy Diagram

The pressure-enthalpy diagram is often a more convenient way to represent thermodynamic cycles than the T-s diagram. Its main attraction is that the enthalpy at any point can be read off directly. It is used considerably in refrigeration applications, and we shall follow the same traditions in this course.

A typical example of a p-h diagram for a refrigerant is shown in figure 2.6, while the reversed cycle on a p-h diagram is shown in figure 2.7.

Example 2.3 A refrigerating plant using R134a as the refrigerant operates between saturation pressures of 1.6393 bar and 7.700 bar. Vapour is dry saturated at entry to compressor, but leaves

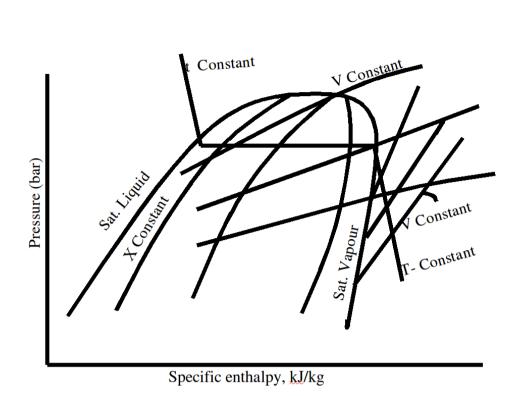


Figure 2.6: Sketch of a pressure-enthalpy chart for a refrigerant

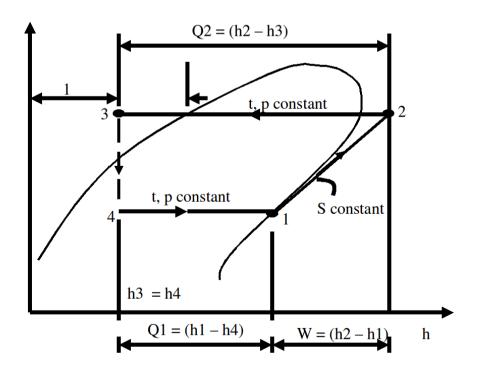


Figure 2.7: Reversed Cycle on a ph-diagram

the compressor at 40°C. Saturated liquid leaves the condenser and is throttled to the evaporator pressure. Assuming isentropic compression, calculate:

- 1. The refrigerating effect
- 2. The specific work input
- *3.* The COP_{ref} .

Solution

The cycle is shown on the p-h diagram in figure 2.8

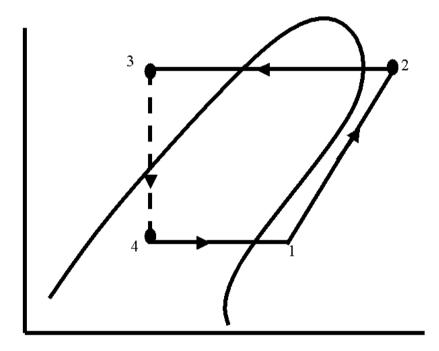


Figure 2.8: Cycle for example 2.3 on a ph-diagram

From the tables, at 7.6752 bar, 40° C, $h_2 = 324.12$ kJ/kg. The Liquid is saturated at 3, hence

$$P_4 = 1.6389 bar$$
 $h_4 = h_{f3} = 140.96 kJ/kg$ $h_1 = h_q \ at \ 1.6389 bar = 285.94 \ kJ/kg$

- 1. The refrigerating effect, $Q_{41} = h_1 h_4 = 285.94 140.96$, or $Q_{41} = 144.98$ kJ/kg.
- 2. The Specific work input, $W_{12} = h_2 h_1 = 324.12 285.94$, hence $W_{12} = 38.18$ kJ/kg
- 3. The coefficient of performance, $COP_{ref} = Q_{41}/W_{12} = 144.98/38.18$, hence $COP_{ref} = 3.797$

2.6 Compressors in Refrigeration Plants

Refrigerators employ the same basic types of compressors in common use, namely centrifugal and reciprocating compressors. However, the working fluid is the refrigerant vapour, rather than air. For very low power applications, especially for domestic plants and small air-conditioning units, vane-type compressors are used. For general applications up to 600kW, reciprocating compressors are employed. On the other hand, for higher power applications, centrifugal compressors are normally used for power in the range 300KW to 15MW. Where high volume flow rates are required, centrifugal compressors are usually used; several stages are employed for high pressure ratios. In this way, the diameter of the impeller is kept low, thus avoiding high centripetal stresses. The range of speeds covered by such compressors is in the range 300rev/min to 20000rev/min.

If reciprocating compressors are employed, the speeds are kept in the range 200 to 600 rev/min. If v is the specific volume of the refrigerant at entry to the compressor, the volume flow rate is given by

$$\dot{V} = \dot{m}v \tag{2.8}$$

For a reciprocating compressor of volumetric efficiency η_v , the swept volume is given by

$$V_s = \frac{\dot{V}}{FnN\eta_n} \tag{2.9}$$

where n is the number of cylinders and N the rotational speed. F=1 for a single acting machine, while F=2 for a double-acting machine.

Substituting for \dot{V} , it follows that

$$V_s = \frac{\dot{m}v}{FnN\eta_v} \tag{2.10}$$

It is clear from equation (2.10) that the swept volume, V_s varies directly with the specific volume, v

To keep V_s within required limits, the number of cylinders n can be increased for a given value of the rotational speed, N.

Example 2.4 For the refrigeration plant of Example 2.3, the compressor is a double-acting reciprocating compressor with a bore of 250 mm, stroke of 300 mm, speed of 200 rev/min and volumetric efficiency of 0.85. Determine

- 1. The mass flow rate of refrigerant;
- 2. The refrigeration capacity;
- 3. The required power input to the electric motor if the mechanical efficiency is 90%.

Solution

1. From thermodynamic tables, at inlet to the compressor (p=1.6389 bar), $v = v_g = 0.118m^3/kg$ Hence, the swept volume of the compressor is given by

$$V_s = \pi D^2 L/4 = \pi (0.25)^2 (0.3)/4$$
$$= 0.014726m^3/cycle$$

From equation (2.10), it follows that

$$\begin{split} \dot{m} &= \frac{V_s FnN\eta_v}{v} \\ &= \frac{0.014726\times2\times1\times(200/60)\times0.85}{0.118} \end{split}$$
 hence $\dot{m} = 0.7072$ kg/s

- 2. The refrigerating capacity is given by $Q_{41} = \dot{m}(h_1 h_4) = 0.6848 \times 144.98 = 99.28kW$.
- 3. Power input from Motor is given by

$$W = \frac{\dot{m}(h_2 - h_1)}{\eta_m}$$
$$= \frac{0.6848 \times 38.18}{0.9}$$
$$= 29.05 \text{ kW}$$

2.7 Use of the Flash Chamber

Consider again the basic practical cycle of the refrigerator plant shown in figure 2.9.

The liquid at state 3 is throttled to a wet vapour at state 4. and then passed through the evaporator. Within the evaporator, only the liquid fraction of the refrigerant contributes to the refrigerating effect by evaporating. The saturated vapour fractions entering the evaporator do not contribute to the refrigerating effect.

Consequently, the performance of the refrigerator can be improved if the vapour leaving the throttle valve is separated and passed directly to the compressor, while only the liquid fraction is passed through the evaporator. This separation may be achieved by the use of a flash chamber as shown in figure 2.10.

Here, we consider a unit mass of the refrigerant through the condenser where it is condensed to the liquid phase. In the first throttle valve, the refrigerant is throttled to an intermediate pressure, P_i , and fed into the flash chamber at state 8. in the flash-chamber, the vapour fraction of the fluid is bled off and mixed with vapour leaving the first stage compressor so that both fluid streams now enter the second stage compressor.

The liquid fraction of mass (1-x) leaving the flash-chamber flows through a second throttle valve before entering the evaporator. The mixture at 8 consists of x kg of dry saturated vapour and (1-x) kg of saturated liquid per unit mass of the refrigerant in the condenser. Since the enthalpy before throttling is known, we have

$$h_{fi} + x h_{fgi} = h_7$$
, hence
$$x = (h_7 - h_{fi})/h_{fgi}$$
 (2.11)

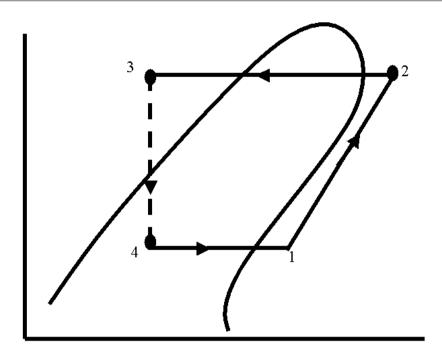


Figure 2.9: Schematic p-h diagram showing use of flash chamber

The total work input is given by

$$W = W_{12} + W_{56}$$

= $(h_2 - h_1)(1 - x) + (h_6 - h_5)$ (2.12)

Similarly, the refrigerating effect, Q_{ref} is given by

$$Q_{ref} = (h_1 - h_4)(1 - x) (2.13)$$

The heat rejected at the condenser, Q_{cond} is given by $Q_{cond} = h_7 - h_6$

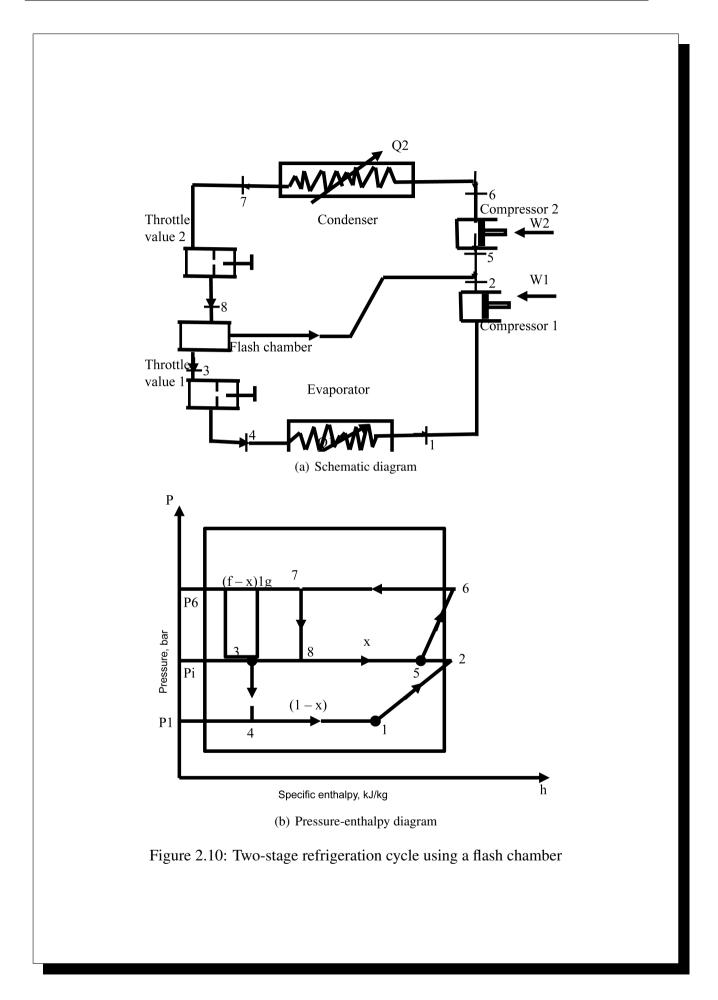
Hence,

$$COP_{ref} = \frac{Q_{ref}}{W} = \frac{(h_1 - h_4)(1 - x)}{(h_2 - h_1)(1 - x) + (h_6 - h_5)}$$
 (2.14)

Example 2.5 A vapour-compression refrigerator using ammonia as the refrigerant operates between evaporating and condensing temperatures of -16 and 50° C. The vapour is dry saturated at the compressor inlet, and there is no undercooling of the condensate. A flash-chamber is used with an interstage pressure of 5.346bar.

1. Determine

- (a) The refrigerating effect
- (b) The mass flow rate per kilowatt of refrigeration capacity
- (c) The power input per kW of refrigeration capacity



(d) The COP_{ref}

2. Recalculate sections (1) to (4) if the flash-chamber is not used.

Solution

Using figure 2.10, we have $h_1 = h_g$ at $-16^{\circ}C = 1425.3$ kJ/kg. Further, $s_2 = s_1 = s_g = 5.563$ kJ/kg K. Also, $h_2 = h$ at 5.346 bar, and s = 5.563.

Thus, from tables at P=5.346 bar, we have

$$\frac{h_2 - h_g}{h_{50K} - h_g} = \frac{s_2 - s_g}{s_{50K} - s_g}$$

This implies that

$$h_2 = h_g + (h_{50K} - h_g) \frac{s_2 - s_g}{s_{50K} - s_g}$$

$$= 1450.6 + (1577 - 1450.6) \left(\frac{5.563 - 5.263}{5.682 - 5.263}\right)$$

$$= 1450.6 + 25.0387$$
or $h_2 = 1541.10kJ/kg$

Also, we have $h_7 = h_f$ at $50^{\circ}C = 421.9$ kJ/kg. In addition, we have $h_8 = h_7 = h_{fi} + x_i h_{fgi}$, hence

$$x_i = \frac{h_7 - h_{fi}}{h_{fai}} = \frac{421.9 - 209.1}{1450.6 - 209.1}$$

hence $x_i = 0.1714$.

Now, $xh_{gi} + (1-x)h_2 = h_5$. Thus, 0.1714(1450.6) + (1-0.1714)1450 = 248.6328 + 1222.714, hence $h_5 = 1525.59$ kJ/kg. This shows that the fluid is superheated at state 5.

By interpolation at P=5.346bar and h=1525.59 kJ/kg, we have

$$s_5 = s_g + \frac{1525.59 - h_g}{h_{50K} - h_g} (s_{50K} - s_g)$$

$$= 5.263 + \frac{1525.59 - 1450.6}{1577 - 1450.6} (5.682 - 5.263)$$

$$= 5.263 + 0.06877$$

or
$$s_5 = 5.33177kJ/kg K$$

The process 5-6 is isentropic, hence interpolating at P_6 , we have (using T_7 =50°C)

$$h_6 = h_{50K} + (h_{100K} - h_{50K}) \left(\frac{s_6 - s_{50K}}{s_{100K} - s_{50K}} \right)$$
$$= 1633.1 + (1766.8 - 1633.1) \left(\frac{5.33177 - 5.23}{5.567 - 5.23} \right)$$

Hence,

$$h_6 = 1633.1 + 40.3758 = 1744.98$$
kJ/kg

Also, $h_4 = h_{fi} = 209.1 \text{kJ/kg}$.

