Cumulative Distribution Functions
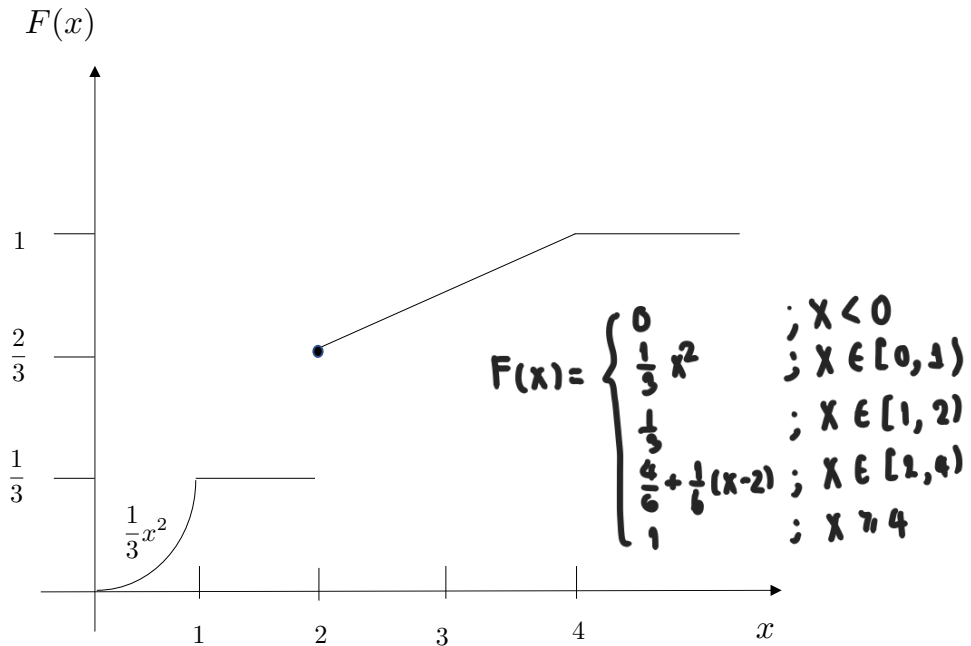
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1. Let X be a random variable with the cdf shown below.

$F(x)$

$$F(x)= \begin{cases} 0 & ; X < 0 \\ \frac{1}{3}x^2 & ; X \in [0,1) \\ \frac{1}{3} & ; X \in [1,2) \\ \frac{4}{6} + \frac{1}{6}(x-2) & ; X \in [2,4) \\ 1 & ; X \geq 4 \end{cases}$$

a) Is X a continuous random variable? If not, is X a discrete random variable? **X is mixed**

b) Find the probability of the following events:
  i. $\{X = 2\}$ **1/3**
  ii. $\{X < 2\}$ **1/3**
  iii. $\{X = 2\} \cup \{0.5 \leq X \leq 1.5\} = \frac{1}{3} + F(1.5) - F(0.5) = \frac{2}{3} - \frac{1}{12} = \frac{7}{12}$
  iv. $\{X = 2\} \cup \{0.5 \leq X \leq 3\} = F(3) - F(0.5) = \frac{5}{6} - \frac{1}{12} = \frac{9}{12}$

c) Derive an expression for the pdf of X. You can use the indicator function and the delta function (or unit impulse), $\delta(x)$. Note that $\delta(x)$ satisfies two properties: $\delta(x) = 0$ for all $x \neq 0$, and $\int_{-\infty}^{\infty} \delta(x) = 1$.

$$f(x) = F'(x) = \begin{cases} 0 & ; X < 0 \\ \frac{2x}{3} & ; X \in [0,1) \\ 0 & ; X \in [1,2) \\ \delta(x)/3; & X = 2 \\ 1/6 & ; X \in [2,4) \\ 0 & ; X \geq 4 \end{cases}$$

→ so f integrates to 1