Stats 170 - Time Series Analysis

Homework 2

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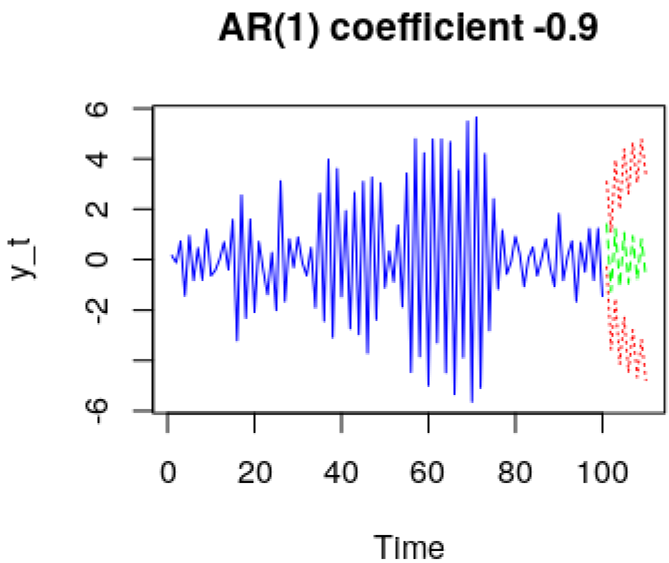
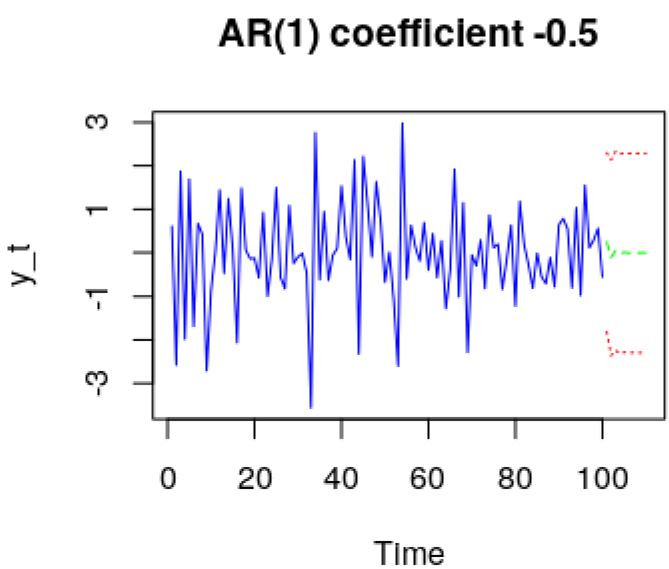
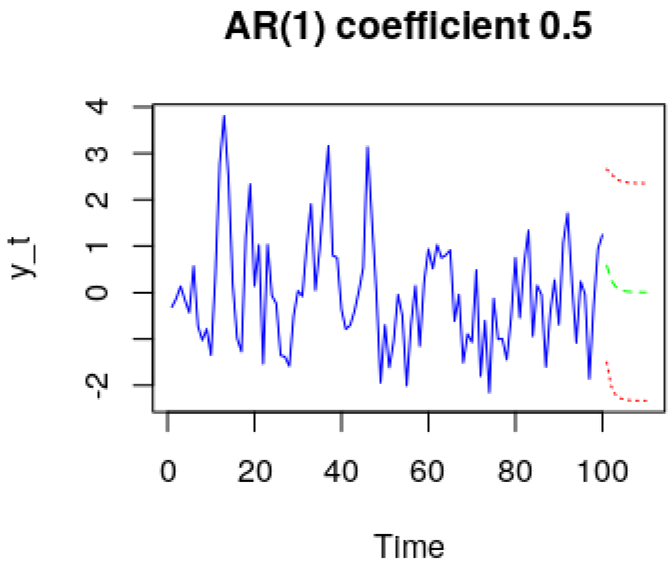
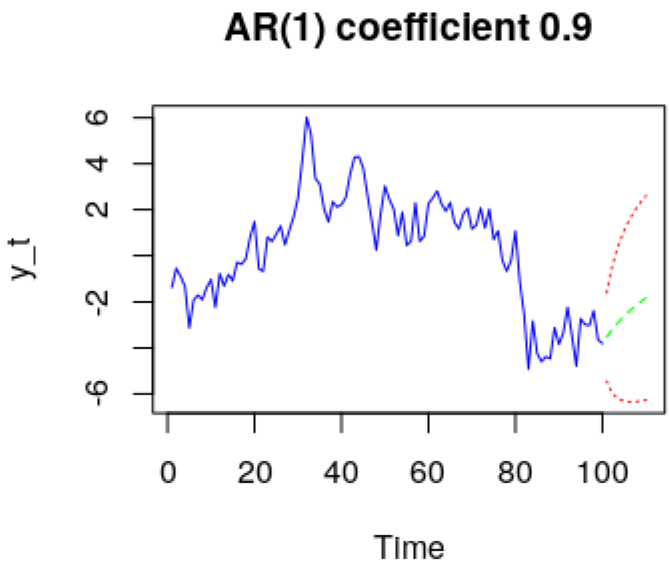
1

a. Simulate time series of length 100 for an AR(1) model with α equal to -0.9,-0.5,0.5, and 0.9. Estimate the parameter of each model and make predictions for 1 to 10 steps ahead.