1. The refrigerating effect is given by

$$Q_{ref} = (h_1 - h_4)(1 - x)$$

$$= (1425.3 - 209.1)(1 - 0.1714)$$
and $Q_{ref} = 1007.74 h \text{ M/s}$

hence, $Q_{ref} = 1007.74 k J/kg$

2. The mass flow rate \dot{m} is given by

$$\dot{m} = 1/1091.5976 = 0.0009223 \, \text{kg/s} = 3.2979 \, \text{kg/h}$$

3. The power input W is given by

$$W = (h_2 - h_1)(1 - x) + (h_6 - h_5)$$

= (1475.6387 - 1425.3)(1 - 0.1714) + (1673.476 - 1471.347)
= 41.7106 + 202.129

or
$$W = 315.34$$
kJ/kg

4. The coefficient of performance, COP_{ref} is given by

$$COP_{ref} = \frac{Q_{ref}}{W} = \frac{1007.74}{315.34}$$

or $COP_{ref} = 3.196$

Exercise 2.2 Recalculate example 2.5 if the flash chamber is not used.

Example 2.6 A vapour-compression refrigerator using ammonia as the refrigerant operates between evaporating and condensing temperatures of -18 and 48°C. The vapour is dry saturated at the compressor inlet, and there is no undercooling of the condensate. A flash-chamber is used with an interstage pressure of 4.975bar. Determine

- 1. The refrigerating effect
- 2. The mass flow rate per kilowatt of refrigeration capacity
- 3. The power input per kW of refrigeration capacity
- 4. The COP_{ref}

If the flash-chamber is not used, in questions 5 to 8, determine

- 5. The refrigerating effect
- 6. The mass flow rate per kilowatt of refrigeration capacity
- 7. The power input per kW of refrigeration capacity
- 8. The COP_{ref}
- 9. From the foregoing, what is the benefit of using the flash chamber.

Solution

Basic data

First, the schematic diagram of the plant is drawn as shown in figure 2.10(a), while the pressure-enthalpy diagram is shown in part (b) of the figure.

A number of basic data required for the solution of the problem are first determined. With the help of figure 2.10, we have $h_1 = h_q$ at $-18^{\circ}C = 1423.7$ kJ/kg. Further, we note that

- $s_2 = s_1 = s_g = 5.593 kJ/kgK$
- Also, $h_2 = h$ at 4.975 bar and $s_2 = 5.593$ kJ/kgK

Thus, from tables at P = 4.975 bar, we have

$$\frac{h_2 - h_g}{h_{50K} - h_g} = \frac{s_2 - s_g}{s_{50K} - s_g}$$

But, from tables,

$$s_{50K} = 5.706$$

 $s_{g2} = 5.288$
 $h_{50K} = 1574$
 $h_{g2} = 1448.5$

The interpolation formula then becomes

$$h_2 = h_g + (h_{50K} - h_g) \frac{s_2 - s_g}{s_{50K} - s_g}$$

$$= 1448.5 + (1574 - 1448.5) \left(\frac{5.593 - 5.288}{5.706 - 5.288} \right)$$

$$= 1540.073 kJ/kg$$

- In addition, we have $h_7 = h_f$ at $48^{\circ}C = 411.9$ kJ/kg.
- The specific entropy of the refrigerant entering the flash chamber, $h_8 = h_7 = h_{8f} + xh_{8fg}$. But,

$$h_{8f} = 199.7 \text{kJ/kg}$$

 $h_{8fg} = 1448.5 - 199.7 = 1248.8 \text{kJ/kg}$

Thus,

$$x = \frac{h_7 - h_{8f}}{h_{8g} - h_{8f}} = \frac{411.9 - 199.7}{1448.5 - 199.7}$$
$$= 0.1699$$

- Now, $xh_{8f} + (1-x)h_2 = h_5$. Thus, 0.1699(1448.5) + (1-0.1699)1540.5 = 1524.5. Thus, $h_5 = 1524.5$ kJ/kg. Since, from tables, $h_5 > h_{8g}$, it follows that the refrigerant is superheated at point 5.
- To determine s_5 , we employ linear interpolation at p = 5.bar and h = 1524.5bar. From tables at the saturated vapour condition and 50K of superheat, we have

$$s_{5g} = 5.288 = s_{2g}$$

 $s_{5.50K} = 5.706 = s_{2.50K}$

We employ the interpolation formular

$$s_5 = s_{5g} + \frac{1524.5 - h_{5g}}{h_{5,50K} - h_g} (s_{5,50K} - s_{5g})$$

$$= 5.288 + \frac{1524.5 - 1448.5}{1574 - 1448.5} (5.706 - 5.288)$$

$$= 5.5416$$

• The process 5–6 is isentropic, hence $s_6 = s_5 = 5.5416$. The data from the tables are

$$s_{6,50K} = 5.248$$

 $s_{6,100K} = 5.584$
 $h_{5,50K} = 1631.1$
 $h_{6,100K} = 1764$

Extrapolating at p_6 , we have, using the formula

$$h_6 = h_{6,50K} + (h_{6,100K} - h_{6,50K}) \left(\frac{s_6 - s_{6,50K}}{s_{6,100K} - s_{6,50K}} \right)$$
$$= 1631.1 + (1764.1 - 1631.1) \left(\frac{5.5416 - 5.248}{5.584 - 5.248} \right)$$
$$= 1747.06$$

• Also, we note that $h_4 = h_{8f} = 199.7$.

The solution of the remaining portions of the problem then follows easily.

1. The refrigerating effect is given by

$$Q_{ref} = (h_1 - h_4)(1 - x)$$

$$= (1423.7 - 199.7)(1 - 0.1699)$$

$$= 1016.014kJ/kg$$

- 2. The mass flow rate of the refrigerant is given by $\dot{m}=1/1016.014=0.0009842kg/s=3.5433$ kg/h
- 3. The power input to the compressors is given by

$$w_{12} = (h_2 - h_1)(1 - x) = 96.60$$

 $w_{56} = h_6 - h_5 = 222.548$
 $w = w_{12} + w_{56} = 96.6 + 222.548 = 319.146$

4. The coefficient of performance, COP_{ref} is given by

$$COP_{ref} = \frac{Q_{ref}}{w} = \frac{1016.014}{319.146}$$

= 3 1835

If the flash chamber is ommitted, then the plant schematic diagram becomes as that shown in figure 2.4.

The pressure-enthalpy diagram for this case is given in figure 2.7.

The value of h_1 and s_1 remain unchanged. The entropy of the vapour leaving the compressor, $s_2' = s_1 = 5.593$. However, the pressure is that corresponding to the saturation temperature of 48°C, namely 19.29 bar. From tables, the specific entropy at 50K and 100K degree of superheat are 5.248 and 5.584 kJ/kgK respectively. Hence, the vapour is above 100K of superheat. By extrapolation, we have

$$h'_{2} = \frac{s'_{2} - s_{2,50K}}{s_{2,100K} - s_{2,50k}} (h_{2,100K} - h_{2,50K})$$
$$= 1767.56kJ/kg$$

Also, we have $h_{3p}=h_f$ at $48^{\circ}C=411.9$ kJ/kg. Similarly, $h_{4p}=h_{3p}=411.9$. The other parameters can then be readily calculated as follows:

1. The refrigerating effect is given as

$$Q'_{ref} = h_1 - h'_4$$

$$= 1423.7 - 411.9$$

$$= 1011.8kJ/kW$$

- 2. The refrigerant mass flow rate is given by $\dot{m}=1/Q_{ref}'=1/1011.8=0.0009883$.
- 3. The compressor work input is given by

$$W'_{12} = h'_2 - h_1$$

= 1767.56 - 1423.7
= 343.86kJ/kg

4. The new coefficient of performance becomes

$$COP'_{ref} = \frac{Q'_{ref}}{W'_{12}} = \frac{1011.8}{343.86}$$

= 2.9425

- 5. The value of the coefficient of performance without the use of the flash chamber is lower than that with the use of the flash chamber. Hence the use of the flash chamber results in improved coefficient of performance.
- **Exercise 2.3** 1. An ammonia refrigeration cycle works between a condenser saturation temperature of 50° C and an evaporator temperature of -50° C. The fluid entering the throttle valve is saturated. The vapour entering the compressor has an entropy of 5.365 kJ/kg K, ie it will reach saturation at -2° C. Calculate the refrigerating effect and the coefficient of performance.

The cycle is modified as shown in figure 13.5 on page 280 of Rogers & Mayhew (2002). The temperature in the flash chamber is -2° C, and the saturated vapour bled off is mixed with the saturated vapour leaving the first-stage compressor. Calculate the correct amount to be bled, the refrigerating effect, and the coefficient of performance, all quantities to be based on 1 kg passing through the condenser. (Answer: 774.2 kJ/kg, 1.58; 0.197 kg, 822.4 kJ/kg, 1.86)

2. A refrigerator operating on ammonia, is shown diagrammatically in figure 2.11. Saturated liquid at a temperature of -30° C flows at station 1 and saturated liquid at a temperature of 10° C at stations 3, 4, and 7. saturated vapour flows at station 5, at a temperature of -30° C. There are no pressure lossess except through the spray nozzle. The flash chamber, compressor and liquid pump are all adiabatic and the last two both operate with an isentropic efficiency of 0.80. The specific volume of liquid ammonia, which can be regarded as incompressible, is 0.0015 m³/kg. Determine the coefficient of performance, and the mass flow rate of ammonia through the liquid pump for a refrigerating effect of 10 kW. (Answer: 1.97, 0.00549 kg/s)

Exercise 2.4 Attempt each of the following exercises:

- 1. Discuss any one improvement to the basic Carnot cycle to realise a practical vapour-compression refrigeration cycle.
- 2. If a refrigerator operating in a hotter environment is transferred to a colder environment, would its performance improve, deteriorate or remain the same.
- 3. State one reason why vapour superheating at the compressor inlet is desirable.
- 4. State one important application of a gas refrigeration cycle.
- 5. State one reason why the flash chamber is used in a vapour compression system.

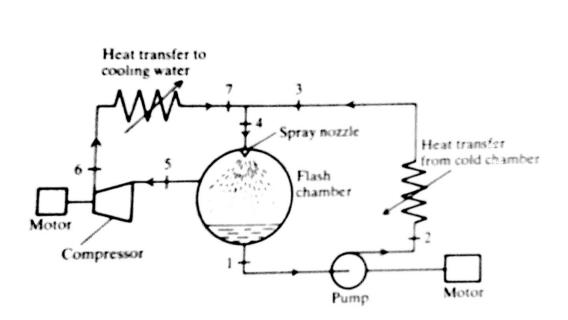


Figure 2.11: Ammonia refrigeration plant for example 2

Chapter 3

OTHER REFRIGERATION SYSTEMS

3.1 Absorption Refrigeration Cycles

In a vapour-absorption refrigerator, the work of compression is considerably reduced or eliminated altogether by absorbing the refrigerant vapour in a suitable material called an absorbent. The absorbent could be a liquid or a solid. A typical example is shown in figure 3.1 using a liquid absorbent.

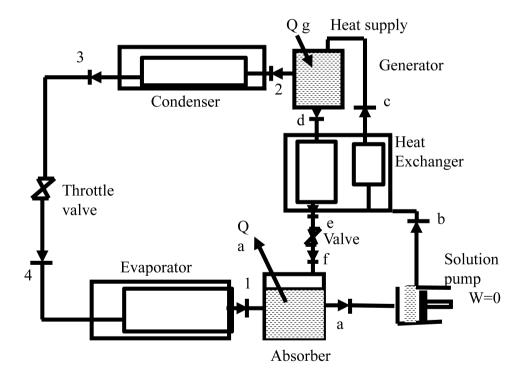


Figure 3.1: Schematic diagram of a vapour absorption refrigerator cycle with a liquid absorbent

The basic process occurring in the plant may be described as follows:-

2-3 Condensation

3-4 Throttling

4-1 Evaporation

These processes occur in the usual way and can use conventional devices. The usual process of compression (1-2) is modified and replaced with a number of processes, which result in the effective compression of the refrigerant. These processes may be described as follows:

- **1-a Absorption:** Here, the refrigerant is absorbed in a fluid such as water or other suitable absorbent. The absorbent temperature is only a little above atmospheric and the solubility of the refrigerant vapour is very high. This is accompanied by a rejection of heat Q_a .
- **a-b Pumping:** The liquid solution is pumped to the required condenser pressure Using a suitable liquid pump. Since a liquid is being pumped, the required work input is very small compared with other energy transfers in the system. In some systems, the pump is dispensed with entirely, resulting in a refrigerator with no moving parts. A good example of this is the Electrolux refrigerator.
- **b-c Preheating in heat exchanger:** The solution is heated partially in the heat exchanger. In some systems, the heat exchanger is elliminated for simplicity, but with reduction in performance.
- **c-d, c-2 Heating in generator:** The solution is heated in the generator which is kept at temperature T_g . At the higher temperature of the generator, the solubility of the refrigerant is very much reduced; hence the heat supplied at the generator Q_g results in the removal of the refrigerant vapour from the solution. The generator may be heated from any suitable heat source, such as steam coils, gas, electricity, solar energy, waste heat, etc. The vapour released from the generator flows into the condenser to complete the main cycle, while the remaining weak solution (so called because it contains a lower concentration of the refrigerant) is returned to the absorber through a throttle valve. Note that the heat exchanger is merely used to reduce the quantities of heat Q_g and Q_a .

The plant may be representing approximately on a TS diagram as shown in figure 3.2. Note that the points 1 and 2 are not joined because the exact path is complicated.

The work required to compress the liquid solution is very small compared with what would be required to compress the same quantity of the refrigerant as a vapour between the same end states. The required increase in enthalpy is supplied by heating the generator from an external heat source.

3.1.1 Theoretical Analysis

Heat transfer in the system occurs at three temperature levels: First, the atmospheric temperature T_a at which heat is rejected in the condenser and absorber; next the evaporator or cold chamber temperature T_e ; and finally the temperature T_g at which heat is received in the generator. The energy transferred as work in the solution pump is usually negligible compared to the other energy transfers.

To determine the maximum possible COP of the system, let us imagine that two reversible engines arranged as shown in figure 3.3 replace the process 1-2.

