UNIVERSITY OF AMSTERDAM

Wealth Distribution Using Lattice Gas Automata (LGA)

Investigating economic dynamics through computational modeling

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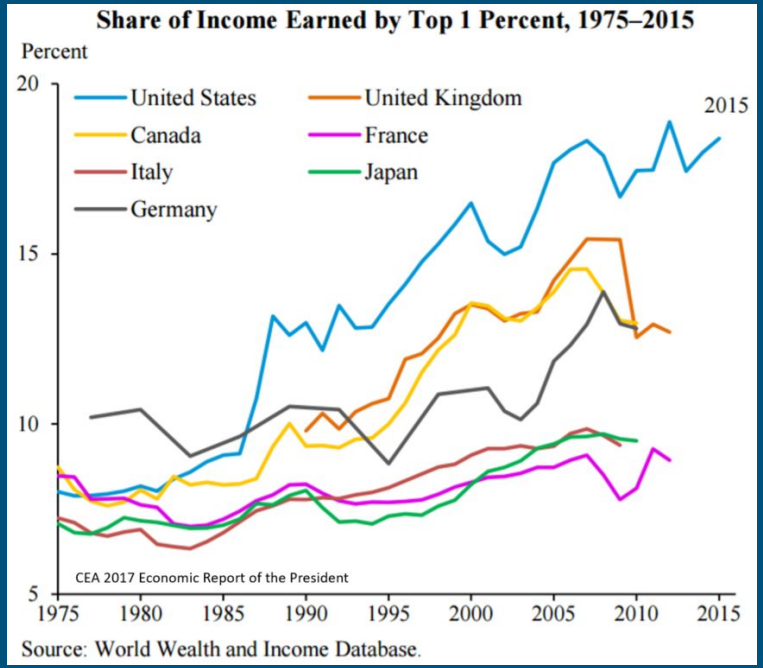
Agenda

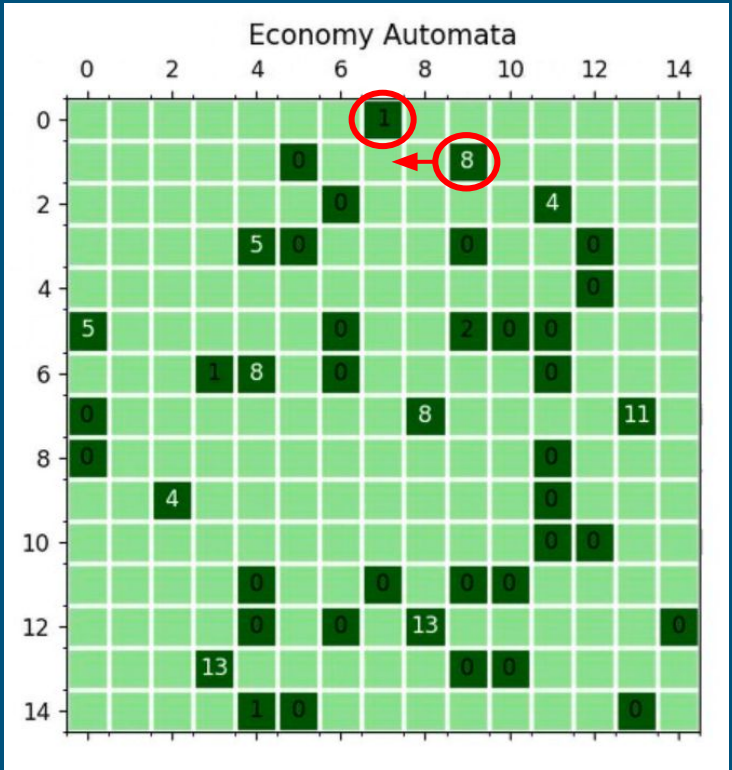
- Theoretical Background
- Model:
  - Lattice gas automata (LGA)
  - Transaction rules, charity and taxation
- Investigation focus
- Research question
- Hypothesis

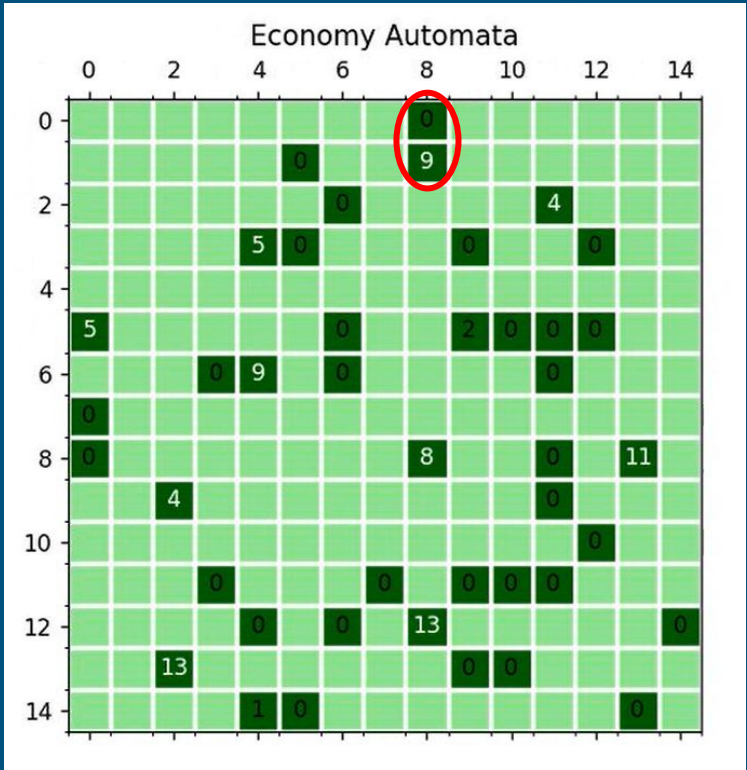
Theoretical Background

- Relevance
  - Income inequality
  - Knock-on effect of disparity
- Matthew effect
  - Individuals with more money higher probability to earn more and vice versa
  - Skewed income distributions
  - Gini coefficient
    - 0 perfectly equal
    - 1 perfectly unequal

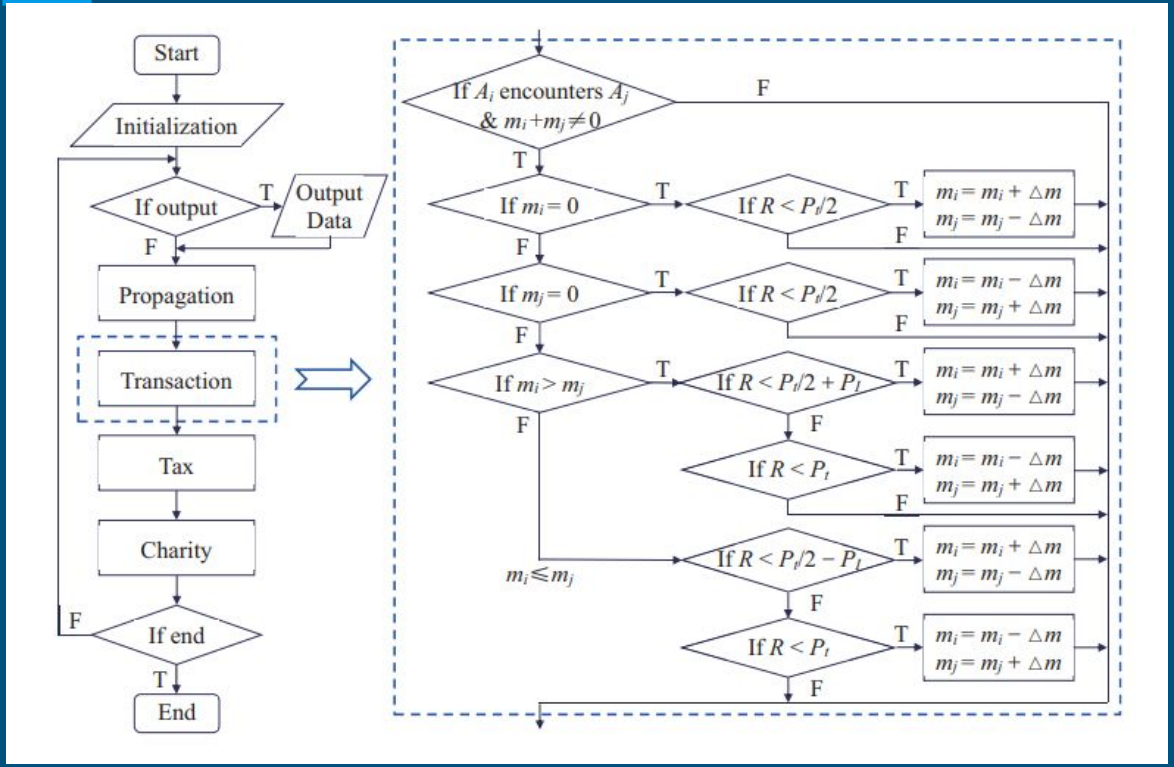


Movement of agents





Simulation rules



Tax:

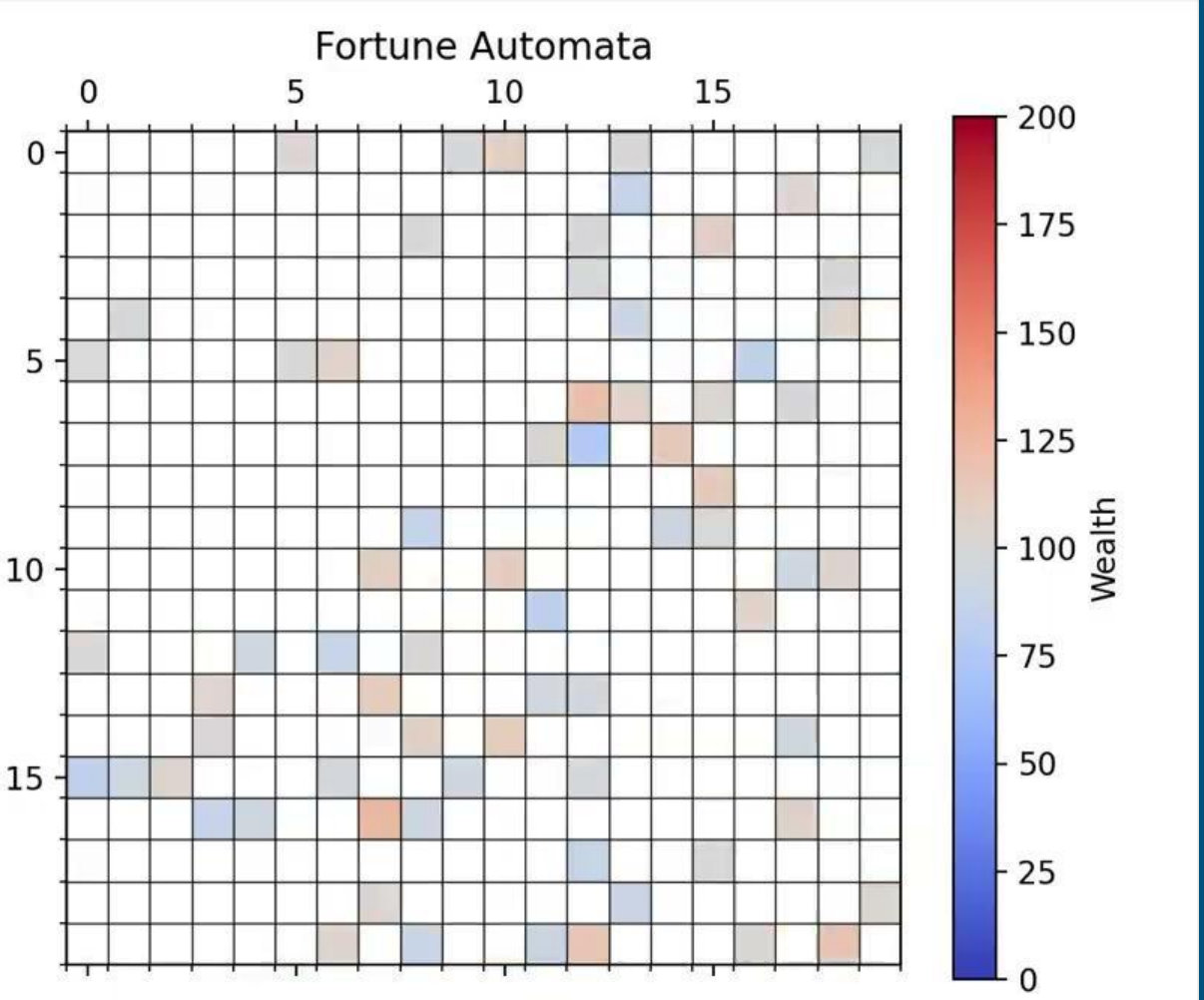
$$\Psi_i = \psi_i \Delta m,$$

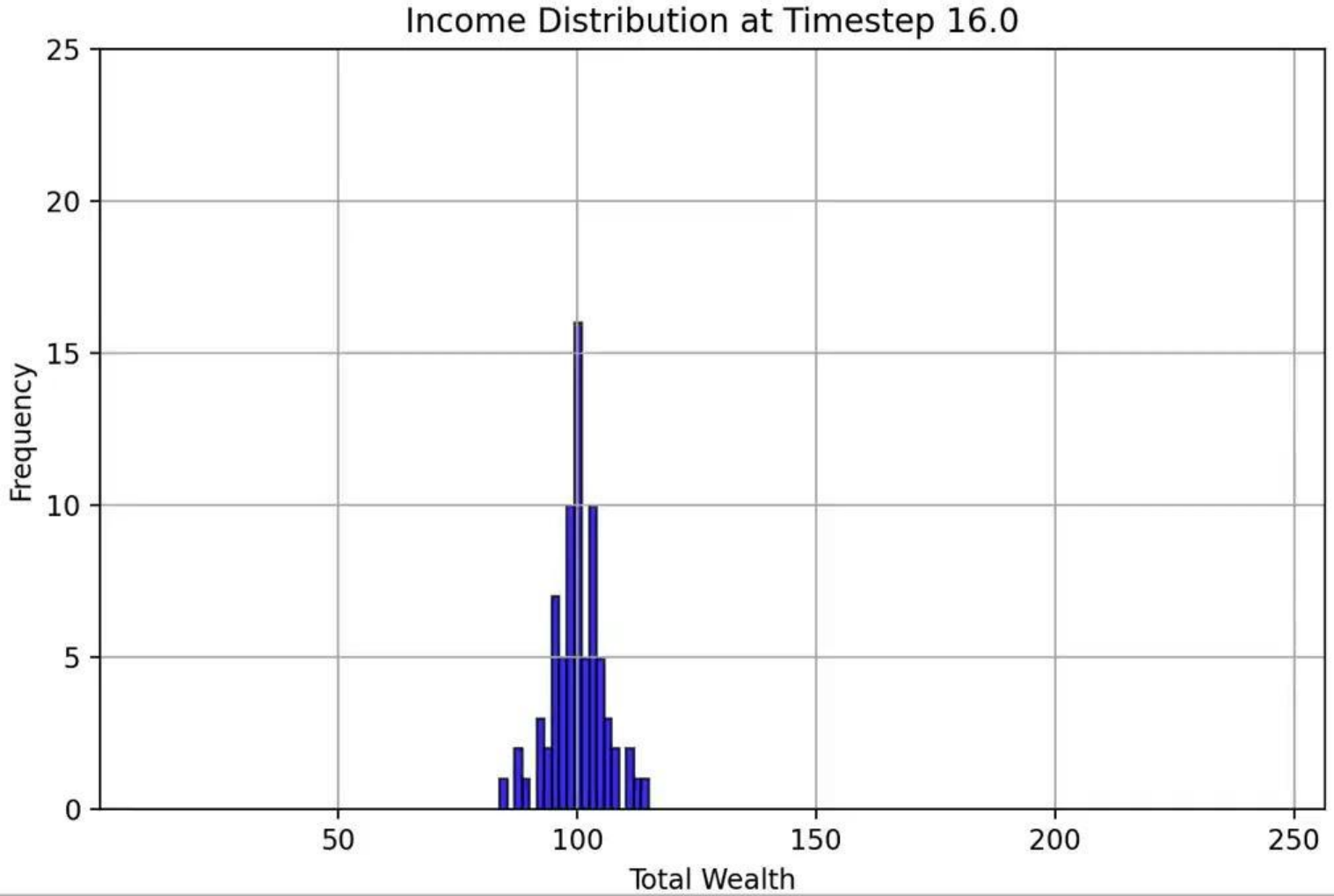
$$\psi_i = \left(\frac{m_i}{m_{max}}\right)^{\omega} \psi_{max},$$

Charity:

$$m_i \leftarrow m_i - m_c,$$

$$m_i \leftarrow m_i + \frac{\Phi_c}{N_p},$$


Fortune Automata
0
5
10
15
0
5
10
15
200
175
150
125
100
75
50
25
0
Wealth


Income Distribution at Timestep 16.0
Frequency
Total Wealth
0
5
10
15
20
25
50
100
150
200
250

Investigation Focus

- Focus on effects of taxation and charity:
  - Most directly applicable to real-life economic reality
  - Possibility for policy to be adjusted and implemented based on findings
- Emergent behaviour:
  - Taxation effectiveness
  - Charity effectiveness
  - Phase transition

Research Objectives

How does varying charity and taxation rates affect inequality in an economy?