Model Equations:

$\alpha = -0.9$:

$$\hat{y_t^\star} = 0.0290 - 0.9329\hat{y_{t-1}^\star} + w_t$$

$\alpha = -0.5$:

$$\hat{y_t^\star} = -0.0036 - 0.4420\hat{y_{t-1}^\star} + w_t$$

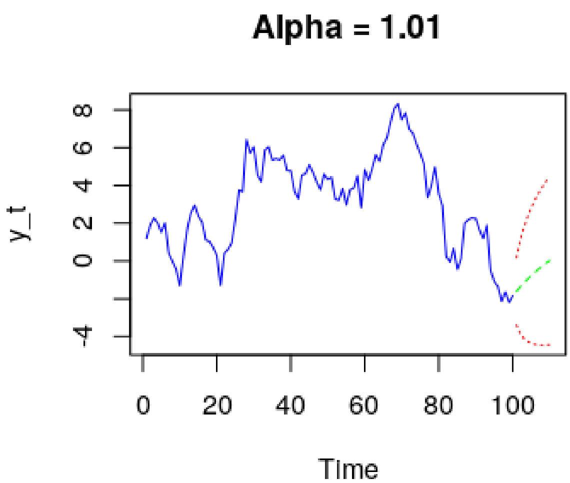
$\alpha = 0.5$:

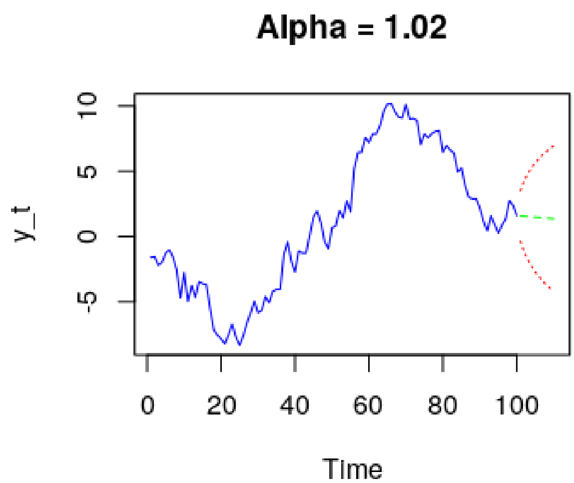
$$\hat{y_t^\star} = 0.0120 + 0.4619\hat{y_{t-1}^\star} + w_t$$

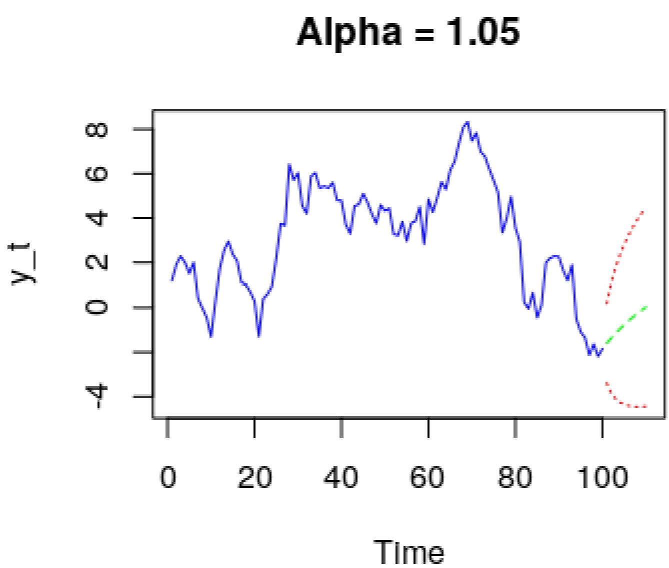
$\alpha = 0.9$:

$$\hat{y_t^\star} = -0.3094 + 0.9204\hat{y_{t-1}^\star} + w_t$$

b. Simulate time series of length 100 from an AR(1) model with α equal to 1.01, 1.02, and 1.05. Determine the roots of the polynomial in each case. Comment on your results.







For $\alpha = 1.01$, I achieve a root of 0.99099. Here I obtain an equation of

$$\hat{y_t^{\star}} = 2.2662 + 0.9414\hat{y_{t-1}^{\star}} + w_t$$

For $\alpha = 1.02$, I achieve a root of 0.9803922.Here I obtain an equation of

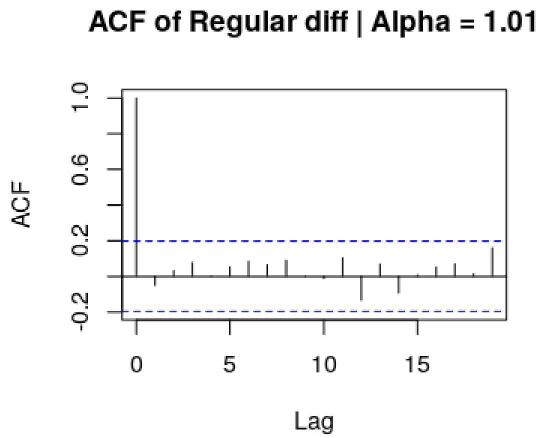
$$\hat{y_t^{\star}} = 0.5206 + 0.9766\hat{y_{t-1}^{\star}} + w_t$$

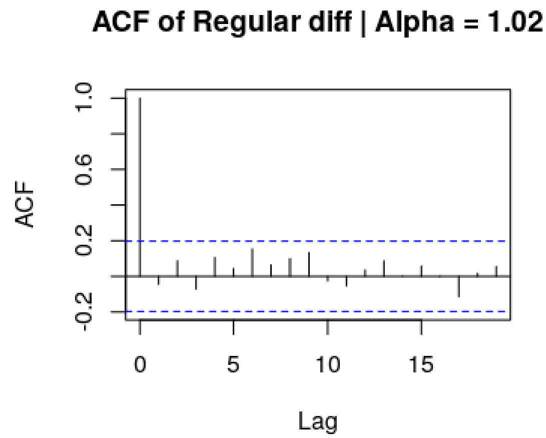
For $\alpha = 1.05$, I achieve a root of 0.952381. Here I obtain an equation of