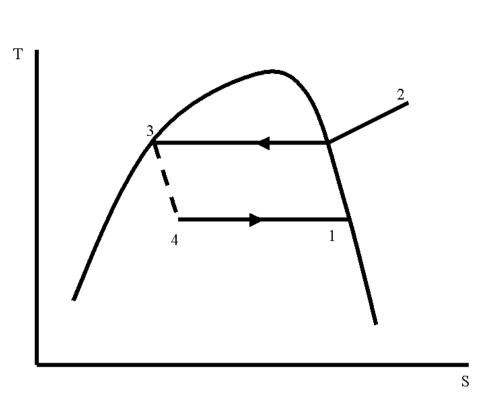


Figure 3.2: TS diagram of vapour absorption refrigerator with liquid absorbent

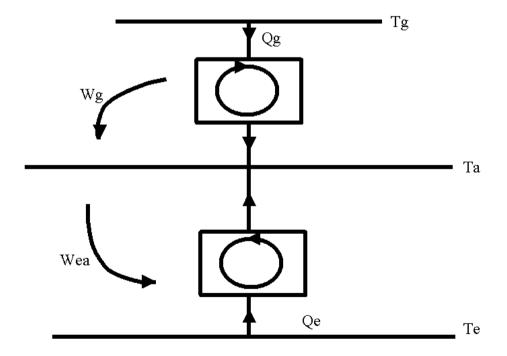


Figure 3.3: Two-reversible engines equivalent to an absorption refrigerator

The first reversible engine receives a quantity of heat Q_g at T_g and rejects heat at T_a while producing work of amount W_{qa} . The Carnot efficiency of this first engine is given by

$$\eta_1 = \frac{|W_{ga}|}{Q_g} = \frac{T_g - T_a}{T_g} \tag{3.1}$$

The second reversible engine operates as a refrigerator and receives a quantity of heat Q_e at T_e while rejecting heat at T_a , the work input in this case being W_{ea} . The coefficient of performance of the refrigerator is given as

$$\frac{Q_e}{W_{eq}} = \frac{T_e}{T_a - T_e} \tag{3.2}$$

Assuming that $|W_{ga}|$ is made equal to $|W_{ea}|$, the plant of figure 3.3 will be equivalent to an absorption refrigerator with zero solution pump work. Let the coefficient of performance of the combined plant be defined as

$$COP_{ref} = \frac{\text{cold source}}{\text{Heat energy supplied at the}} = \frac{Q_e}{Q_g}$$
 generator (3.3)

Substituting the expressions for Q_e and Q_q , we have

$$COP_{ref} = \frac{Q_e}{Q_g} = \frac{W_{ea}T_e}{T_a - T_e} \frac{T_g - T_a}{|W_{ga}|T_g}$$

This may be rewritten as

$$COP_{ref} = \frac{W_{ea}}{|W_{aa}|} \frac{T_e(T_g - T_a)}{T_a(T_a - T_e)}$$
 (3.4)

If $W_{ea} = |W_{qa}|$, then we have

$$COP_{ref} = \frac{T_e(T_g - T_a)}{T_g(T_a - T_e)}$$
 (3.5)

This will be the maximum COP of an absorption refrigerator.

Absorption/Concentration Diagrams

Example 3.1 A vapour-absorption refrigerator cycle using lithium bromide-water solution operates between pressures of 0.01 and 0.05bar. The refrigerator capacity is 150KW, while the temperature at the generator and absorber are respectively 80 and 30°C. The solution may be taken to be saturated at exit from both the generator and absorber. A heat exchanger transfers heat from the weak solution leaving the generator to the strong solution pumped from the absorber such that the solution entering the generator is saturated at the generator temperature. Properties of lithium-bromide-water solution are shown in the enthalpy-concentration diagram of figure 3.4.

Assuming that the specific enthalpy of superheated steam is approximately equal to the specific enthalpy of saturated steam at the same temperature, and neglecting pressure losses, heat losses, and pump work, calculate:

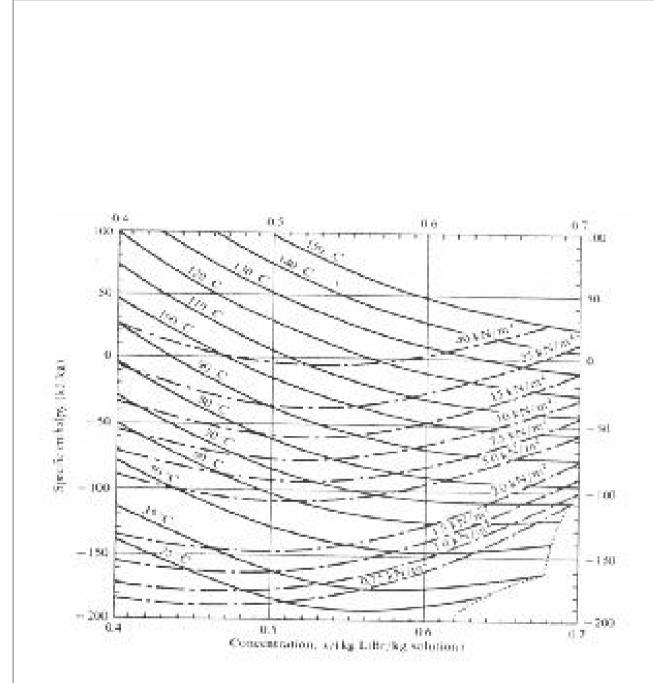


Figure 3.4: Enthalpy-concentration chart for lithium-bromide-water solution Source: Eastop & McConkey (2003)

- 1. The maximum COP_{ref} of the refrigerator. (You may assume that the ambient temperature $T_a = T_{absorber}$)
- 2. The mass flow rate of refrigerant through the evaporator and condenser
- 3. The heat supplied in the generator, Q_q
- 4. The heat rejected at the absorber, Q_a
- 5. The COP_{ref}
- 6. the specific enthalpy of the solution entering the absorber
- 7. The heat to be supplied in the generator if the heat exchanger is not used.
- 8. The percentage change of the COP_{ref} without the heat exchanger

Solution

A schematic diagram of the plant is given in figure 3.1 and reproduced in figure 3.5.

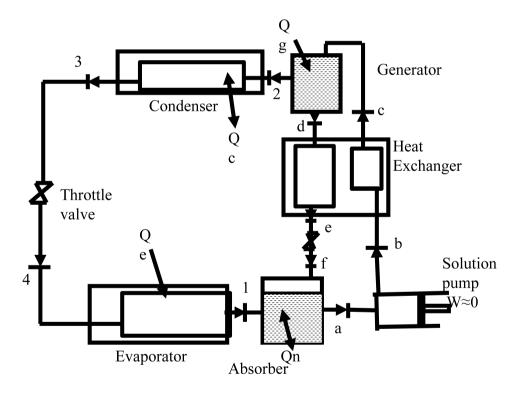


Figure 3.5: Schematic diagram for example 3.1

Now, the evaporator pressure is given as

$$T_e = T_{sat}$$
 at 0.01 bar = $7^{\circ}C = 273 + 7 = 280K$

The ambient temperature and generator temperature are given as

$$T_a = 30^{\circ}C = 273 + 30 = 303K$$

 $T_g = 80^{\circ}C = 80 + 273 = 353K$

1. The maximum coefficient of performance, COP_{max} is given by

$$COP_{ref(max)} = \frac{T_e(T_g - T_a)}{T_g(T_a - T_e)} = \frac{280(353 - 303)}{353(303 - 280)} = 1.73$$

2. The refrigeration capacity is 150 kW and the evaporator pressure is 0.01bar. From the stream tables, h_{fg} at 0.01bar = 2485kJ/kg

$$Q_{evap} = \dot{m}_1(h_1 - h_4)$$

Now, $h_4 = h_3 = h_f$ at 0.05bar = 138 kJ/kg.

Thus, the mass flow rate \dot{m}_1 is given as

$$\dot{m}_1 = \frac{150 \times 10^3}{(2514 - 138)} = 0.06313 \, \text{kg/s}$$

3. Now, applying a mass balance to the refrigerant at the absorber, we have

$$\dot{m}_1 = \dot{m}_a - \dot{m}_f$$

$$\Rightarrow \dot{m}_a - \dot{m}_f = 0.06313$$
(3.6)

Now, considering a mass balance of the lithium bromide alone we have,

$$y_1 \dot{m}_1 = y_a \dot{m}_a - y_f \dot{m}_f \tag{3.7}$$

where y represents the concentration of lithium bromide/water solution. But at exit from the evaporator, only the pure refrigerator flows; hence $y_1 = O$.

Since the absorber/evaporator pressure is 0.01bar, = 1 KN/m², we note from the chart that $y_a = 0.53$. Also, $h_a = -170$ KJ/Kg.

Further, $y_f = y_e = y_d$. From the chart, at 0.05bar and 80°C, $y_d = 0.615$ and $h_d = -90kJ/kg$.

Hence,

$$0.53\dot{m}_a = 0.615\dot{m}_f \tag{3.8}$$

Solving equations (3.7) and (3.8), we have

$$\begin{split} \dot{m}_a &= 0.06313 + \dot{m}_f = \frac{0.615 \dot{m}_f}{0.53} \\ 0.06313 + \dot{m}_f &= 1.160377 \dot{m}_f \\ \therefore 0.06313 = 0.160377 \dot{m}_f \end{split}$$

Hence,

$$\dot{m}_f = 0.06313/0.160377 = 0.393634 \, \text{kg/s}$$

Hence, we have

$$\dot{m}_a = 1.160377 \dot{m}_f = 0.456764 \, \text{kg/s}$$

In the generator, we have

$$\dot{m}_2 = \dot{m}_1 = 0.06313$$
 kg/s $\dot{m}_c = \dot{m}_b = \dot{m}_a = 0.456764$ kg/s

Since the steam specific enthalpy is approximately equal to the saturation value at the given temp, we have $h_2 = h_g$ at $80^{\circ}C = 2643.2$ kJ/kg. Further, $h_d = -90$ kJ/kg, $h_a = -170$ kJ/kg, $h_c = h$ at 5 kN/m³, concentration y = 0.53, hence $h_c = -105$ kJ/kg.

We now apply an energy balance to the generator. This gives,

$$Q_a + \dot{m}_c h_c = \dot{m}_2 h_2 + \dot{m}_d h_d$$

Hence, we have

$$Q_g = (0.06313 \times 2643.2) + 0.393634 \times (-90) - 0.456764 \times (-105)$$
$$= 166.8652 - 31.49 + 47.96$$
$$\therefore Q_g = 179.395kW$$

4. Heat rejected in the absorber can now be determined.

An energy balance of the condenser gives $-Q_c + \dot{m}_3 h_3 = \dot{m}_2 h_2$. But $\dot{m}_2 = \dot{m}_3 = \dot{m}_1$, hence $Q_c = -\dot{m}_1(h_2 - h_3) = -0.06313(2643.2-138)$, or $Q_c = -158.153276kW$.

Neglecting the heat pump work, and performing an energy balance for the complete system, we have

$$\dot{Q}_q + \dot{Q}_e + \dot{Q}_c + \dot{Q}_A = 0$$

where $\dot{Q}_e = 150kW$ is the heat supplied in the evaporator. Hence,

$$183.333 + 150 - 158.153276 + \dot{Q}_A = 0$$
$$\therefore \dot{Q}_A = -175.17972 \ kW$$

5. The COP_{ref} is defined as

$$COP_{ref} = \frac{\dot{Q}_e}{\dot{Q}_q} = \frac{150}{179.395} = 0.836$$

6. Noting that $h_f = h_e$, an energy balance on the absorber gives

$$\dot{Q}_A + \dot{m}_1 h_1 + \dot{m}_f h_f = \dot{m}_a h_a$$

$$\therefore h_f = (\dot{m}_a h_a - \dot{m}_1 h_1 - \dot{Q}_A) / \dot{m}_f$$

But $h_1 = h_g$ at 0.01bar = 2514 kJ/kg. Hence,

$$h_f = [0.456764 \times (-170) - 0.06313 \times 2614 - (-175.17972)]/0.393634$$

$$= \frac{-61.17898}{0.393634} = -155.42097$$

$$\therefore h_f = -155.42097 \text{ kJ/kg}$$

Alternatively, by performing an energy balance on the heat exchanger, we have

$$\dot{m}_d(h_d - h_e) = \dot{m}_a(h_c - h_b)$$

For throttling, $h_c = h_f$. Also, neglecting heat pump work, $h_b = h_a$. Hence

$$0.393634(-80 - h_f) = 0.456764(-105 - (-170)) = 29.68966$$

Hence.

$$h_f = -\frac{29.68966}{0.393634} - 80$$
or $h_f = -155.42453 \, \text{kJ/kg}$

as obtained previously.

- 7. Heat gain in the heat exchanger, Q_x is given by $Q_x = \dot{m}_c(h_c h_b)$. Also, neglecting heat pump work, $h_b \approx h_a$, hence $Q_x = 0.456764(-105 (-170)) = 29.68966kW$. Hence heat to be supplied to the generate without the heat exchanger is $Q_g' = Q_g + Q_x = 183.333 + 29.6894 = 213.02266kW$.
- 8. Without the heat exchanger, the COP_{ref} would be

$$COP'_{ref} = \frac{150}{213.02266} = 0.70415$$

% change in COP_{ref} is the given by

$$\Delta COP_{ref} = \frac{(COP'_{ref} - COP_{ref})}{COP_{ref}} \times 100$$
$$= \left(\frac{0.70415}{0.8181833} - 1\right) \times 100$$
$$= -13.94\%$$

3.2 Gas Cycles

In some applications, it may be more convenient to use a gas as a refrigerant, rather than a condensable vapour. The basic cycle remains the same as described previously, the notable exception

being that the throttling process is replaced by an expansion engine. This is necessary because the throttling process result in a temperature reduction <u>only</u> if the starting temperature is less than the <u>inversion temperature of the gas</u>. If the expansion process is adiabatic and reversible (i.e. isentropic), then the temperature at the end of the expansion process is given by the equation

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(\gamma - 1)/\gamma} \tag{3.9}$$

A simple example of such a plant is illustrated in figure 3.6(a), while the TS diagram is shown in figure 3.6(b). We observe that heat supply and heat rejection occur at constant pressures, much like the constant pressure or Joule heat engine cycle. Since the heat transfer processes do not occur at constant temperatures, it is clear that the efficiency of the cycle will be lower than that of a corresponding cycle using a condensable vapour as refrigerant. From figure 3.6(b), we note that the temperature range for the gas cycle is from t_4 to t_2 , while that for a vapour cycle is t_1 to t_3 . (Shown by the dotted lines).

Another disadvantage of a gas refrigerator cycle is the larger volume of refrigerant to be handled, and larger heat transfer areas required.