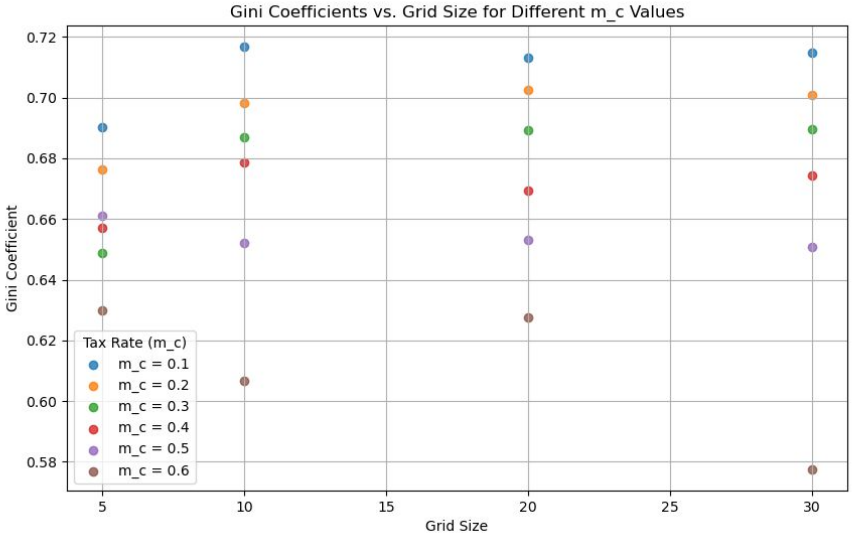
1. How does the ratio of rich over poor change over time?
2. Can we observe a phase transition in the number of rich agents?
3. How does grid size impact the fraction of rich agents?

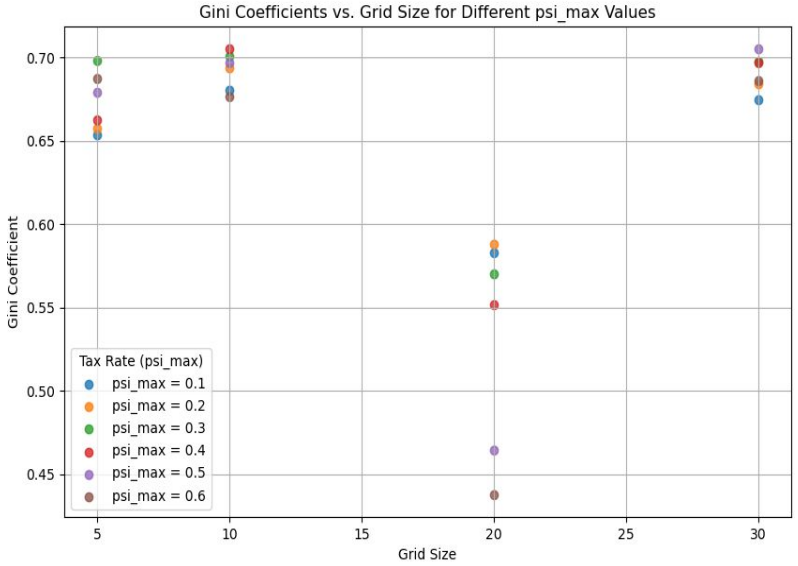
Can we make policy recommendations for curbing income inequality growth?

Hypothesis

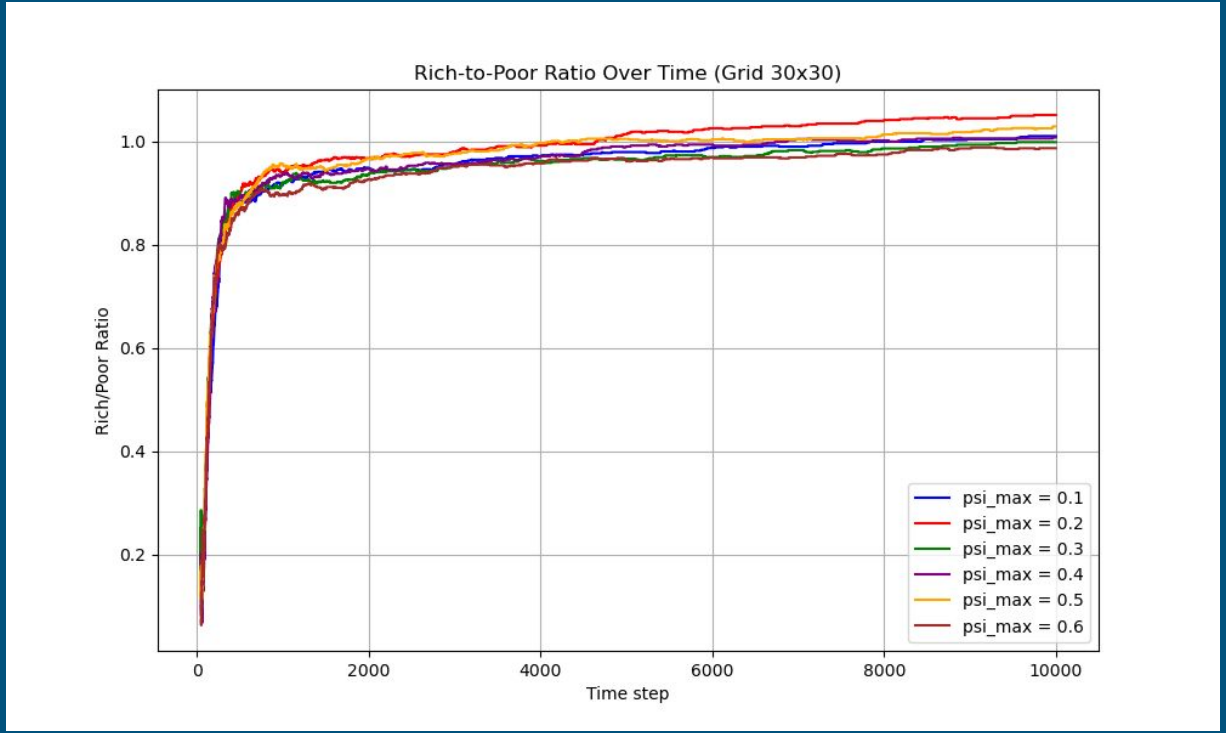
1. We expect taxation to have a more pronounced effect on the income than charity contributions, since charity is not fully mandatory.
2. We expect to see a sharp decline in the number of rich agents as the tax contribution increases but the charity contribution should not have a significant effect.
3. We expect that increasing the grid size would lead to a lessening of income inequality, due to the redistributive effect of taxation and charity to is not localised.

Inequality


Gini Coefficients vs. Grid Size for Different m_c Values
Gini Coefficient
Grid Size
Tax Rate (m_c)
m_c = 0.1
m_c = 0.2
m_c = 0.3
m_c = 0.4
m_c = 0.5
m_c = 0.6


Gini Coefficients vs. Grid Size for Different psi_max Values
Gini Coefficient
Grid Size
Tax Rate (psi_max)
psi_max = 0.1
psi_max = 0.2
psi_max = 0.3
psi_max = 0.4
psi_max = 0.5
psi_max = 0.6

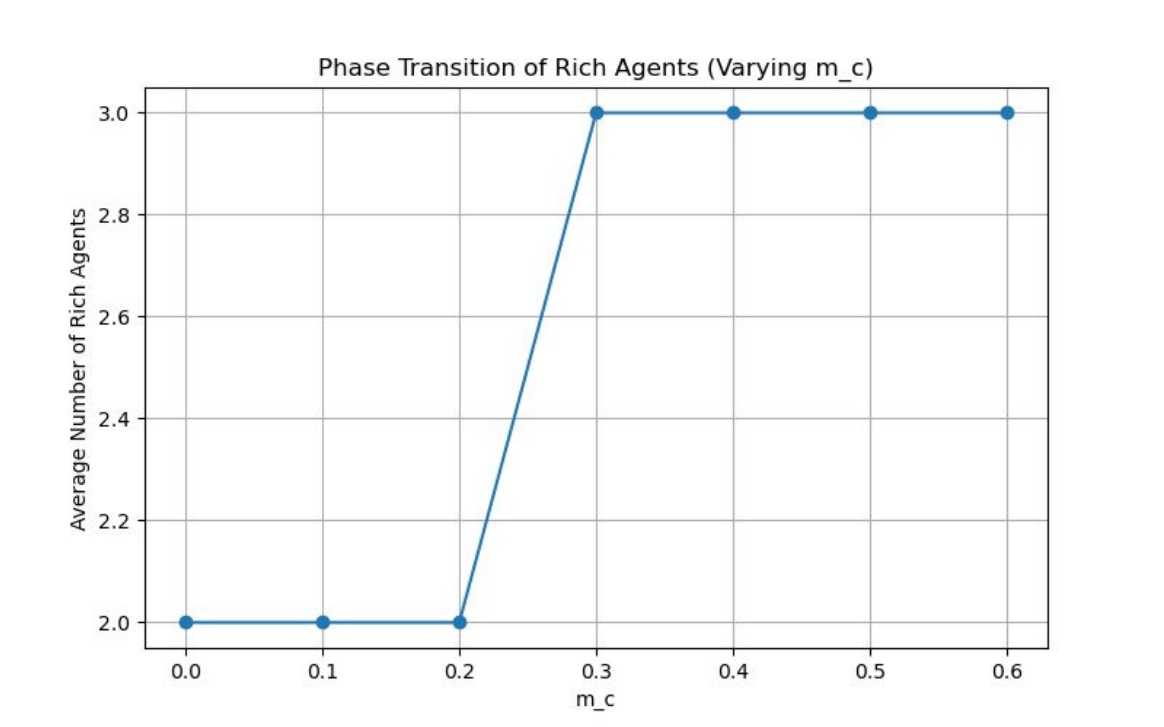
Distributions



Phase Transition (What happens to the rich?)

Grid size = 5x5

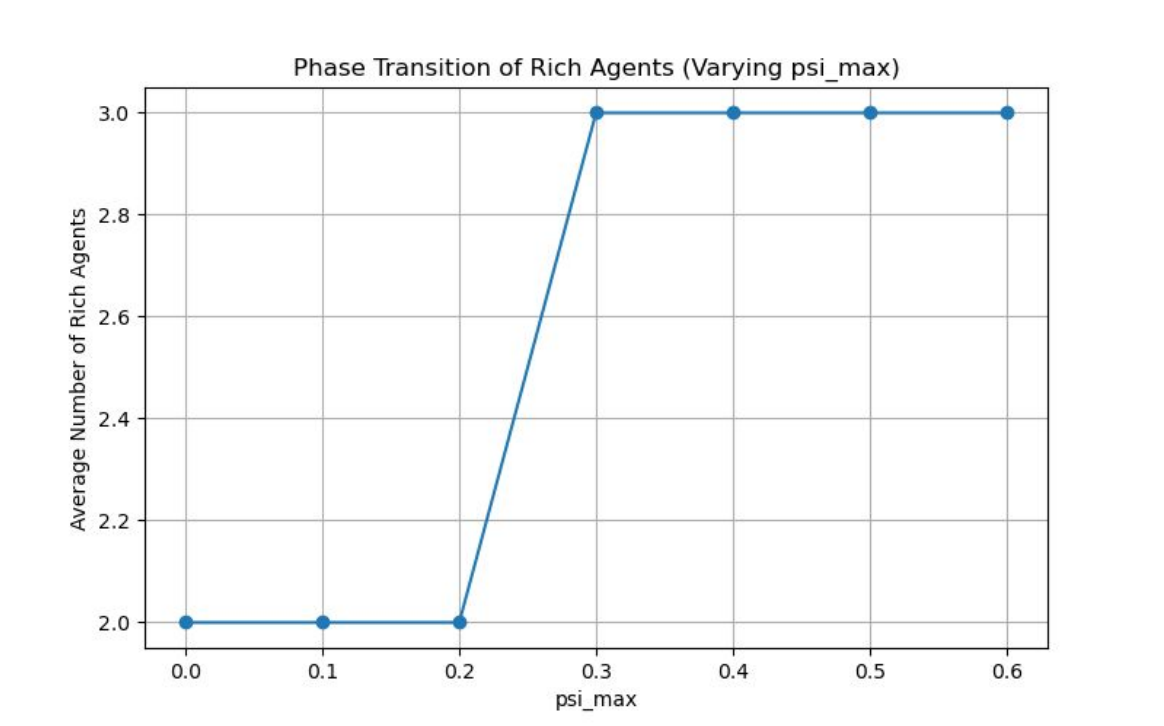
Tax contribution (psi_max) = 0.3



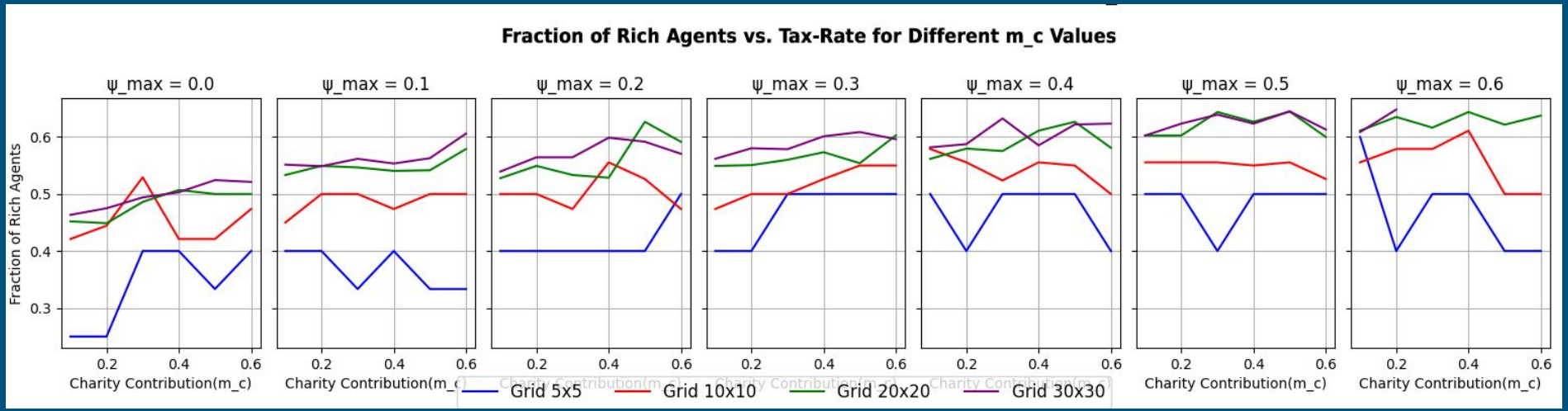
Phase Transition (What happens to the rich?)

Grid size = 5x5

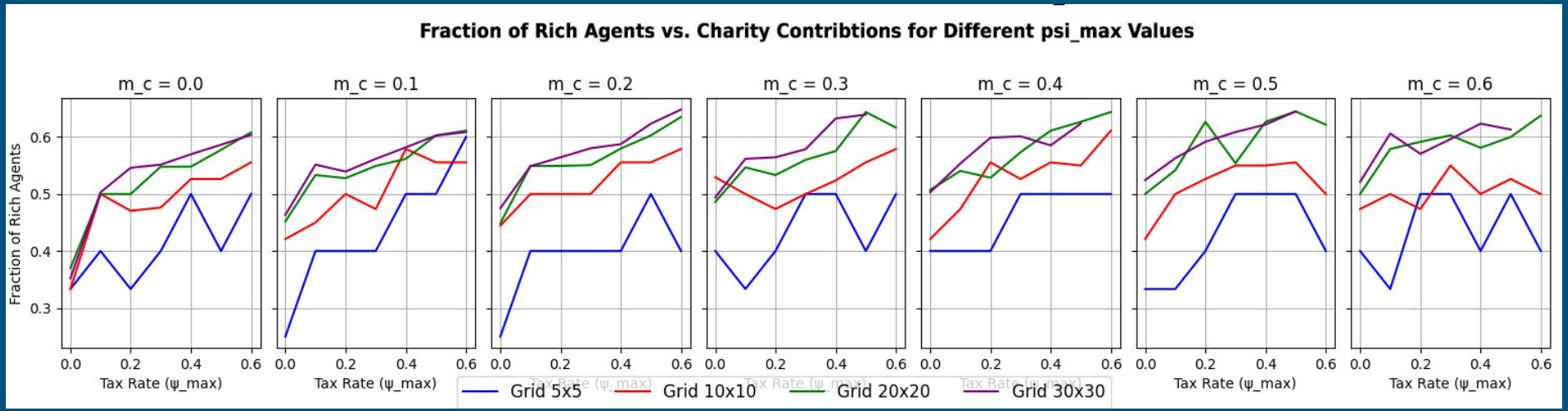
Charity contribution m_c = 0.4



Finite-Size Scaling



Finite-Size Scaling



Conclusion

- We find that charity contributions bring about a lesser inequality in large systems.
- The effect of the tax rate on the system is inconclusive, a clear effect is not to be seen.

Suggested Improvements/Limitations

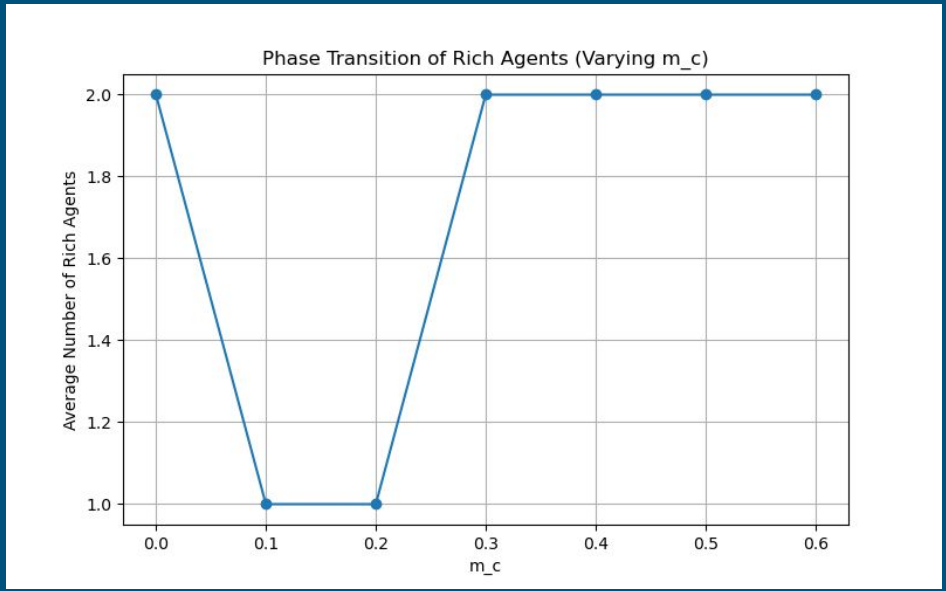
- Implement dynamic thresholds for ‘richness’ and ‘poorness’ of agents such that the categorization of agents changes with increase in the wealth of the economy.
- Implement local charity and tax redistributions to compare with our current implementation and contrast with our current model.
- Run many simulations for each parameter set to get confidence interval for each result.
- Look at how hexagonal grids affect agent dynamics.