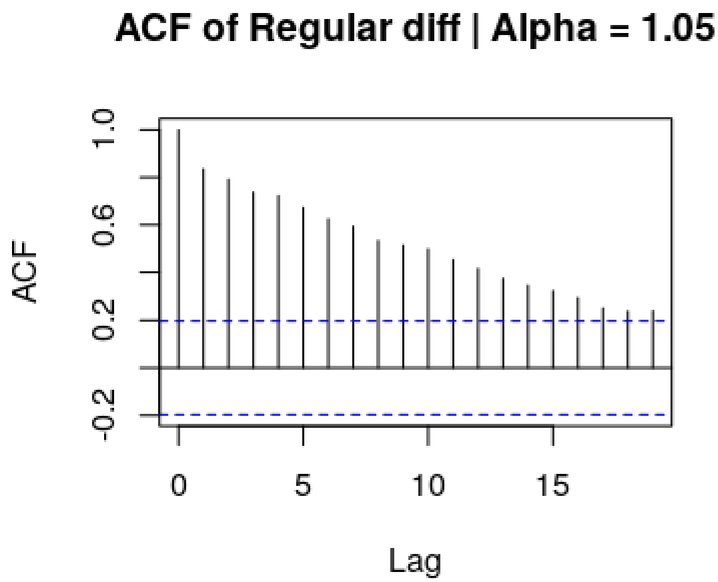
$$\hat{y_t^{\star}} = -6.9478 + 0.9909\hat{y_{t-1}^{\star}} + w_t$$

Since the roots are all under 1, we can conclude that we are dealing with all nonstationary processes. The polyroot appears to decrease in value as α increases.

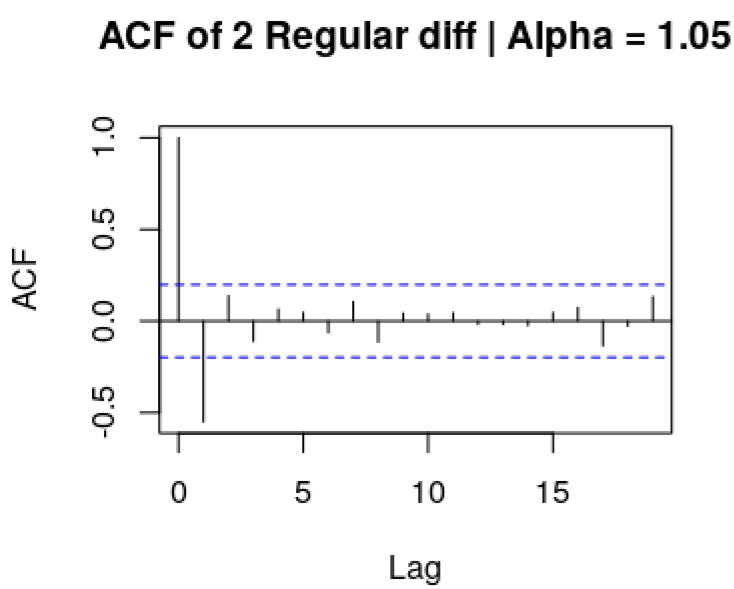
c. Now we perform differencing on the nonstationary series an check the acf's.





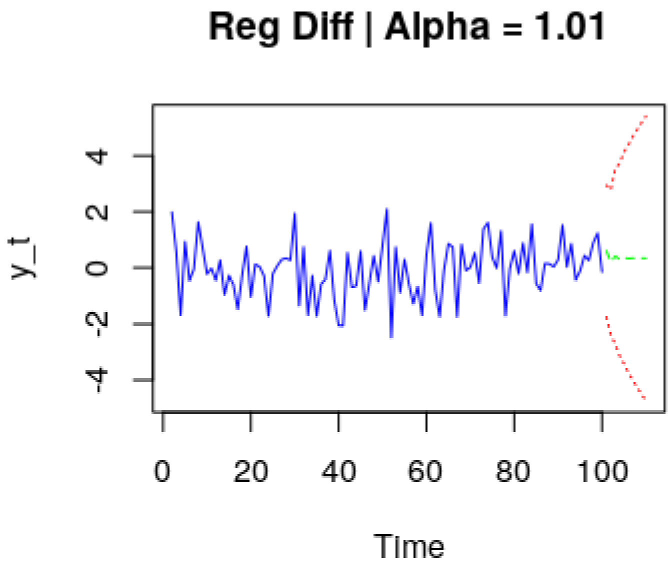


I now apply second regular differencing to estimate a better model for $\alpha = 1.05$. Here is the updated ACF after second regular differencing.



Let's start our analysis at $\alpha = 1.01$, we obtain the following after ftting an ARIMA(1,1,0).

$$\hat{y_t^\star} = -0.5399 y_{t-1} + w_t$$



Here are the next 3 steps provided mathematically.

