Feuille de notes
24 février 2022 22:45

$$TOS = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$
$$= V^+ e^{j\beta d}\left(1 + |\Gamma_L| e^{j\angle\Gamma} e^{-j2\beta d}\right.$$

$$Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) \qquad Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$\Rightarrow z_L = \frac{1+\Gamma_L}{1-\Gamma_L}$, si $\Gamma_L \in \mathbb{R}^+$ ↳ $TOS = z_L$

$$\Delta d_{max} = \Delta d_{min} = \frac{\lambda}{2}$$

Puissance moyenne: $p = \frac{1}{2} Re\{V I^*\}$

$$\Gamma(d) = \Gamma_L e^{-2\gamma d}$$

$$\Gamma(d) = \frac{Z(d)-Z_0}{Z(d)+Z_0} \quad ; \quad Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$$

• Matrice ABCD:

$$\begin{bmatrix} V(d) \\ I(d) \end{bmatrix} = \begin{bmatrix} \cos\beta d & j Z_0 \sin\beta d \\ \frac{j}{Z_0}\sin\beta d & \cos\beta d \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} \quad \text{SANS PERTES}$$

$$Z(d) = \frac{Z_L \cos\beta d + j Z_0 \sin\beta d}{j\frac{Z_L}{Z_0}\sin\beta d + \cos\beta d} = Z_0\,\frac{(Z_L + j Z_0 \tan\beta d)}{j Z_L \tan\beta d + Z_0}$$

$$z(d) = \frac{z_L + j\tan\beta d}{j z_L \tan\beta d + 1}$$

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

$$\begin{bmatrix} V(d) \\ I(d) \end{bmatrix} = \begin{bmatrix} \cosh\gamma d & Z_0 \sinh\gamma d \\ \frac{1}{Z_0}\sinh\gamma d & \cosh\gamma d \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} \quad \text{AVEC PERTES}$$

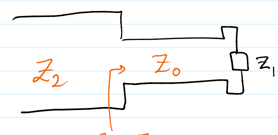
$$Z(d) = \frac{Z_L \cosh\gamma d + Z_0 \sinh\gamma d}{\frac{Z_L}{Z_0}\sinh\gamma d + \cosh\gamma d}$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \beta = \omega\sqrt{LC} \quad \alpha = \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right)$$

$$Z(d) = \frac{Z_0 Z_L + j Z_0 \tan\beta d}{j Z_L \tan\beta d + Z_0}$$

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$



But: $Z_{in} = Z_2$

$$Z_0 = \sqrt{Z_1 Z_2}$$

$$\Gamma_V = \frac{R_L - Z_0}{R_L + Z_0}$$

$$\Gamma_I = \frac{G_L - Y_0}{G_L + Y_0} = -\Gamma_V$$

$$1 + \Gamma = T$$

Condition de Heaviside

$$RC = LG$$
$$\alpha = \sqrt{RG}$$
$$c = \frac{1}{\sqrt{LC}} = \lambda f = \frac{\omega}{\beta}$$
$$\beta = \omega\sqrt{LC}$$
$$\alpha = \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right) = 0 \text{ si pas pertes}$$

$\xrightarrow{+s}$ $\xleftarrow{-s}$

$$V(s) = A e^{-\gamma s} + B e^{\gamma s}$$
$$= A e^{-\alpha s} e^{+j\beta s}$$

ONDES

$\bar{\mathcal{E}}, \bar{\mathcal{H}}, \bar{\mathcal{J}}$: vecteurs réels dépendent de x, y, z et t en général

$\bar{E}_c, \bar{H}_c, \bar{J}_c$: vecteurs complexes
e.g. $\bar{\mathcal{E}} = Re\{\bar{E}_c\}$

$\bar{E}, \bar{H}, \bar{J}$: vecteurs phaseurs
e.g. $\bar{E}_c = \bar{E} e^{j\omega t}$

$\underline{\gamma}, \underline{\varepsilon}$: constantes complexes, scalaires

<u>formules utiles</u> :

$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{\beta} = \lambda f$, $\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m

$\beta = \|\bar{k}\| = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$, $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$c = 3 \cdot 10^8$ m/s

$\bar{H} = \bar{k} \times \bar{E}$ $\quad P_{moy} = \frac{1}{2} \mathrm{Re}\{\bar{E} \times \bar{H}^*\}$, $\quad \bar{E} = \eta \bar{H}$

$\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$\bar{E} = \bar{A} e^{-j(-j\gamma \hat{n}\cdot\bar{R})} + \bar{B} e^{+j(-j\gamma \hat{n}\cdot\bar{R})}$

$= \bar{A}\, e^{-\alpha \hat{n}\cdot\bar{R}}\, e^{-j\beta\hat{n}\cdot\bar{R}}$

$+ \bar{B}\, e^{+\alpha \hat{n}\cdot\bar{R}}\, e^{+j\beta\hat{n}\cdot\bar{R}}$

$\underline{\varepsilon} = \varepsilon_0 (\varepsilon_r - j\frac{\sigma_0}{\omega\varepsilon_0})$

$\bar{k} = -j\gamma\, \hat{n}$

$P_{moy} = \frac{\|\bar{E}\|^2}{2\eta} = \frac{\eta\|\bar{H}\|^2}{2}$

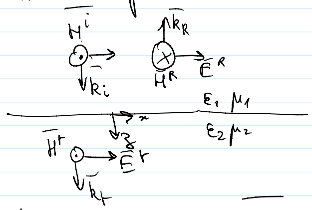
Bon conducteur : $p = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma_e}{\omega\varepsilon_r\varepsilon_0} > 100$

$\bar{P}_{moy} = \hat{n}\, \frac{|A_x|^2 + |A_y|^2 + |A_z|^2}{2\eta}$

$\sin x = \cos(x - \frac{\pi}{2})$ $\quad \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

$\alpha = \beta = \sqrt{\frac{\sigma\omega\mu}{2}}$

Plane uniforme incidence normale : pol linéaire



$\bar{H} = \frac{\bar{k} \times \bar{E}}{\omega\mu}$

$$\begin{cases} P^i = P^i_z = \bar{P}^i \cdot \hat{z} = \frac{1}{2}\mathrm{Re}\left\{\frac{E^i_x E^{i*}_x}{\eta_1^*}\right\}\cdot\hat{z} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta_1^*}\right\}|E^i_x|^2_{z=0} \\ P^R = P^R_z = P^R\cdot(-\hat{z}) = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta_1^*}\right\}|E^R_x|^2_{z=0} \\ P^t = P^t_z = P^t\cdot(\hat{z}) = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{\eta_2^*}\right\}|E^t_x|^2_{z=0} \end{cases}$$