To reduce the external work input for compression, the expander and compressor are coupled as shown in figure 3.6(a). In this way, the net work input to the plant is reduced to $\sum W = W_{12} + W_{34}$ But

$$W_{12} = h_2 - h_1$$
$$W_{34} = -(h_3 - h_4)$$

Also for a perfect gas, $dh = c_n dT$, hence

$$W_{12} = Cp(T_2 - T_1)$$

$$W_{34} = -Cp(T_3 - T_4)$$

The refrigerating effect is then given by

$$Q_{41} = h_1 - h_4 = c_p(T_1 - T_4)$$

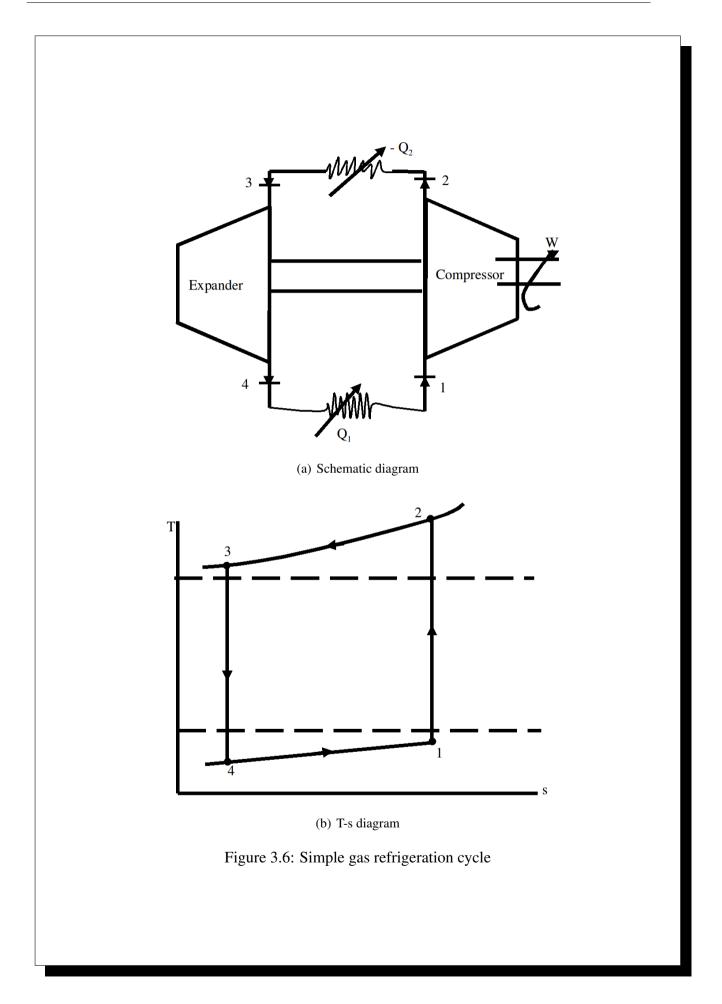
In an actual plant, there would be irreversibilities in the compression and expansion processes, as shown in figure 3.7.

In such a case, the temperature T₂ and T₄ are recalculated using the expressions

$$T_2 = T_1 + (T_{2s} - T_1)/\eta_c (3.10)$$

$$T_4 = T_3 - \eta_T (T_3 - T_{4s}) \tag{3.11}$$

where η_c and η_T are the isentropic efficiencies of the compressor and turbine respectively. Gas refrigeration cycles have important applications in aircraft air conditioning and gas liquifaction.



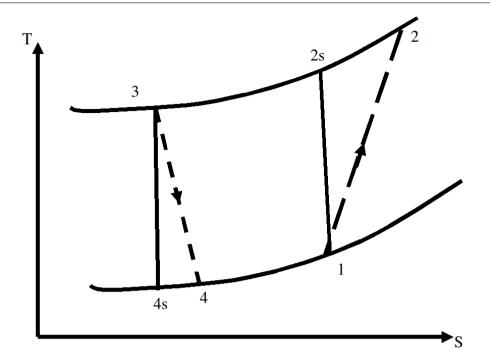


Figure 3.7: T-s diagram showing irreversibility in the expansion and compression processes

Example 3.2 An air refrigeration plant operates with a centrifugal compressor and an air turbine mounted coaxially such that the power output of the turbine contributes to the work required to drive the compressor. The temperature of the air at the compressor inlet is 15°C and the presure ratio is 2.5. The air during its passage from the compressor to the turbine passes through an intercooler and enters the turbine at 40°C. The cold space temperature is required to be maintained at 15°C. Taking the isentropic efficiency of the compressor and turbine to be 84%, and the mechanical efficiency of the turbine-compressor drive as 90%, calculate

- 1. the temperature of the air leaving the compressor, T_2 ;
- 2. the temperature of the air leaving the turbine, T_4 ;
- 3. the refrigerating effect;
- 4. the mass flow rate per kilowatt of refrigerating capacity;
- 5. the driving power required per kilowatt of refrigerating capacity.

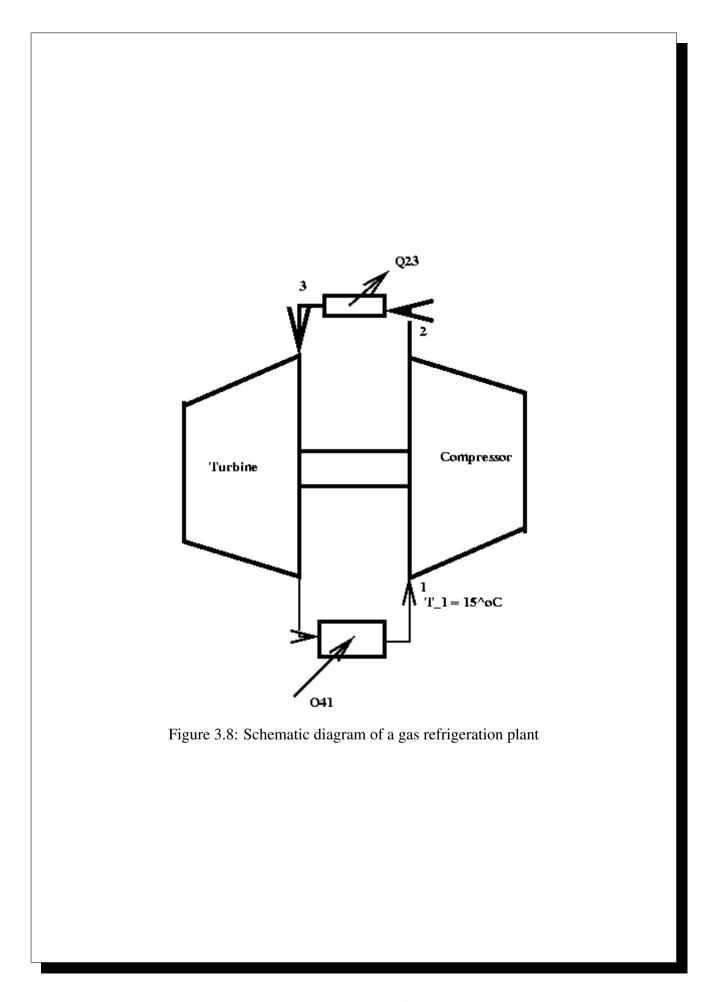
Solution

The plant may be represented as shown in figure 3.8. We first determine the temperatures T_2 , T_3 .

1. Now, we have

$$T_{2s} = T_1(r_p)^{(\gamma - 1)/\gamma}$$

 $r_p = 2.5; \gamma = 1.4; T_1 = 15 + 273 = 288$
 $\therefore T_{2s} = 288(2.5)^{0.4/1.4} = 374.186K$



Hence,

$$\frac{T_{2s} - T_1}{T_2 - T_1} = 0.84$$

$$\therefore T_2 = T_1 + (T_{2s} - T_1)/0.84 = 288 + (374.186 - 288)/0.84$$

$$= 288 + 102.60 = 390.6$$

$$\therefore T_2 = 390.6K$$

2. Now, $T_3 = 40 + 273 = 313K$. Assuming isentropic expansion in the turbine, we have

$$T_{4s} = T_3 r_p^{(\gamma - 1)/\gamma} = 313(1/2.5)^{0.28571}$$

= 240.907K

Hence, we have

$$T_3 - T_4 = 0.84$$

$$T_3 - T_{4s} = 0.84$$

$$T_4 = T_3 - 0.84(T_3 - T_{4s})$$

$$= 313 - 0.84(313 - 240.907)$$

$$or T_4 = 252.44K$$

- 3. The refrigerating effect, Q_{41} is given by $Q_{41} = c_P(T_1 T_4) = 1.005(288 252.44)$, or $Q_{41} = 35.74$ kJ/kg.
- 4. The mass flow rate of refrigerant per kW of refrigerating capacity may now be determined. For one kW, we have $\dot{m}Q_{41}=1$, hence $\dot{m}=1/Q_{41}=1/35.74=0.02798kg/s=1.678kg/min$.
- 5. The driving power required per kW of refrigerating capacity may also be obtained. We have $W_{12}=c_p(T_2-T_1)=1.005(390.6-288)=103.113$ kJ/kg. Turbine power output, W_{34} is given by $W_{34}=c_p(T_3-T_4)=1.005(313-252.44)=60.8628$ kJ/kg. Hence, net power input is $W_{net}=103.113-60.8628=42.2502$ kJ/kg. Multiplying by the refrigerant mass flow rate, we have $W_{net}=42.2502\times0.02798=1.18216$ kW. Allowing for 90% mechanical efficiency of the turbine-compressor drive, we have net power input = 1.18216/0.9=1.314kW.
- Exercise 3.1 1. In an aircraft refrigerator unit, air is bled from the engine compressor at 3.5 bar and 270°C and is passed through an air-cooled heat excahnger. The refrigerant air bleed leaves the exchanger at 3.5 bar and 75°C and is exapnade through a turbine to 0.76 bar. The isentropic efficiency of the turbine is 85%. The air is then delivered to the aircraft cabin and leaves the aircraft at 16°C.
 - (a) Draw a schematic diagram of the plant.
 - (b) Show the key processes in a Ts diagram.
 - (c) What is the refrigerating effect in kJ/kg of the air?
 - (d) Calculate the power developed by the air turbine per unit mass flow rate of the air.

3.3 Gas liquefaction

Common substances which exist as gases at atmospheric conditions are often required in the liquid state for special applications. Typical examples include air, nitrogen, oxygen, ammonia and methane.

In order to liquify a gas, its temperature must first be reduced below the critical value. Critical temperatures of some gases are shown in table 3.1.

Table 3.1: Critical temperature of some common gases

Gas	Ammonia NH ₃	Nitrogen N ₂	Oxygen O ₂	Methane CH ₄
Temperature, °C		-147		

In general, cooling water or a refrigeration process is first employed to cool the gas below its *temperature of inversion*, after which the gas itself is employed as the refrigerant to complete the process.

The simplest gas liquifaction process was first developed by Linde for liquifying air, and it is illustrated in figure 3.9.

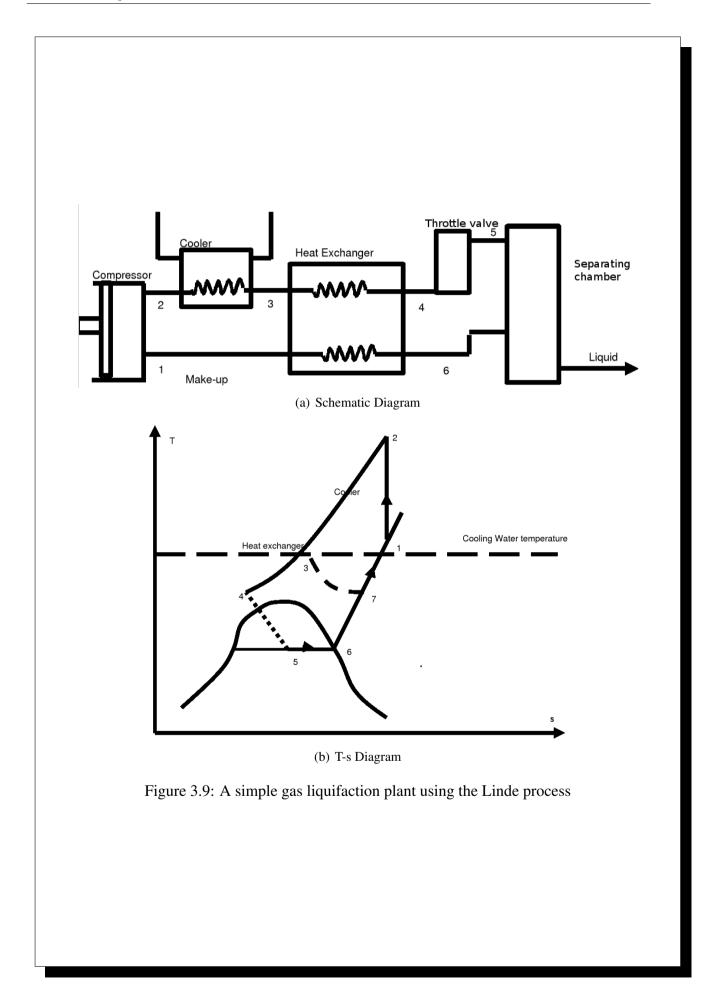
Air at atmospheric pressure, state 1, is compressed to state 2, and is then cooled at constant pressure back to atmospheric temperature, state 3, in a water cooler. The air is further cooled to state 4 in a heat exchanger before being throttled to state 5. In this state, the liquid portion 5_f is drained off and the vapour portion 5_g is taken to the heat exchanger and back to the compressor. To keep the mass of air in the system constant, make-up air which is equal in mass to the liquid air drained off, is introduced into the system at state 1.

At the start of the process, it is clear that there is no cold gas in the system, hence the throttling process will follow path 3–6. If the gas is below its inversion temperature, a little cooling will occur. This cooling effect is cumulative since it is utlised in the heat exchanger, and the throttling curve 3–6 will shift towards the path 4–5. The ultimate steady state position of state 4 will depend on the rate at which the liquid is drained off, and rate of heat leakage into the system.

If the gas to be liquified has a sub-atmospheric inversion temperature, it has to be precooled below the inversion temperature using a conventional refrigeration system. Alternatively, the throttle valve could be replaced with an expansion machine.

Exercise 3.2 *Now, consider the problem of gas liquifaction.*

- 1. State the major advantage of liquifying a gas before use.
- 2. What is the temperature of inversion?
- 3. Using a sinple schematic diagram, describe the Linde process for liquifying air. (2 points).



