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Data Structures And Algorithms Assignment 1

Question 3

(i) $\lim_{n \to \infty} \frac{\ln(n) + 4}{5n^4 + 7n^2 + 6}$

Using L'hopital's rule,

$$\lim_{n \to \infty} \left(\frac{\frac{d}{dn}(\ln(n) + 4)}{\frac{d}{dn}(5n^4 + 7n^2 + 6)} \right)$$

$$\lim_{n \to \infty} \left(\frac{\frac{1}{n}}{20n^3 + 14n} \right)$$

$$\lim_{n \to \infty} \left(\frac{1}{n(20n^3 + 14n)} \right)$$

$$\lim_{n \to \infty} (i)$$

$$\lim_{n \to \infty} (n(20n^3 + 4n))$$

$$= \underline{0}$$

(ii) $\lim_{n \to \infty} \frac{2n}{\log_2(n)}$

Using L'hopital's Rule,

$$\lim_{n \to \infty} \left(\frac{\frac{d}{dn}(2^n)}{\frac{d}{dn}(\log_2(n))} \right)$$

$$\lim_{n \to \infty} \left(\frac{\frac{\ln(2) \times 2^n}{1}}{\ln(2n)} \right)$$

$$\lim_{n \to \infty} \left(\ln(2)^2 \times 2^n \times n \right)$$

$$\lim_{n \to \infty} \left(\ln(2)^2 \times 2^n \right) = \infty$$

$$\lim_{n \to \infty} (n) = \infty$$

$$\infty \times \infty = \infty$$

$$= \infty$$

Approximation

(i) $$\sum_{k=0}^{30} k^2 = \sum_{k=0}^{n} k^2 = \frac{n^3}{3}$$

$$\therefore \sum_{k=0}^{30} k^2 = \frac{30^3}{3}$$

$$= \frac{27000}{3}$$

$$\therefore \sum_{k=0}^{30} k^2 = 9000$$

(ii) $$\sum_{k=0}^{100} k^3$$

$$\sum_{k=0}^{100} k^3 = \sum_{k=0}^{n} k^3 \approx \frac{n^4}{4}$$

$$= \frac{(100)^4}{4} = \frac{100{,}000{,}000}{4}$$

Approximation

$$\sum_{k=0}^{100} k^3 = 25,000,000$$

Proving by induction

(1) $$2\sum_{k=0}^{n} 3^k = 3^{n+1} - 1$$

$k=0$

$I_1 \quad n=0$

$$2\sum_{k=0}^{i} 3^k = 3^{i+1} - 1$$

show $i+3$

$$3\sum_{k=0}^{i+1} 3k = 3^{(i+1)+1} - 1$$

$$2\sum_{k=0}^{i+1} 3^k = 2(3^0 + 3^1 + 3^2 + \dots 3^i + 3^{i+1})$$

$$= 3^{(i+1)+1} - 1$$

$$2(3^0 + 3^1 + 3^2 \cdots 3^i) = 2\sum_{k=0}^{i} 3^k$$

$$2\sum_{k=0}^{i+1} = 2\sum_{k=0}^{i} 3^k + (3^{i+1}) = 3^{(i+1)+1} - 1$$

By induction,

$$3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$$

$$2\sum_{k=0}^{i+1} 3^k = 3^{i+1} - 1 + 3^{i+1} = 3^{(i+1)+1} - 1$$

$$= 3^{i+1} + 3^{i+1} - 1 = 3^{(i+1)+1} - 1$$

$$= 3^{(i+1)+1} - 1 = 3^{(i+1)+1} - 1$$

$$\therefore 2\sum_{k=0}^{n} 3^k = 3^{n+1} - 1$$

(i) $\sum_{k=0}^{n} k\binom{n}{k} = n2^n - 1$

Question 4

Master Theorem

(i) $T(n) = 7T(\frac{n}{2}) + n^2$

Using the master theorem,

$aT(n/b) + fn = 7T(n/2) + n^2$

$a = 7, \; b = 2, \; fn = 2$

$a = 7, \; b^k = 4$

$a > b$; applies to case 3

$T(n) \in \Theta(n^{\log_b a}) = \Theta(n^{\log_n 7})$

$\log_2 7 = 2.80735$

$= \Theta(n^{2.80735})$

(ii) $T(n) = 5T(\frac{n}{3}) + O(n)$

~~aT(n~~

$aT(n/b) + fn = 5T(n/3) + O(n)$

$= 5T(n/3) + n$

$= 5T(n/3) + n^1$

$a = 5, \; b = 3, \; f(n) = n^\gamma \Rightarrow D = 1, \; k = 0$

check if $D >, <$ or $= \log_3 5$

$\log_3 5 = 1.465$

$D = 1$

$D < \log_3 5$

$a > b^a$

$\Theta(n^{\log_b a})$

$= \cancel{\Theta(n^b}$

$= \Theta(n^{\log_3 5})$

$= \Theta(n^{1.465})$

(iii) $T(n) = 3T(\frac{n}{2}) + \frac{3}{4}n + 1$

$a = 3, b = 2$

$\log_b a = \log_2 3 \simeq 1.58$

$\therefore T(n) = \Theta(n^{\log_2 3})$