N2. Try to find a set of circumstances in which it would make sense to use Aitken's $\Delta^2$ method of accelerating convergence.

N3. Once you have built your Laguerre (sub)routine, use it to compute several iterations (example of your choosing) and see if you can identify convergence of order three.

Coding type question

C1. Build a polynomial root finding routine, designed to find all roots of a polynomial of degree $n$ with $n$ real distinct roots all lying in the interval with endpoints $-10$, and 10. Suppose the polynomial is $P(x)$. Your routine should be meet the following specifications:

(i) All polynomials (including polynomials that arise as derivatives of polynomials) should be evaluated by Horner's method (you are not allowed to have Matlab/Octave evaluate the polynomials for you other than by performing the computations Horner's method entails and which you need to code).

(ii) It first attempts to find $n$ approximate roots using the method of deflation. Whenever it needs to be seeded with a new starting point, it begins at a point achieving $\min_j |Q_k(x_j)|$, where the $x_j$ are evenly spaced points dividing the interval into subintervals of length 1/8, and the $Q_k$ are the polynomials that arise through deflation. Define an approximate root at this stage to be $x \in \mathbb{R}$ such that $|Q_k(x)| \leq 10^{-6}$.

(iii) It then 'polishes' the approximate roots of part (ii) by using each in turn as the starting point for a root finding algorithm working with the original polynomial. Define a polished root to be $x \in \mathbb{R}$ such that $|P(x)| \leq 10^{-24}$.

(iv) The root finding in parts (ii) and (iii) is to be done using Laguerre's method. As a reminder this is as follows:

Given starting point $x_0$, and polynomial $Q(x)$ of degree $m$, the iteration is

$$x_{j+1} = x_j - a_j,$$

where

$$a_j = \frac{n}{G(x_j) \pm \sqrt{(n-1)(nH(x_j) - G(x_j)^2)}},$$

with

$$G(x) = \frac{Q'(x)}{Q(x)},$$

$$H(x) = G(x)^2 - \frac{Q''(x)}{Q(x)},$$

and the sign in the denominator is chosen to give the larger absolute value of the denominator.