CPSC 578: Assignment #5

Due on Mon, Nov 2, 2015

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Problem 1

You want to interpolate between two similar colors. They're represented in RGB space. You've heard that YIQ space is more naturally related to the human eye, so you convert to YIQ, interpolate there, and convert back. Explain why you get exactly the same result as if you'd interpolated in RGB. For which other color-description systems in this chapter will this turn out to be true, and why?

Assume we have two similar colors represented as C_1 and C_2 in RGB, to convert them to YIQ, we calculate:

$$C' = LC \tag{1}$$

in which, L can be represented as follows:

$$L = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix}$$

$\therefore$ to interpolate in YIQ space, we have the interpolated color $C_i'nt$ as:

$$C'_{int} = kC'_1 + (1-k)C'_2 \tag{2}$$

That is,

$$C'_{int} = L(kC_1 + (1-k)C_2) \tag{3}$$

To convert back to RGB space,

$$C_{int} = L^{-1}L(kC_1 + (1-k)C_2) = kC_1 + (1-k)C_2 \tag{4}$$

in which,

$$L^{-1} = \begin{pmatrix} 1 & 0.956 & 0.621 \\ 1 & -0.272 & -0.647 \\ 1 & -1.106 & 1.703 \end{pmatrix}$$

$\therefore$ we prove that the whole process is the same to directly interpolating in RGB space.

Problem 2

The chapter claims that if you interpolate colors using whatever color triples they're represented by, and if the colors are nearby, then it won't matter what color system you're using, in the sense that the results will be very similar. Verify this in the case of RGB and $L^*u^*v^*$versions of the colors (r,g,b) = (0.7,0.4,0.3) and (r,g,b) = $(0.7+\epsilon, 0.4-2\epsilon, 0.3-\epsilon)$, by finding the 50-50 mix of the two colors in both RGB and $L^*u^*v^*$and comparing. Do this for ϵ = 0.01, 0.05, and 0.25.

To prove that when two colors are nearby, it won't matter what color system is used, we calculate the average of the Luv value of the two RGB colors and the Luv value of the average of the two RGB colors.

When $\epsilon = 0.01$:

$Luv_1 = rgbToLuv(0.7, 0.4, 0.3) = (51.3, 57.3, 26.8)$ and $Luv_2 = rgbToLuv(0.7 + \epsilon, 0.4 + \epsilon, 0.3 + \epsilon) = (50.5, 63.4, 26.2)$

$\therefore Luv_{int} = (Luv_1 + Luv_2)/2 = (50.9, 60.35, 26.5)$

while on the other hand $Luv'_{int} = rgbToLuv(0.7 + \epsilon/2, 0.4 - 2\epsilon/2, 0.3 - \epsilon/2) = (50.9, 60.34, 26.52)$

The difference between Luv_{int} and Luv'_{int} is trivial.

When $\epsilon = 0.05$:

$Luv_1 = rgbToLuv(0.7, 0.4, 0.3) = (51.3, 57.3, 26.8)$ and $Luv_2 = rgbToLuv(0.7 + \epsilon, 0.4 - 2\epsilon, 0.3 - \epsilon) = (47.9, 87.9, 24.9)$

$\therefore Luv_{int} = (Luv_1 + Luv_2)/2 = (49.6, 72.6, 25.9)$

while on the other hand $Luv'_{int} = rgbToLuv(0.7 + \epsilon/2, 0.4 + \epsilon/2, 0.3 + \epsilon/2) = (49.4, 72.6, 25.6)$

The difference between Luv_{int} and Luv'_{int} is trivial.

When $\epsilon = 0.25$:

Assuming when $0.4 - 2\epsilon$, it equals to zero.

$Luv_1 = rgbToLuv(0.7, 0.4, 0.3) = (51.3, 57.3, 26.8)$ and $Luv_2 = rgbToLuv(0.7 + \epsilon, 0.4 - 2\epsilon, 0.3 - \epsilon) = (50.6, 165.6, 34.7)$

$\therefore Luv_{int} = (Luv_1 + Luv_2)/2 = (50.95, 111.45, 30.8)$

while on the other hand $Luv'_{int} = rgbToLuv(0.7 + \epsilon/2, 0.4 - 2\epsilon/2, 0.3 - \epsilon/2) = (47.38, 122.84, 28.46)$

The difference between Luv_{int} and Luv'_{int} is not trivial any more as ϵ is quite large.

Problem 3

(a) Suppose that the sensitivities of the receptors in the eye were not shaped like Gaussian bumps, but were instead triangular, the graph of the red receptor being an equilateral triangle with base between 600 nm, and 700 nm, the green having its base between 500 nm and 600 nm, and the blue having its base between 400 nm and 500 nm (all three equilateral triangles having the same heights). What would the CIE diagram look like? How many primaries would be needed for perfect color reproduction? (b) Suppose instead that the domains overlapped so that red was defined on [500,600], green on [450,550], and blue on [400,500]. What would the chromaticity diagram look like? How many primaries would be needed to faithfully reproduce every color percept?

So I am assuming that when constructing a CIE diagram, we find all the possible wavelength combinations and plot the corresponding (x, y, z) values to the diagram, while the combination is arbitrary, that is, it doesn't need to be consecutive?

(a)The diagram would be a triangle with vertices on (0, 0), (0, 1) and (1, 0).

The light with wavelength of between 400nm to 500nm are located in (0, 0).

The light with wavelength of between 500nm to 600nm are located in (0, 1).

The light with wavelength of between 600nm to 700nm are located in (1, 0).

The others should be inside the triangle.

Three primitive colors should be needed for perfect color reproduction.

(b)According to my understanding, the diagram should have the same shape as (a) does, even though there is overlaps in the receptors.

Three primitive colors should be needed for perfect color reproduction.