- $P^i = P^t + P^R \Rightarrow P^t = P^i - P^R \Rightarrow |T|^2 \mathrm{Re}\left\{\frac{1}{\eta_2^*}\right\} = \mathrm{Re}\left\{\frac{1}{\eta_1^*}\right\}(1 - |\Gamma|^2)$

$$\begin{cases} \bar{E}^i = \hat{x}\, E^i_x \\ \bar{E}^R = \hat{x}\, E^R_x \\ \bar{E}^t = \hat{x}\, E^t_x \end{cases} \quad \begin{cases} \bar{k}_i = \hat{n}_i \beta_1 = \hat{z}\beta_1 & ; \beta_1^2 = \omega^2\mu_1\varepsilon_1 \\ \bar{k}_R = \hat{n}_R \cdot \beta_1 \\ \bar{k}_t = \hat{n}_t \beta_2 = \hat{z}\beta_2 & ; \beta_2^2 = \omega^2\mu_2\varepsilon_2 \end{cases}$$

$$\Rightarrow \begin{cases} \bar{H}^i = \frac{\bar{k}_i \times \bar{E}^i}{\omega\mu_1\varepsilon_1} = \frac{\beta_1 E^i_x\, \hat{z}\times\hat{x}}{\omega\mu_1} = \hat{y}\, E_x^i/\eta_1 \\ \bar{H}^R = \frac{\bar{k}_R \times \bar{E}^R}{\omega\mu_1\varepsilon_1} = \frac{\beta_1(-\hat{z})\times\hat{x}\, E^R_x}{\omega\mu_1} = -\hat{y}\, E^R_x/\eta_1 \\ \bar{H}^t = \frac{\bar{k}_t \times \bar{E}^t}{\omega\mu_2\varepsilon_0} = \frac{\beta_2(\hat{z})\times\hat{x}\, E^t_x}{\omega\mu_2} = \hat{y}\, E^t_x/\eta_2 \end{cases}$$

$$\overline{H^t} = \frac{\overline{k_t} \times \overline{E^t}}{\omega \mu_2 \varepsilon_2} = \frac{\beta_2(\hat{z}) \times \hat{x}\, E_x^t}{\omega \mu_1} = \hat{y}\, E_x^t / \eta_2$$

$$\overline{E^i_{tan}} + \overline{E^R_{tan}} = \overline{E^r_{tan}}$$

$$\Rightarrow E_x^i(z=0) \cdot e^{-j\overline{k_i} \cdot \overline{R}}\, \hat{x} + E_x^R(z=0) \cdot e^{-j\overline{k_R} \cdot \overline{R}}\, \hat{x} = E_x^t(z=0) \cdot e^{-j\overline{k_t} \cdot \overline{R}}\, \hat{x}$$

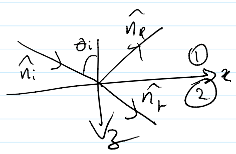
$\rightarrow$ À l'interface $R = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

$$\Rightarrow \boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \qquad \boxed{T = 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

$$Z \approx \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon_0 \varepsilon_R \left(1 + j \frac{\sigma}{\omega \varepsilon_0 \varepsilon_R}\right)}}$$

$$|Z| = \sqrt{\frac{\omega \mu}{\sigma}}$$

Onde plane incidence oblique



$$\hat{n}_i = \begin{bmatrix} \sin\theta_i \\ 0 \\ \cos\theta_i \end{bmatrix} \rightarrow \overline{n_{i,tan}} = \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{E^i} = \overline{A}\, e^{-j\beta_1 \hat{n}_i \cdot \overline{R}}$$

$$\overline{E^R} = \overline{B}\, e^{-j\beta_1 \hat{n}_R \cdot \overline{R}}$$

$$\overline{E^t} = \overline{C}\, e^{-j\beta_2 \hat{n}_t \cdot \overline{R}} \quad \text{ou } \overline{C}\, e^{-\overline{\gamma_2} \cdot \overline{R}}$$

$$\boxed{\begin{array}{l} \overline{\gamma} = (\alpha + j\beta) \cdot \hat{n} \\ e^{-\overline{\gamma} \cdot \overline{R}} = e^{-\alpha \hat{n} \cdot \overline{R}}\, e^{-j\beta \hat{n} \cdot \overline{R}} \end{array}}$$

Pour $\hat{n}_i$, $\hat{n}_R$, $\hat{n}_t$, on a $\rightarrow$ $\hat{n} = 1\overline{n_{tan}} + \hat{z}\, n_z$
à l'interface $z = 0$

De même, $\overline{R} = 1\overline{R_{tan}} + z\hat{z}$
avec $\overline{R_{tan}} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

À $z=0$, on a $\overline{E^i_{tan}}(x,y,0) + \overline{E^R_{tan}}(x,y,0) = \overline{E^t_{tan}}(x,y,0)$
De même pour $\overline{H^{i,R,t}_{tan}}$

Composantes du champs à différents endroits
$$f(x,y,z)\,\hat{x} + g(x,y,z)\,\hat{y} + h(x,y,z)\,\hat{z}$$

$$\overline{F}_{tan} = f(x,y,z)\,\hat{x} + g(x,y,z)\,\hat{y} + 0\,\hat{z}$$

$$\text{tan}_{,i,0}\ldots = f(x,y,0)\,\hat{x} + g(x,y,0)\,\hat{y} + 0\,\hat{z}$$

$$\overline{r}_{tan} = f(x,y,z)\,\hat{x} + g(x,y,z)\,\hat{y} + 0\,\hat{z}$$

$$r_{tan} \underset{\substack{\hookrightarrow \text{frontière} \\ z=0}}{} = f(x,y,0)\,\hat{x} + g(x,y,0)\,\hat{y} + 0\,\hat{z}$$

i.e. : $\overline{E}^{i}_{tan} = \begin{bmatrix} cte_x \\ cte_y \\ 0 \end{bmatrix} e^{-j\beta_1\left(\overline{R_{tan}}\cdot\overline{n_{i,tan}} + z\, n_{i,z}\right)}$

$$= -\overline{\gamma_{tan}} \cdot \overline{R_{tan}}$$

↳ De même pour $\overline{E}^{t}$ et $\overline{E}^{R}$

$$\Rightarrow \overline{E}^{i}_{tan}\Big|_{z=0} = \begin{bmatrix} cte_x \\ cte_y \\ 0 \end{bmatrix} e^{-j\beta_1 \overline{R_{tan}}\cdot\overline{n_{i,tan}}}$$

Condition frontière : $\overline{E}^{i}_{tan}\Big|_{z=0} + \overline{E}^{R}_{tan}\Big|_{z=0} = \overline{E}^{t}_{tan}\Big|_{z=0}$

$$\Rightarrow \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_i e^{-j\beta_1 \overline{R_{tan}}\cdot\overline{n_{i,tan}}} + \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_R e^{-j\beta_1 R_{tan}\cdot n_{R,tan}} = \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_t e^{-j\beta_2 \overline{R_{tan}}\cdot n_{t,tan}}$$

$$\Rightarrow \beta_1 \overline{R_{tan}}\cdot\overline{n_{i,tan}} = \beta_1 \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \beta_1 \sin\theta_i \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}}_{\text{dépendance du terme } E_{tan}|_{z=0} \text{ par rapport à } x \text{ et } y}$$

↳ équation satisfaite partout sur interface

⇒ les termes doivent avoir la même dépendance par rapport à x et y (i.e. la même combinaison de x et y)

$$\Rightarrow \boxed{\beta_1 \overline{n_{i,tan}} = \beta\, \overline{n_{R,tan}} = \beta\, \overline{n_{t,tan}}}$$

Reflexion

$\Rightarrow \overline{n_{i,tan}} = \overline{n_{R,tan}}$

$\Rightarrow \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix} = \overline{n_{R,tan}}$