References

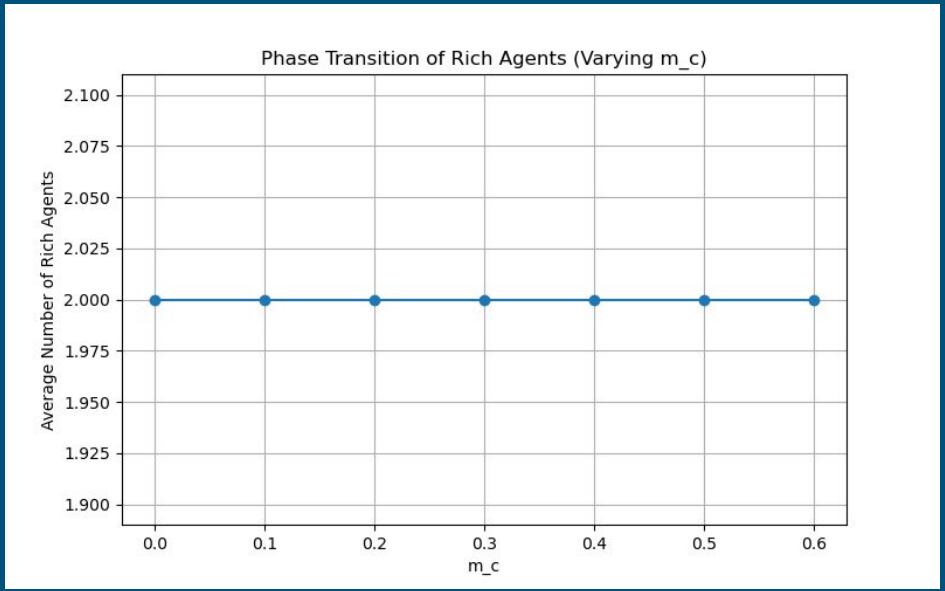
1. Cui, L., & Lin, C. (2020). Lattice–Gas–Automaton Modeling of Income Distribution. Entropy, 22(7), 778. https://doi.org/10.3390/e22070778
2. Lo, Shih-Ching. "Cellular Automata Simulation for Wealth Distribution." In *AIP Conference Proceedings*, vol. 1148, no. 1, pp. 476-479. American Institute of Physics, 2009.
3. J. Cerdá, C. Montoliu, R.J. Colom, LGEM: A lattice Boltzmann economic model for income distribution and tax regulation, Mathematical and Computer Modelling, Volume 57, Issues 7–8, 2013, Pages 1648-1655, ISSN 0895-7177, https://doi.org/10.1016/j.mcm.2011.10.051.
4. Champernowne, D. G., & Cowell, F. A. (1998). *Economic Inequality and Income Distribution*. Cambridge University Press.

Appendix

Phase Transition (5x5, varying charity, rich)

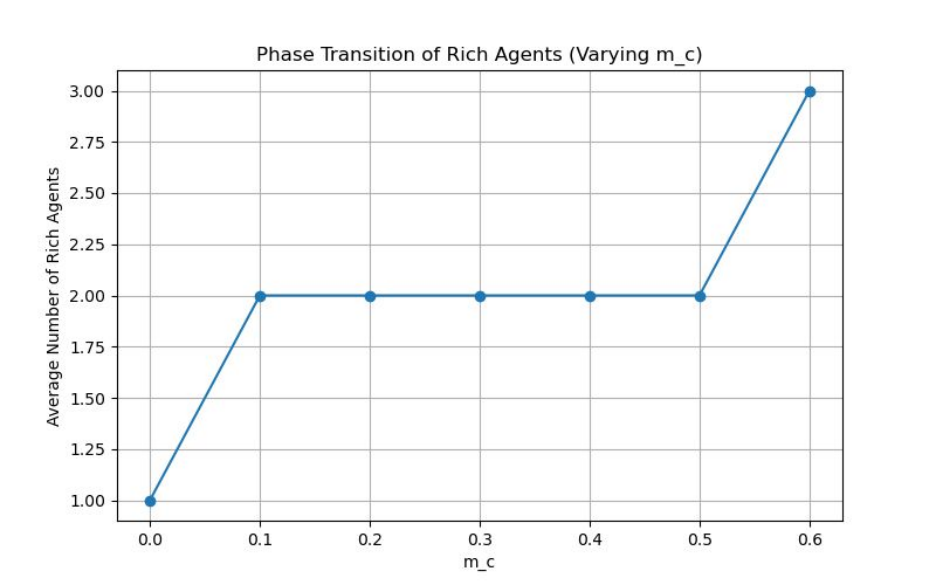


psi_max = 0.0

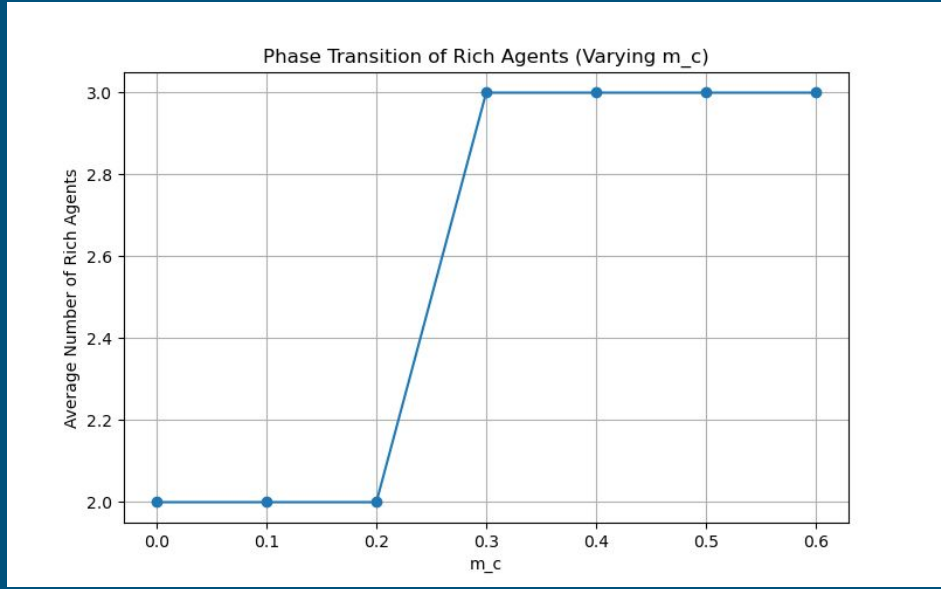


psi_max = 0.1

Phase Transition (5x5, varying charity, rich)

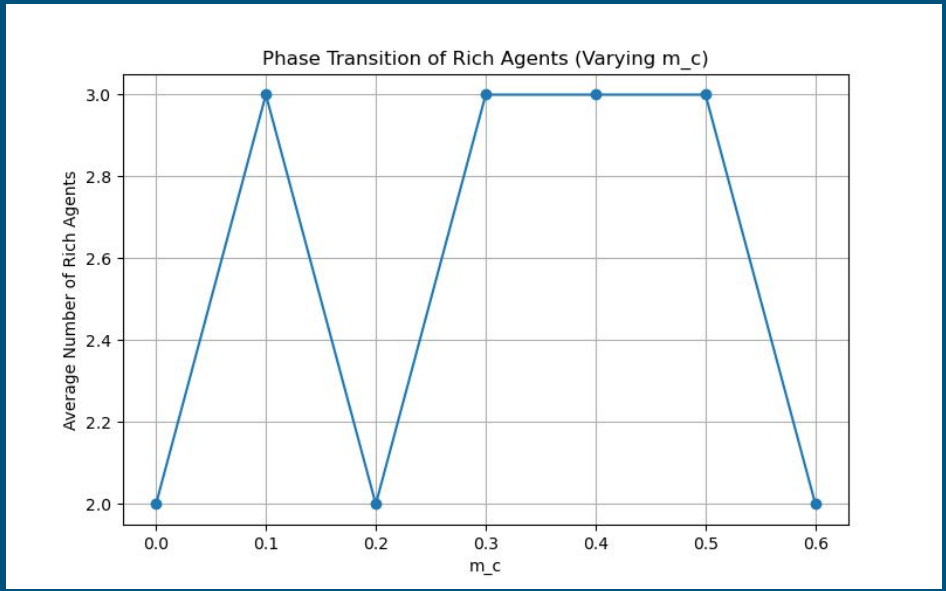


psi_max = 0.2

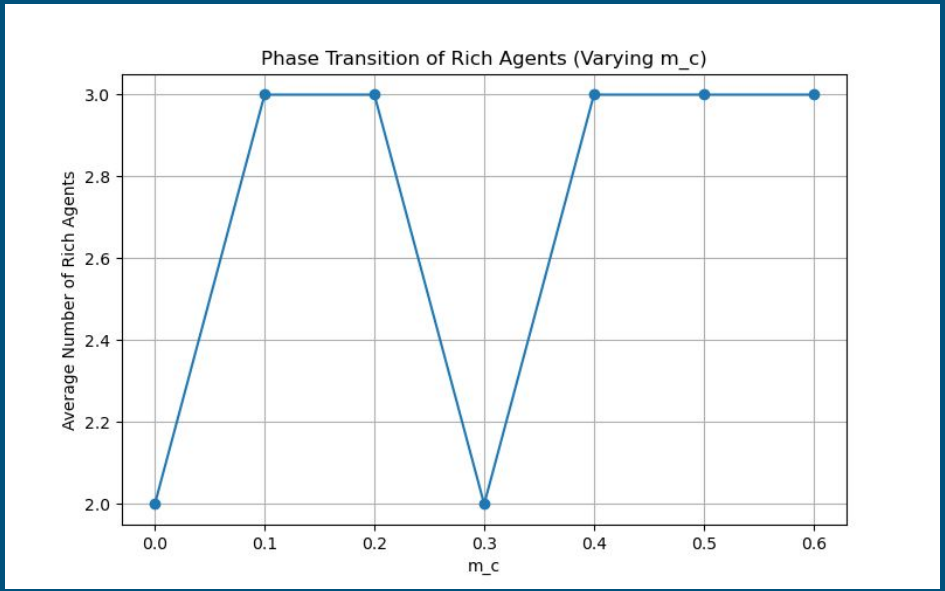


psi_max = 0.3

Phase Transition (5x5, varying charity, rich)

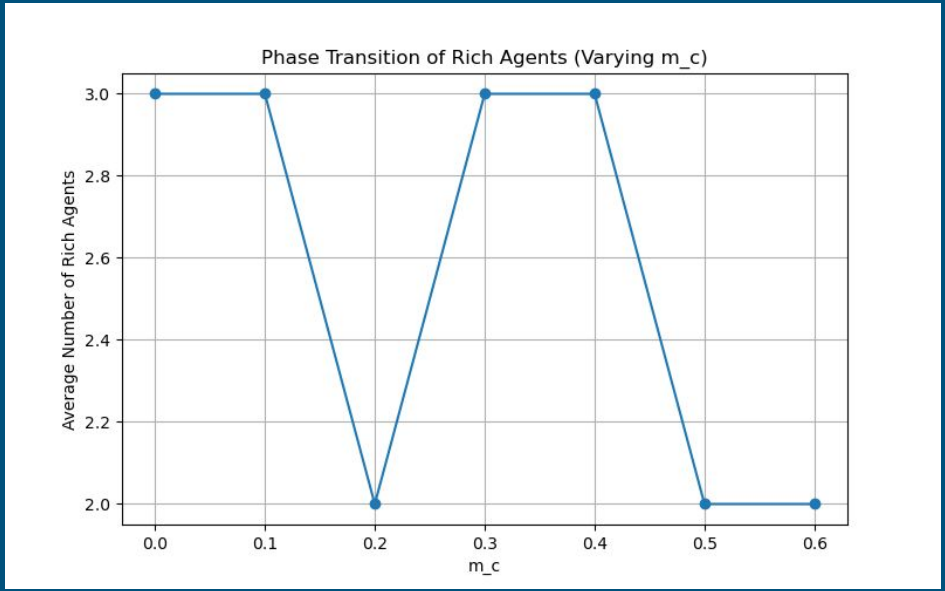


psi_max = 0.4



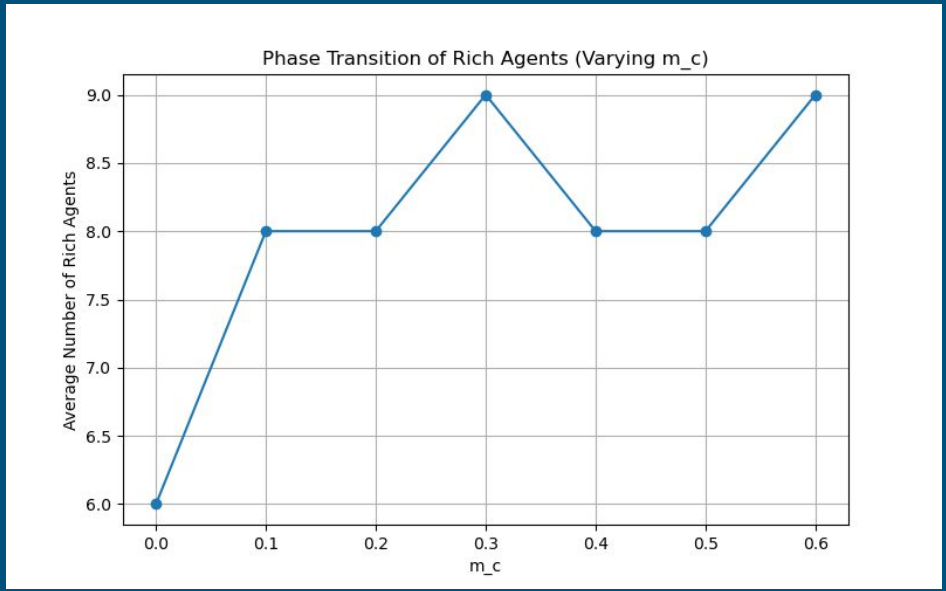
psi_max = 0.5

Phase Transition (5x5, varying charity, rich)

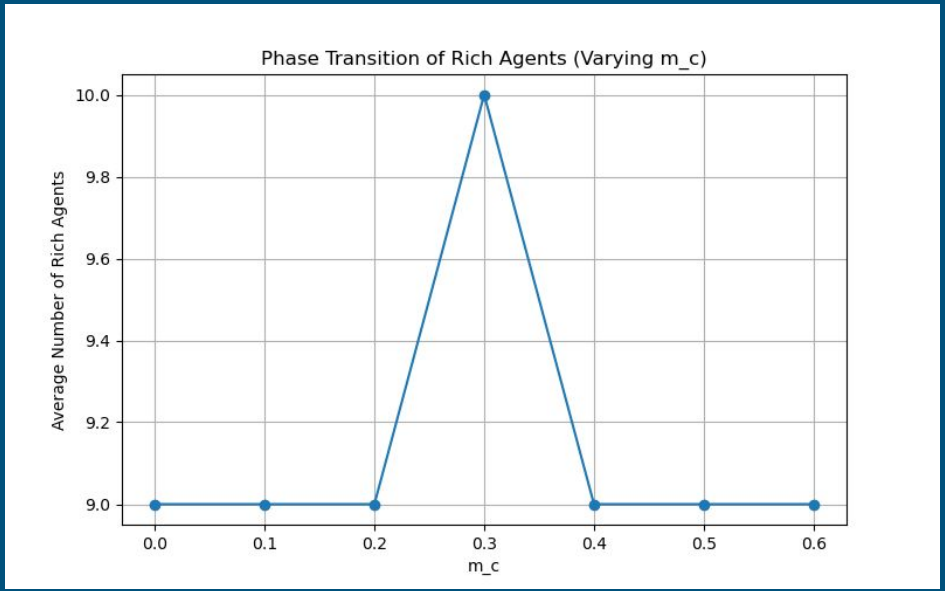


psi_max = 0.6

Phase Transition (10x10, varying charity, rich)

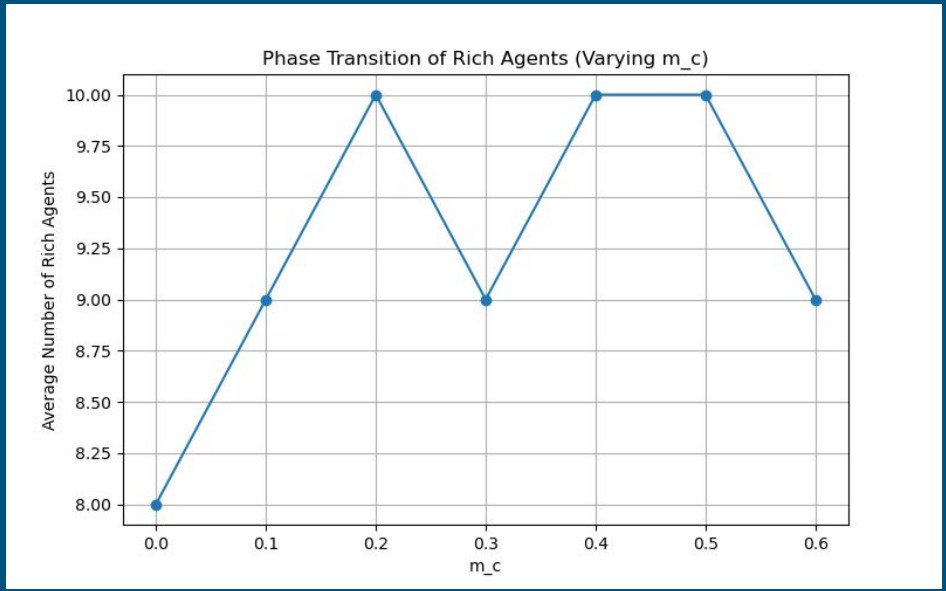


psi_max = 0.0

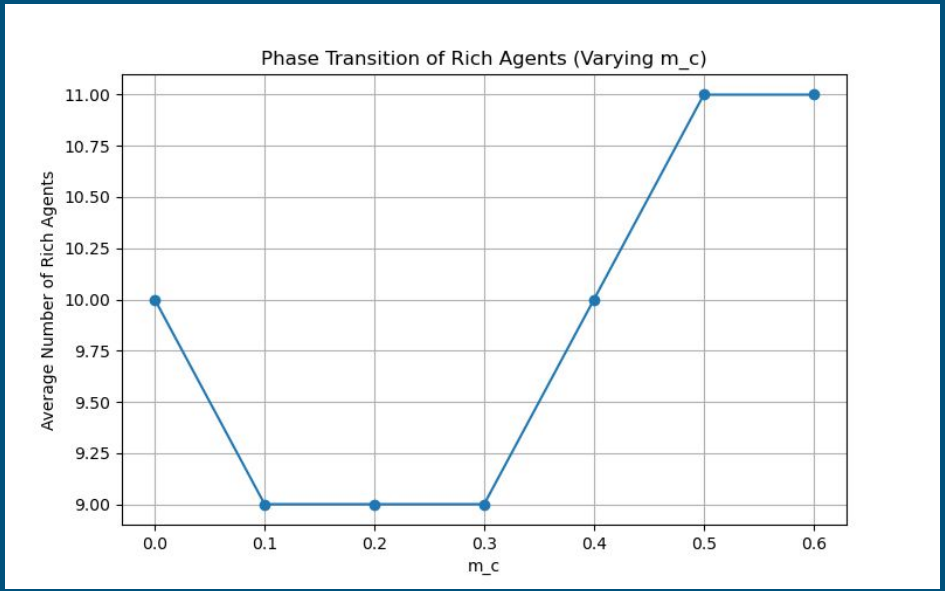


psi_max = 0.1

Phase Transition (10x10, varying charity, rich)

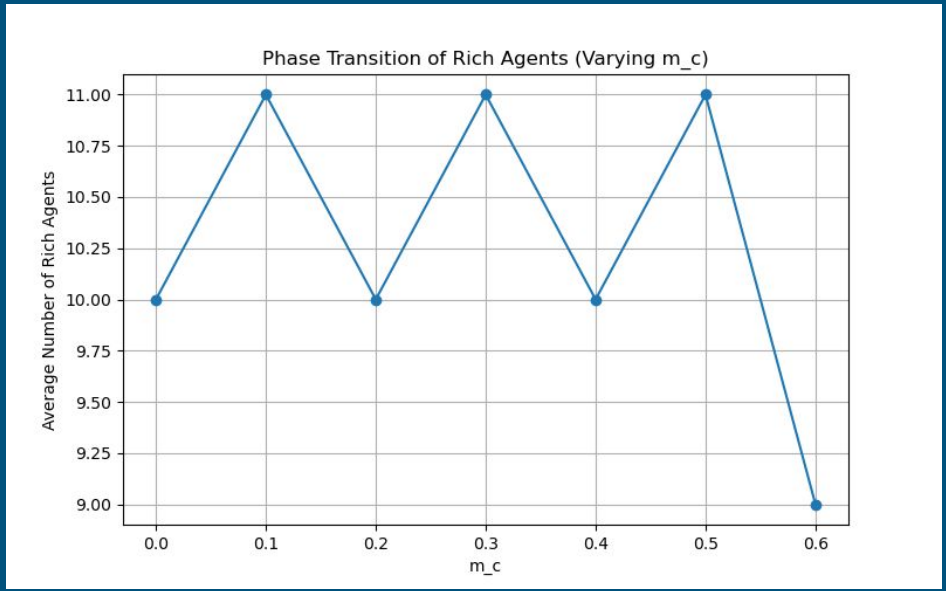


psi_max = 0.2

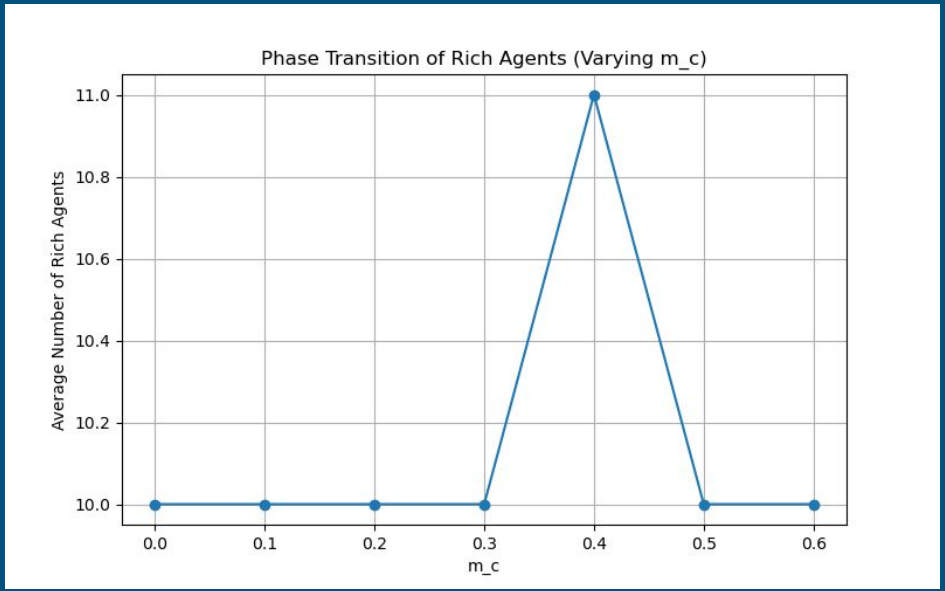


psi_max = 0.3

Phase Transition (10x10, varying charity, rich)

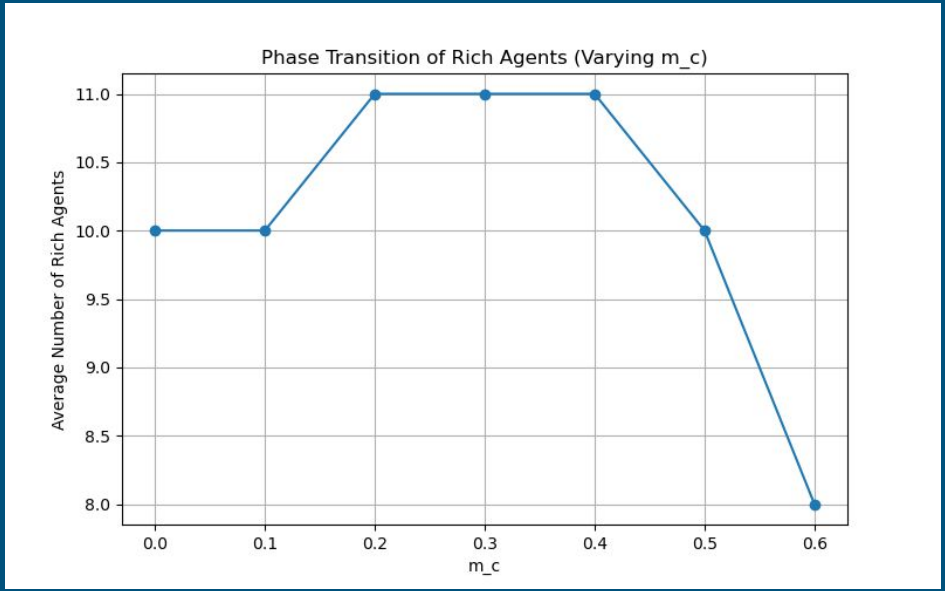


psi_max = 0.4



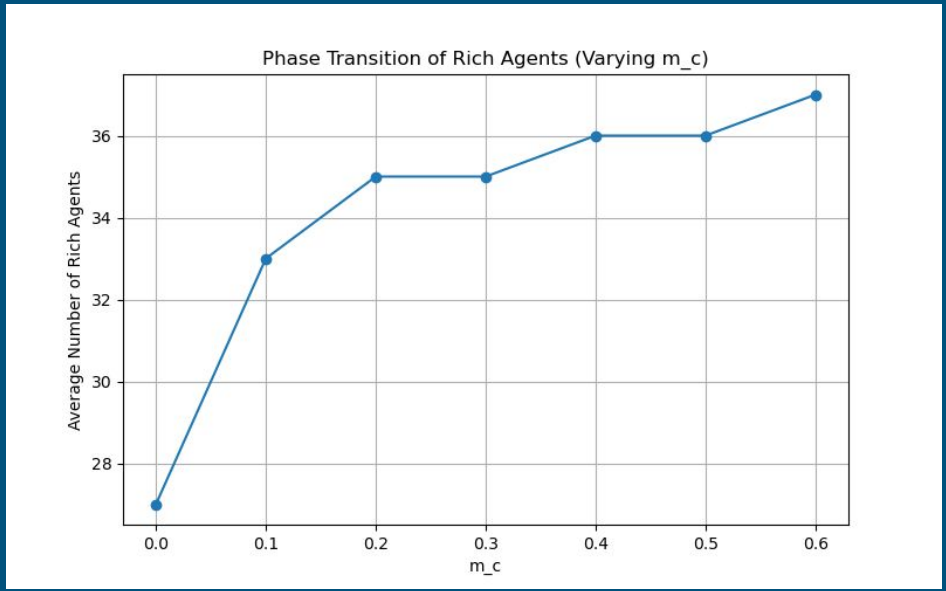
psi_max = 0.5

Phase Transition (10x10, varying charity, rich)

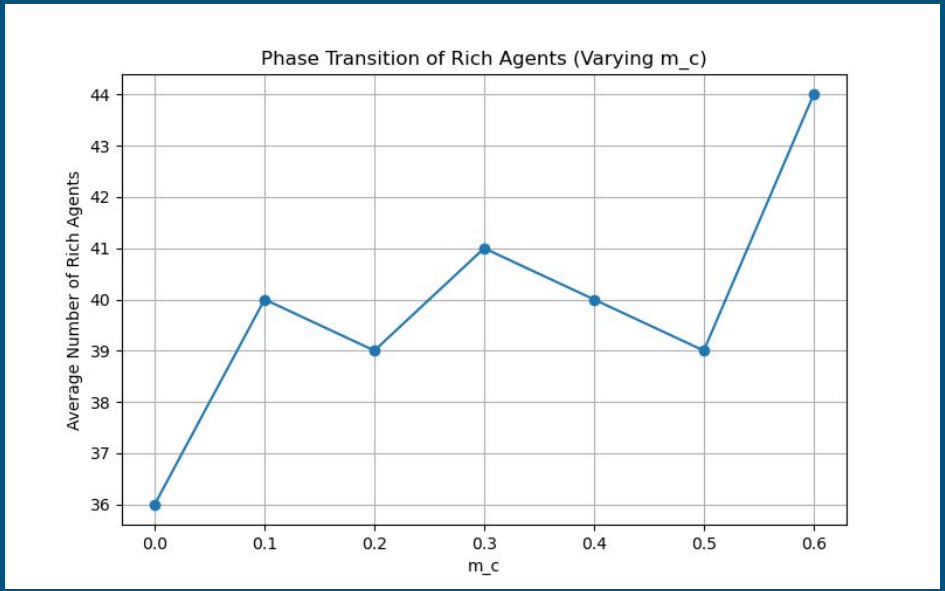


psi_max = 0.6

Phase Transition (20x20, varying charity, rich)

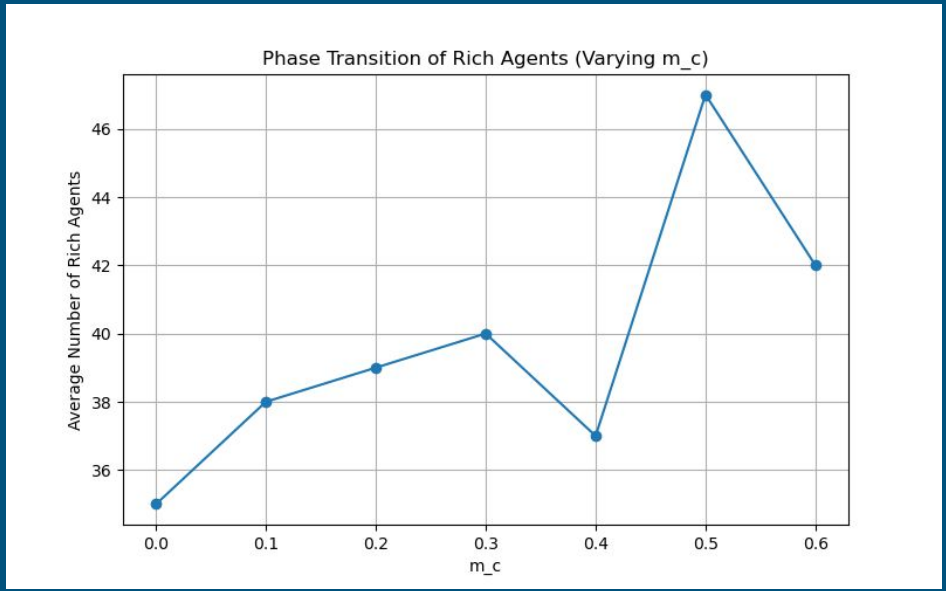


psi_max = 0.0

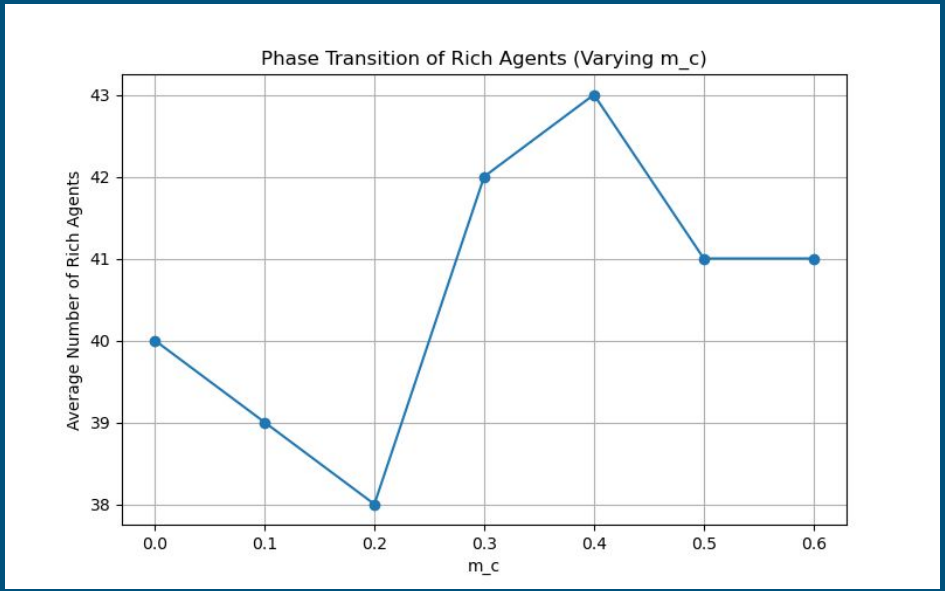


psi_max = 0.1

Phase Transition (20x20, varying charity, rich)

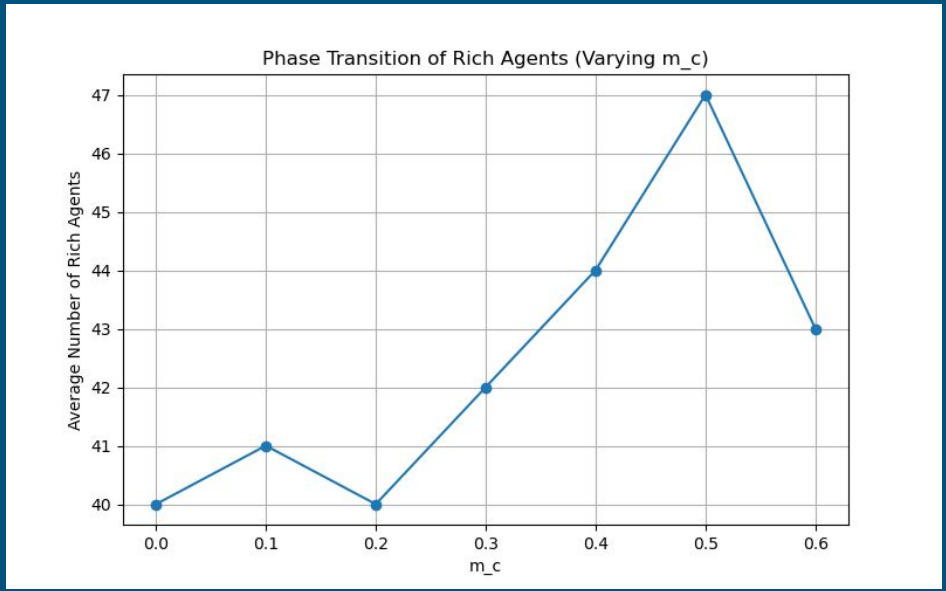


psi_max = 0.2

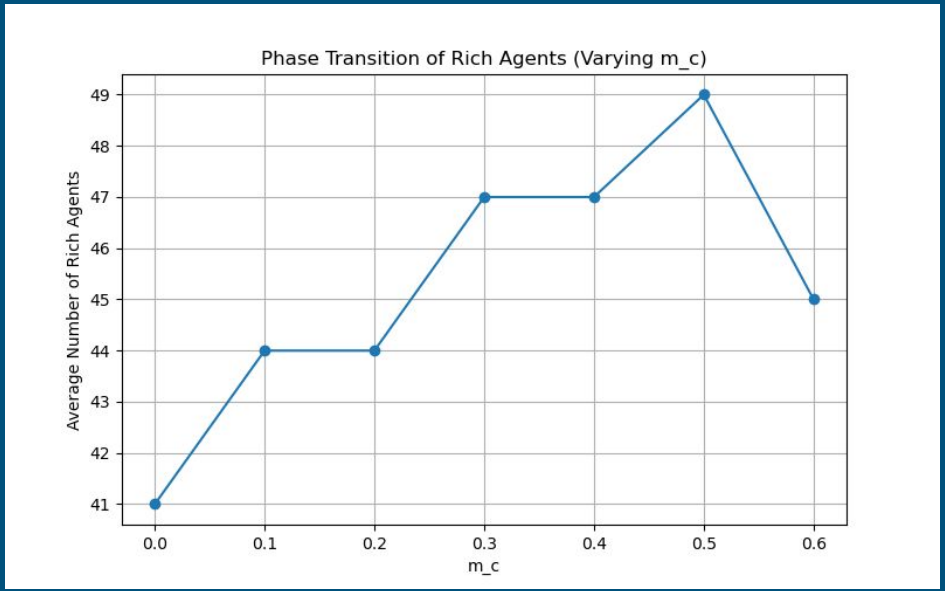


psi_max = 0.3

Phase Transition (20x20, varying charity, rich)