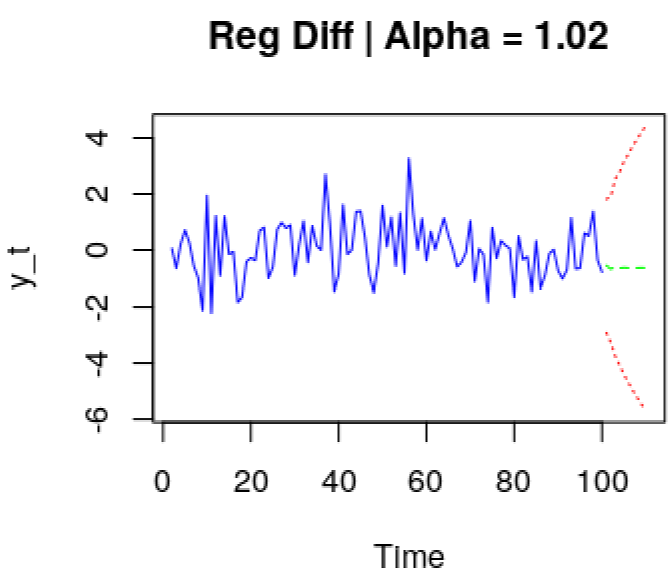
$$\hat{y_{t+1}^\star} = -0.5399 \hat{y_t^\star} + w_{t+1}$$

$$\hat{y_{t+2}^\star} = -0.5399 \hat{y_{t+1}^\star} + w_{t+2}$$

$$\hat{y_{t+3}^\star} = -0.5399 \hat{y_{t+2}^\star} + w_{t+3}$$

Now, let's focus on $\alpha = 1.02$, we again sufficed with just an ARIMA(1,1,0) model. Here is our estimated model and next three time steps as a forecast.

$$\hat{y_t^\star} = -0.5593 w_{t-1} + w_t$$



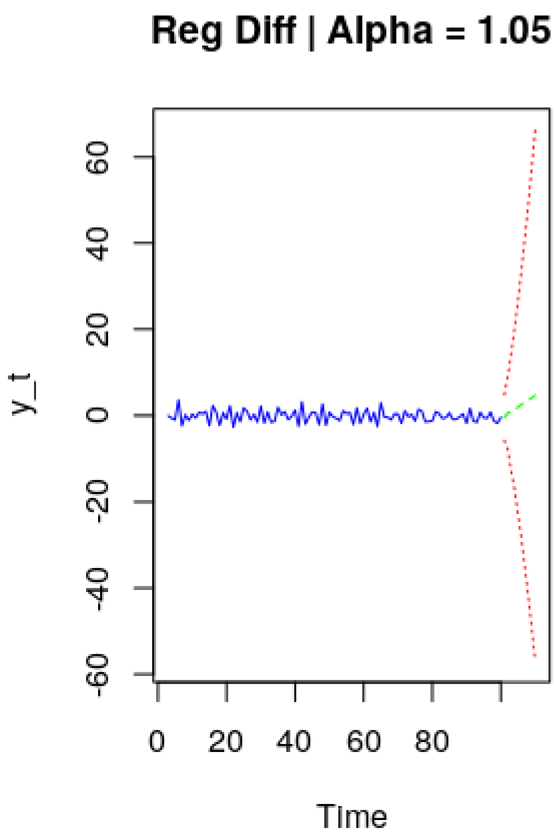
$$\hat{y^\star_{t+1}} = -0.5593\hat{y^\star_t} + w_{t+1}$$

$$\hat{y^\star_{t+2}} = -0.5593\hat{y^\star_{t+1}} + w_{t+2}$$

$$\hat{y^\star_{t+3}} = -0.5593\hat{y^\star_{t+2}} + w_{t+3}$$

Finally, let's check $\alpha = 1.05$. We had to perform second regular differencing, making it ARIMA(1,2,0). Here is the model and the next three time steps as a forecast.

$$\hat{y^\star_t} = -0.5593w_{t-1} + w_t$$

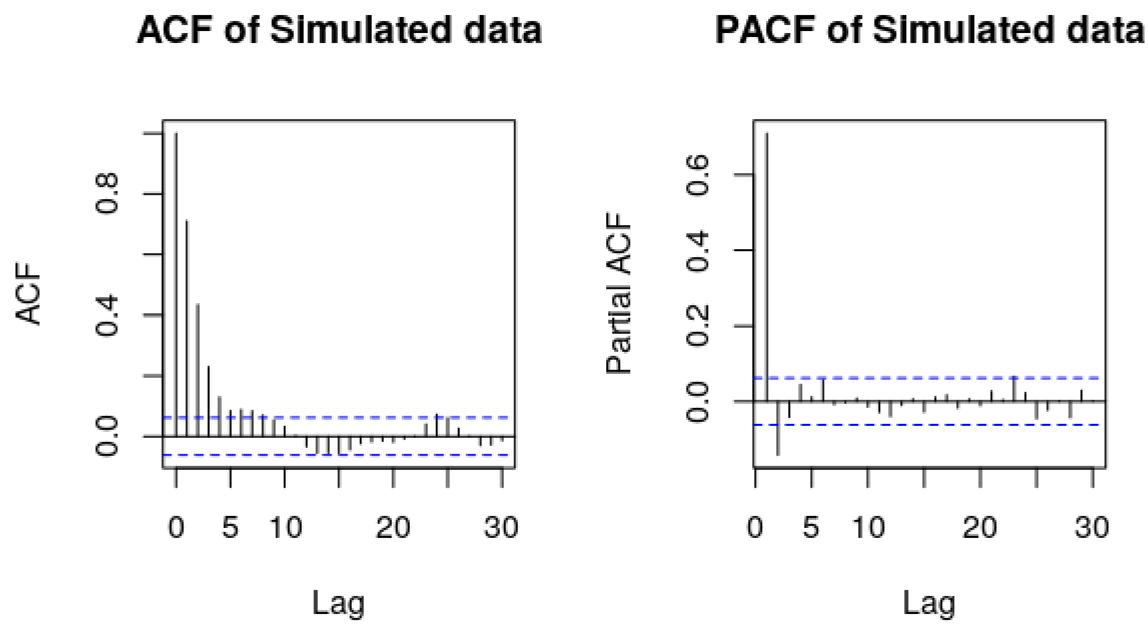


$$\hat{y^\star_{t+1}} = -0.7903\hat{y^\star_t} + w_{t+1}$$

$$\hat{y^\star_{t+2}} = -0.7903\hat{y^\star_{t+1}} + w_{t+2}$$

$$\hat{y^\star_{t+3}} = -0.7903\hat{y^\star_{t+2}} + w_{t+3}$$

b. Plot the correlogram and partial correlogram for the simulated data.