$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ n_{R,z} \end{bmatrix}$

or $\|\hat{n}_R\| = 1$

$\Rightarrow \sin^2\theta_i + n_{R,z}^2 = 1$

$\Rightarrow n_{R,z} = \pm\cos\theta_i$

$\Rightarrow \hat{} \ \Big[\ \sin\theta_i \ \Big] = \Big[\ x\ \Big]$

Transmission

$\beta_1 \overline{n_{i,tan}} = \beta_2 \overline{n_{t,tan}}$

$\Rightarrow \begin{bmatrix} \beta_1 \sin\theta_i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \beta_2 n_{t,x} \\ \beta_2 n_{t,y} \\ 0 \end{bmatrix}$

$\Rightarrow \beta_1 \sin\theta_i = \beta_2 n_{t,x}$

$\Rightarrow n_{t,x} = \frac{\beta_1}{\beta_2}\sin\theta_i$

et $n_{t,y} = 0$

$\Rightarrow \hat{n}_t = \begin{bmatrix} \frac{\beta_1}{\beta_2}\sin\theta_i \\ 0 \\ n_{t,z} \end{bmatrix}$

or $\|\hat{n}_t\| = 1$

$\Rightarrow n_{t,z}^2 + \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i = 1$

$\Rightarrow n_{t,z} = \pm\sqrt{1 - \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}$

on choisit ⊕ car $z > 0$

$\Rightarrow \sin^2\theta_i + n_{R,z}^2 = 1$

$\Rightarrow n_{R,z} = \pm\cos\theta_i$

$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ \pm\cos\theta_i \end{bmatrix}$ et $\bar{R} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\|\bar{E}^R\| \propto e^{-\bar{\gamma}_1\cdot\bar{R}} = e^{-(\alpha_1+j\beta_1)\hat{n}_R\cdot\bar{R}} = e^{-\alpha_1\hat{n}_R\cdot\bar{R}}\, e^{-j\beta_1\hat{n}_R\cdot\bar{R}}$$

$$= e^{-\alpha_1(x\sin\theta_i \pm z\cos\theta_i)}\, e^{-j\beta_1(x\sin\theta_i \pm z\cos\theta_i)}$$

$$= e^{-\alpha_1 x\sin\theta_i}\, e^{\mp\alpha_1 z\cos\theta_i}\, e^{-j\beta_1 x\sin\theta_i}\, e^{\mp j\beta_1 z\cos\theta_i}$$

L'onde réfléchit se propage

en $z<0 \Rightarrow \alpha_1 z\cos\theta_i < 0$

$\Rightarrow -\alpha_1 z\cos\theta_i > 0$

il faut choisir $-\cos\theta_i$

et $n_{t,y} = 0$

$\Rightarrow n_{t,z} = \pm\sqrt{1-\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}$

on choisit $+$ car $z>0$ pour le rayon transmit

$\Rightarrow \hat{n}_t = \begin{bmatrix} \frac{\beta_1}{\beta_2}\sin\theta_i \\ 0 \\ \sqrt{1-\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)^2} \end{bmatrix}$

or $\hat{n}_t = \begin{bmatrix} \sin\theta_t \\ 0 \\ \cos\theta_t \end{bmatrix}$

$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i$

$\Rightarrow \boxed{N_2\sin\theta_t = N_1\sin\theta_i}$

avec $N = \sqrt{\mu_r\varepsilon_r}$

$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ -\cos\theta_i \end{bmatrix}$

$\Rightarrow \boxed{\theta_i = \theta_R}$

pour tout angle d'incidence entre 0 et 90°

Si on a des pertes dans milieu de transmission :

$$\varepsilon_2 = \varepsilon_0\varepsilon_{R_2}\left(1 - j\frac{\sigma}{\omega\varepsilon_0\varepsilon_{R_2}}\right)$$

$$\bar{\gamma}_t = \begin{bmatrix} \gamma_{x,t} \\ 0 \\ \gamma_{z,t} \end{bmatrix} = \begin{bmatrix} \beta_1\sin\theta_i \\ 0 \\ \gamma_{z,t} \end{bmatrix}$$

on sait que $\bar{\gamma}_t\cdot\bar{\gamma}_t = -\omega^2\mu_2\varepsilon_2$

$\Rightarrow \gamma_{x,t}^2 + \gamma_{z,t}^2 = -\omega^2\mu_2\varepsilon_2$

$\Rightarrow \gamma_{z,t} = \pm\sqrt{-\omega^2\mu_2\varepsilon_2 - \gamma_x^2}$

$= \alpha_{z,t} + j\beta_{z,t}$

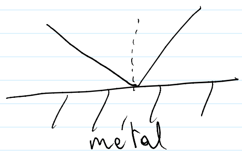
il faut calculer $\gamma_{z,t}$ et choisir la racine $\alpha_{z,t} > 0$ pour atténuation car $\bar{E}^t = Cte\; e^{-\gamma_{x,t}x}\, e^{-\alpha_{z,t}z}\, e^{-j\beta_{z,t}z}$

→ Si le milieu de transmission est un bon conducteur, alors :

$$\gamma_{z,t} \approx \sqrt{-\omega^2\mu_2\left(-j\frac{\sigma}{\omega}\right)} = \sqrt{\frac{\sigma\omega\mu_2}{2}}(1+j)$$

$\Rightarrow \alpha_z = \beta_z = \sqrt{\frac{\sigma\omega\mu}{2}} \gg \beta_x$

$$\Rightarrow \alpha_z = \beta_z = \sqrt{\frac{\sigma \omega \mu}{2}} \gg \beta_x$$



L'onde propagée selon $\hat{z}$ pratiquement

$$\bar{\gamma}_r = \bar{\alpha}_r + \bar{\beta}_r$$

$$\bar{\alpha}_r = \begin{bmatrix} 0 \\ 0 \\ \alpha_z \end{bmatrix}$$

$$\bar{\beta}_r = \begin{bmatrix} \beta_x \\ 0 \\ \beta_z \end{bmatrix}$$

$\hat{/\!/}$ et $\hat{\perp}$: Caractéristiques

- $$\frac{E_\perp^R}{E_\perp^i} = \Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \quad , \quad \frac{E_\perp^t}{E_\perp^i} = T_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

et $1 + \Gamma_\perp = T_\perp$ $\qquad$ $\eta_2 \cos\theta_i = \eta_1 \cos\theta_t + 1$

$\perp \Rightarrow$ TE

- $$\frac{E_{/\!/}^R}{E_{/\!/}^i} = \Gamma_{/\!/} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \quad , \quad \frac{E_{/\!/}^t}{E_{/\!/}^i} = T_{/\!/} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$\Gamma_{/\!/} = 0$ si $\boxed{\tan\theta_i = \frac{N_2}{N_1}}$

Brewster

Ne veut pas forcément dire $T_{/\!/} = 1$ car impédance interface non nulle

Reflexion totale arrive pour

$$\Rightarrow \sin\theta_t = \frac{N_1}{N_2} \sin\theta_i > 1$$

$$\rightarrow \cos\theta_t = \pm\sqrt{1 - \sin^2\theta_t}$$

$$= \pm j \underbrace{\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}}_{\in \mathbb{R}}$$

on sait que $\bar{E}^t = \overline{Cte}\, e^{-j\beta_2 \sin\theta_t x}\, e^{-j\beta_2 \cos\theta_t z}$

$$\Rightarrow E^t \propto e^{-j\beta_2 \cos\theta_t z}$$

$z > 0$

$$\Rightarrow E^t \propto e^{\pm \beta_2 \left(\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}\right) z}$$

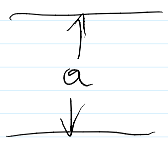
$\Rightarrow E^r \propto e^{\dots}$

il faut donc

$\cos\theta_r < 0$

$$\Rightarrow \cos\theta_r = -j\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}$$

Chapitre 4 : Guides d'onde



$a < \frac{\lambda}{2} \Rightarrow$ déphasage d'une même onde entre 2 point de son trajet dans le guide d'onde.