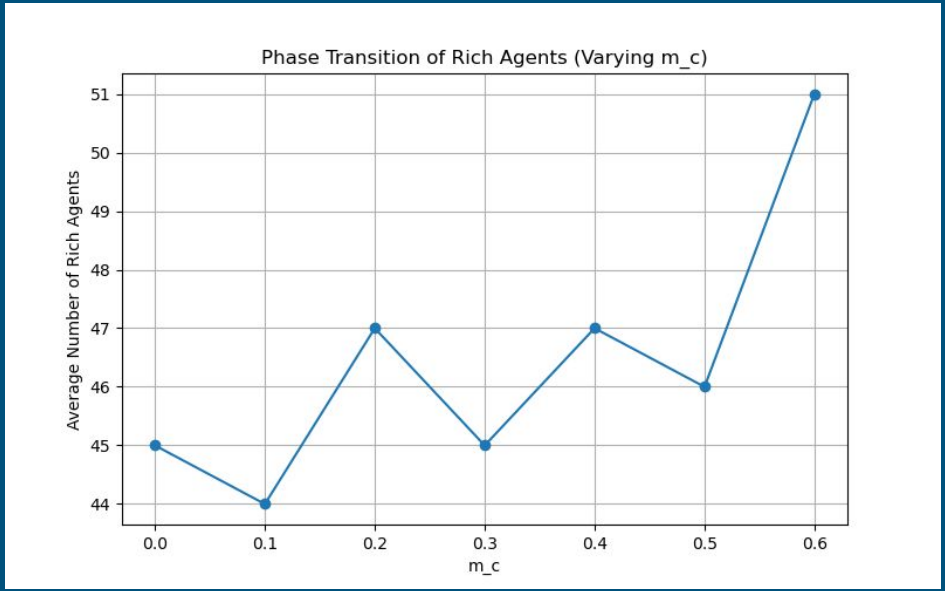


psi_max = 0.4



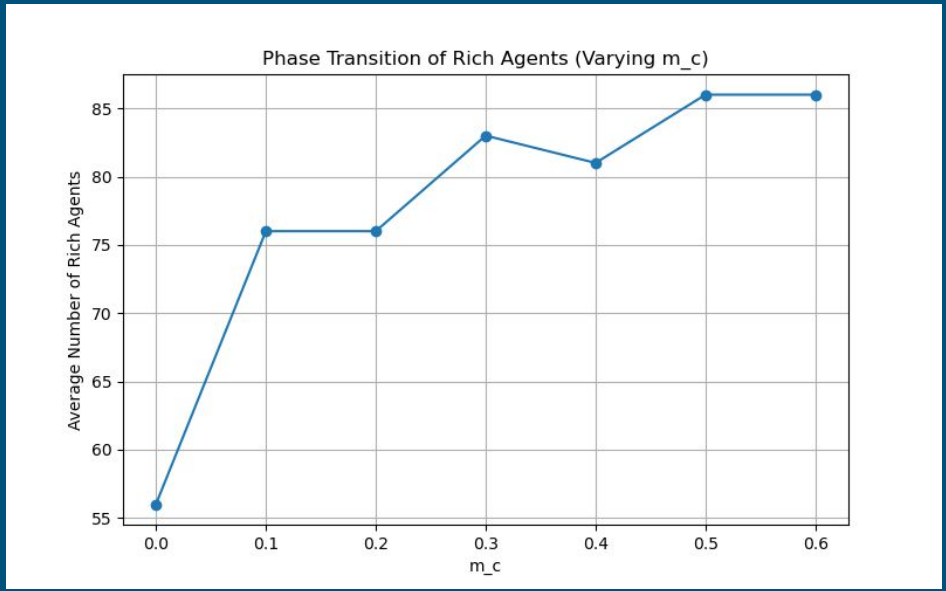
psi_max = 0.5

Phase Transition (20x20, varying charity, rich)

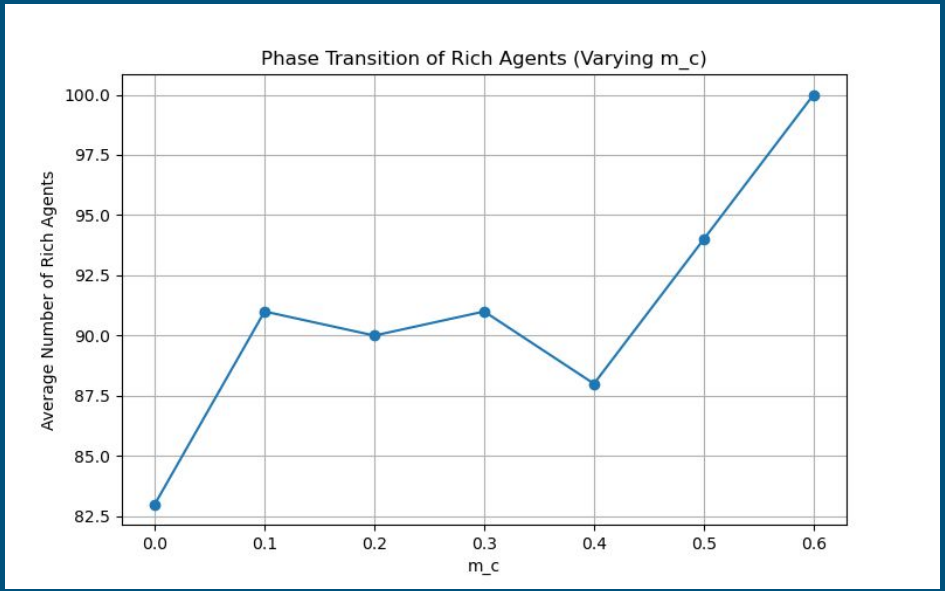


psi_max = 0.6

Phase Transition (30x30, varying charity, rich)

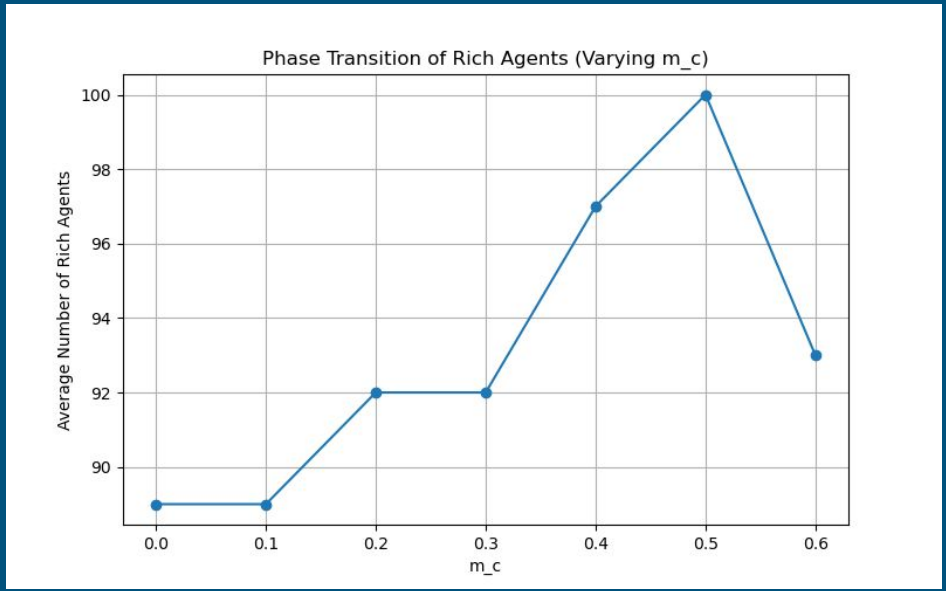


psi_max = 0.0

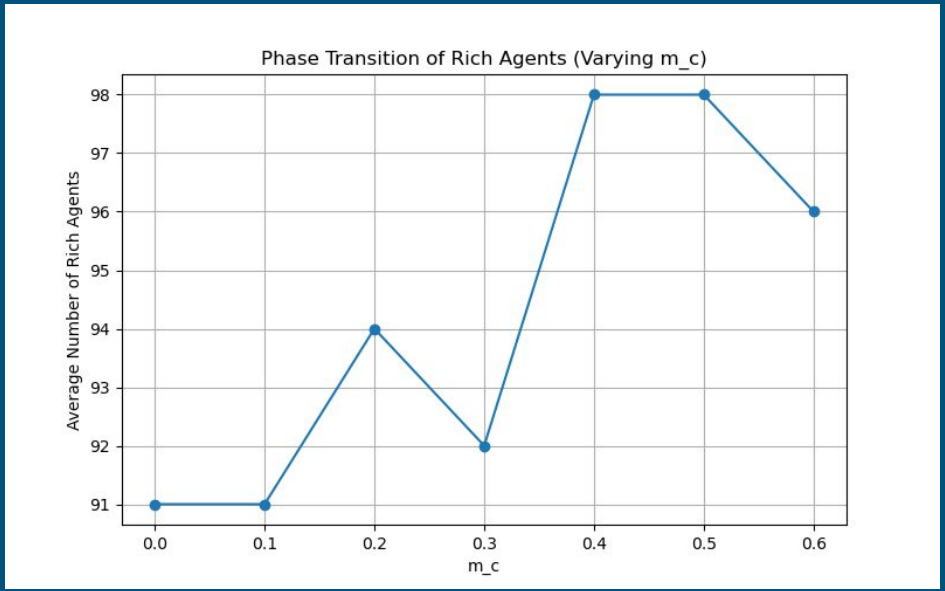


psi_max = 0.1

Phase Transition (30x30, varying charity, rich)

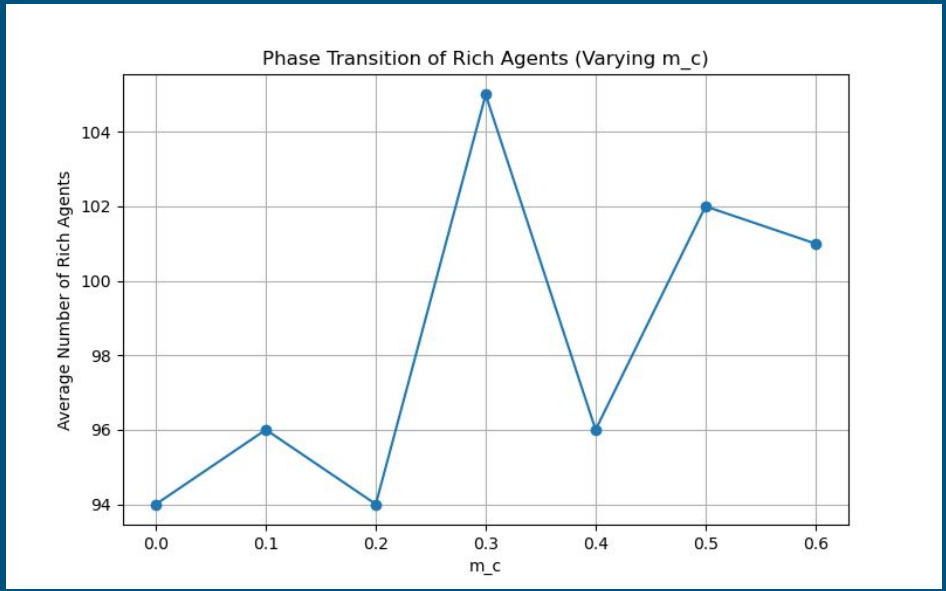


psi_max = 0.2

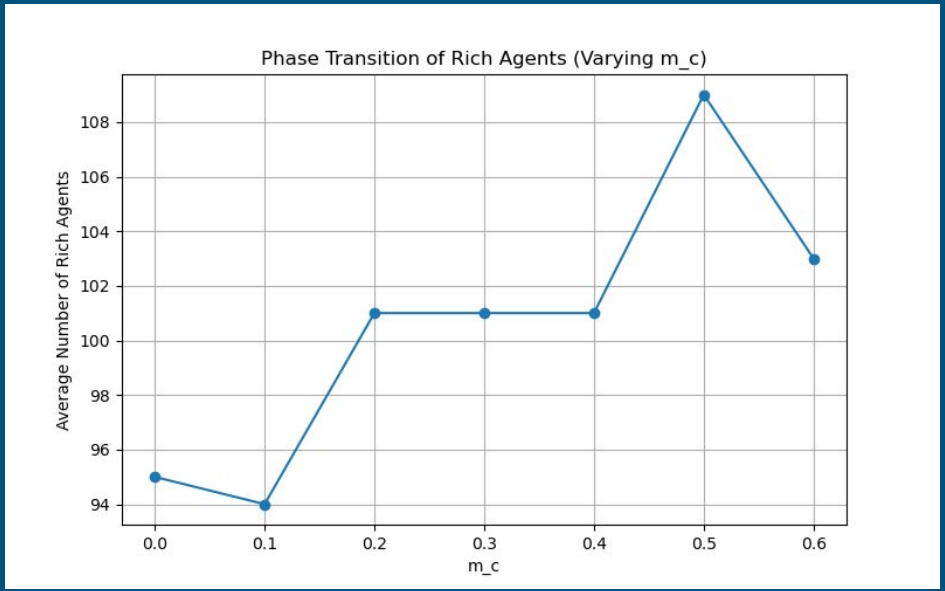


psi_max = 0.3

Phase Transition (30x30, varying charity, rich)

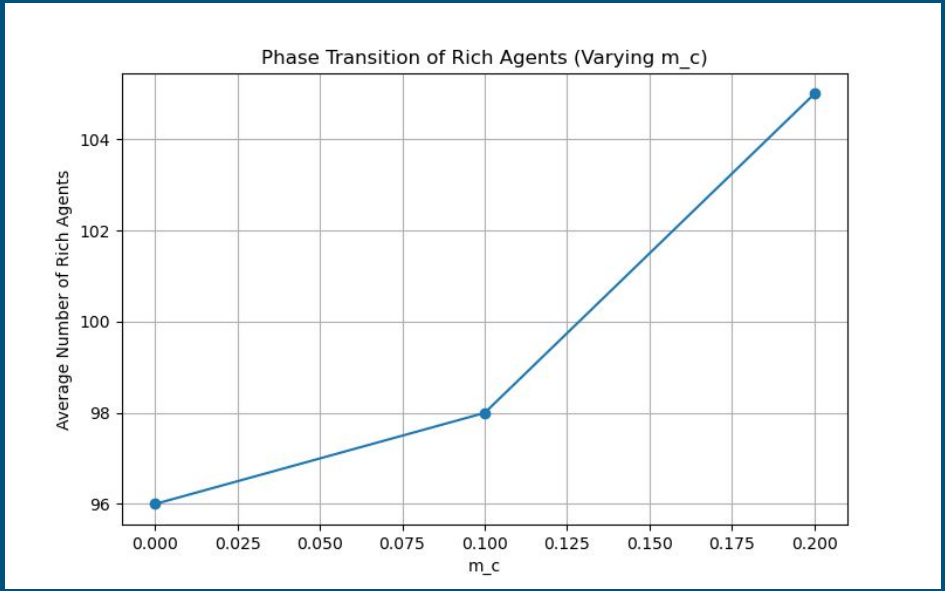


psi_max = 0.4



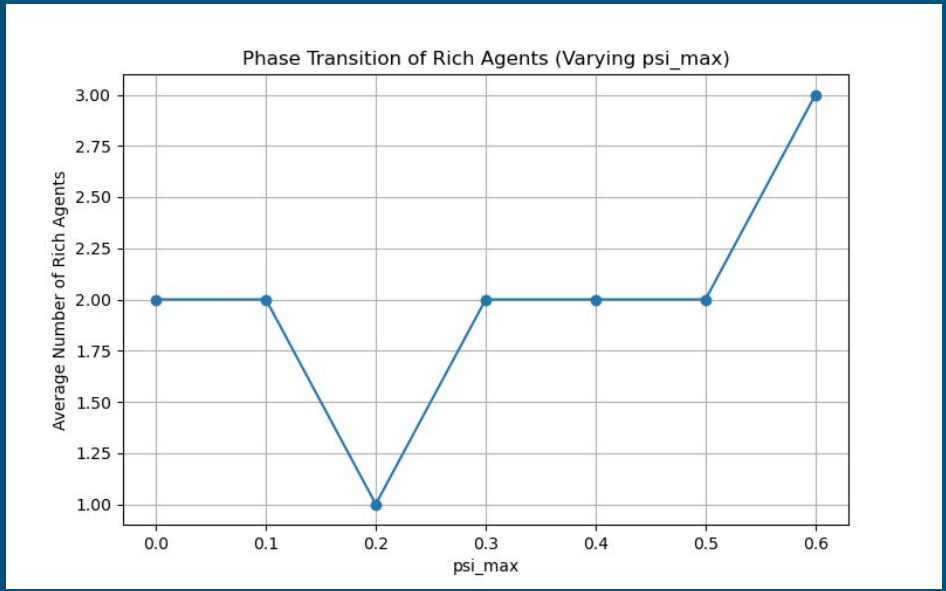
psi_max = 0.5

Phase Transition (30x30, varying charity, rich)

