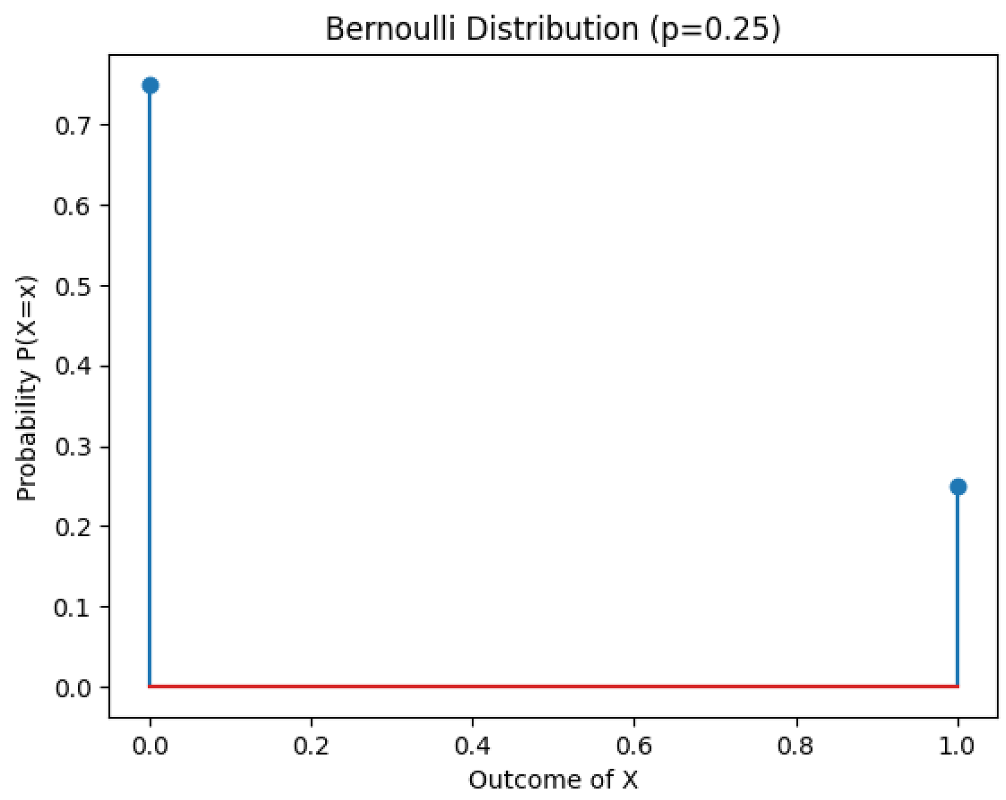
1 (a)

```
In [9]: import numpy as np
import matplotlib.pyplot as plt

# generate pmf
p = 0.25
x = np.arange(2)
pmf = np.array([1-p,p])

# plot
fig, ax = plt.subplots()
ax.stem(x, pmf)
ax.set_xlabel('Outcome of X')
ax.set_ylabel('Probability P(X=x)')
ax.set_title(f'Bernoulli Distribution (p={p})')
plt.show()
```



1 (b)

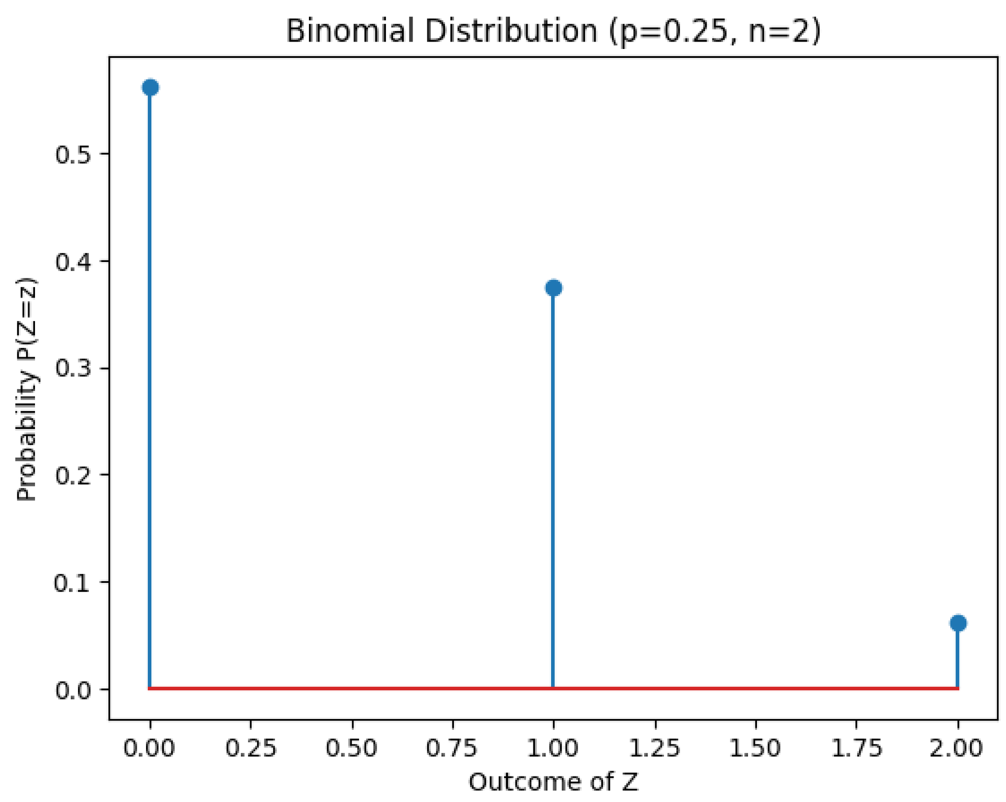
We can compute the entropy by summing up -P(x)log2(P(x)) over all values P(x) take, which is 3/4 and 1/4

```
In [12]: entropy = 0
for px in pmf:
    entropy -= px * np.log2(px)
print(f"Entropy = {entropy}")
```

```
Entropy = 0.8112781244591328
```