$a \geq \frac{\lambda}{2}$ condition nécessaire à propagation de 1 onde sans déphasage (vérifié pour mode TE)

mode TE $\longrightarrow$ $E_\perp$ et $H_{/\!/}$

mode TM $\longrightarrow$ $E_{/\!/}$ et $H_\perp$

$$\bar{E} = \bar{C}\, e^{-\bar{\gamma}\cdot\bar{R}}$$

$$k_r^2 + k_z^2 = \beta^2$$

$$\text{Rot}\,\bar{H} = \begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \begin{bmatrix} j\omega\varepsilon E_x \\ j\omega\varepsilon E_y \\ j\omega\varepsilon E_z \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} \frac{\partial H_z}{\partial y} + jk_z H_y \\ -jk_z H_x - \frac{\partial H_z}{\partial x} \\ \dots \end{bmatrix} = \begin{bmatrix} j\omega\varepsilon E_x \\ j\omega\varepsilon E_y \\ \dots \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} \cdots \\ -jk_z H_x - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \begin{bmatrix} \cdots \\ j\omega\varepsilon E_y \\ j\omega\varepsilon E_z \end{bmatrix}$$

$$\begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = j\omega\varepsilon E_x & (1) \\ -jk_z E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y & (2) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = \frac{\cancel{j}\omega\varepsilon}{-\cancel{j}k_z}\left(-j\omega\mu H_y + \frac{\partial E_z}{\partial x}\right) \\ E_x = \frac{\left(-j\omega\mu H_y + \frac{\partial E_z}{\partial x}\right)}{-jk_z} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = \frac{\omega\varepsilon}{k_z} j\omega\mu H_y - \frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x} \\ \hookrightarrow H_y\left(jk_z - j\frac{\beta^2}{k_z}\right) = -\frac{\partial H_z}{\partial y} - \frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x} \end{cases}$$

$$\hookrightarrow \Rightarrow jH_y\left(k_t^2\right) = -k_z\frac{\partial H_z}{\partial y} - \omega\varepsilon\frac{\partial E_z}{\partial x}$$

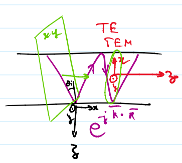
$$\Rightarrow H_y = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial y} - j\frac{\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow \Bigg\{ \quad H_y = \frac{-\frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x} - \frac{\partial H_z}{\partial y}}{jk_z - \frac{j\omega^2\varepsilon\mu}{k_z}} = \frac{-\omega\varepsilon\frac{\partial E_z}{\partial x} - k_z\frac{\partial H_z}{\partial y}}{-jk_t^2} = \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x} - j\frac{k_z}{k_t^2}\frac{\partial H_z}{\partial y}$$

$$k_t^2 = \omega^2\mu\varepsilon - k_z^2$$

$E_z = 0$

$$\bar{E} = \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}$$



On obtient 6 équations impliquant les composantes des champs E et H.

$\frac{\partial H_z}{\partial y} + jk_z H_y = j\omega\varepsilon E_x$ A	$\frac{\partial E_z}{\partial y} + jk_z E_y = -j\omega\mu H_x$ B
$-jk_z H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$ B	$-jk_z E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$ A
$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$	$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$

Avec paire A : $H_z = 0$ et $E_z = 0$ avec paire B :

$$E_x = \frac{-j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial y} - \frac{jk_z}{k_t^2}\frac{\partial E_z}{\partial x} \quad (1)$$

$$E_y = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial x} - \frac{jk_z}{k_t^2}\frac{\partial E_z}{\partial y} \quad (3)$$

$$H_y = \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x} - \frac{jk_z}{k_t^2}\frac{\partial H_z}{\partial y} \quad (2)$$

$$H_x = \frac{j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial y} - \frac{jkz}{k_t^2}\frac{\partial H_z}{\partial x} \quad (4)$$

Modes Transverses Électriques ou TE $(E_z = 0,\ H_z \neq 0)$	Modes Transverses Magnétiques ou TM $(H_z = 0,\ E_z \neq 0)$
$E_x = \frac{-j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial y}$	$E_x = \frac{-jk_z}{k_t^2}\frac{\partial E_z}{\partial x}$
$H_y = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial y}$	$H_y \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x}$
$E_y = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial x}$	$E_y = \frac{-jk_z}{k_t^2}\frac{\partial E_z}{\partial y}$
$H_x = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial x}$	$H_x = \frac{j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial y}$

$$\underbrace{k_x^2 + k_y^2}_{k_t^2} + k_z^2 = \beta^2 = w^2\mu\varepsilon$$

Guide Rectangulaire :

$\boxed{TE} \rightarrow E_z = 0$

$$H_z = X(x)\,Y(y)\,e^{-jk_z z} = \left[A\cos(k_x x) + B\sin k_x x\right]\left[C\cos(k_y y) + D\sin(k_y y)\right] e^{-jk_z z}$$

en appliquant conditions frontières $\overline{E}_{tan} = 0$

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}$$

et $H_z = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$, m et $n \in \mathbb{N}$

H_0 constante

$\boxed{TM} \rightarrow H_z = 0$

$$E_z = \underbrace{\left[A\cos(k_x x) + B\sin(k_x x)\right]}_{X(x)}\underbrace{\left[C\cos(k_y y) + D\sin(k_y y)\right]}_{Y(y)}\underbrace{e^{-jk_z z}}_{Z(z)}$$

Pour $\overline{E}_{tan} = 0$, on obtient avec guide rectangulaire :

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}$$

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) e^{-jk_z z}$$

$k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-jk_z z}$$

avec $m, n \in \mathbb{N}$ et E_0 une constante

$\boxed{F_{coupure} :}$

$$k_x^2 + k_y^2 + k_z^2 = \beta^2$$

$$\Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = \beta^2$$

$$\Rightarrow k_z^2 = \beta^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$k_z^2 < 0 \Rightarrow k_z$ réel $\Rightarrow$ propagation

$k_z^2 > 0 \Rightarrow k_z$ imaginaire $\Rightarrow$

$$k_r = \frac{\omega_{coupure}}{u_{TEM}}$$

$$f > \frac{u_{TEM}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u_{TEM} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{k_z} = \frac{c}{N}$$

$$k_r^2 + k_z^2 = \beta^2 = \omega^2\mu\varepsilon$$

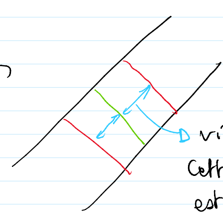
Terminologie : $k_z = \beta_g$ nombre d'onde guidé