Here we notice good evidence that we are observing an AR(2) process. The autocorrelations within the acf decays over time, and the pacf becomes zero after lag k = 2.

c. Fit an AR model, write the equation of the fitted model.

$$\hat{x_t^\star} = -0.0076 + 0.8108x_{t-1}^\star - 0.1425x_{t-2}^\star$$

Here is my model after applying ARIMA(2,0,0)

d. Construct 95% confidence intervals of the fitted model. Do the model parameters fall within the confidence intervals?

I obtain a confidence interval of $[-1.518, 2.386]$. We notice that all the parameter estimates fall within the confidence intervals.

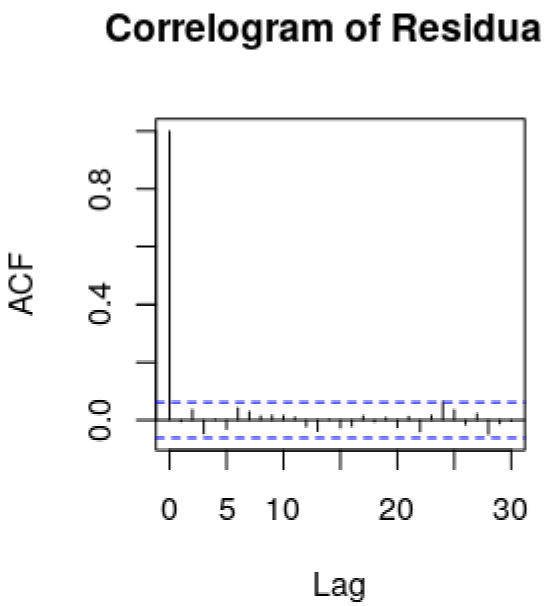
e. The model is stationary. Justification:

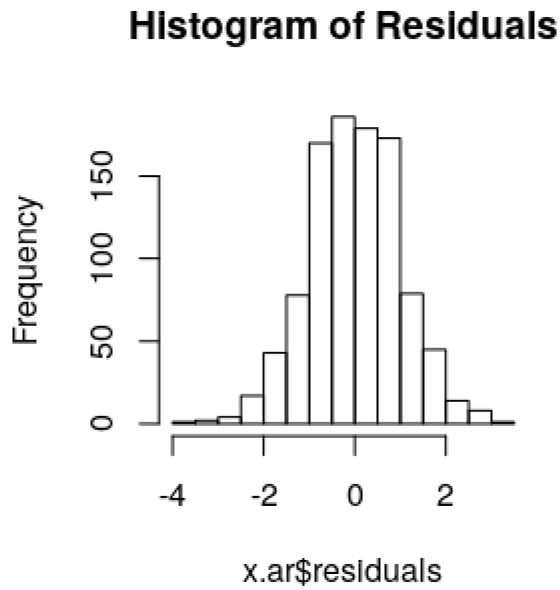
We can translate

$$x_t = \frac{5}{6}x_{t-1} - \frac{1}{6}x_{t-2} + w_t \Rightarrow (1 - \frac{5}{6}B + \frac{1}{6}B^2)x_t = w_t$$

Looking at the roots, of $(1 - \frac{5}{6}B + \frac{1}{6}B^2)$, we observe that $B = 2$ or $B = 3$. Since these root are OUTSIDE of the unit circle, the model is stationary.

f. Plot the correlogram of the residuals of the fitted model





The ACF indicates that the residuals follow the pattern of i.i.d. white noise. This means that the estimated model we were working on did very well.

3

a. Show that the series $\{x_t\}$ is nonstationary.
Given

$$x_t = \frac{3}{2}x_{t-1} - \frac{1}{2}x_{t-2} + w_t \Rightarrow (1 - \frac{3}{2}B + \frac{1}{2}B^2)x_t = w_t$$

This gives roots of $B = 1$ and $B = 2$. Since one of the roots is within the unit circle, we decleare that the series $\{x_t\}$ is nonstationary.

b. Show that the series $\{y_t\}$ is stationary.

Observe

$$x_t = \frac{3}{2}x_{t-1} + \frac{1}{2}x_{t-2} + w_t \Rightarrow \frac{1}{2}(x_{t-1} + x_{t-2}) + w_t$$

Thus,

$$y_t = \frac{1}{2}y_{t-1} + w_t$$

$$w_t = (1 - \frac{1}{2}B)y_t$$

Looking at $(1 - \frac{1}{2}B)$, we notice the root of $B = 2$ is outside of the unit circle. This means, that $\{y_t\}$, where $y_t = \triangledown x_t$, is stationary.

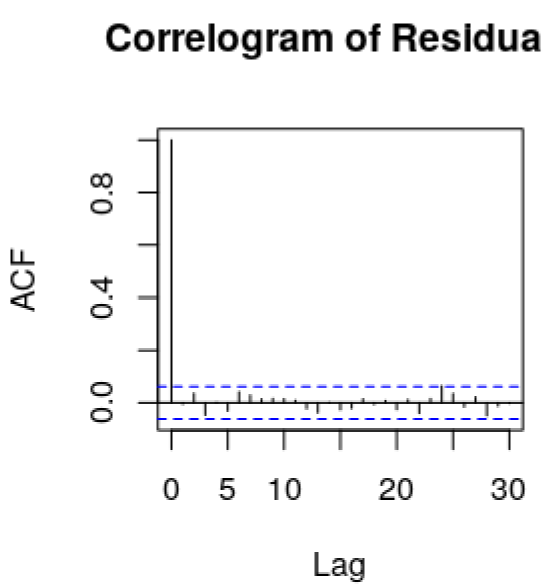
d. Fit your AR Model to y. Give model paramter estimates and a 95% confidence interval. Compare the confidence intervals to the parameters used to simulate the data and explain the results.

