Expectation of the Sample Mean, Joint pdfs

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1. ***Sample mean.*** Given $X_i \overset{i.i.d.}{\sim} \mathcal{N}(\mu, \sigma^2)$, let $Y := \sum_{i=1}^{n} X_i$ denote their sum.

a) Find $\mathbb{E}[Y]$ using the properties of expectation. $E[Y] = \sum_{i=1}^{n} E[X_i] = n\mu$

b) The sample mean is defined as $\frac{1}{n}\sum_{i=1}^{n} X_i$. What is the expected value of the sample mean? $\frac{1}{n}(n\mu) = n$

Note: The sum of Gaussian random variables is also Gaussian (a property will show later in the course). Next time we will find var(Y) using the properties of expectation and independence.

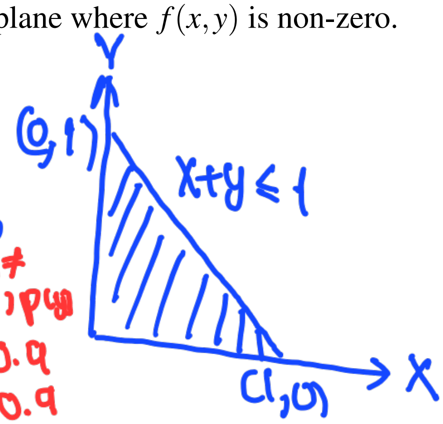
2. ***Joint pdf.*** Let $(X,Y) \sim f(x,y)$ where

$$f(x,y) = \begin{cases} c & \text{if } x,y \geq 0 \text{ and } x+y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Sketch the region of the x,y plane where $f(x,y)$ is non-zero.

b) Find c. $C = \frac{1}{\text{Area}} = 2$

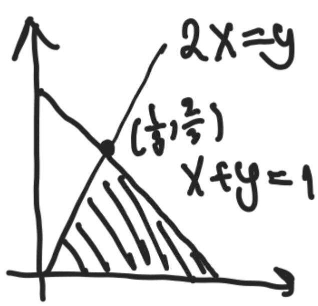
c) Find $f(y)$.



d) Are X,Y independent? No

i.e. $p(x,y) \neq p(x)p(y)$
for $x = 0.9$
$y = 0.9$
$(p(x,y) = 0)$

e) Find $\mathbb{P}(2X \geq Y)$.



$P(2X \geq Y) = \frac{\text{Area}(\triangle)}{\text{Area}(\triangle)} = \frac{1/3}{1/2} = \frac{2}{3}$