Chapter 4

PYCHROMETRY & AIR-CONDITIONING

In many modern industrial processes, it is often essential to provide air at a controlled temperature and moisture content. This is usually achieved through air-conditioning. Air-conditioning is also required in many modern private and public buildings. The study will therefore be introduced by considering the properties of moist atmospheric air in what is known as psychrometry or hydrometry.

4.1 Properties of moist atmospheric air, or psychrometry

Consider atmospheric air at 1.013bar and 25°C. From the steam tables ,the saturation pressure of steam at 25°C is 0.03166bar. If the water vapour in the air is not in contact with the liquid, its pressure will be most probably below the saturation condition. As a result, the vapour could be considered to behave as a perfect gas, and the properties of the mixture could be determined from the Gibbs-Dalton law considered previously.

Suppose that in a given quantity of atmospheric air at 25° C and 1.013bar, the partial pressure of the vapour, P_s is 0.02bar. Then it follows that

$$P = P_a + P_s \tag{4.1}$$

Where P_a is the partial pressure of the dry air.

Thus
$$P_a = P - P_s = 1.013 - 0.02 = 0.993$$
 bar.

From the tables, the saturation temperature corresponding to 0.02bar is 17.5° C. Thus, the vapour in the atmospheric air is superheated by 25-17.5 = 7.5K.

This is illustrated in figure 4.1, where state 1 indicates the state of the superheated vapour.

If the vapour is cooled to state 2, the saturation temperature corresponding to the partial pressure (say by contact with a cold surface), condensation of the vapour will commence. This temperature is often called the dew point temperature, t_d .

4.1.1 Moisture Content or Specific Humidity, w

The moisture content or specific humidity, w, is the ratio of the mass of water vapour to the mass of dry air in a given volume of the mixture. Thus,

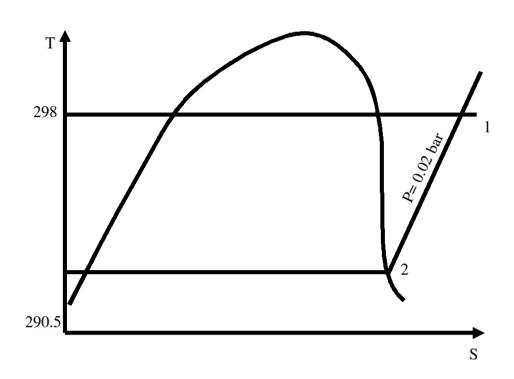


Figure 4.1: T-s diagram for superheated water vapour in atmospheric air

$$w = \frac{m_s}{m_a} \tag{4.2}$$

But

$$m_s = V/v_s$$
$$m_a = V/v_a$$

where V is the volume of the mixture and v_s , v_a are the respective specific volumes for the steam and dry air. Substituting in equation (4.2), we have

$$w = \frac{V/v_s}{V/v_a} = \frac{v_a}{v_s} \tag{4.3}$$

The specific humidity may also be expressed in terms of partial pressures by considering the vapour and air as perfect gases. Now,

$$m=\frac{pV}{RT}$$
 But $R=\frac{\bar{R}}{\bar{m}}$ Hence, $m=\frac{pV\bar{m}}{\bar{R}T}$

Substituting into equation (4.2), we have

$$w = \frac{p_s V \bar{m}_s}{\bar{R}T} \frac{\bar{R}T}{p_a V \bar{m}_a} = \frac{\bar{m}_s}{\bar{m}_a} \frac{p_s}{p_a}$$

Now, $\bar{m}_s = 18$; $\bar{m}_a = 28.96$, $P_a = P - P_s$

Hence,

Thus,

$$w = \frac{18}{28.96} \frac{p_s}{p_a} = 0.622 \frac{p_s}{p - p_s} \tag{4.4}$$

4.1.2 Relative Humidity, ϕ

The relative humidity, ϕ , of a volume of the mixture is defined as the ratio of the mass of water vapour to the mass that would make the mixture saturated at the same temperature.

$$\phi = \frac{m_s}{m_{s_{sat}}}$$
 Since $m_s = \frac{p_s V}{R_s T}$
$$m_{s_{sat}} = \frac{P_{sat} V}{R_s T}$$

It follows that

$$\phi = \frac{p_s V}{R_s T} \frac{R_s T}{p_{sat} V} = \frac{p_s}{p_{sat}} \tag{4.5}$$

hence

$$p_s = \phi p_{sat}$$

where p_{sat} is the saturation pressure at the mixture temperature.

4.1.3 Percentage Saturation, ψ

The percentage saturation, ψ is often defined as the ratio of the specific humidity of a mixture to the saturation specific humidity at the same temperature. Thus

$$\psi = \frac{w}{w_{sat}} \tag{4.6}$$

From equation (4.4), it follows that

$$\psi = 0.622 \frac{p_s}{p - p_s} \frac{p - p_{sat}}{0.622 p_{sat}}$$
$$= \frac{p_s}{p_{sat}} \frac{p - p_{sat}}{p - p_s}$$

But $p_s/p_{sat} = \phi$. Hence

$$\psi = \phi \frac{p - p_{sat}}{p - p_{s}} \tag{4.7}$$

This may also be expressed in percentages as

$$\psi = 100\phi \frac{p - p_{sat}}{p - p_s} \tag{4.8}$$

Example 4.1 Air at 30°C and 1.013bar barometric pressure is saturated with water vapour, Determine.

- 1. The partial pressures of the vapour and dry air
- 2. The volume of the mixture per unit mass of the vapour.
- 3. The mass of dry air per unit mass of the vapour
- 4. The specific humidity of the mixture
- 5. The relative humidity of the mixture

Solution

- 1. We have that p=1.013bar. At 30°C, from steam tables, we have , $p_s=0.04242\ bar$, $p_a=1.013-p_s=0.97058bar$.
- 2. The volume of the mixture per unit mass of the vapour = V/m_s . Now, we have

$$m_a = \frac{p_a V}{R_a T}$$
$$m_s = \frac{p_s V}{R_s T}$$

Thus, we have

$$V/m_s = \frac{R_s T}{p_s} = \frac{\bar{R}}{\bar{m}_s} \frac{T}{p_s}$$

$$= \frac{8.3145 \times 10^3}{18} \times \frac{303}{0.04242 \times 10^5}$$

$$= 3299.4048 \times 10^3 / 10^5$$

$$= 32.994 m^3 / kg$$

3. The mass of dry air per unit mass of the vapour is given by

$$\begin{split} m_a/m_s &= \frac{(p_a V)/(R_a T)}{(p_s V)/(R_s T)} \\ &= \frac{p_a V}{p_s V} \frac{R_s T}{R_a T} = \frac{p_a R_s}{p_s R_a} \\ &= \frac{p_a}{p_s} \frac{\bar{R}}{\bar{m}_s} \frac{\bar{m}_a}{\bar{R}} = \frac{p_a \bar{m}_a}{p_s \bar{m}_s} = \frac{0.97058 \times 28.96}{0.04242 \times 18} \\ &= 36.811772 \end{split}$$

4. The specific humidity of the mixture, w is given by

$$w = 0.622 \frac{p_s}{p - p_s}$$
$$= 0.622 \frac{(0.04242)}{0.97058} = 0.027185$$

5. Since the air is saturated, the relative humidity, $\phi = p_s/p_{sat} = p_{sat}/p_{sat} = 1.0$

Example 4.2 In an atmosphere at 30°C and 1.013bar, the partial pressure of the water vapour is 0.02063bar. Determine

- 1. The saturation temperature
- 2. The degree of superheat of the water vapour
- 3. The specific humidity
- 4. The relative humidity
- 5. The mass of condensate formed per unit mass of dry air if the air is cooled to 10°C from its initial condition.

Solution

- 1. From steam tables, the saturation temperature corresponding to a partial pressure of 0.02063 b $t_s = 17.95^{\circ}$ C.
- 2. The degree of superheat of the water vapour = $t t_s = 30 17.95 = 12.05K$.
- 3. The specific humidity is given as

$$w = 0.622 \frac{p_s}{(p - p_s)} = \frac{0.622 \times 0.02063}{1.013 - 0.02063} = 0.0129306$$

- 4. The relative humidity, $\phi = p_s/p_{sat}$. From steam tables, the saturation pressure at the temperature of $30^{\circ}\text{C} = 0.04242$ bar. Hence $\phi = 0.02063/0.04242 = 0.4863$.
- 5. If the air is cooled to 10° from its initial condition, since its temperature is now less than the initial saturation value, the air remians saturated with water vapour. Hence,

$$w_1 = 0.622 \frac{p_s}{p - p_s}$$

$$= \frac{0.622 \times 0.02063}{1.013 - 0.02063}$$

$$= 0.0129306$$

Since $T_2 < T_{sat}$, it follows that the air is saturated at T_2 . But, $P_{sat} @ 10^{\circ}C = 0.01227 bar$. Now,

$$w_2 = 0.622 \frac{p_{s2}}{p - p_{s2}} = 0.622 \frac{0.01227}{1,013 - 0.01227}$$
$$= 0.007626$$

Hence, water condensed is given by

$$w_1 - w_2 = 0.012906 - 0.007626 = 0.005646$$

4.2 Measurement of Relative Humidity

Wet & dry bulb thermometers

4.3 Psychometric chart

4.4 Thermodynamic properties of moist air

A number of thermodynamic properties of moist air are important for air conditioning applications. The most important of these, namely, specific enthalpy, specific heat capacity and specific volume will be considered.

4.4.1 Specific Enthalpy of Moist Air

The specific enthalpy h of a mixture of mass m is determined from the sum of the products of the mass and specific enthalpies of the constituents.

Thus,

$$mh = \sum_{i=1}^{n} m_i h_i \tag{4.9}$$

But $m = \sum m_i$, hence

$$h = \frac{\sum m_i h_i}{\sum m_i} \tag{4.10}$$

In the case of a mixture of air and water vapour, the equation reduces to $mh = m_s h_s + m_a h_a$, hence

$$h = \frac{m_s h_s + m_a h_a}{m_s + m_a} \tag{4.11}$$

Similarly, the specific volume, v, is given by

$$v = \frac{m_a v_a + m_s v_s}{m_a + m_s} \tag{4.12}$$

4.4.2 Specific heat capacity

If the superheated vapour in the mixture is treated as a perfect gas, then if follows that $h = c_p T$. Substituting in equation (4.11), we have $mc_p T = m_a c_{pa} T_a + m_s c_{ps} T_s$, hence

$$c_p = \frac{m_a c_{pa} T_a + m_s c_{ps} T_s}{(m_a + m_s)T}$$

If $T = T_a = T_s$, then

$$c_p = \frac{m_a c_{pa} + m_s c_{ps}}{m_a + m_s} \tag{4.13}$$

4.5 Air- Conditioning

Air-conditioning involves the supply of air at a regulated temperature and relative humidity for human comfort or to provide a suitable environment for industrial processes. Depending upon the season and the environmental conditions, it may require heating/cooling and/or humidification/dehumidification of the air inlet. In many cases, it is usual to draw some one-third of the total air supply from the atmosphere and to recirculate the remainder from the controlled space.

A detailed analysis of the process involved requires an understanding of the associated heat and mass transfer processes. This is presented in a simplified form in Rogers and Mayhew (1992). An alternative approach, which obscures some of the fundamental principles involved, is employed by Eastop and McConkey (1993). The subject will be considered in this course using the basic approach of Rogers and Mayhew (1992).

4.5.1 Heating Loads

In the design of an air-conditioning system, it is important to consider all energy inputs into the room or space being conditioned. This is often called heating load calculations. Typical sources of energy input are solar radiation, human occupants, lighting, machinery, etc.

In addition, energy transfer associated with the air circulation and moisture transfer with or without change of phase (vaporization or condensation) have to be considered.

In the building services industry in some countries, the terms "sensible heat gain" and "latent heat gain" are often used in heating load calculations, but these shall not be adopted in this course. Instead. The usual thermodynamic concepts of balancing all mass and energy inputs shall be employed.

4.5.2 Air-Conditioning for Summer or Hot Season

In summer or the hot season, recirculated air is mixed with fresh air, cooled and dehumidified before being supplied to the room. A typical plant is shown in figure 4.2.

In the cooler unit, the relative humidity of the air increase as the air is cooled. A dehumidifier is therefore introduced to extract the excess moisture. The air leaving the cooler may therefore be saturated and at a lower temperature than is actually required. In this case, the air may be heated at constant pressure to the final temperature. This will also reduce the relative humidity to the required value.

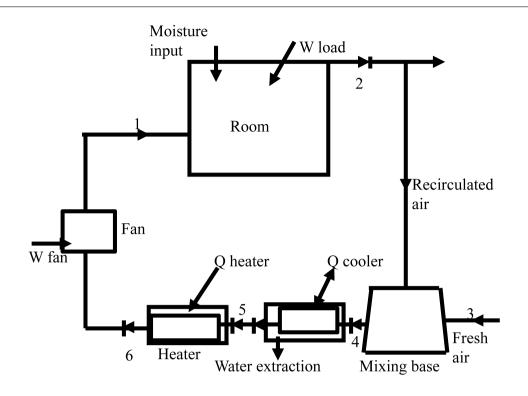


Figure 4.2: Typical air-conditioning plant for heating season or summer

4.5.3 Air-Conditioning for Winter or Cold Season

A typical air conditioning plant for use in the winter or cold season is shown in figure 4.3. The system is basically the same as that of the summer, except that a humidifier is introduced.

Example 4.3 An air-conditioning plant is to deliver 30.m³/min of air at 20°C and relative humidity of 50%. The ambient air pressure, temperature and relative humidity are 1.013bar, 36°C and 90% respectively. The air leaves the cooling coil at the dew point and is reheated to the delivery temperature. The axial-flow fan is located before the cooler and is rated at 1.1kW. Assuming that the pressure remains constant throughout at 1.013bar and that there is no air recirculation, determine

- 1. Temperature of the air leaving the cooling coils
- 2. Rate of heat transfer in the heater
- 3. Rate of heat transfer in the cooler.

Solution

The system may be represented schematically as in figure 4.4

Along the heater, we note that the composition of the mixture between states 4 and 5 remains unchanged, hence the vapour partial pressure is constant.

Now, $P_{g5} = P_{sat}$ at $20^{\circ} = 0.02337$ bar. Thus, $P_{s4} = P_{s5} = \phi_5 P_{g5} = 0.5 \times 0.02337 = 0.011685$ bar

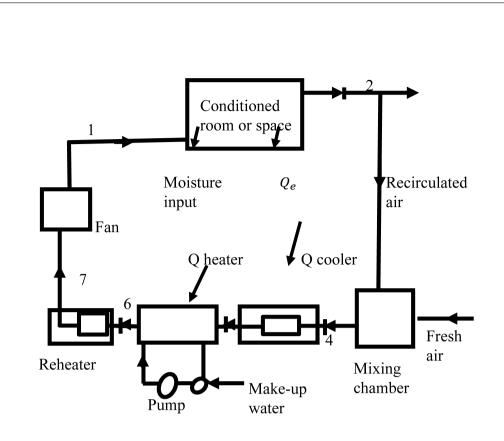


Figure 4.3: Air-conditioning system for winter or cold season

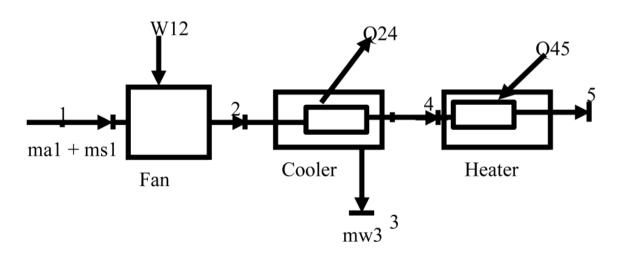


Figure 4.4: Schematic diagram for example 4.3

At section 4, the air leaving the cooler is saturated with vapour. The temperature must therefore be the saturation temperature at $p_{s4} (= p_{s5})$. From steam tables,

$$T_4 = 7 + \frac{0.011685 - 0.01}{0.015 - 0.01}(13 - 7) = 7 + 0.337 \times 6$$
 or $T_4 = 7 + 2.25 = 9.25^{\circ}C$

To determine the rate of heat transfer in the heater, Q_{45} , we apply energy balance equation between sections 4 and 5 to get $m_{a5}h_{a5} + m_{s5}h_{s5} = Q_{45} + m_{a4}h_{a4} + m_{s4}h_{s4}$

$$\rightarrow Q_{45} = (m_{as}h_{s5} + m_{s5}h_{s5}) - (m_{a4}h_{a4} + m_{s4}h_{s4})$$

Changes in kinetic and potential energies have been neglected.

The mass balance of the air and moisture streams is also applied, yielding $m_{a5} = m_{a4}$, $m_{s5} = m_{s4}$ Treating the air and vapour streams as perfect gases, the equation becomes

$$Q_{45} = m_{a5}c_{pa}(T_5 - T_4) + m_{s5}c_{ps}(T_5 - T_4)$$

= $(T_5 - T_4)(M_{a5}Cpa + M_{s5}c_ps)$

At low pressure, the value of $c_{ps} \approx 1.864$. (See p. 17 of Mayhew & Rogers (1977)). Also, $c_{pa} = 1.005$.

Since the volume flow rate of the air delivered is known, we have

$$\begin{split} m_{s5} &= \frac{p_{s5}V_5}{R_sT} = \frac{0.011685 \times 10^5 \times (30/60)}{(8.314/18) \times 293 \times 10^3} \\ or \, m_{s5} &= \frac{100 \times 0.011685 \times 0.5}{0.46186 \times 293} = 0.004317 \, \text{kg/s} \end{split}$$

Similarly, the dry air mass flow rate is given as

$$m_{a5} = \frac{(p-p_{s5})V_s}{R_aT} = \frac{100(1.013-0.011685)\times0.5}{0.287\times293}$$
 or $m_{a5} = 0.59538$ kg/s

Thus, the heat transferred in the heater is given by

$$Q_{45} = (20 - 9.022)(0.5938 \times 1.005 + 0.004317 \times 1.864)$$
$$= 10.978 \times (0.59836 + 0.008047)$$
$$= 6.6571 \, kW$$

Heat transferred to the Cooler, Q_{24}

To determine Q_{24} , we apply the energy equation to the fan and cooler, yielding.

$$\dot{m}_{a4}h_{a4} + \dot{m}_{s4}h_{s4} = \dot{m}_{a1}h_{a1} + \dot{m}_{s1}h_{s1} + W_{12} - \dot{m}_{w3}h_{w3} - Q_{24}$$

Hence,

$$Q_{24} = -(\dot{m}_{a4}h_{a4} + \dot{m}_{s4}h_{s4} + \dot{m}_{w3}h_{w3}) + (\dot{m}_{a1}h_{a1} + \dot{m}_{s1}h_{s1}) + W_{12}$$

Now,

$$\dot{m}_{a4} = \dot{m}_{a1} = \dot{m}_{a5} = 0.59538 \text{ kg/s}$$

 $\dot{m}_{s1} = \dot{m}_{s4} + \dot{m}_{w3}$

Hence,

$$Q_{24} = \dot{m}_a c_{pa} (T_4 - T_1) + (\dot{m}_{s4} h_{s4} + \dot{m}_{w3} h_{w3} - \dot{m}_{s1} h_{s1}) + W_{12}$$

$$(4.14)$$

Also, from the inlet condition, we have

$$p_{g1} = p_g$$
 at 36°C = 0.05940 bar, $p_{s1} = \phi_1 p_{g1} = 0.9 \times 0.0594 = 0.05346$.

Also, from equation (4.4), we have

$$w_1 = 0.622 \frac{p_s}{p - p_s} = \frac{0.622 \times 0.05346}{1.013 - 0.05346}$$
$$= \frac{0.0361155}{0.95954}$$

or $w_1 = 0.03465$ kg steam/kg dry air

Hence,

$$\dot{m}_{s1} = w_1 \dot{m}_a = 0.03465 \times 0.59538$$

= 0.0206 kg/s

Thus,

$$\dot{m}_{w3} = \dot{m}_{s1} - \dot{m}_{s4}$$

$$= 0.0206 - 0.004317$$

$$= 0.016283 \, kg/s$$

The temperature of the condensed vapour is T_4 . = 9.25°C. From the steam tables, h_{w3} = h_f at 9.25°C = 37.8KJ/kg, Similarly, $h_{s4} = h_g$ at 9.022°C ≈ 2517.4 kJ/kg.

Also, since the vapour at state 1 is superheated, we obtain h_{s1} by interpolating around $p_{s1} \approx 0.05$ bar since h is nearly independent of pressure at low pressure.

Hence

$$h_{s1} = 2561 + \frac{36 - 32.9}{50 - 32.9} (2594 - 2561)$$

$$= 2561 + \frac{3.1}{17.1} (33)$$

$$= 2561 + 5.9825$$

$$= 2566.98 \, kJ/kg$$

Substituting into equation (4.14), we have

$$Q_{24} = -0.59538 \times 1.005(9.022 - 36)$$

$$- (0.004317 \times 2517.4 + 0.016283 \times 37.8 - 0.0206 \times 2566.98) + 1.1$$

$$= 16.1424 - (10.8676 + 0.6496 - 55.19) + 1.1$$

$$= 59.73 \text{ kW}$$

Example 4.4 An air-conditioning (AC) unit of a pressurized aircraft receives its air from the jet engine compressor at a pressure of 1.22 bar. The atmospheric pressure and temperature are 0.227 bar and 217 K respectively. The air-conditioning unit consists of a freewheeling compressor and turbine mounted on the same shaft as shown in figure 4.5.

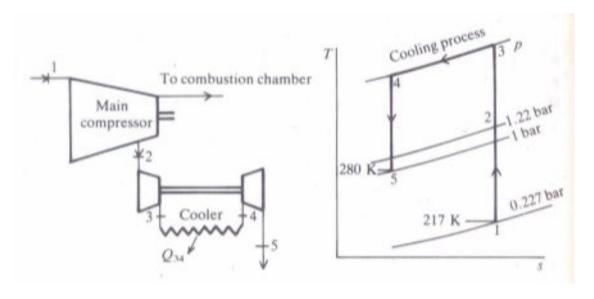


Figure 4.5: Schematic and T-s diagrams for an aircraft air-conditioning system

The air coming into the unit is compressed to a pressure P_3 , cooled at constant pressure to T_4 , and expanded in the turbine to a pressure of 1 bar and temperature of $T_5 = 280$ K. Assume that all processes in the system are reversible

- 1. Determine the temperature of the air leaving the main compressor, T_2
- 2. State the expression for calculating the temperature of the air leaving the compressor of the AC unit, T_3
- 3. State the expression for calculating the work done by the AC compressor, W_{23}
- 4. Determine the expression for calculating the cooler outlet tempearture, T_4
- 5. Determine the expression for the turbine work, W_{45}
- 6. Assume that the turbine produces sufficient work to drive the compressor, and hence $W_{23}+W_{45}=0$
 - (a) For values of P_3 in the first column of table 4.1, compute the values of W_{23} , W_{45} and $f(P_3) = W_{23} + W_{45}$ and hence complete columns 2, 3 and 4 of the table.

Table 4.1: Data for question 4.4						
P_3 (bar)	W_{23}	W_{45}	$f(P_3) = W_{23} + W_{45}$			
2.95						
3.00						
3.05						
3.10						

(b) Using the data for $f(P_3)$ in column 4 of table 4.1, determine, by linear interpolation

Table 4.1. Data for question 4.4

- or otherwise, the approximate value of P_3 for which $f(P_3) = W_{23} + W_{45} = 0$.
- (c) Determine the required cooler exit tempearture, T_4

Solution

1. To determine the tempearture of the air leaving the main compressor T_2 , we note that he engine compressor intake conditions are $p_1 = 0.227$ bar, $T_1 = 217K$. The air extracted (or bled) from the main compressor at state 2 is $p_2 = 1.22$ bar. Hence, T_2 is given by

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{0.4/1.4} = 217 \left(\frac{1.22}{0.227}\right)^{0.4/1.4} = 315 \text{ K}$$

2. The temperature of the air leaving the freewheeling compressor, T_3 , is given by

$$T_3 = T_2 \left(\frac{p}{p_2}\right)^{0.4/1.4}$$

3. The work done by the freewheeling compressor is given by

$$W_{23} = (h_3 - h_2) = c_p(T_3 - T_2) = c_pT_2 \left\{ \left(\frac{p}{p_2}\right)^{0.4/1.4} - 1 \right\}$$

4. Now, the turbine outlet state 5 is defined by $p_5 = 1$ bar, $T_5 = 280$ K. The outlet temperature of the cooler, T_4 , is then given by

$$T_4 = T_5 \left(\frac{p}{p_5}\right)^{0.4/1.4}$$

5. The turbine work, T_{45} is then given by

$$W_{45} = (h_5 - h_4) = c_p(T_5 - T_4) = c_pT_5 \left\{ 1 - \left(\frac{p}{p_5}\right)^{0.4/1.4} \right\}$$

6. Since the turbine produces just sufficient work to drive the compressor, it follows that W_{23} + $W_{45} = 0$

Table 4.2: Computed data for question 4.4						
P_3 (bar)	W_{23}	W_{45}	$f(P_3) = W_{23} + W_{45}$			
2.9500	101.1799	-101.9178	-0.7379			
3.0000	103.3642	-103.7629	-0.3987			
3.0500	105.5227	-105.5862	-0.0635			
3.1000	107.6561	-107.3883	0.2678			
3.1500	109.7650	-109.1697	0.5953			
3.2000	111.8502	-110.9310	0.9191			
3.2500	113.9122	-112.6728	1.2394			
3.3000	115.9516	-114.3956	1.5561			

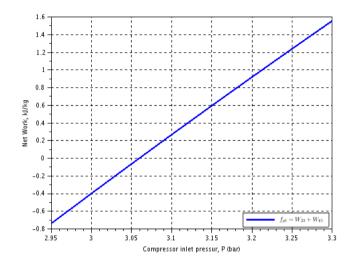


Figure 4.6: Plot compressor power requirement and turbine power output against pressure for example 4.4

(a) The functions, W_{23} , W_{45} and $f(p_3) = W_{23} + W_{45}$ are determined as shown in columns 2, 3 and 4 of table 4.2.

Using the data from the table, the net power input f_{p3} was plotted against the compressor inlet pressure as shown in figure 4.6.

From the figure, it is clear that $f_{p3} = 0$ slightly above 3.0 bar. Using a numerical root finding technique, the intercept was determined to be 3.06 bar. Hence, the inlet pressure to the compressor is 3.06 bar.

The computation was done using the SCILAB script shown in listing 4.1

Listing 4.1: Programme to determine solve an aircraft air-conditioning problem //Aircraft air-conditioning system

```
_{2} function [fp3, w23, w45] = powerout(p)//determine the power
     w23 = cpa * T2 * ((p/p2) .^{(0.4/1.4)} - 1); //work required by
     w45 = cpa * T5 * (1 - (p/p5) .^{(0.4/1.4)}); //power output of
     fp3 = w23 + w45
```

6 endfunction

reew urbin

```
scpa = 1.005; //specific heat of air
    p1 = 0.227; //inlet air presure in bar
10 T1 = 217; // inlet air temperature in degree Kelvin
    p2 = 1.22; // pressureat compressor inlet (bar)
12 T2 = T1 * (p2/p1)^{(0.4/1.4)}; // temperature of air entering compres
                                                                                                                                                                         or in
    T5 = 280; //outlet temperature of air from turbine in degree Kelvi
_{14} p5 = 1; //pressure at turbine exit
    p3 = [2.95 \ 3.0 \ 3.05 \ 3.1 \ 3.15 \ 3.2 \ 3.25 \ 3.3]; // pressures in bar
_{16} [fp3, w23, w45] = powerout(p3); //determine the power
    result = [p3' w23' w45' fp3']; // output of data
18 [wrow, wcol] = size (result); // determine dimensions
                         resultdata = myprintf(result, ["%10.4f" "%10.4f" "%10.4f'
                                                                                                                                                                          "%10.
20 \text{ } ww = scf(); // create \text{ } new \text{ } graphic \text{ } window
    plot (p3, fp3)
_{22} //plot(p3,-w45, "+-");//plot second graph with lines and points
    aa = gca(); // get current graphic handle
24 aa.children.children.thickness = 3;//use 3 poits for line thickne
    aa.grid = [0.05, 0.05]; // set grid lines
26 xtitle ("", "Compressor inlet pressur, P (bar)", "Net Work, kJ/kg'
                                                                                                                                                                        ; // t
    legend("\$ f_{p3}) = W_{23} + W_{45}\$", "in_lower_right"); // place legend("$ f_{p3}) = W_{10} + W_{1
28 filename = "aircraft-ac.sce"; //name of current file
    wdir = get_absolute_file_path(filename); // get full path for the j
                                                                                                                                                                         le
30 outfile1 = basename(filename) + "-table.tex"; // output file name
    string2latex(wdir, outfile1, resultdata); //save as LaTeX file
32 outfile 2 = wdir + basename (filename) + "-table-plot.png"; // output file
    xs2png(0, outfile2)
[x1, x2] = rootsearch(powerout, 3.05, 3.06, 0.001)
```

Exercise 4.1 In a simple air-conditioning plant, a fan of power 3.12kW takes in dry air at $T_1 = 10^{\circ}$ C and $P_1 = 1.013$ bar at a rate of 2.3 kg/s and blows it into a humidifying duct. Water at 10° C is sprayed into the duct at section 3 and the air-water mixture is heated between sections 4 and 5 until it is dry saturated and leaves at $P_5 = 1.021$ bar and $T_5 = 75^{\circ}$ C.