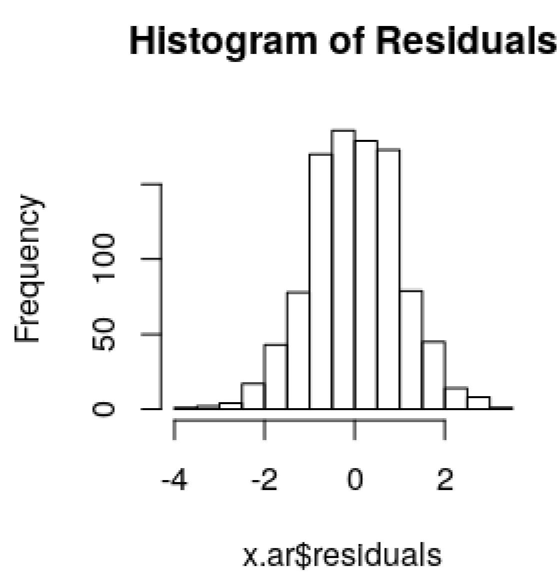
I obtain the following model estimates after applying ARIMA(2,0,0):

$$\hat{x_t^\star} = 0.1080 + 0.5031x_{t-1}^\star + 0.0073x_{t-2}^\star$$

I obtain a confidence interval of $[-1.189, 2.845]$. The paramaters used to simulate the data lie within the confidence interval.

e. Comment on residuals.

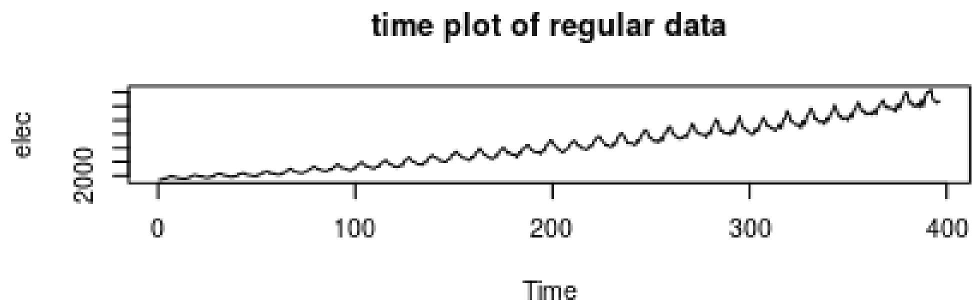


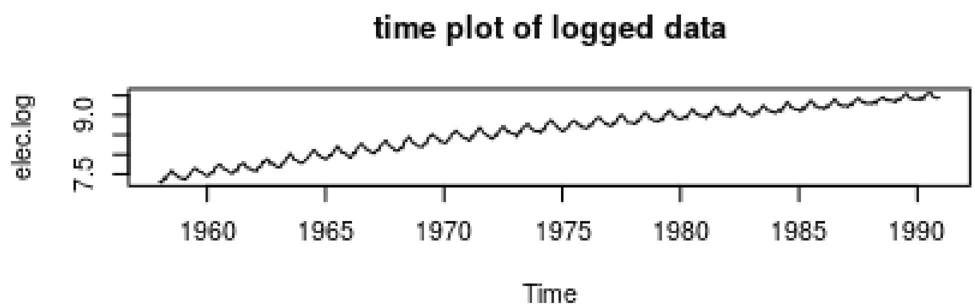


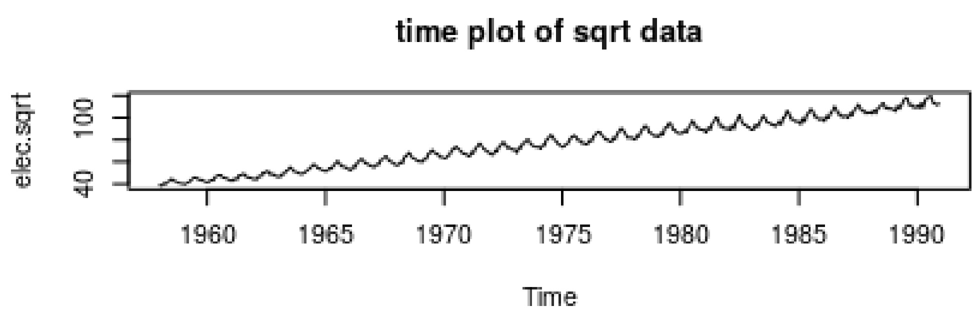
We notice the residuals appear to follow the pattern of i.i.d. white noise, so the model did an adequate job.

4

Reading in the electricity data, I then took a look at the time plot to see whether a transformation should be performed.