2

```
In [19]: p = 0.25
x_pmf = {
    0 : 1-p,
    1 : p
}
z_pmf_dict = {}
# convolution
for i in x_pmf.keys():
    for j in x_pmf.keys():
        if not i+j in z_pmf_dict.keys():
            z_pmf_dict[i+j] = x_pmf[i]*x_pmf[j]
        else:
            z_pmf_dict[i+j] += x_pmf[i]*x_pmf[j]
z = np.array(list(z_pmf_dict.keys()))
z_pmf = np.array(list(z_pmf_dict.values()))
# plot
fig, ax = plt.subplots()
ax.stem(z, z_pmf)
ax.set_xlabel('Outcome of Z')
ax.set_ylabel('Probability P(Z=z)')
ax.set_title(f'Binomial Distribution (p={p}, n=2)')
plt.show()
```



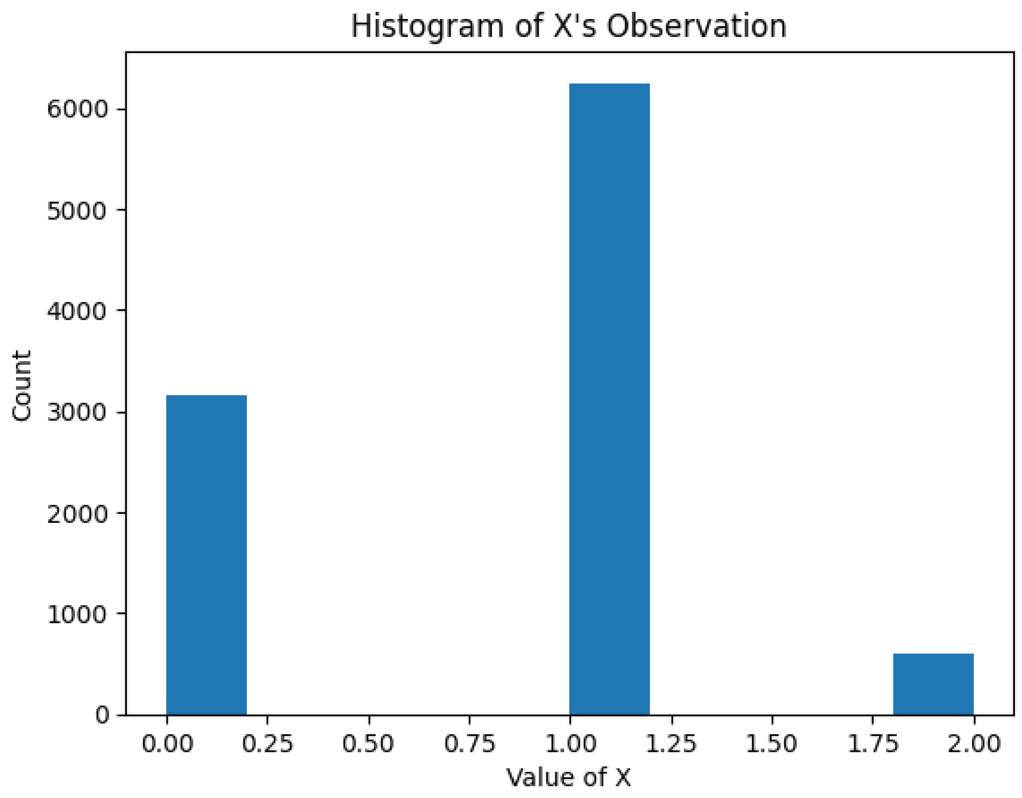
3 (a)

```
In [14]: A = [[]]
for _ in range(4):
    B = []
    for i in [0,1]:
        for item in A:
            B.append(item+[i])
    A = B
pmf = {}
for item in A:
    count = 0
    for i in range(len(item)-1):
        if item[i]==0 and item[i+1]==1:
            count+= 1
    try:
        pmf[count] += 1/len(A)
    except:
        pmf[count] = 1/len(A)

print(f"pmf of X = {pmf}")
```

```
pmf of X = {0: 0.3125, 1: 0.625, 2: 0.0625}
```

3 (b)

```
In [15]: def getX():
    item = [np.random.choice([0,1]),np.random.choice([0,1]),np.random.choice([0,1]),np.random.choice([0,1])]
    count = 0
    for i in range(len(item)-1):
        if item[i]==0 and item[i+1]==1:
            count+= 1
    return count

plt.hist([getX() for _ in range(10000)])
plt.xlabel("Value of X")
plt.ylabel("Count")
plt.title("Histogram of X's Observation")
plt.show()
```


Histogram of X's Observation
Count
Value of X
0
1000
2000
3000
4000
5000
6000
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

In [ ]: