Devoir Q1
25 janvier 2022 13:21

Devoir 1
Question 1
ELE3500

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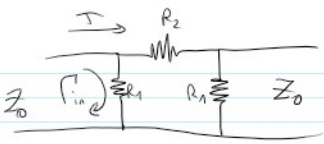
$$\frac{17,25}{20}$$

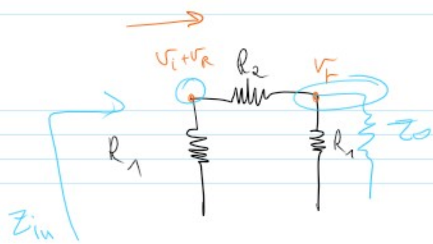


$Z_0 = 100\,\Omega$

$\Gamma_{in} = 0$

$T = 0,5$

a)



$$\frac{V_T}{V_i} = 0,5$$

$$Z_{in} = R_1 // (R_2 + R_1 // Z_0)$$

$$= R_1 // \left(R_2 + \left(R_1^{-1} + Z_0^{-1}\right)^{-1} \right)$$

$$R_1 // \left(\frac{R_2(Z_0 + R_1) + Z_0 R_1}{Z_0 + R_1} \right)$$

$$\left(\frac{1}{R_1} + \frac{Z_0 + R_1}{R_2 Z_0 + R_2 R_1 + Z_0 R_1} \right)^{-1}$$

$$= \frac{R_1 R_2 Z_0 + R_1^2 R_2 + R_1^2 Z_0}{R_2 Z_0 + R_2 R_1 + Z_0 R_1 + R_1 Z_0 + R_1^2} = Z_0$$

Car $\Gamma_{in} = 0$

$$\Rightarrow \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0 \Rightarrow Z_{in} = Z_0$$

$$\cancel{R_1 R_2 Z_0} + R_1^2 R_2 + \cancel{R_1^2 Z_0} = Z_0^2 R_2 + \cancel{Z_0 R_2 R_1} + 2 Z_0^2 R_1 + \cancel{Z_0 R_1^2}$$

$$\Rightarrow \boxed{R_1^2 R_2 = Z_0^2 (R_2 + 2 R_1)}$$

$R_1 // Z_0$

$$V_{out} = \frac{R_1 // Z_0}{R_2 + R_1 // Z_0} V_{in}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{\frac{R_1 Z_0}{R_1 + Z_0}}{\frac{R_2 R_1 + R_2 Z_0 + R_1 Z_0}{R_1 + Z_0}} = \frac{R_1 Z_0}{\cancel{R_1 + Z_0}} \times \frac{\cancel{R_1 + Z_0}}{R_2 R_1 + R_2 Z_0 + R_1 Z_0}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \boxed{\frac{R_1 Z_0}{R_2 R_1 + (R_2 + R_1) Z_0} = 10^{-6/20} = 0,5}$$

En résolvant le système, on a :

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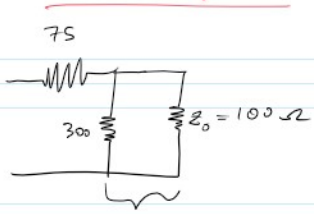
$$\boxed{\begin{aligned} R_1 &= Z_0 \frac{1 + 10^{-6/20}}{1 - 10^{-6/20}} \approx 300,85\ \Omega \\ R_2 &= Z_0 \frac{1 - \left(10^{-6/20}\right)^2}{2 \cdot 10^{-6/20}} \approx 74,7\ \Omega \end{aligned}}$$

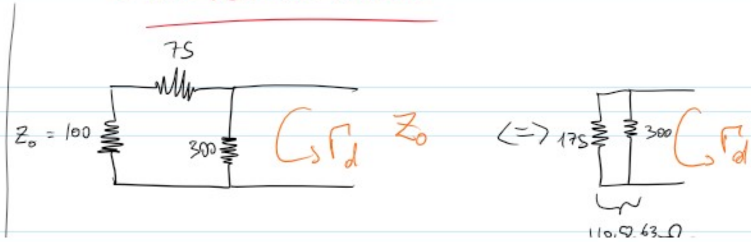
b)

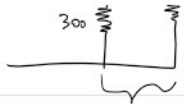
- $R_G = 10\ \Omega = 0,1 Z_0$
- $R_L = 100 = Z_0$
- $V_S = 200V$ à $t = 0s$
  - ↳ dure $100ns$
- $R_1 = \infty$
- $0 \leq t \leq 75ns$

Onde vient de gauche :

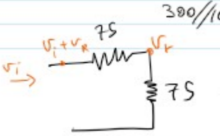


Onde vient de droite :