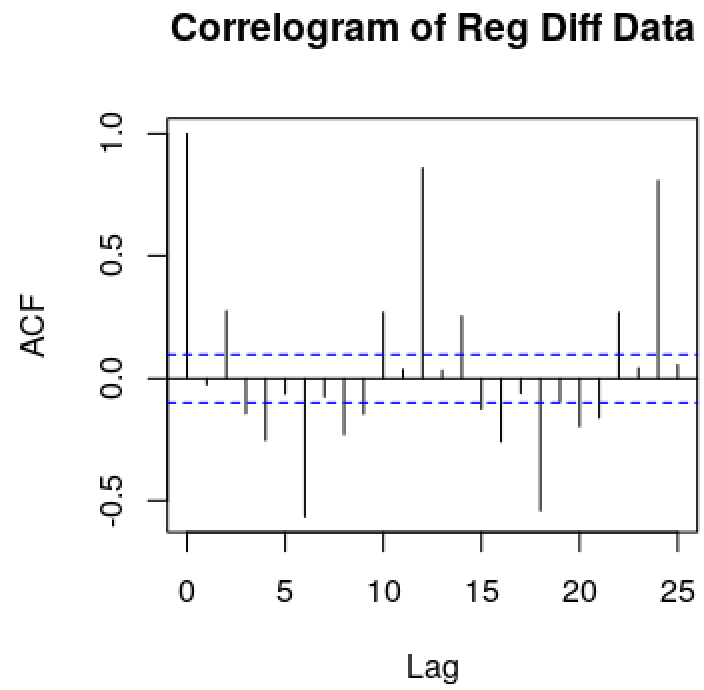




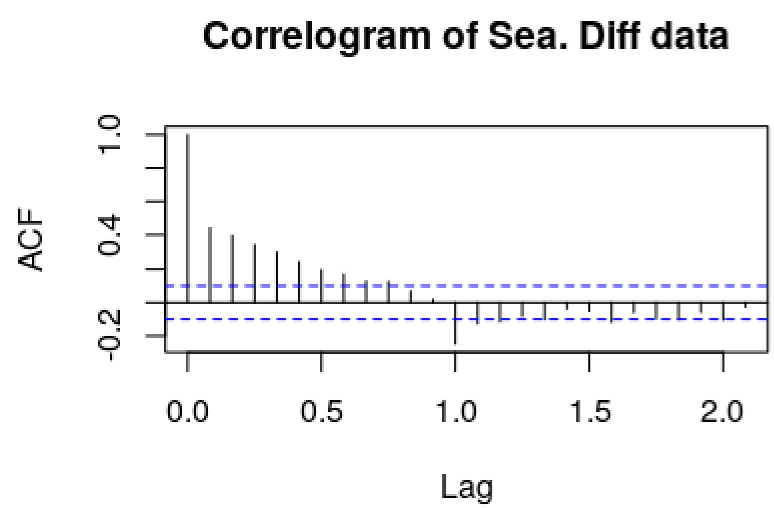


Looking at these three time plots, I decided that the sqrt transformation seemed to the do the best job of stabilizing the data, with a constant variance.

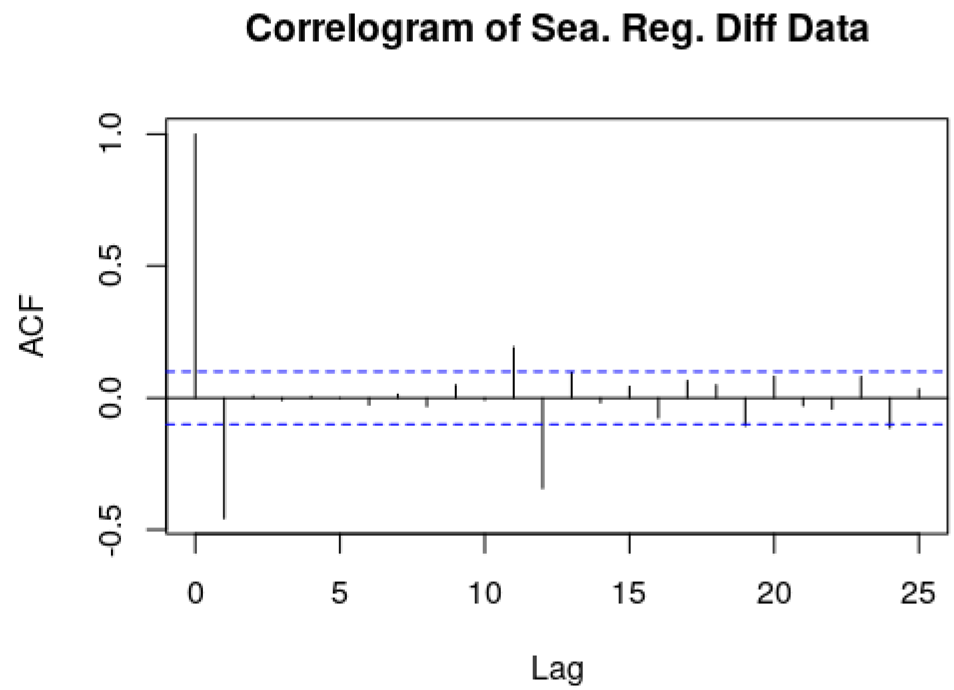
Next we apply regular differencing, let's take a look at the ACF.



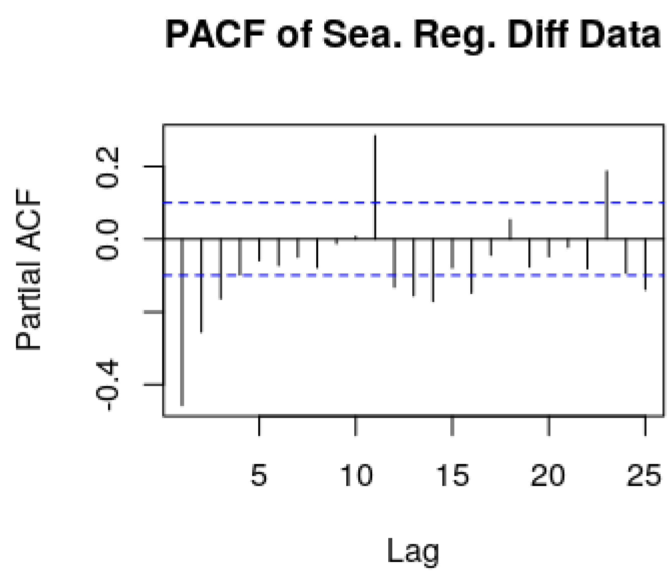
Before we apply seasonal differencing to this regularly differenced data, let's first look at just seasonally differenced data of 12 time steps.