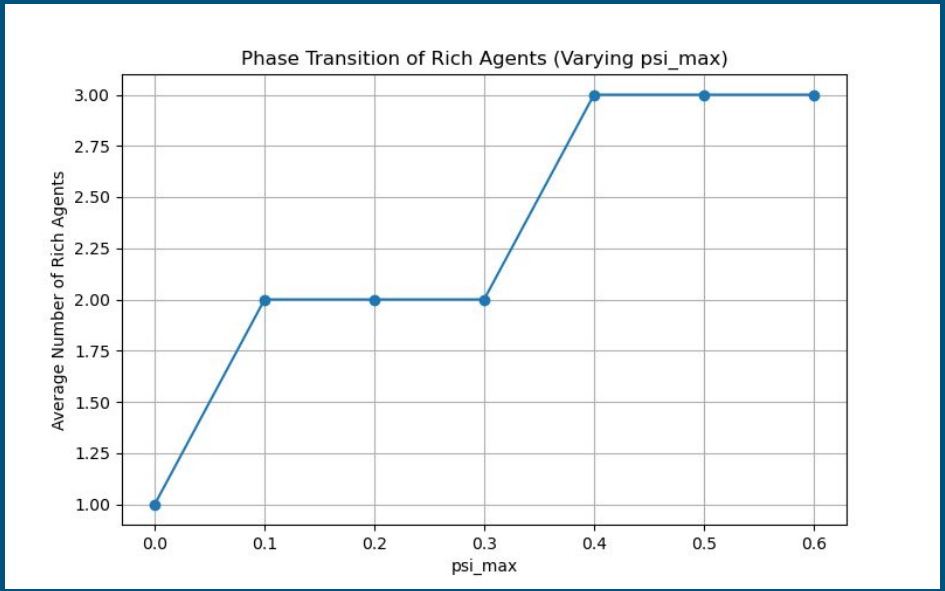


psi_max = 0.6

Phase Transition (5x5, varying tax, rich)

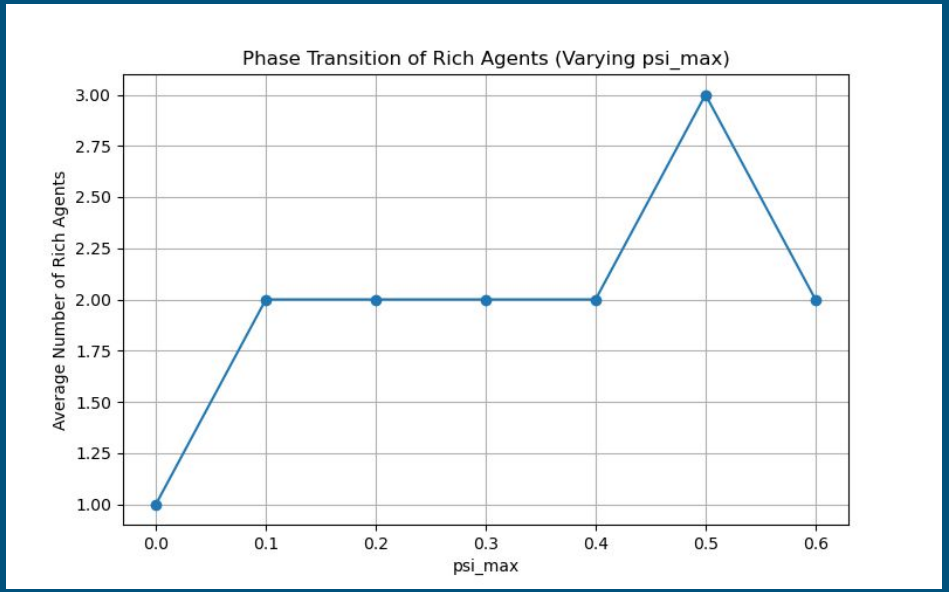


m_c = 0.0

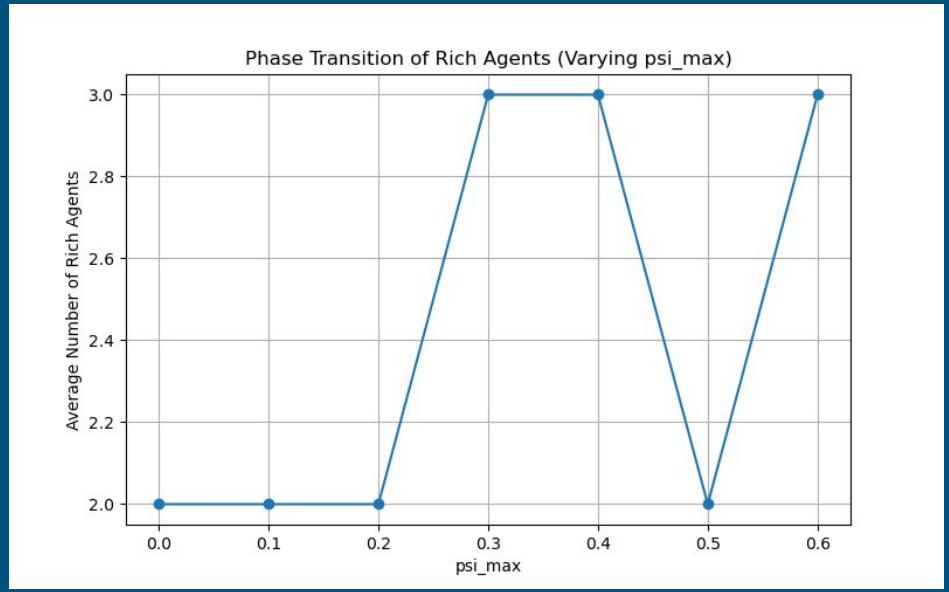


m_c = 0.1

Phase Transition (5x5, varying tax, rich)

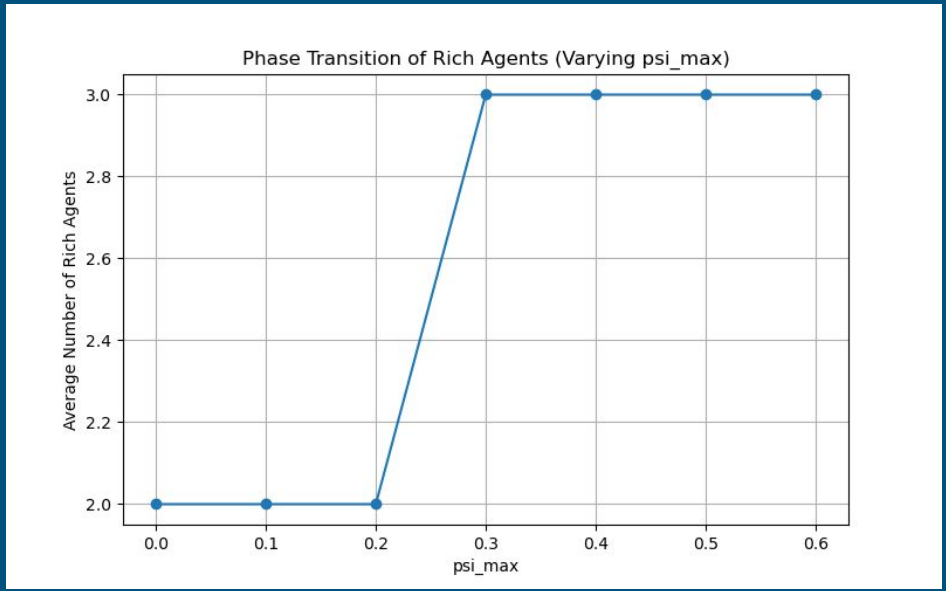


m_c = 0.2

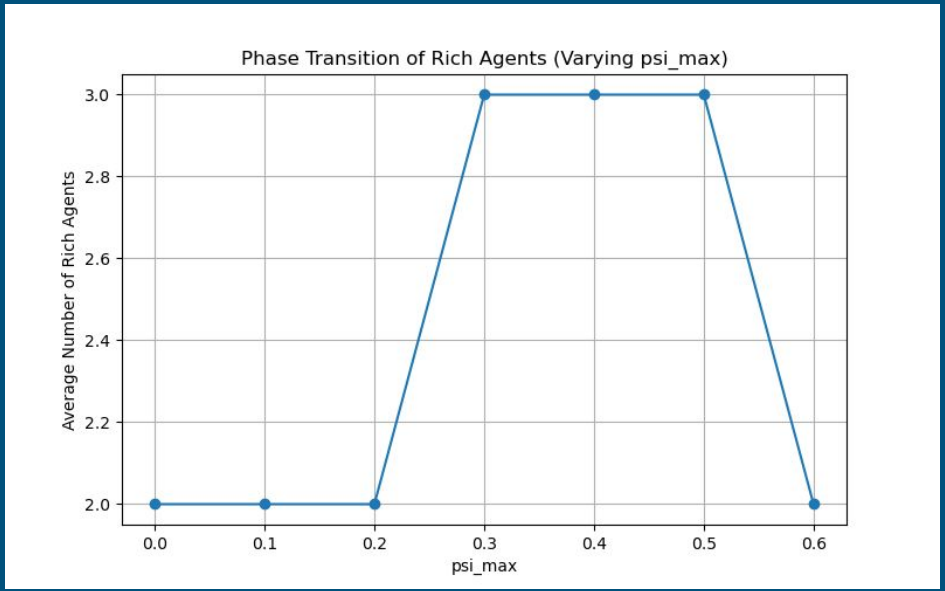


m_c = 0.3

Phase Transition (5x5, varying tax, rich)

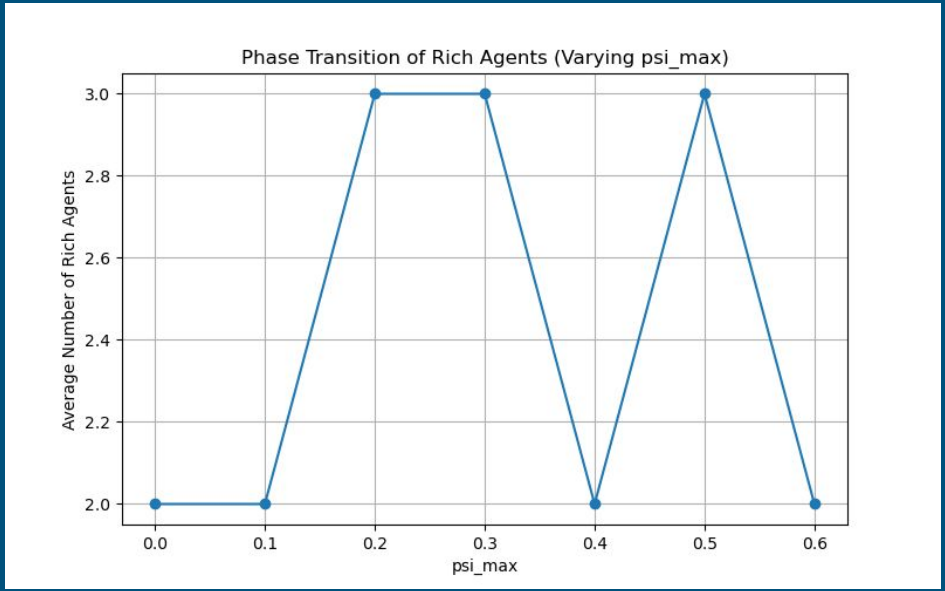


m_c = 0.4



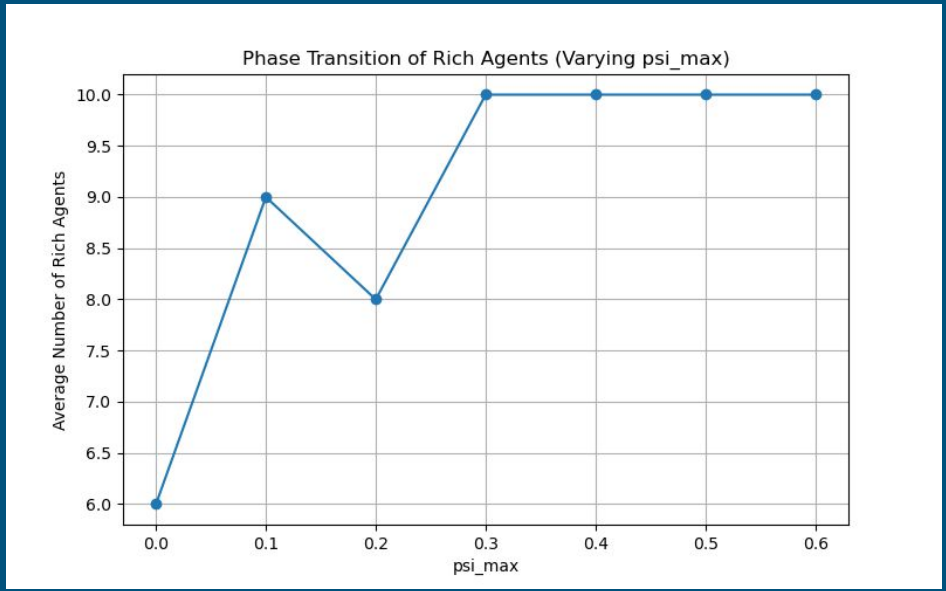
m_c = 0.5

Phase Transition (5x5, varying tax, rich)

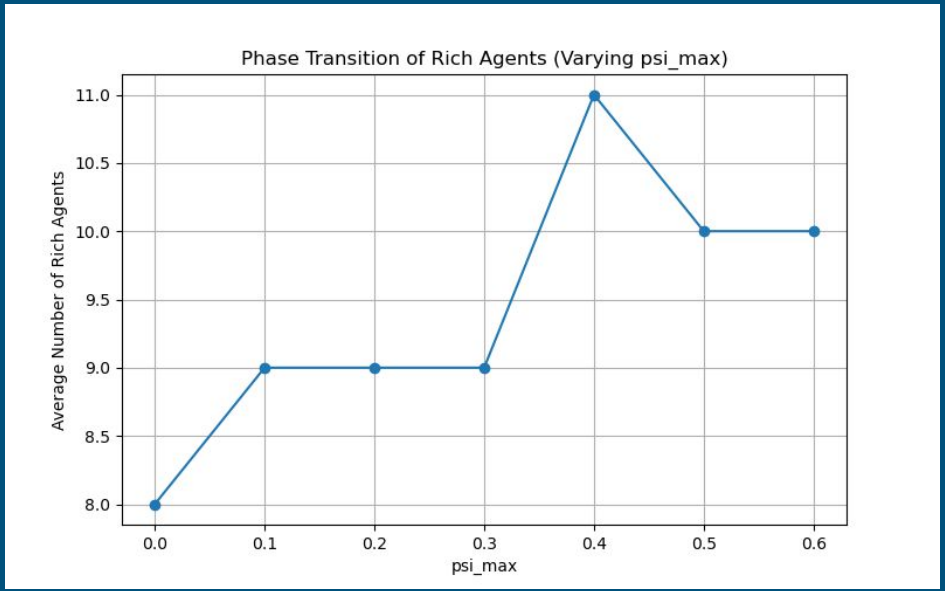


m_c = 0.6

Phase Transition (10x10, varying tax, rich)

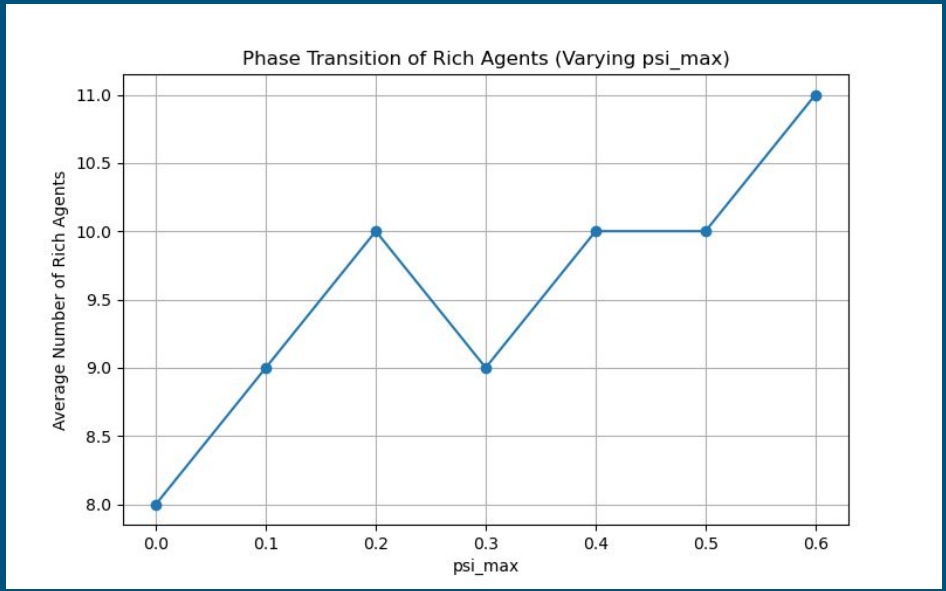


m_c = 0.0

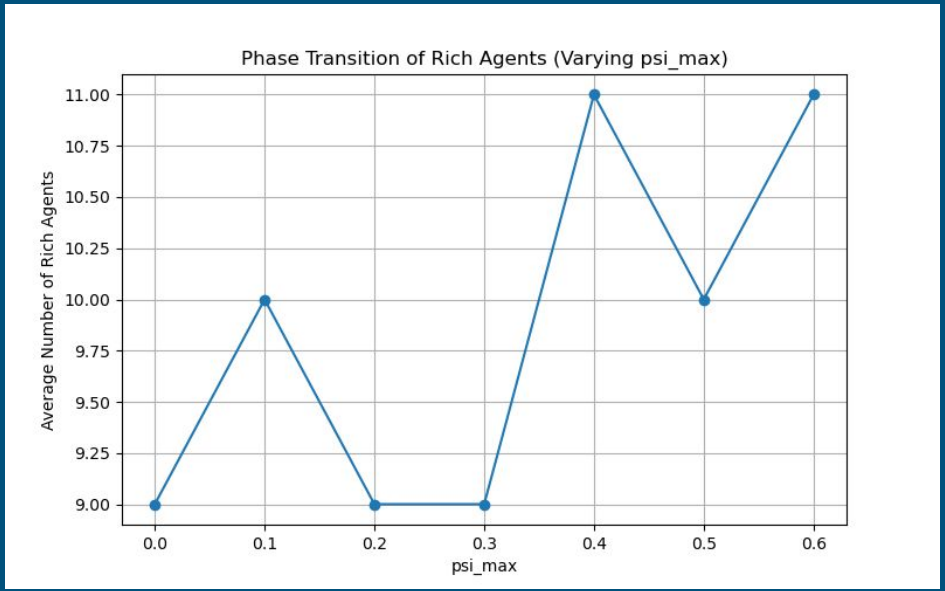


m_c = 0.1

Phase Transition (10x10, varying tax, rich)