$300 // 100 = 75\ \Omega$

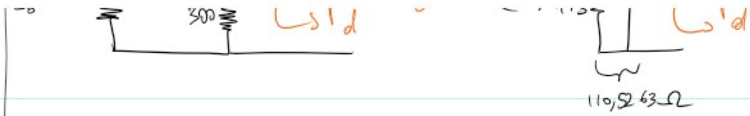


$\rightarrow v_r = (v_i + v_R) \times \frac{75}{75+75}$

$\Rightarrow \frac{v_r}{v_i} = \left(1 + \frac{v_R}{v_i}\right) \times \frac{1}{2}$

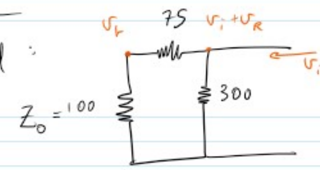
$\Rightarrow T_g = \frac{1}{2}(1 + \Gamma_g)$

$$\boxed{\Gamma_g = \frac{150-100}{150+100} = \frac{50}{250} = 0{,}2 \qquad \Rightarrow T_g = 0{,}6}$$



$$\Rightarrow \Gamma_d = \frac{110{,}5263 - 100}{110{,}5263 + 100} = 0{,}05$$

T_d :



$$v_r = (v_i + v_R)\,\frac{100}{100+75}$$

$$\Rightarrow \frac{v_r}{v_i} = \left(1 + \frac{v_R}{v_i}\right)\frac{100}{175}$$

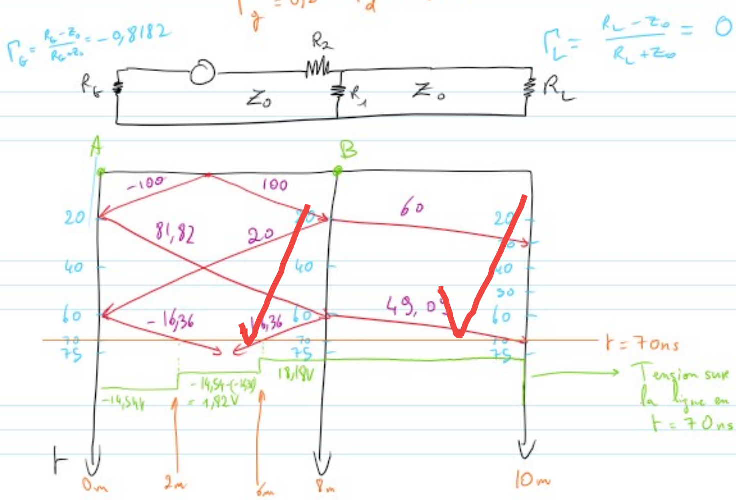
$$\Rightarrow T_d = (1 + \Gamma_d)\,\frac{100}{175}$$

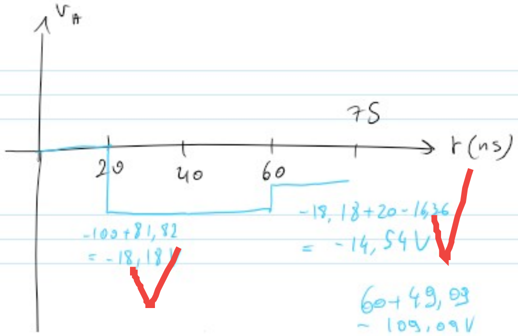
$$\boxed{\Rightarrow T_d = 0{,}6 \qquad \Gamma_d = 0{,}05}$$

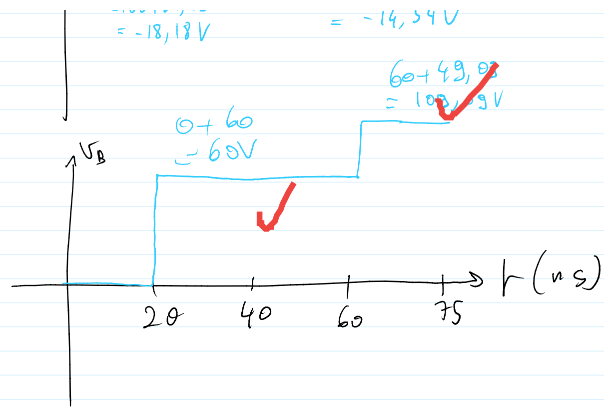
$T_g = 0{,}6$ $\quad T_d = 0{,}6$

$\Gamma_g = 0{,}2$ $\quad \Gamma_d = 0{,}05$

$\Gamma_G = \frac{R_G - Z_0}{R_G + Z_0} = -0{,}8182$ $\qquad \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 0$







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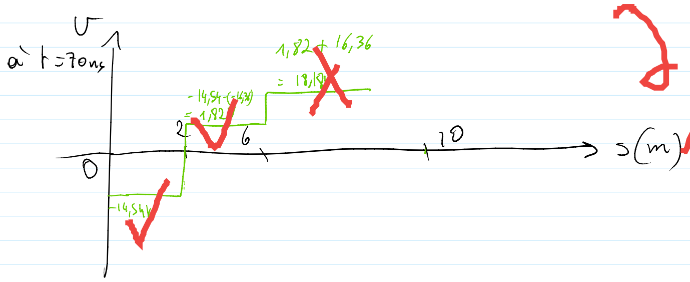
c) À $t = 70$ ns, on trace dans le diagramme de rebond une ligne horizontale, on a alors :