$\beta_g = \frac{2\pi}{\lambda_g}$ Longueur d'onde guidée

↳ période de l'onde le long de l'axe z

u_p : vitesse de phase

$$u_p^{m,n} = \lambda_g f = \frac{\omega}{k_z} = \frac{\omega}{\frac{1}{u_{TEM}}\sqrt{\omega^2 - \omega_c^2}}$$



$$u_p^{m,n} = \frac{u_{TEM}}{\sqrt{1 - \left(f_c^{m,n}/f\right)^2}}$$

$$\beta_g^{m,n} = \beta_{libre}\sqrt{1 - \left(f_c^{m,n}/f\right)^2}$$

↳ avec $\beta_{libre} = \frac{\omega}{u_{TEM}}$

$$k_z = \beta\sqrt{1 - \frac{f_c^2}{f^2}} = \sqrt{\beta^2 - \frac{\beta^2 f_c^2}{f^2}} = \sqrt{\frac{\beta^2 f^2 - \beta^2 f_c^2}{f^2}} = \sqrt{\frac{\omega^2\mu\varepsilon f^2 - \omega^2\mu\varepsilon f_c^2}{f^2}} = \sqrt{\frac{\omega^2}{f^2 u_{TEM}^2}\left(f^2 - f_c^2\right)}$$

$$\lambda_g^{m,n} = \frac{\lambda_{libre}}{\sqrt{1 - \left(f_c^{m,n}/f\right)^2}}$$

$L \dots \lambda \dots = u_{TEM} = \frac{1}{\dots}$

$$\lambda_g = \frac{\quad}{\sqrt{1-\left(f_c^{mn}/f\right)^2}}$$

$\hookrightarrow$ avec $\lambda_{libre} = \dfrac{u_{TEM}}{f} = \dfrac{1}{\sqrt{\mu\varepsilon}\, f}$

$$u_g = \frac{d\omega}{dk_z} = u_{TEM}\sqrt{1-\left(f_c/f\right)^2}$$

$$= \sqrt{\frac{\omega^2}{f^2 u_{TEM}^2}\left(f^2 - f_c^2\right)}$$

$$\left(\frac{\omega}{f}\right)^2 = (2\pi)^2$$

$$= \sqrt{\frac{(2\pi f)^2 - (2\pi f_c)^2}{u_{TEM}^2}}$$

$$= \sqrt{\frac{\omega^2 - \omega_c^2}{u_{TEM}^2}}$$

$$u_g \cdot u_p = u_{TEM}^2$$

$$\rightarrow P_{tot}^{TE} = \left(\frac{1}{4} \text{ ou } \frac{1}{8}\right)|H_0|^2\, \eta\, ab \left(\frac{f}{f_c}\right)^2 \sqrt{1-\left(f_c/f\right)^2}$$

avec $\begin{cases} 1/4 \text{ si } m=0 \text{ ou } n=0 \text{ (pas les deux)} \\ 1/8 \text{ si } m\neq 0 \text{ et } n\neq 0 \end{cases}$

$$\rightarrow P_{tot}^{TM} = \frac{1}{8}\,\frac{1}{\eta}\left(\frac{f}{f_c}\right)^2 \sqrt{1-\left(f_c/f\right)^2}\,|E_0|^2\, ab$$

$$\eta_g^{TE} = \left.\frac{\sqrt{|E_x|^2+|E_y|^2}}{\sqrt{|H_x|^2+|H_y|^2}}\right|_{\substack{x=a/2\\ y=b/2}} = \frac{\eta_{libre}}{\sqrt{1-\left(f_c/f\right)^2}}$$

$$\eta_g^{TM} = \eta_{libre}\sqrt{1-\left(f_c/f\right)^2}$$

$$\bar{J}_s = \hat{n}\times\bar{H}$$

Guides circulaires

TE

$$H_z = J_m(k_r\rho)\left[C\cos m\phi + D\sin m\phi\right]e^{-jk_z z} \quad \text{et} \quad J_m'(k_r a) = 0$$

$$E_\rho = -\frac{j\omega\mu}{k_r^2}\,\frac{J_m(k_r\rho)}{\rho}\left[-C\sin m\phi + D\cos m\phi\right]e^{-jk_z z}$$

$$E_\phi = \frac{j\omega\mu}{k_r^2}\,\frac{\partial H_z}{\partial\rho} = \frac{j\omega\mu}{k_r^2}\,J_m'(k_r\rho)\left[C\cos m\phi + D\sin m\phi\right]$$

$$E_\rho = -\frac{j\omega\mu}{k_t^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \varphi}$$

$$= \frac{-j\omega\mu m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$C\cos(m(\phi-\phi_0))$

$$E_\varphi = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial \rho} = \frac{j\omega\mu}{k_t} J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$H_\rho = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial \rho} = \frac{-jk_z}{k_t} J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$H_\varphi = \frac{-jk_z}{k_t^2}\frac{1}{\rho}\frac{\partial Hz}{\partial \varphi} = \frac{-jk_z m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

TM

$$E_z = J_m(k_t \rho)\left[C\cos m\phi + D\sin m\phi\right]e^{-jk_z z} \quad \text{et} \quad J_m(k_t a) = 0$$

ii) Modes TM ($E_z \neq 0$, $H_z = 0$)

$$\nabla^2 E_z + \beta^2 E_z = 0$$

$$\boxed{E_z = J_m(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}}$$

$$E_\rho = \frac{-jk_z}{k_t} J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$E_\varphi = \frac{-jk_z m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$$H_\rho = \frac{j\omega\varepsilon m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$$H_\varphi = \frac{-j\omega\varepsilon}{k_t} J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

conditions aux frontières : $\left.\begin{matrix} E_z(\rho = a) = 0 \\ E_\varphi(\rho = a) = 0 \end{matrix}\right\} \rightarrow \boxed{J_m(k_t a) = 0}$

On a donc que $k_t a$ est un zéro de $J_m(\mathrm{X})$ i.e. $k_t^{mn} a = \mathrm{X}_{mn}$.

Dérivées de Bessel

TE

Zéros X_{mn}' de la dérivée $J_{mn}'(\mathrm{X}_{mn}')$ ($n = 1, 2, 3, \ldots$) de la fonction de Bessel $J_m(\mathrm{X})$
(Tiré de C. Balanis, Advanced Engineering Electromagnetics, John Wiley)

	m=0	m=1	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10	m=11
n= 1	3.8318	1.8412	3.0542	4.2012	5.3175	6.4155	7.5013	8.5777	9.6474	10.7114	11.7708	12.8264
n= 2	7.0156	5.3315	6.7062	8.0153	9.2824	10.5199	11.7349	12.9324	14.1155	15.2867	16.4479	17.6003
n=3	10.1735	8.5363	9.9695	11.3459	12.6819	13.9872	15.2682	16.5294	17.7740	19.0046	20.2230	21.4309
n=4	13.3237	11.7060	13.1704	14.5859	15.9641	17.3129	18.6375	19.9419	21.2291	22.5014	23.7607	25.0085
n=5	16.4706	14.8636	16.3475	17.7888	19.1960	20.5755	21.9317	23.2681	24.5872	25.8913	27.1820	28.4609

Fonction de Bessel

TM

Zeros X_{mn} de la function de Bessel $J_m(\mathrm{X}_{mn})$ (n = 1, 2, 3, ….). Tiré de C. Balanis, Advanced Engineering Electromagnetics, John Wiley.

	m=0	m=1	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10	m=11
n=1	2.4049	3.8318	5.1357	6.3802	7.5884	8.7715	9.9361	11.0864	12.2251	13.3543	14.4755	12.8264
n=2	5.5201	7.0156	8.4173	9.7610	11.0647	12.3386	13.5893	14.8213	16.0378	17.2412	18.4335	19.6160
n=3	8.6537	10.1735	11.6199	13.0152	14.3726	15.7002	17.0038	18.2876	19.5545	20.8071	22.0470	23.2759
n=4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801	20.3208	21.6415	22.9452	24.2339	25.5095	26.7733
n=5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178	23.5861	24.9349	26.2668	27.5838	28.8874	80.1791

Exemple d'application numérique : Soit un guide rempli d'air ayant un rayon de 5 cm. Quelle est la fréquence de coupure du mode TM_{44}?

$m = 4$, $n = 4$ $\qquad$ $\mathrm{X}_{mn} = 17.616 = k_t a$

$k_t = 17.616/5\text{cm} = 352.32\ \text{m}^{-1}$

$\omega_c = ck_t = 3\times10^8 \times 352.32$

On trouve, $f_c^{44} = \omega_c/(2\pi) = 16.82$ GHz

Dans guide circulaire, mode TE_{11} dominant !

$$\boxed{\text{Guide coaxial}}$$

$\bar{E} = \quad V_2 - V_1 \quad 1 \quad e^{-j\beta z} \; \hat{}$

$$\bar{E} = \frac{V_2 - V_1}{\ln b/a} \frac{1}{\rho} e^{-j\beta z} \hat{\rho}$$

mode TEM dominant

Impédance caractéristique

$$Z_0 = \frac{\eta}{2\pi} \ln(b/a)$$

en posant $Z_0 = \sqrt{\frac{L}{C}}$ et $u_{TEM} = \frac{1}{\sqrt{LC}}$

on peut extraire L et C

- Extraction de α, atténuation dûe au métal

$$\alpha_{TEM} = \frac{R_s}{d\,\eta} \quad \mu \qquad R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

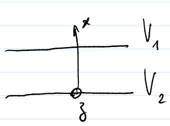
$$\alpha_{TE_{10}} = \frac{R_s \left[1 + \frac{2b}{a}\left(f_c/f\right)^2\right]}{b\,\eta\sqrt{1-\left(f_c/f\right)^2}}$$

$$P_z(z) = P_z(0)\,e^{-2\alpha z}$$

Conditions TEM:

1. 2 conducteurs
2. milieu homogène
3. $\nabla \cdot \bar{E} = 0 \Rightarrow \nabla_t^2 V = 0$

Plaques parallèles :



Conditions frontières : $V(x=0) = V_2$
$V(x=d) = V_1$

on a $\nabla_t^2 V = 0$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

puisque $\frac{\partial(\cdot)}{\partial y} = 0$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = 0$$

$$\Rightarrow V = Ax + B$$

$$\Rightarrow \begin{cases} x = 0 \Rightarrow B = V_2 \\ x = d \Rightarrow A = \frac{V_1 - V_2}{d} \end{cases}$$

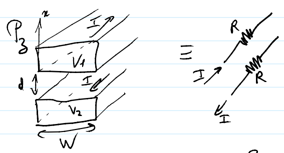
$$\Rightarrow V = \frac{V_1 - V_2}{d} x + V_2$$

$$\Rightarrow \bar{E} = -\nabla_t V e^{-j\beta z} = -\frac{\partial V}{\partial x} \hat{x} e^{-j\beta z}$$

$$\Rightarrow \bar{E} = \frac{V_2 - V_1}{d} e^{-j\beta z} \hat{x}$$

$$\vec{H} = \frac{\nabla \times \bar{E}}{-j\omega\mu} = \begin{bmatrix} 0 \\ H_y \\ 0 \end{bmatrix}$$

avec $H_y = \frac{1}{\eta} \frac{V_2 - V_1}{d} e^{-j\beta z} = \frac{E_x}{\eta}$



$$E_x = \frac{V_2 - V_1}{d} e^{-j\beta z}$$

$$H_y = \frac{E_x}{\eta}$$

$$P_z = \frac{1}{2} \operatorname{Re}\{E_x H_y^*\} \times d \times W$$

$$= \frac{|E_x|^2}{2\eta} W \times d$$

ΔP_z = − puissance dissipée

$$= -\frac{1}{2} R |I|^2 \times 2$$

avec :

$R = \frac{R_s \times \Delta z}{W}$ et $|I| = \|\bar{J}_s\| \cdot W$

$$= \|\hat{n} \times \bar{H}\| \cdot W$$

$$= |H_y| \cdot W$$

$$= E_x / \eta$$

$$\Rightarrow \Delta P_z = -\sqrt{\frac{\omega\mu}{2\sigma}} \frac{\Delta z}{W} W^2 \frac{|E_x|^2}{\eta^2}$$

$$\Rightarrow \Delta P_z = -\sqrt{\omega\mu} \; \underline{W} \; |E_x|^2$$

$$\Rightarrow \frac{\Delta P_z}{\Delta z} = -\sqrt{\frac{\omega\mu}{2\sigma}}\,\frac{W}{b}\,\underbrace{\frac{|E_x|^2}{b}}_{=\frac{2P_z}{W.d}}$$

$$\Rightarrow \frac{dP_z}{dz} = -\sqrt{\frac{\omega\mu}{2\sigma}}\,\frac{W}{b}\,\frac{2P_z}{Wd} = -\frac{2R_s}{bd}P_z$$

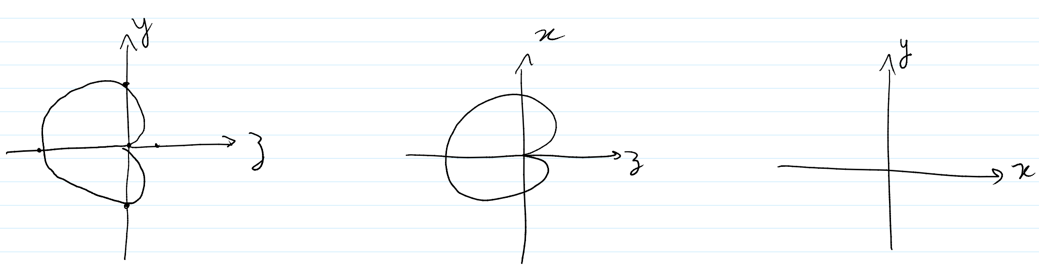
or $\dfrac{dP_z}{dz} = -2\alpha P_z$

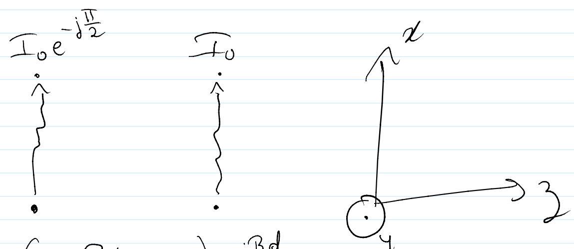
$$\Rightarrow \quad \alpha = \frac{R_s}{db}$$

Pour guide coaxial :

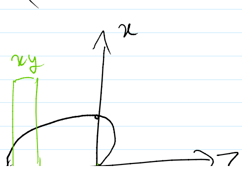
$$\alpha = \frac{R_s\left(\frac{1}{a}+\frac{1}{b}\right)}{4\pi Z_0}$$

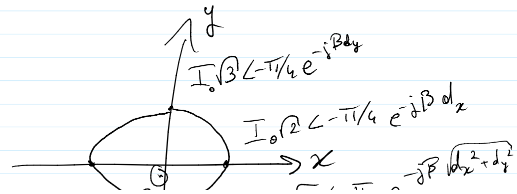
avec $Z_0 = \dfrac{b}{2\pi}\ln(b/a)$

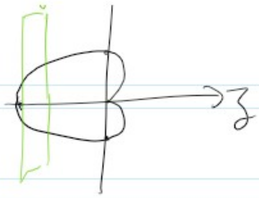


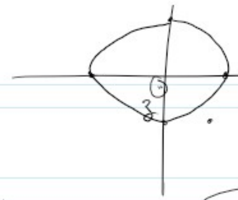


$$AF = \Sigma = \left(I_0\sqrt{2}\angle -\pi/4\right)e^{-j\beta d_x}$$









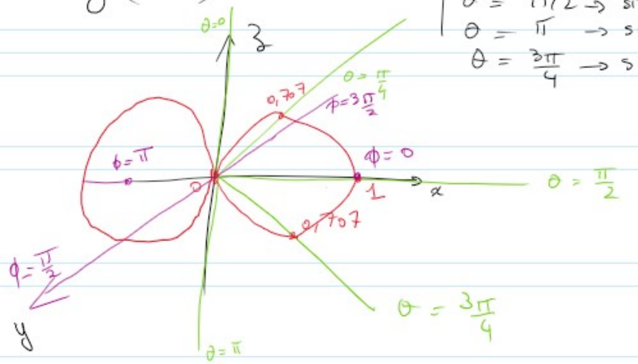
$I_0\sqrt{2}\angle -\pi/4 \; e^{-j\beta d_x}$

$I_0\sqrt{2}\angle -\frac{\pi}{4} \; e^{-j\beta\sqrt{d_x^2+d_y^2}}$

$$d_{xy} = \sqrt{d_x^2+d_y^2}$$

$$g(\theta,\phi) = \sin\theta$$

$\theta = 0 \rightarrow \sin\theta = 0$
$\theta = \pi/4 \rightarrow \sin\theta = 0{,}707$ $\qquad 0 < \theta < \pi$
$\theta = \pi/2 \rightarrow \sin\theta = 1$
$\theta = \pi \rightarrow \sin\theta = 0$
$\theta = \frac{3\pi}{4} \rightarrow \sin\theta = 0{,}707$



$$0 < \phi < 2\pi$$

En coordonnées sphériques :

$$\hat{R} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \qquad \hat{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

5.2 Antenne de type « dipôle élémentaire »

- Champs électriques et magnétiques du dipôle <u>valides pour toute distance r entre le dipôle et l'observateur</u>:

$$d\bar{H} = \hat{\varphi} \; -\frac{I\,dz\;\beta^2}{4\pi}\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]\sin\theta\, e^{-j\beta r}$$

$$d\bar{E} = -\eta\frac{I\,dz\,\beta^2}{4\pi}\times\left\{\hat{r}\cos\theta\left[\frac{2}{(j\beta r)^2} + \frac{2}{(j\beta r)^3}\right] + \hat{\theta}\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]\right\}e^{-j\beta r}$$

- Si le dipôle n'est pas situé à l'origine on remplace r par R ($- \|\mathbf{r} - \mathbf{r}'\|$)

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5.2.1 Champs lointains

- Si on est loin de l'antenne r est très grand, alors $1/r \gg 1/r^2 \gg 1/r^3$, donc:

$$d\mathbf{E} \approx \hat{\boldsymbol{\theta}} j\eta\beta I dz \sin\theta \frac{e^{-j\beta r}}{4\pi r}$$

$$d\mathbf{H} \approx \hat{\boldsymbol{\varphi}} j\beta I dz \sin\theta \frac{e^{-j\beta r}}{4\pi r}$$

- On appelle les termes en $1/r$ « le champ lointain »
- Concept applicable à toutes les antennes

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Final H2020

Question 1 :

$E_\perp = 1\ V/m$

$E_{/\!/} = 1\ V/m$

pol. linéaire

a)

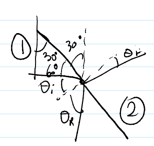
couche $P^t = P^i$

$|\Gamma| = 0$

$$P^i_\perp = \frac{1}{2}\,\frac{1}{377} \times |E^i_\perp|^2$$

$$P^t_\perp = P^i_\perp = \frac{1}{2}\,\mathrm{Re}\left\{\frac{1}{\eta_2^*}\right\}|E^t_\perp|^2$$

$$N_2 \sin\theta_t = N_1 \sin\theta_i$$



$\theta_i = 30° = \theta_B$

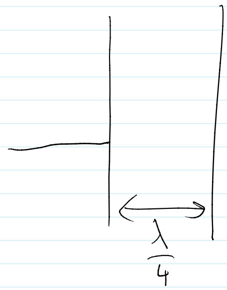
$$\tan\theta_B = \frac{N_2}{N_1} \qquad N_2 = \sqrt{1} = 1$$

$$\Rightarrow N_1 = \sqrt{\varepsilon_{R_1}} = \frac{N_2}{\tan(30°)} = \frac{1}{\tan 30°} = 1{,}73$$

$$\Rightarrow \varepsilon_{R_1} = \left(\frac{1}{\tan 30°}\right)^2 = 3$$

b)

b)



$$P^r = P^i = \frac{1}{2}\frac{1}{\eta_{air}} \times \left(|E^i_\perp|^2 + |E^i_{/\!/}|^2\right)$$

$$= \frac{1}{2}\frac{1}{\eta_{prisme}} \times \left(|E^r_\perp|^2 + |E^r_{/\!/}|^2\right)$$

$$\frac{\eta_{prisme}}{\eta_{air}} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{1}{3}}}{\sqrt{\frac{\mu_0}{\varepsilon_0}}} = \frac{1}{\sqrt{3}}$$

$$\frac{\eta_{air}}{\eta_{prisme}}\left(2|E^r_\perp|^2\right) = 2|E^i_\perp|^2$$

$$\Rightarrow |E^r_\perp| = \sqrt{\frac{\eta_{prisme}}{\eta_{air}}|E^i_\perp|^2} = 0{,}76 \text{ V/m}$$