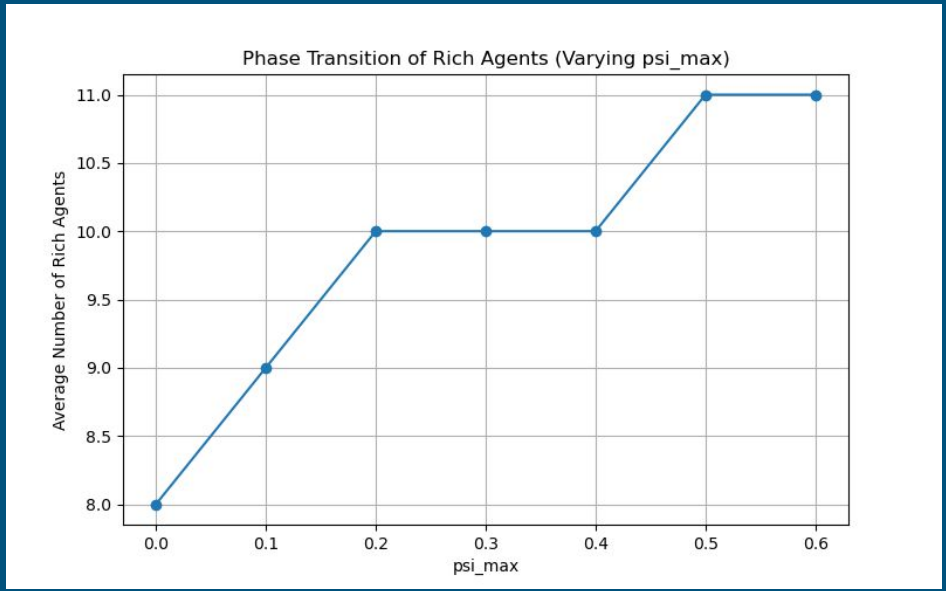


m_c = 0.2

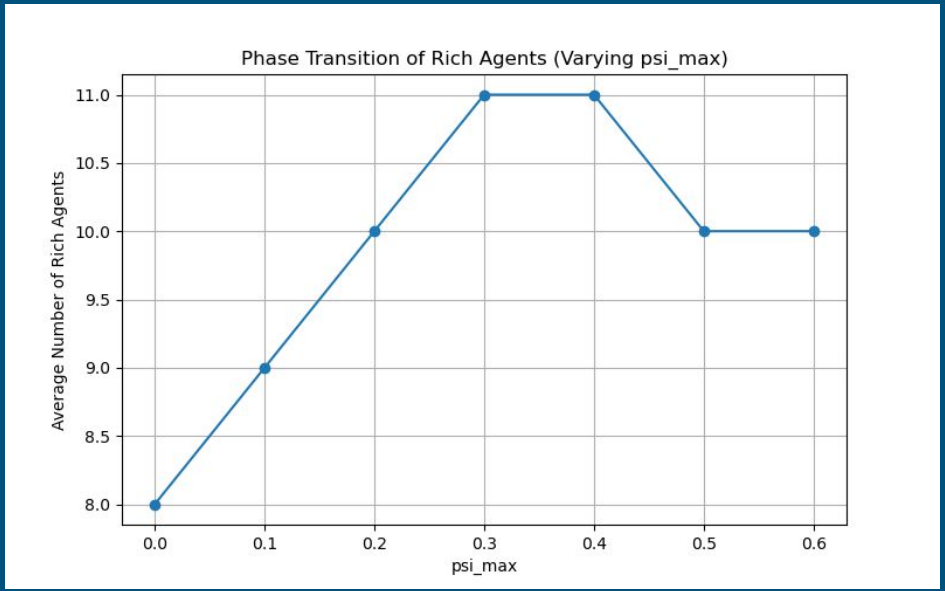


m_c = 0.3

Phase Transition (10x10, varying tax, rich)

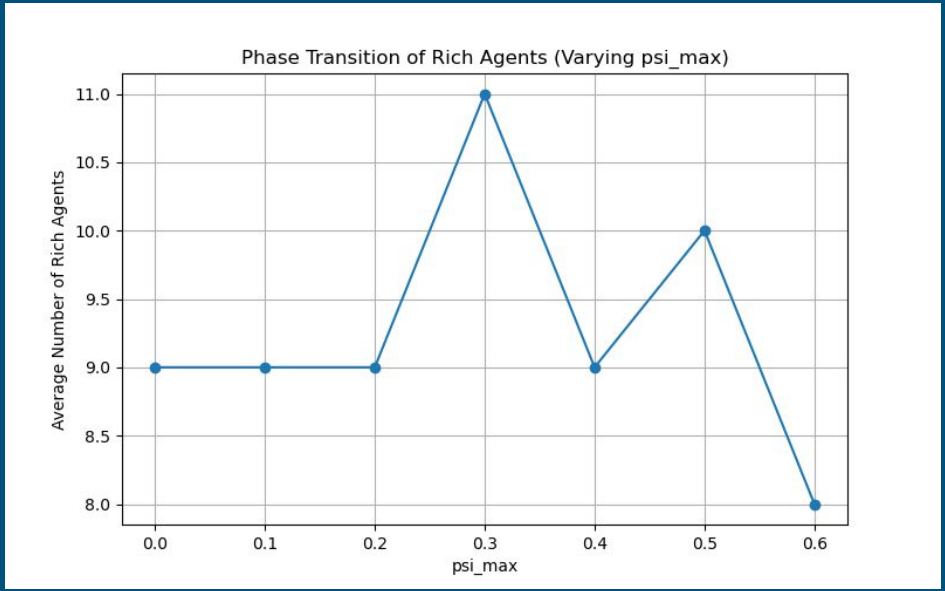


m_c = 0.4



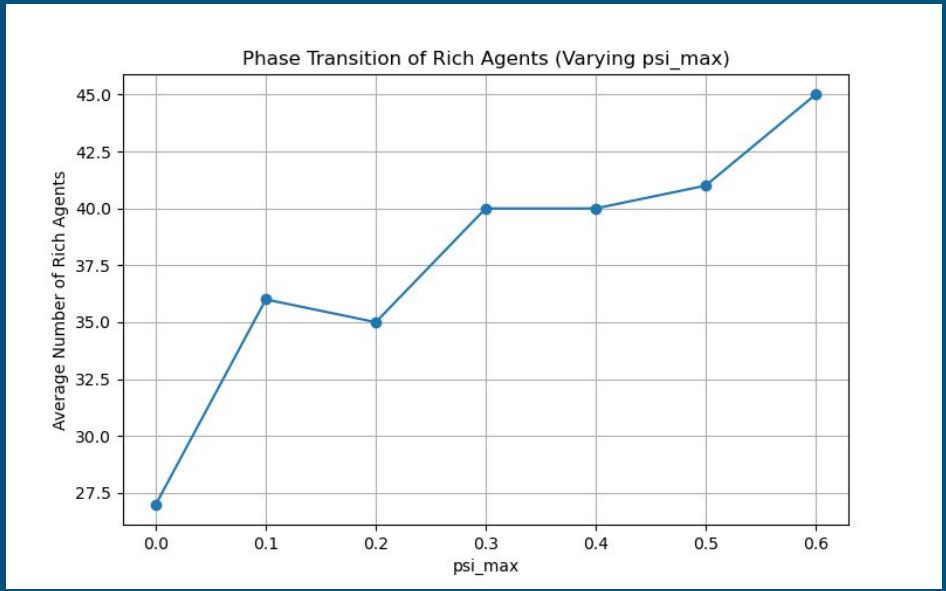
m_c = 0.5

Phase Transition (10x10, varying tax, rich)

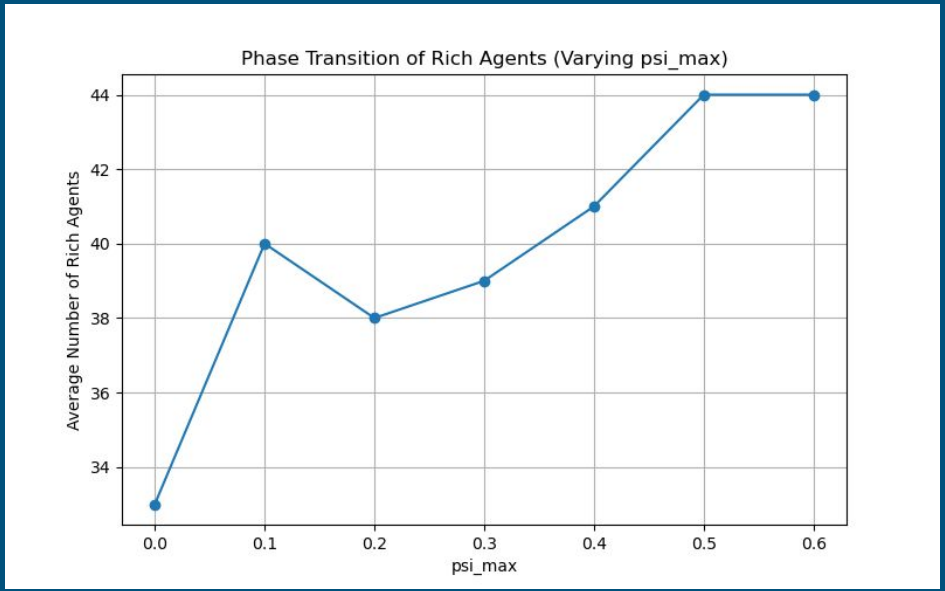


m_c = 0.6

Phase Transition (20x20, varying tax, rich)

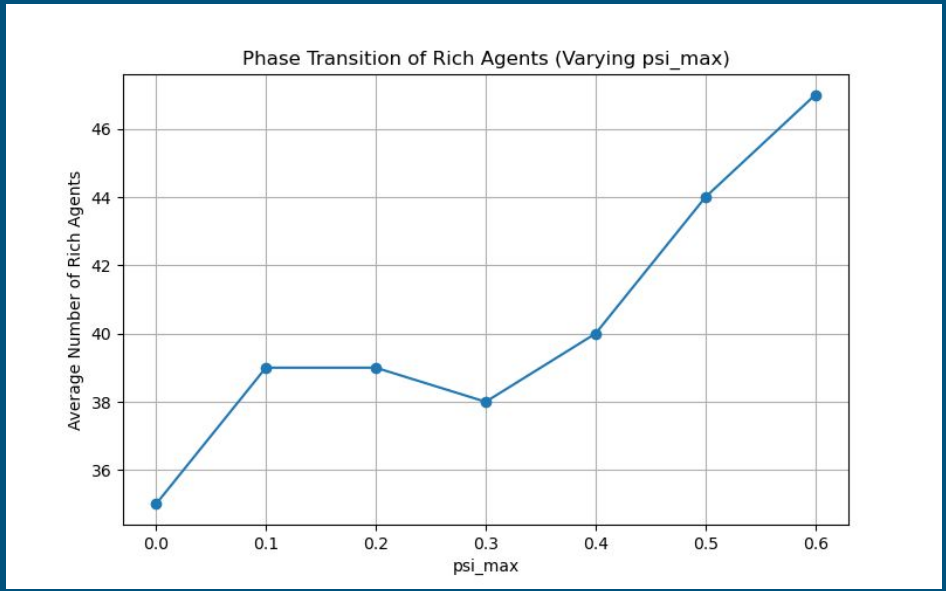


m_c = 0.0

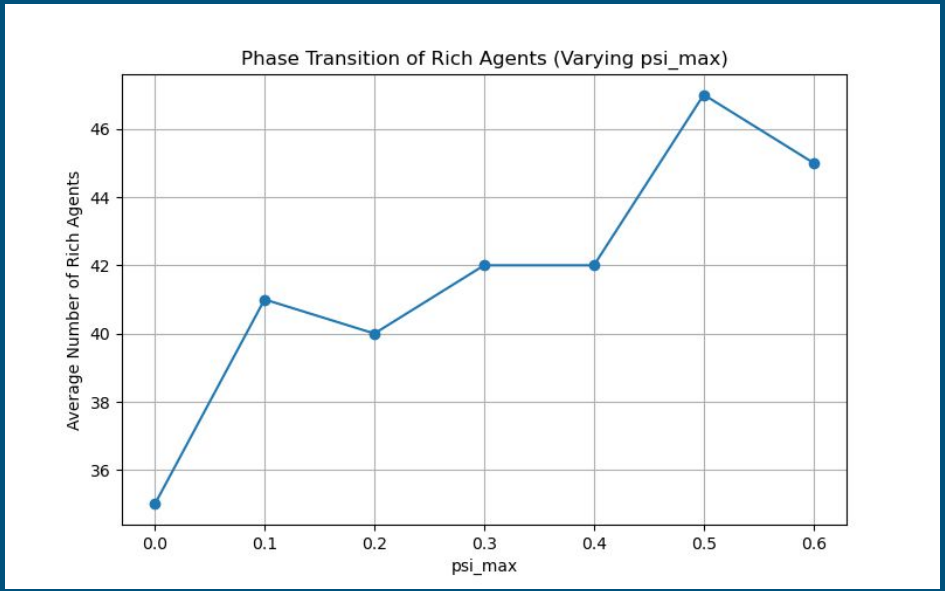


m_c = 0.1

Phase Transition (20x20, varying tax, rich)

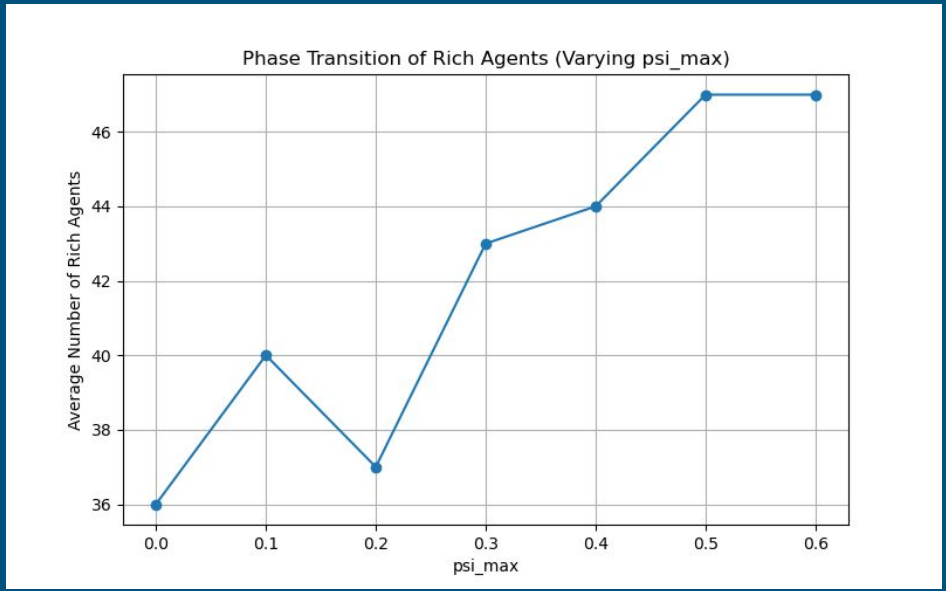


m_c = 0.2

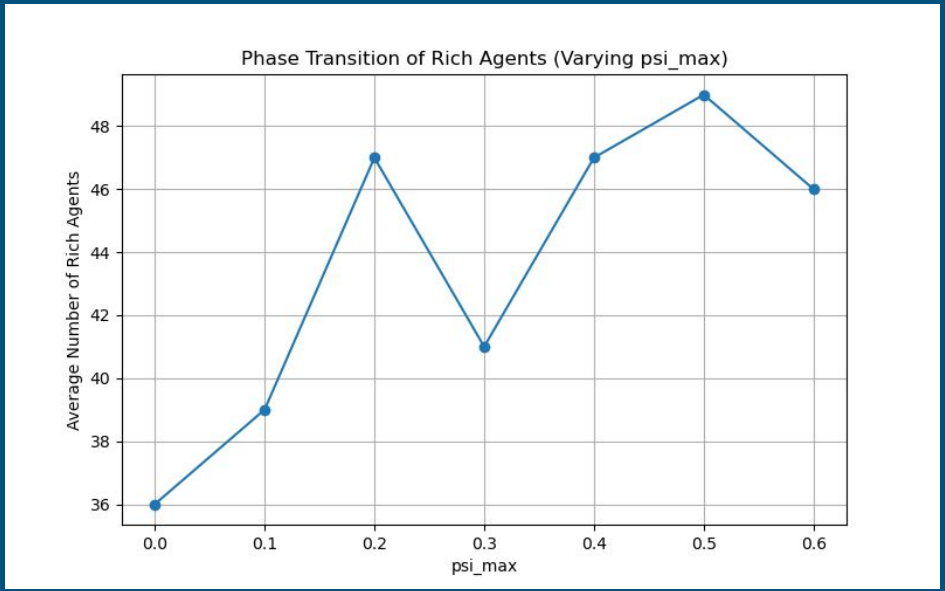


m_c = 0.3

Phase Transition (20x20, varying tax, rich)