Looking at both of these ACFs, we still have some work to do. Let's now perform seasonal differencing after regularly differencing the data.



We now notice only two significant lags, the latter due to some form of seasonality. Let's identify a model. First, let's take a look at the pacf.



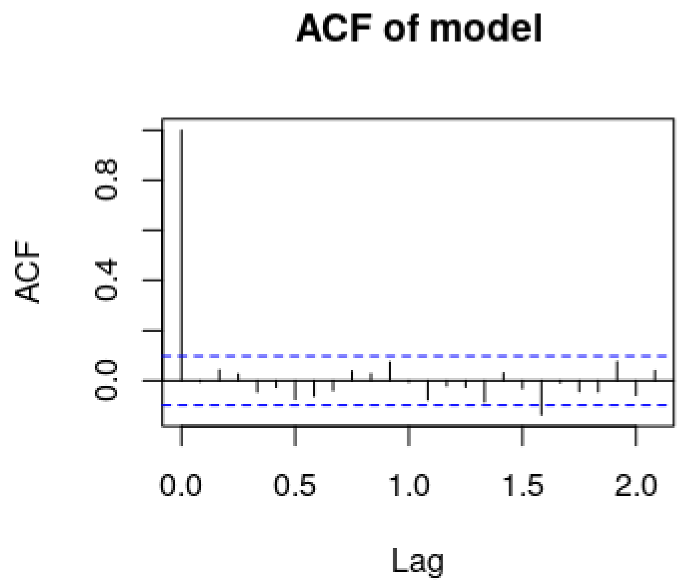
So we notice that based on the acf only having one significant lag and the fact that the pacf slowly changes we now identify a model. Due to the aforementioned patterns in the acf and pacf, I decide to fit the following model in R:

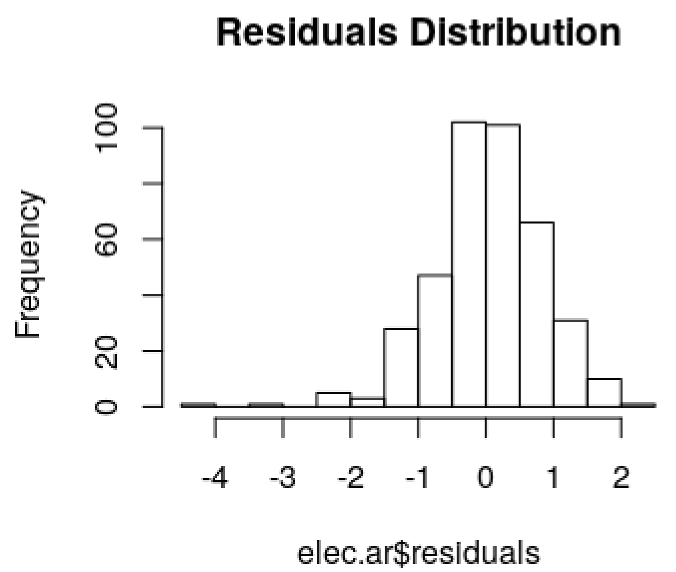
```
arima(elec.sqrt, order = c(1,1,1), seas = list(order=c(1,1,1), 12))
```

Here is our model:

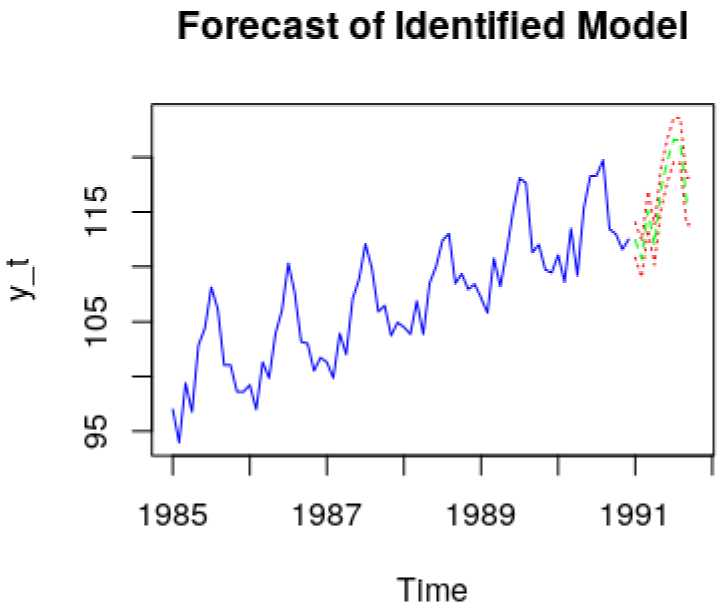
$$\hat{y_t^\star} = 0.0568y_{t-1}^\star + 0.0673y_{t-13}^\star + w_t - 0.7125w_{t-1} - 0.6682w_{t-13}$$

Let's look at the residuals of our model.





The residuals appear to look like i.i.d. white noise. Let's now forecast 10 periods ahead. Here is a plot with the prediction intervals.



Our forecast seems to do a great job of keeping the pattern we were noticing previously. The red lines are there to indicate our confidence interval.