CPSC 578: Assignment #3

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Problem 1

Consider the rendering shown below. A line segment has intensities of 100 and 200 at its two ends, and the intensity on the segment varies linearly between the end values. What is the error in estimating the intensity of pixel B by interpolating between the intensities computed for pixels A and C?

A: (The line has 26 dots vertically, while 58 dots horizontally)

According to the dots and the line provided, if we consider horizontal axis as x axis, while the center of the coordinate system is the start of the line, we can generalize the formula as follows:

$$I(x) = 100 + 100 * \frac{\sqrt{x^2 + (\frac{13}{29}x)^2}}{\sqrt{58^2 + 26^2}} \tag{1}$$

$\therefore$ A point represents an intensity of $I(A) = 113.8$
while B point represents an intensity of $I(B) = 139.7$
while C point represents an intensity of $I(C) = 179.3$

if A and C are used to estimate B, then:

$$I`(B) = \frac{I(A) + I(B)}{2} = 146.5 \tag{2}$$

$\therefore$ we can calculate the error of estimating the intensity of B as:

$$E_B = |I`(B) - I(B)| = 6.8 \tag{3}$$

Problem 2

Consider a virtual camera in a world coordinate system with the eye point at (1,10,2) and looking in the direction towards point (2, 10, -4), with an up direction given by the unit vector (0,1,0). The virtual camera has a view frustum that is 45 degrees both horizontally and vertically. A 5 by 5 pixel image is to be computed.
i. Give an equation in terms of a parameter t for all points P that lie on the ray from the camera origin through the center of the center pixel (pixel C in the diagram below) of the image.
ii. Find the angle between the rays that go through the centers of pixel A and pixel B. Is the angle between the rays that go through the centers of pixel B and pixel C the same, larger or smaller. Justify your answer.

A(i):

$\because$ Pixel C represents the point where the ray directly points into
$\therefore$ Pixel C can be denoted by:

$$\begin{aligned} P_c &= (1, 10, 2) + t[(2, 10, -4) - (1, 10, 2)] && (0 \leq t \leq 1) \qquad (4)\\ &= (1, 10, 2) + t(1, 0, -6) && (0 \leq t \leq 1) \qquad (5) \end{aligned}$$

A(ii):

$\because$ The view frustum is 45 degrees vertically

$\because$ A, B and C are centers of each the pixels

If we denote $\angle APC$, $\angle BPC$ and $\angle APB$ as γ, α and β accordingly, we can get following relations:

$$0^\circ \leq \alpha + \beta = \gamma \leq 45^\circ \tag{6}$$
$$AB = BC \tag{7}$$

Now, let's prove $\alpha > \beta$, note that as long as $\alpha > \frac{\gamma}{2}$, the non-equation stands:

$\because \tan\alpha = \frac{1}{2}\tan\gamma$

The above non-equation turns into:

$$\arctan(\frac{1}{2}\tan\gamma) > \tan\frac{\gamma}{2} \tag{8}$$

Thanks to the fact that tan has a positive derivative in the range of $(0, 45^\circ)$, we can derive the above non-equation to:

$$\frac{1}{2}\tan\gamma > \frac{\gamma}{2} \tag{9}$$

Compute the derivative of tan function, we get:

$$(\tan x)` = 1 + \tan^2 x > 1 \tag{10}$$

$\therefore \frac{1}{2}\tan\gamma > \frac{\gamma}{2}$ stands

$\therefore \alpha > \beta$

Thus, we have proved that $\angle BPC > \angle APB$.