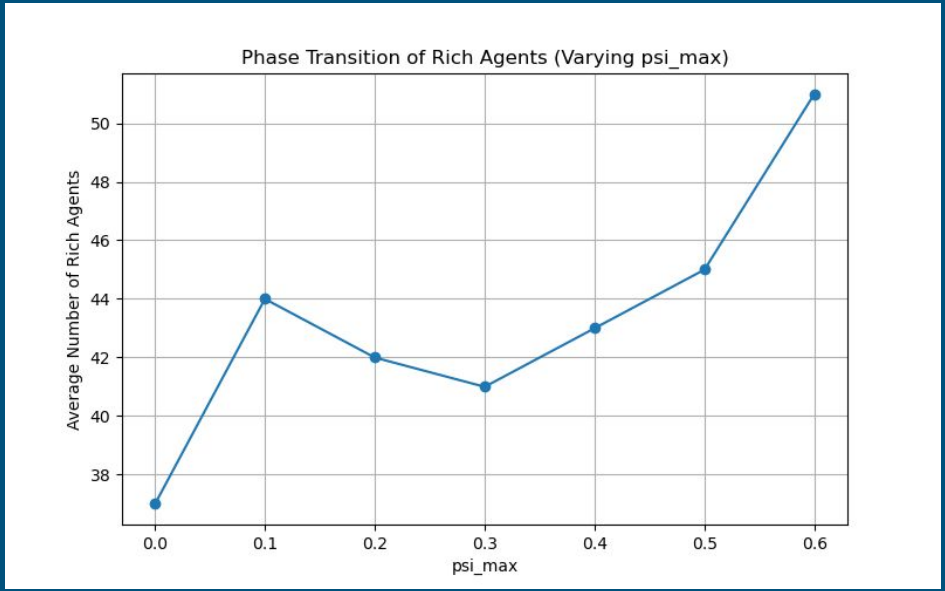


m_c = 0.4



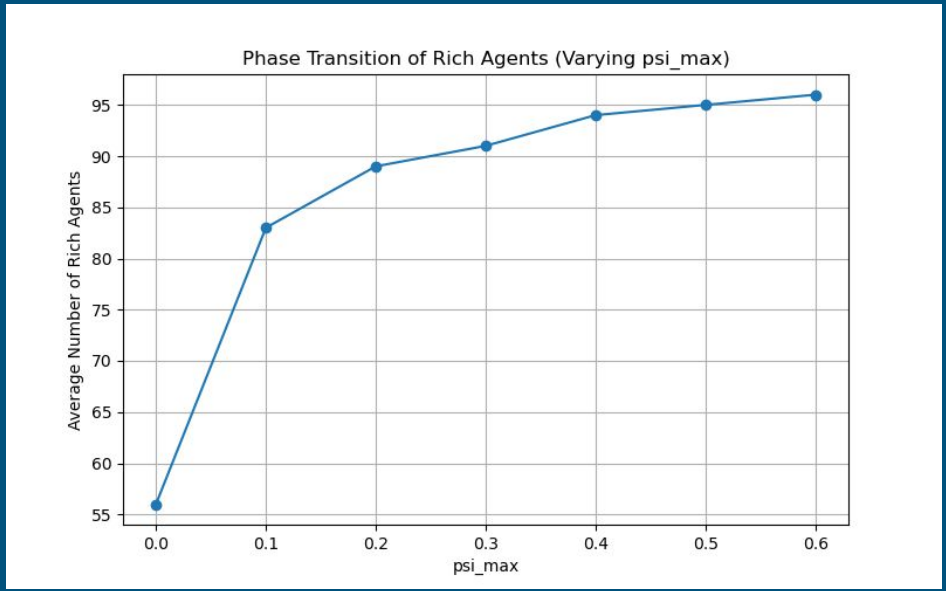
m_c = 0.5

Phase Transition (20x20, varying tax, rich)

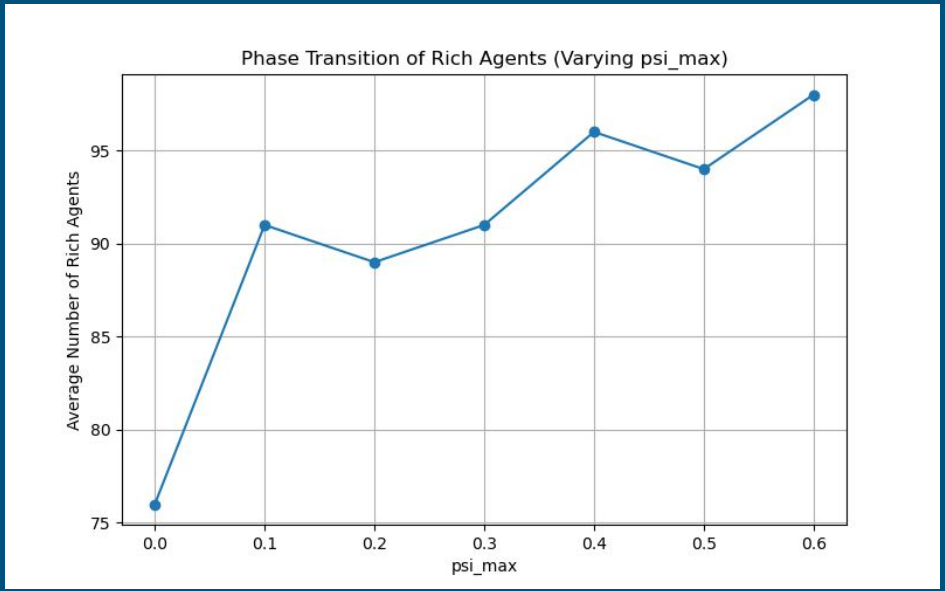


m_c = 0.6

Phase Transition (30x30, varying tax, rich)

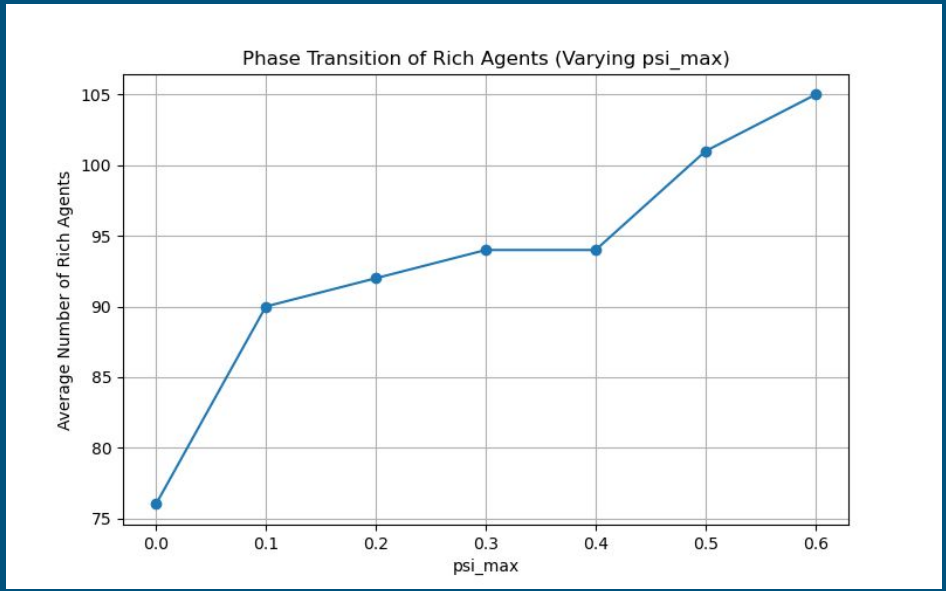


m_c = 0.0

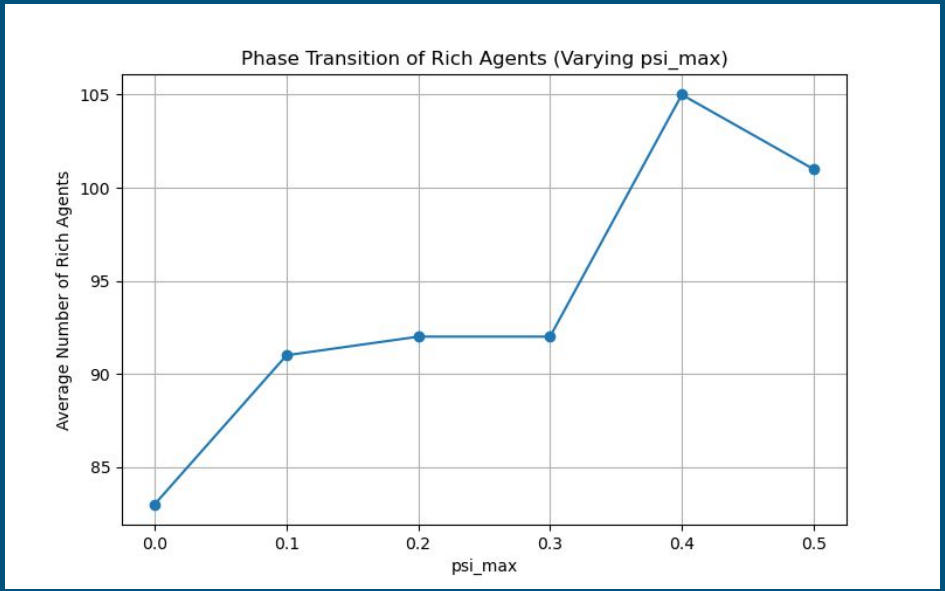


m_c = 0.1

Phase Transition (30x30, varying tax, rich)

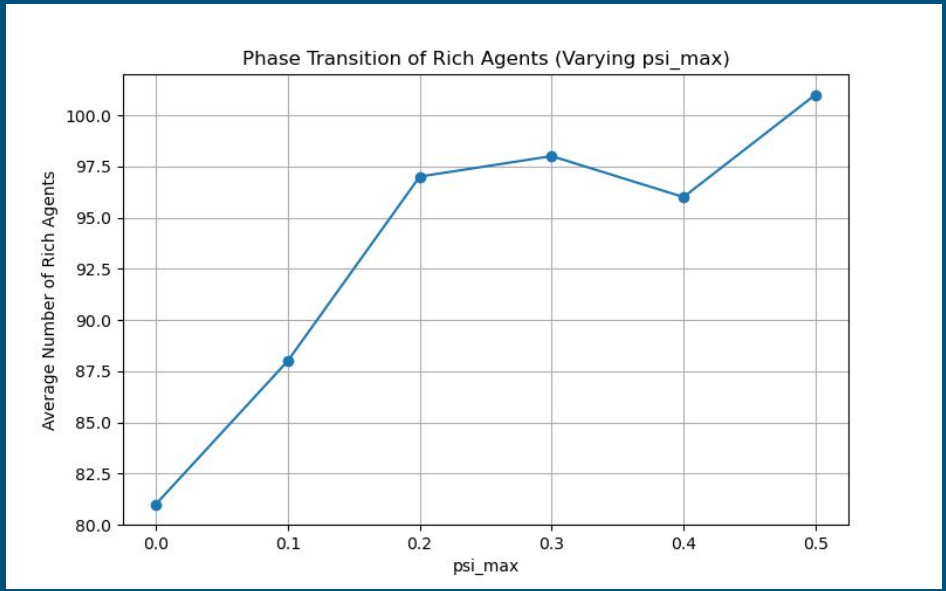


m_c = 0.2

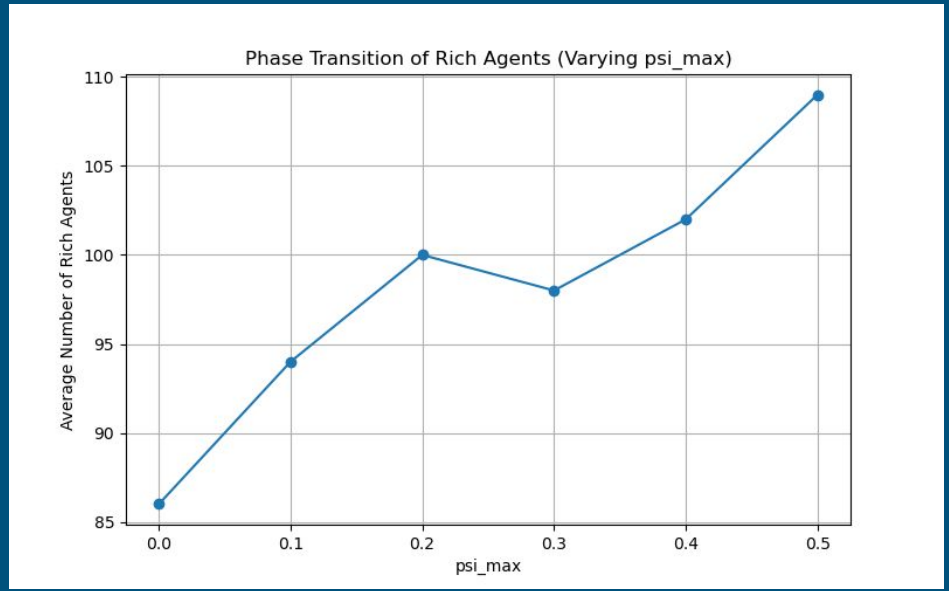


m_c = 0.3

Phase Transition (30x30, varying tax, rich)

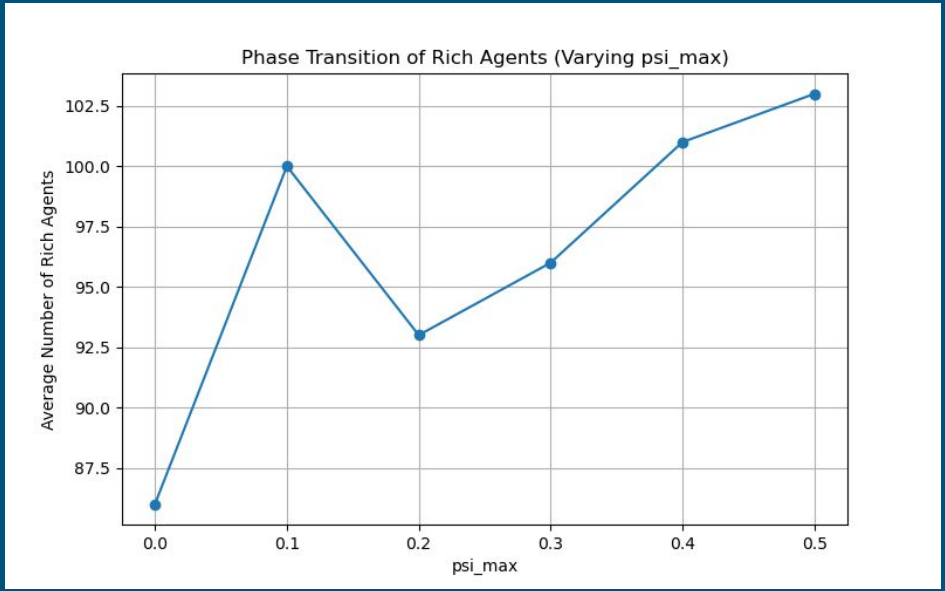


m_c = 0.4



m_c = 0.5

Phase Transition (30x30, varying tax, rich)



m_c = 0.6

Simulation Design

LGCA framework

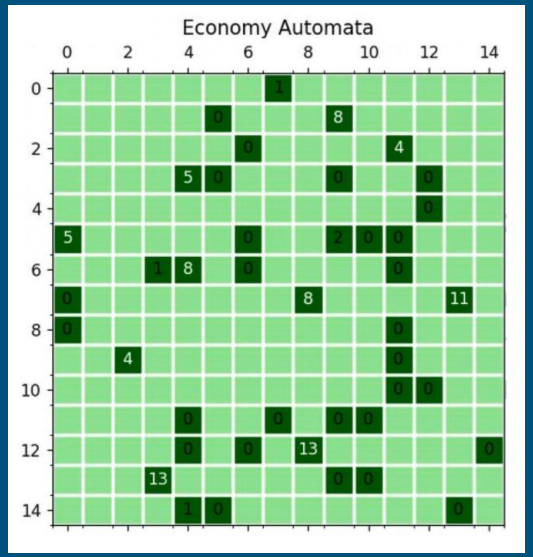
- Various parameters which are included in the model
- Key focus on charity and tax contributions

Key parameters

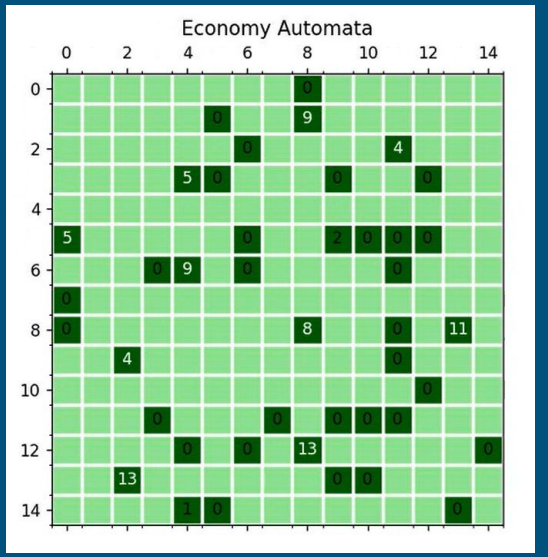
- Ψ_{max} - maximum taxation rate
- m_c - contribution to charity

Other parameters

- P_m - constant at 0.7
- P_t - constant at 0.8
- P_i - varied
- Δm - constant at 1
- m_r - 1.5 times the initial wealth
- m_p - 0.7 times the initial wealth



Step n



Step n+1