Entropy and Mutual Information Basics

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1. Consider the following pmf.

		y			
	$p(x,y)$	fish	cat	dog	
	1	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	→ 7/16
x	2	$\frac{1}{16}$	0	$\frac{1}{4}$	→ 5/16
	3	0	$\frac{1}{8}$	$\frac{1}{16}$	→ 3/16
	4	$\frac{1}{16}$	0	0	→ 1/16
		↓ 5/8	↓ 2/8	↓ 5/8	

Find the following:

a) $H(X,Y) = \sum_{x,y} -p(x,y)\log(p(x,y)) = \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{4}{16} + \frac{2}{8} + \frac{3}{8} + \frac{4}{16} + \frac{4}{16} = 2.75$

b) $H(X) = \sum_{x} -p(x)\log p(x) = 4 - \frac{1\log 1 + 3\log 3 + 5\log 5 + 7\log 7}{16} = 1.56$

c) $H(Y) = \sum_{y} -p(y)\log p(y) = 3 - \frac{3\log 3 + 2\log 2 + 3\log 3}{8} = 1.25$

d) $H(X|Y) = \sum_{y} H(X|Y=y) = \sum_{x,y} p(x,y)\log\frac{p(x,y)}{p(x)} = 1.19$ $\quad (2.75 - 1.56)$

e) $H(Y|X) = \sum_{x,y} p(x,y)\log\frac{p(x,y)}{p(y)} = 1.06\ (2.75 - 1.25)$

f) $I(X;Y) = H(X) - H(X|Y) = 0.75$ $\quad = H(X) + H(Y) - H(X,Y)$

g) $I(Y;X) = H(Y) - H(Y|X) = 0.75$

h) Draw a Venn diagram with two partially overlapping circles, and label the diagram with the quantities above.