- 1. Draw a schematic diagram of the plant, and hence determine:-
- 2. the specific enthalpy of the water sprayed into the duct (Hint: Assume that the specific ethalpy of liquid water is equal to that of the saturated liquid at the same temperature);
- 3. the state of the mixture at the duct exit (section 5) as indicated by
 - (a) the vapour partial pressure;
 - (b) the dry air partial pressure;

- (c) the moisture content of the mixture;
- (d) the specific enthalpy of the vapour;
- (e) the amount of water sprayed into the duct:
- (f) the mass flow rate of the mixture;
- 4. an expression for calculating the heat supplied into the duct, Q_{45} (Hint: Take an energy balce of the entire duct, neglecting changes in kinetic energy);
- 5. the value of Q_{45} in kW.

Example 4.5 In a certain location, the barometric pressure, ambient air temperature and relative humidity are 1.013 bar, 38°C and 90% respectively. An air conditioning plant is designed to deliver 35 m³/min. of air to a room at 22 °C and relative humidity of 50%. The air leaves the cooling coil at the dew point and is reheated to the delivery temperature. The axial-flow fan is located before the cooler and is rated at 1.2 kW. Pressure remains constant throughout the system at 1.013 bar and there is no air recirculation. Specific heat of water vapour at low pressure may be taken as $c_{ps} = 1.864 \text{ kJ/kg}$.

- 1. Draw a schematic diagram of the system showing the basic energy and mass flow terms.
- 2. Evaluate the saturation vapour pressure of moisture in the ambient air
- 3. Compute the partial pressure of moisture in the ambient air.
- 4. Obtain the specific enthalpy of the water vapour in the ambient air.
- 5. Calculate the moisture content of the ambient air.
- 6. Evaluate the mass flow rate of moisture in the ambient air.
- 7. Determine the saturation vapour pressure of the moisture in the room air
- 8. Determine the partial pressure of the moisture in the room air.
- 9. Determine the temperature of the air leaving the cooling coil and entering the reheater.

- 10. Obtain an expression for calculating the rate of heat input by the reheater.
- 11. Determine the mass flow rate of moisture supplied to the room.
- 12. Calculate the mass flow rate of dry air supplied to the room.
- 13. Calculate the value of the rate of heat input by the heater.
- 14. Obtain an expression for calculating the rate of heat extracted by the cooling coil.
- 15. Calculate the mass flow rate of water condensed on the cooling coil.
- 16. Calculate the specific enthalpy of the condensed water leaving the cooling coil.
- 17. Determine the specific enthalpy of the water vapour leaving the cooling coil.
- 18. Calculate the value of the heat extracted by the cooling coil

Solution

- 1. The schematic diagram of the cooling tower is given in figure 4.7.
- 2. The follwing parameters are determined

$$P_1 = 1.013bar$$

$$T_1 = 38^{\circ}C$$

$$\phi_1 = 90\%$$

$$P_{1g} = P_g@38^{\circ}C = 0.06624 \ bar.$$

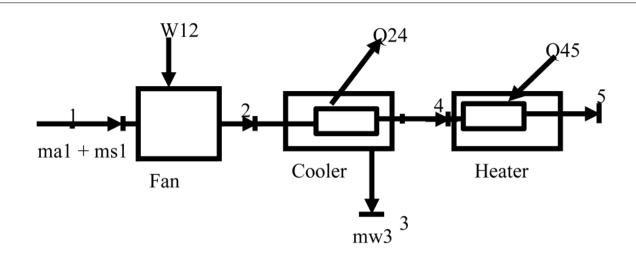


Figure 4.7: Schematic diagram of the air-conditioning system in question 4.5

- 3. Since the relative humidity is 90%, it follows that $P_{1s}=\phi_1P_{1g}=0.9\times0.06624=0.059616$ bar.
- 4. Specific enthalpy of the water vapour in the ambient air, $h_{15} = h_q@38^{\circ}C = 2570.1 \text{ kJ/kg}$
- 5. The moisture content of the ambient air is given by

$$w_1 = 0.622 \frac{P_{1s}}{P - P_{1s}} = 0.622 \frac{0.059616}{1.013 - 0.059616}$$
$$= 0.0388942$$

6. The mass flow rate of moisture in the ambient air is given by $m_{1s} = w_1 m_a$. But

$$m_a = \frac{P_{1a}V_a}{R_aT_1}$$

$$P_{1a} = (1.013 - 0.059616) \times 10^5$$

$$V_a = 35m^3/min = (35/60) = 0.5833333m^3/s$$

$$R_a = 287J/(kqK)$$

Hence, $m_a = 0.6230779 kg/s$. Thus,

$$m_{1s} = w_1 m_a$$

= 0.0388942 × 0.6230779
= 0.0242341 kg/s

- 7. The stauration vapour pressure of the moisture in the room is given by $P_{5g} = P_{sat}$ at $22^{\circ}C = 0.02642$ bar.
- 8. The relative humidity of the room air, $\phi_5 = 0.5$. Hence, partial pressure of the moisture in the room air is given by

$$P_{5s} = \phi_5 P_{5g} = 0.5 \times 0.02642$$
$$= 0.01321bar$$

- 9. The temperature of the air leaving the cooling coil and entering the reheater, T_4 =dew point temperature corresponding to the partial pressure of the moisture in the room air. Hence, $T_4 = T_{sat}$ at $P_{5s} = 11.1^{\circ}C$.
- 10. To determine the rate of heat transfer in the heater, Q_{45} , we apply energy balance equation between sections 4 and 5 to get $m_{a5}h_{a5} + m_{s5}h_{s5} = Q_{45} + m_{a4}h_{a4} + m_{s4}h_{s4}$

$$\rightarrow Q_{45} = (m_{as}h_{s5} + m_{s5}h_{s5}) - (m_{a4}h_{a4} + m_{s4}h_{s4})$$

Changes in kinetic and potential energies have been neglected.

The mass balance of the air and moisture streams is also applied, yielding $m_{a5} = m_{a4}$, $m_{s5} = m_{s4}$

Treating the air and vapour streams as perfect gases, the equation becomes

$$Q_{45} = m_{a5}c_{pa}(T_5 - T_4) + m_{s5}c_{ps}(T_5 - T_4)$$

= $(T_5 - T_4)(M_{a5}Cpa + M_{s5}c_ps)$ (4.15)

11. The mass flow rate of the moisture in the room air is now determined. Now, the moisture content of the room air is given by

$$w_5 = 0.622 \frac{P_{5s}}{P - P_{5s}}$$
$$= 0.622 \frac{0.01321}{1.013 - 0.01321}$$
$$= 0.0082183$$

Thus, we have

$$m_{5s} = w_5 m_a$$

= 0.0082183 × 0.6230779
= 0.0056557 kq/s

- 12. Mass flow rate of the dry air supplied to the room is the same as $m_a = 0.6230779$ kg/s.
- 13. The heat input by the heater is then obtained by substituting the relevant details in equation 4.15 to have

$$Q_{45} = (T_5 - T_4)(m_a c_{pa} + m_{5s} c_{ps})$$

= 6.9372561kW

14. Heat transferred to the cooler, Q_{24}

To determine Q_{24} , we apply the energy equation to the fan and cooler, yielding.

$$\dot{m}_{a4}h_{a4} + \dot{m}_{s4}h_{s4} = \dot{m}_{a1}h_{a1} + \dot{m}_{s1}h_{s1} + W_{12} - \dot{m}_{w3}h_{w3} - Q_{24}$$

Hence,

$$Q_{24} = -(\dot{m}_{s1}h_{s1} - \dot{m}_{a4}h_{a4} - \dot{m}_{w3}h_{w3}) + (\dot{m}_{a1}h_{a1} + \dot{m}_{s1}h_{s1}) + W_{12}$$

Now,

$$\dot{m}_{a4} = \dot{m}_{a1} = \dot{m}_{a5} = 0.59538 \text{ kg/s}$$

 $\dot{m}_{s1} = \dot{m}_{s4} + \dot{m}_{w3}$

Hence,

$$Q_{24} = \dot{m}_a c_{pa} (T_1 - T_4) + (\dot{m}_{s1} h_{s1} - \dot{m}_{s4} h_{s4} - \dot{m}_{w3} h_{w3}) + W_{12}$$
(4.16)

15. We now determine the mass flow rate of the condensed water leaving the cooling coil. The mass balance of the moisture species around the cooler gives $m_{2s} = m_{3w} + m_{4s}$, hence $m_{3w} = m_{2s} - m_{4s}$. But,

$$m_{2s} = m_{1s}$$
$$m_{4s} = m_{5s}$$

Hence, we have

$$m_{3w} = m_{1s} - m_{5s}$$

= $0.0242255 - 0.0056557$
= $0.0191066kg/s$

- 16. The specfic enthalpy of condensed water leaving the cooler is that of the saturated liquid at the dew point, T_4 . By linear interpolation in the saturated liquid table between 11 and 12° C, we have $h_{3f} = 46.604494$ kJ/kg.
- 17. Specific enthalpy of the water vapour leaving the cooling coil is obtained by interpolation in the saturated vapour between 11 and 12°C. This gives $h_{4s} = 2521.282$ kJ/kg.
- 18. Substituting in the expression for Q_{24} (equation 4.16, we have

$$Q_{24} = 0.6230779 \times 1.005 \times (38 - 11.1)$$

$$+ (0.0056557 \times 2521.282 - 0.0242341 \times 2570.1 - 0.0001911 \times 46.604494) + 1.2$$

$$= 65.149618kW$$

Exercise 4.2 In a certain location, the barometric pressure, ambient air temperature and relative humidity are 1.013 bar, 40° C and 85% respectively. An air conditioning plant is designed to deliver 38 m^3 /min. of air to a room at $22 ^{\circ}$ C and relative humidity of 50%. The air leaves the cooling coil at the dew point and is reheated to the delivery temperature. The axial-flow fan is located before the cooler and is rated at 1.4 kW. Pressure remains constant throughout the system at 1.013 bar and there is no air recirculation. Specific heat of water vapour at low pressure may be taken as $c_{ps} = 1.864 \text{ kJ/kg}$.

- 1. Draw a schematic diagram of the system showing the basic energy and mass flow terms.
- 2. Determine the saturation vapour pressure of the moisture in the room air
- 3. Determine the partial pressure of the moisture in the room air.
- 4. Determine the temperature of the air leaving the cooling coil and entering the reheater.
- 5. Obtain an expression for calculating the rate of heat input by the reheater.
- 6. Determine the mass flow rate of moisture supplied to the room.
- 7. Calculate the mass flow rate of dry air supplied to the room.
- 8. Calculate the value of the rate of heat input by the heater.
- 9. Evaluate the saturation vapour pressure of moisture in the ambient air

- 10. Compute the partial pressure of moisture in the ambient air.
- 11. Obtain the specific enthalpy of the water vapour in the ambient air.
- 12. Calculate the moisture content of the ambient air.
- 13. Evaluate the mass flow rate of moisture in the ambient air.
- 14. Obtain an expression for calculating the rate of heat extracted by the cooling coil.
- 15. Calculate the mass flow rate of water condensed on the cooling coil.
- 16. Calculate the specific enthalpy of the condensed water leaving the cooling coil.
- 17. Determine the specific enthalpy of the water vapour leaving the cooling coil.
- 18. Calculate the value of the heat extracted by the cooling coil

4.6 Cooling Towers

Many industrial processes require large quantities of cooling water. Typical examples include steam power plants, refrigeration systems and chemical processing plants. If the plant is not located near a natural water source such as a river, the water will have to be cooled after use and recirculated. A cooling tower is one of the most effective methods of achieving the required cooling, and it employs the principle of evaporative cooling (See figure 4.8). Here, the incoming hot water is sprayed near the top of the tower and allowed to fall through a packing of material. The packing breaks up the water stream into small droplets and in this way provides a large wetted surface to facilitate evaporation. The warm water flowing down the tower is cooled mainly by evaporation, while the air flowing up is heated and may became saturated with water vapour. It is usual to design the tower such that the outlet water temperature is above the ambient wet-bulb temperature. To account for water lost by evaporation, make-up water is also provided.

The airflow through the tower may be achieved by forced or natural convection, as shown in figure 4.8.

Example 4.6 Water from a condensing plant is cooled in a forced-convection cooling tower. The airflow through the tower is induced by a fan, which consumes 5kW. The induced airflow rate is $1.85m^3$ /sec at atmospheric conditions of 1.013bar, 30° C and relative humidity of 60%. The air

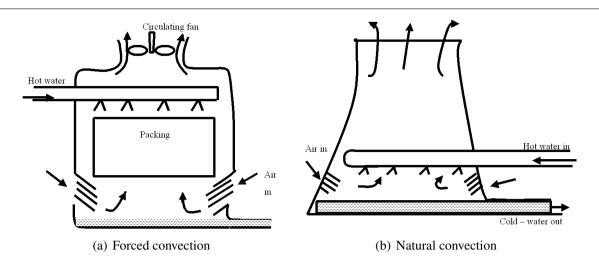


Figure 4.8: Cooling towers

leaves the tower saturated and at a temperature of 42°C. Water enters the tower at 1.38kg/s and a temperature of 65°C. Assuming that the pressure remains constant throughout, determine

- 1. Mass flow rate of make-up water
- 2. Temperature of cold water leaving the tower

Solution

A sketch of the plant is shown in figure 4.9

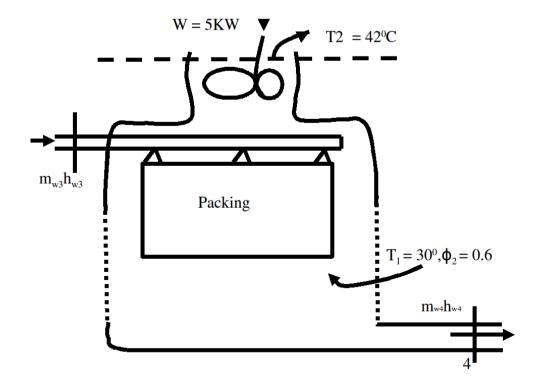


Figure 4.9: Induced draft cooling tower for example 4.6

<u>Air inlet.</u> At the air inlet (section l), We have $p_{g1} = p_g$ at $30^{\circ}C = 0.04242$ bar. Hence

$$p_{s1} = \phi_1 p_{g1} = 0.6 \times 0.04242$$
$$= 0.025452 \ bar$$

Further, we have

$$\dot{m}_{s1} = \frac{p_{s1}\dot{V}_1}{R_sT_1} = \frac{0.025452 \times 10^5 \times 1.85}{0.462 \times 10^3 \times (273 + 30)}$$
$$= 0.03364 \text{ kg/s}$$

Similarly, we determine \dot{m}_{a1} from the expression

$$\begin{split} \dot{m}_{a1} &= \frac{(p-p_{s1})\dot{V}_1}{R_aT} = \frac{(1.013-0.025452)\times 10^5\times 1.85}{0.287\times 303\times 10^3} \\ &= 2.1009 \ \textit{kg/s} \end{split}$$

Air Outlet

At the outlet (section 2), the air is saturated.

Thus, we have $p_{s2} = p_{g2}$ at $42^{\circ}C = 0.08198$ bar. The moisture content, or specific humidity of the air at outlet is given from equation (4.4) as

$$w_2 = 0.622 \frac{p_{s2}}{p_2 - p_{s2}} = 0.622 \frac{0.08198}{1.013 - 0.08198}$$
$$= 0.05477$$

Now, $\dot{m}_{a2} = \dot{m}_{a1} = \dot{m}_a$. Hence,

$$\dot{m}_{s2} = w_2 \dot{m}_{a2} = 0.05477 \times 2.1009$$

= 0.11507 kg/s

The rate of water lost by evaporation (which has to be replaced by make-up water), is given by

$$\Delta \dot{m}_w = \dot{m}_{s2} - \dot{m}_{s1}$$

$$= 0.11507 - 0.03364$$

$$= 0.08143 \text{ kg/s}$$

Now, $\dot{m}_{w3} = 1.38$. Applying a mass balance to the water, we have

$$1.38 + 0.03364 = \dot{m}_{w4} + 0.11507$$

$$\therefore \dot{m}_{w4} = 1.38 + 0.03364 - 0.11507$$

$$= 1.29857 \, kg/s$$

Neglecting changes of kinetic energy and heat loss from the tower to the atmosphere, the energy equation is

$$\dot{m}_{w3}h_{w3} + \dot{m}_{a1}h_{a1} + \dot{m}_{s1}h_{s1} + W
= \dot{m}_{w4}h_{w4} + \dot{m}_{a2}h_{a2} + \dot{m}_{s2}h_{s2}$$
(4.17)

Now, from steam tables

$$h_{s1} = h_g$$
 at $30^{\circ}C = 2555.7$ kJ/kg
 $h_{s2} = h_g$ at $42^{\circ}C = 2577.2$ kJ/kg

For water and air, we take $h=c_pT$, where we take h to be zero at the triple point of water (i.e. $h_f=0$ at $0^{\circ}C$). Also, $h_{w3}=h_f$ at $65^{\circ}C=272kJ/kg$

For air, we take $h = c_p T$, $c_p = 1.005 kJ/Kg$. From equation (4.17), we have

$$h_{w4} = \frac{\dot{m}_{w3}h_{w3} + \dot{m}_a c_p (T_1 - T_2) + (\dot{m}_{s1}h_{s1} - \dot{m}_{s2}h_{s2}) + W}{\dot{m}_{w4}}$$

Hence

$$h_{w4} = \begin{bmatrix} 1.38 \times 272 + 1.005 \times 2.1009(30 - 42) \\ +0.03364 \times 2555.7 - 0.11507 \times 2577.2 + 5 \end{bmatrix} / 1.29857$$

Thus,

$$h_{w4} = (375.36 - 25.33685 + 85.9737 - 296.558 + 5)/1.29857$$

= 144.4389/1.29857 = 111.229 kJ/kg

From the steam tables,

$$T_4 = 26 + \frac{111.229 - 108.9}{113.1 - 108.9} (27 - 26)$$
$$= 26 + .055$$
$$= 26.6^{\circ} C$$

- **Example 4.7** Water from a condensing plant is cooled in a forced-draught cooling tower. The draught is produced by a fan absorbing 4.8 kW and it induces an air flow of 2.05 m^3/s at atmospheric conditions of 1.013 bar, 21° C and relative humidity of 65%. The air leaves the tower saturated and at a temperature of 32° C. The water enters the tower at 62° C and mass flow rate of \dot{m}_{w3} while cooled water leaves at a temperature of T_{w4} and mass flow rate of 1.43 kg/s.
 - 1. Draw a well-labelled schematic diagram of the tower.
- 2. What is the function of the packing used in the tower?

In questions 3 to 7, at the inlet, section 1, determine

- 3. the saturation vapour pressure of the inlet air, P_{g1}
- 4. the partial pressure of the moisture in the inlet air, P_{s1}
- 5. the specific enthalpy of the moisture, h_{s1}
- 6. the moisture mass flow rate at inlet, \dot{m}_{s1}
- 7. the dry air mass flow rate, \dot{m}_{a1}

In questions 8 to 12, at the outlet section 2, determine

- 8. the saturation vapour pressure, P_{g2}
- 9. the partial pressure of the moisture, P_{s2}
- 10. the specific enthalpy of the moisture, h_{s2}
- 11. the moisture content or specific humidity of the air, w_2
- 12. the mass flow rate of the moisture, \dot{m}_{s2}

- 13. Derive a mass balance expression for the moisture species in the tower and hence or otherwise obtain an equation for \dot{m}_{w3}
- 14. Calculate the value of the mass flow rate of water entering the tower, \dot{m}_{w3}
- 15. Determine the rate of water lost by evaporation which has to be replaced by makeup water
- 16. Neglecting changes in kinetic energy and any heat losses from the tower, derive the energy balance equation for the entire tower, and hence or otherwise obtain an expression for determining the specific enthalpy of the water leaving the tower, h_{w4} .

In questions 17 to 18, for the cold water leaving at section 4, determine

- 17. the specific enthalpy, h_{w4}
- 18. the temperature, T_{w4}

Solution

- 1. A sketch of the plant is similar to that shown in figure 4.9
- 2. Function of the cooling tower is to spread the water into small droplets and therefore increase the surface area available for heat and mass transfer.
- 3. Air inlet. At the air inlet (section l), We have $p_{q1} = p_q$ at $21^{\circ}C = 0.02486$ bar.
- 4. $\phi_1 = 0.65$, Hence

$$p_{s1} = \phi_1 p_{g1} = 0.65 \times 0.02486$$
$$= 0.016159 \ bar$$

5. The specfic enthalpy of the water vapour in the ambient air is given by

$$h_{s1} = h_a$$
 at $21^{\circ}C = 2539.4$ kJ/kg

6. Further, we have

$$\dot{m}_{s1} = \frac{p_{s1}\dot{V}_1}{R_sT_1} = \frac{0.016159 \times 10^5 \times 2.05}{0.462 \times 10^3 \times (294)}$$
$$= 0.0243882 \text{ kg/s}$$

7. We determine the air mass flow rate \dot{m}_{a1} from the expression

$$\begin{split} \dot{m}_{a1} &= \frac{(p-p_{s1})\dot{V}_1}{R_aT} = \frac{\left(1.013-0.016159\right)\times10^5\times2.05}{0.287\times294\times10^3} \\ &= 2.4219\ \textit{kg/s} \end{split}$$

Air Outlet

- 8. At the outlet (section 2), the air is saturated.
- 9. Thus, we have $p_{s2} = p_{g2}$ at $32^{\circ}C = 0.04754$ bar.
- 10. The specific enthalpy of the water vapour at tower exit is given by

$$h_{s2} = h_g$$
 at $32^{\circ}C = 2559.3$ kJ/kg

11. The moisture content, or specific humidity of the air at outlet is given as

$$w_2 = 0.622 \frac{p_{s2}}{p_2 - p_{s2}} = 0.622 \frac{0.04754}{1.013 - 0.04754}$$
$$= 0.0306278$$

12. Now, $\dot{m}_{a2} = \dot{m}_{a1} = \dot{m}_a$. Hence,

$$\dot{m}_{s2} = w_2 \dot{m}_{a2} = 0.0306278 \times 2.4219$$

= 0.0741764 kg/s

13. Now, the moisture balance equation is given as

$$m_{w3} + m_{s1} = m_{w4} + m_{s2}$$

Hence, $m_{w3} = m_{w4} + m_{s2} - m_{s1}$

14. Substituting into the above, we have

$$m_{w3} = 1.43 + 0.0741764 - 0.0243882$$

= 1.4797882
= ≈ 1.47979

The rate of water lost by evaporation (which has to be replaced by make-up water), is given by

$$\Delta \dot{m}_w = \dot{m}_{w3} - \dot{m}_{w4}$$

$$= 1.47979 - 1.43$$

$$= 0.04979 \text{ kg/s}$$

Now, $\dot{m}_{w3} = 1.38$. Applying a mass balance to the water, we have

$$1.38 + 0.03364 = \dot{m}_{w4} + 0.11507$$

Hence, $\dot{m}_{w4} = 1.38 + 0.03364 - 0.11507$
 $= 1.29857 \text{ kg/s}$

15. Neglecting changes of kinetic energy and heat loss from the tower to the atmosphere, the energy equation is

$$\dot{m}_{w3}h_{w3} + \dot{m}_{a1}h_{a1} + \dot{m}_{s1}h_{s1} + W
= \dot{m}_{w4}h_{w4} + \dot{m}_{a2}h_{a2} + \dot{m}_{s2}h_{s2}$$
(4.18)

16. For water and air, we take $h = c_p T$, where we take h to be zero at the triple point of water (i.e. $h_f = 0$ at $0^{\circ}C$). Also, $h_{w3} = h_f$ at $62^{\circ}C = (251.1 + 272)/2 = 259.46$ kJ/kg

For air, we take $h = c_p T$, $c_p = 1.005$ kJ/Kg. From equation (4.17), we have

$$h_{w4} = \frac{\dot{m}_{w3}h_{w3} + \dot{m}_a c_p (T_1 - T_2) + (\dot{m}_{s1}h_{s1} - \dot{m}_{s2}h_{s2}) + W}{\dot{m}_{w4}}$$

17. Hence, substituting the numerical values, we have

$$h_{w4} = \begin{bmatrix} 1.43 \times 259.46 + 1.005 \times 2.4219(21 - 32) \\ +0.0243882 \times 2539.4 - 0.0741764 \times 2559.3 + 4.8 \end{bmatrix} / 1.29857$$

Thus,

$$h_{w4} = 163.68 \, kJ/kg$$

18. The temperature of the cold water leaving section 4 is now determine. By interpolation from the steam tables, we have

$$T_4 = 38 + \frac{163.68 - 159.1}{167.5 - 159.1} (40 - 38)$$
$$= 39.09^{\circ} C$$

- Exercise 4.3 1. Turbine condenser cooling water is sprayed at a rate of 1000 kg/min into a natural convection cooling tower at a temperature of 26°C, and the water returns to the condenser at 12°C. Air is drawn into the tower at 15°C, with a relative humidity of 55%, and leaves the tower saturated at 24°C. Calculate the mass flow rate of moist air into the tower and the rate of loss of cooling water by evaporation. It may be assumed that the total pressure throughout the tower is 1 atm. (Answer: 1411 kg/min, 18.31 kg/min).
 - 2. Water from a condensing plant is cooled in a forced-draught cooling tower. The draught is produced by a fan absorbing 4 kW as it induces an air flow of 110 m³/min at atmospheric conditions of 1.013 bar, 17°C and relative humidity 60%. The air leaves the tower saturated and at a temperature of 30°C. If the temperature of the water entering the tower is 50°C and the rate of flow from the tower is 80 kg/min, find the temperature of the water leaving the tower and the rate of flow of water entering the tower. (Answer: 27.2°C, 82.65 kg/min).
 - 3. A fan discharges 300 m³ of moist air per minute at 36°C, 1.10 bar and 92% relative humidity. Find the specific humidity of the mixture, and the mass of water vapour and dry air passing through the fan per minute.
 - The moist air is passed over refrigerator coils in order to dehumidify it, and leaves the refrigerator chamber with a pressure and temperature of 0.98 bar and 7°C. The condensate also leaves the chamber at 7°. Determine the final specific humidity of the mixture, the mass of water condensing on the cooling coils per minute, and the rate of heat transfer to the cooling coils. (Answer: 0.0325, 11.49 kg/min, 353.6 kg/min; 0.00642, 9.22 kg/min, -33820 kJ/min)
 - 4. A room in summer is to be maintained at 18°C, 50% saturation when the outside conditions are 30°C, 80% saturation. The sensible heat gains and latent heat gains are 4.4 kW and 1.89 kW respectively.

The conditioned air is supplied through ducts from a central station consisting of a cooler battery, a reheat battery, and a fan. Fresh air is supplied to a mixing unit where it mixes with a certain percentage of air recirculated from the room, the reminder of the room air being expelled to the atmospher.

The air entering the room is at 12.5° , the air temperature rise in the fan and duct work is 1 K, the air leaving the cooler battery and entering the reheat battery is at 7° C, and the apparatus dew point of the cooler is 1.5° C.

Draw a sketch of the plant, numbering the relevant points, and calculate

- (a) the ratio of the mass rate of flow of recirculated air to the mass rate of air supplied to the room;
- (b) the cooler battery load;
- (c) the reheater battery load;
- (d) the cooler battery bypass factor.

If using the psychrometric chart, take the barometric pressure as 1.01325 bar. (Answer: 0.88; 15.4 kW; 3.5 kW; 0.3)

Chapter 5						
Cooling Load Calculations						

REFERENCES

Dossat R J (1990) Principles of Refrigeration Second Edition. Wiley International, New York

Eastop and McConkey (2003) *Applied Thermodynamics for Engineering Technologists*, **Pearson Education**, New York

Jones W P (1985) Air Condition Engineering. Edward Arnold, London.

Rogers and Mayhew (1978) *Thermodynamic and transport Properties of Fluids*, **Pearson Education**, New York

Rogers and Mayhew (2003) *Thermodynamics, Work and Heta Transfer*, **John Wiley & Sons**, New York

Rajput