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(*) Annexe Question c :

• Calcul des distances le long du fil :

$v = \frac{d}{\Delta t}$ $\Rightarrow d = v \cdot \Delta t$	$\Delta t = 70\,ns - 60\,ns$ $= 10\,ns$ $v = 2.10^8\ m/s$ $\Rightarrow d = 2.10^8 \frac{m}{s} \times 10.10^{-9}\,s$ $\Rightarrow d = 2\ m$	$\Delta t = 40\,ns$ $v = 2.10^8\ m/s$ $d = 2.10^8 \frac{m}{s} \times 40.10^{-9}\,s$ $\Rightarrow d = 8\ m$

0,25

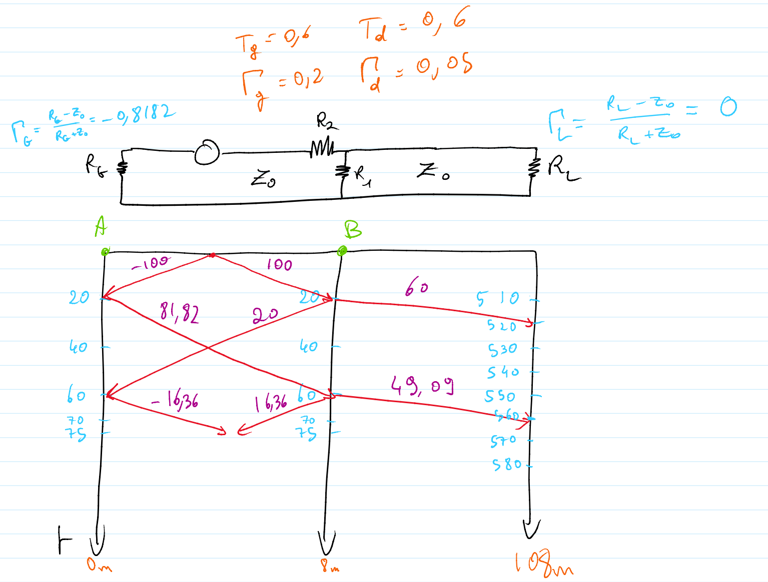
d)

Nous ne changeons que la ligne après l'atténuateur. Donc le diagramme de rebonds fait en b) reste valide pour la partie avant atténuateur.

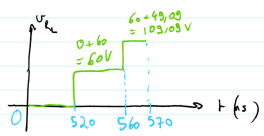
Par ailleurs, le coefficient de reflexion au niveau de la résistance R_L est : $\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 0$

Donc on a le diagramme de rebond suivant :

$T = 0,6$ $T_d = 0,6$



• si aucune atténuation, on a au niveau de R_L :



OR nous avons une atténuation,

→ la ligne est sans distorsion, donc la condition de Heaviside est respectée.

$$\Rightarrow \quad RC = LG \quad (1)$$

De plus, on a $Z_0 = \sqrt{\frac{L}{C}}$ (2)

et $2.10^8 = c = \frac{1}{\sqrt{LC}}$ (3)

En prenant $R = 0,1\ \Omega/m$, on a un système de 3 équations à 3 inconnues.

$$\begin{cases} 0,1\ C = L\ G \\ 100 = \sqrt{\frac{L}{C}} \Rightarrow 100^2 C = L \Rightarrow 100^2 C = \frac{1}{(2.10^8)^2}\frac{1}{C} \Rightarrow C = \sqrt{\frac{1}{(2.10^8)^2} \times \frac{1}{100^2}} \\ 2.10^8 = \frac{1}{\sqrt{LC}} \Rightarrow \left((2.10^8)^2\right)^{-1} = LC \end{cases}$$

$$\Rightarrow C = \frac{1}{2.10^8 \times 100} = 50\ pF$$

$$\hookrightarrow \left(\frac{1}{2.10^8}\right)^2 \times \frac{1}{C} = L \Rightarrow L = 0,5\ mH$$

$$\Rightarrow \quad G = \frac{0,1\ C}{L} = 10^{-5}$$

Ainsi $\alpha = \sqrt{RG} = \sqrt{0,1 \times 2,5.10^{-16}}$

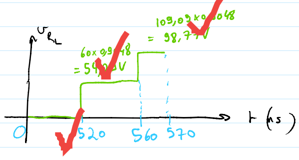
$$\Rightarrow \alpha = 10^{-3}$$

On se situe à $s = 108 - 8 = 100\,m$

Donc l'atténuation vaut

$$e^{-\alpha s} = e^{-100 \cdot 10^{-3}} = 0,9048$$

Ainsi la tension en R_L en fonction du temps vaut :



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