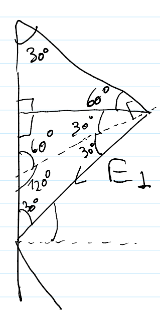
$$\Rightarrow |E^r_{/\!/}| = 0{,}76 \text{ V/m}$$

c) $d \rightarrow /\!/$ et $\perp$

$e \rightarrow \perp$

Brewster

$\Gamma_{/\!/} = 0$



$f \rightarrow E_\perp$

$$\sin\theta_r = \frac{N_1}{}\sin\theta_i = 1$$

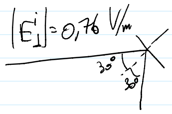
$$\sin \theta_r = \frac{N_1}{N_2} \sin \theta_i = 1$$

$$\frac{1}{\sqrt{3}} \sin 30^\circ = 0{,}28 < 1$$

Donc $\theta_i \neq \theta_c$

d) $E_{/\!/} = 0$

$E_\perp = ?$



$$\Gamma_\perp = \eta_2 \cos 30^\circ - \eta$$

$$\theta_r = \sin^{-1}\left(\frac{N_1}{N_2} \sin 30^\circ\right) \quad \begin{matrix} N_1 = \sqrt{3} \\ N_2 = 1 \end{matrix}$$

$$= 60^\circ$$

$$\Gamma_\perp = \frac{377 \cos 30^\circ - \frac{377}{\sqrt{3}} \cos 60^\circ}{377 \cos 30^\circ + \frac{377}{\sqrt{3}} \cos 60^\circ}$$

$$= 0{,}5$$

$$\Rightarrow \left|E_\perp^R\right| = 0{,}5 \times 0{,}76$$

$$= 0{,}38 \quad V/m$$

e) $E_{/\!/} = 0 \ V/m$

$E_\perp = ?$

$$T_\perp = ? = \frac{2 \eta_{air} \cos 30^\circ}{\eta_{air} \cos 30^\circ + \eta_{prisme} \cos 60^\circ} = 1{,}5$$

$$\theta_r = \sin^{-1}\left(\frac{N_1}{N_2} \sin 60^\circ\right) > 1$$

$N_1 = \sqrt{3}$
$N_2 = 1$

$$\cos \theta_t = -i \sqrt{\frac{N_1^2}{N_2^2} \sin^2 \theta_i - 1}$$

$N_2 = 1$

$$\cos\theta_t = -j\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}$$

$$N_1 = \sqrt{3}$$
$$N_2 = 1$$

$$= -j\,1{,}12$$

$$\Rightarrow T_\perp = \frac{2 \times 377 \cos 60°}{377 \cos 60° + \frac{377}{\sqrt{3}}(-j\,1{,}12)}$$

$$= 1{,}22 \angle 52{,}2°$$

$$\Rightarrow |E_\perp^t| = 1{,}22 \times 0{,}38 = 0{,}4636$$

$$e^{-\beta_2\sqrt{\frac{N_1^2}{N_2^2}\sin\theta_i - 1}\,z} \approx 8{,}9.10^{-4}$$

$$z = 1$$

$$|E_\perp^t|_{z=1} = |E_\perp^t| \times 8{,}9.10^{-4}$$
$$= 4{,}13.10^{-4} \text{ V/m}$$

Question 2 : H20

$$f = 100 \text{ MHz}$$

$$Z = 100\,\Omega$$

$$b = 1.10^{-2}\text{ m}$$

$$\varepsilon_R = 2{,}25$$

$$|E|_{max} = 10^6 \text{ V/m}$$

a) $$Z_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right)$$

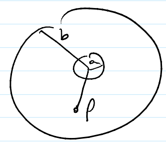
$$\Rightarrow \left(e^{\frac{2\pi Z_0}{\eta}}\right)^{-1} \times b = a = 8{,}21.10^{-4}\text{ m}$$

$$\Rightarrow \left(e^{\frac{2\pi Z_0}{\eta}}\right)^{-1} \times b = a = 8{,}21.10^{-4}\ m$$

b)

$$P_{max} = \frac{|V^+_{max}|^2}{2 Z_0} = \frac{|I^+_{max}|^2 Z_0}{2}$$

$$P_{tot} = \int_a^b \int_0^{2\pi} \frac{\rho |I^+|^2 Z_0^2}{2 \eta \rho^2 (\ln b/a)^2} d\phi\, d\rho = \frac{|I^+|^2 Z_0}{2}$$



$$E_{\rho_{max}} = \frac{(V_2 - V_1)_{max}}{\ln b/a} \times \frac{1}{\rho}\Big|_{\rho = a}$$

$$\Rightarrow (V_2 - V_1)_{max} = E_{\rho_{max}}\, a \ln \frac{b}{a}$$

$$E_{\rho_{max}} = 10^6\ V/m$$

$$\Rightarrow (V_2 - V_1)_{max} = 10^6\, a \ln \frac{b}{a}$$
$$= 2051\ V$$

$$\Rightarrow P_{max} = \frac{2051^2}{2 Z_0} = 21\, kW$$

c) $\sigma = 3{,}5.10^7\ Sm$

$\alpha = .$

$$\alpha_{TEM} = \frac{R_S}{d\eta} \quad \text{avec } R_S = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$d = b - a$

$\eta = \frac{377}{\sqrt{2{,}25}}$

$\omega = 2\pi \times 100.10^6\ rad/s$

$\mu = 4\pi.10^{-7}\ F/m$

$$\Rightarrow \alpha_{TEM} = 1{,}46.10^{-3}\ neper/m$$

$\alpha = 3{,}53 \cdot 10^{-3} = \dfrac{R_s\left(\frac{1}{a}+\frac{1}{b}\right)}{4\pi Z_0}$

$R_s = \sqrt{\dfrac{\omega\mu}{2\sigma}}$

d) mode TEM garanti

guide et non dispersif

TE_{10} $\quad f > f_c = \dfrac{3 \cdot 10^8}{2a}$

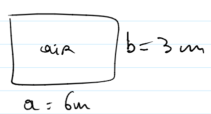
$\Rightarrow a > \dfrac{3 \cdot 10^8}{2f} = 1{,}5\,m$

a

alors qu'ici $b = 1$ cm

Question 3 :

$\sigma = 5{,}8 \cdot 10^7$

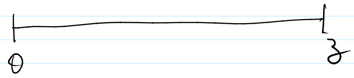


$\dfrac{u_{TEM}}{2}\sqrt{\left(\dfrac{1}{a}\right)^2 + \left(\dfrac{0}{b}\right)^2} < f < 5\,GHz$

2,5 GHz $\qquad$ TE_{01} $\quad$ TE_{20}

$\Delta f = 2{,}5\,GHz$

b) $\quad f = 1{,}5 f_c = 3{,}75\,GHz$



$P(z) = P(0)\, e^{-2\alpha z}$

$\dfrac{P(z)}{P(0)} = e^{-2\alpha z}$

$$\frac{P(z)}{P(0)} = e^{-2\alpha z}$$

$$10 \log \frac{P(z)}{P(0)} = -0{,}5 \text{ dB}$$

$$\Rightarrow 10 \log e^{-2\alpha z} = -0{,}5$$

$$\Rightarrow z = \ln(10^{-0{,}5/10}) \times \left(\frac{1}{-2\alpha}\right)$$

$$\alpha_{TE_{10}} = \frac{R_S \left[1 + \frac{2b}{a}\left(\frac{f_c^{10}}{f}\right)^2\right]}{b\, \eta_{air} \sqrt{1 - \left(\frac{f_c^{10}}{f}\right)^2}}$$

$$f_c = 2{,}5 \text{ GHz}$$
$$f = 3{,}75 \text{ GHz}$$

$$R_s = \sqrt{\frac{\omega \mu}{2\sigma}}$$

$$\Rightarrow \alpha_{TE_{10}} = 2{,}73 \cdot 10^{-3} \text{ neper/m}$$

$$z = 21{,}03 \text{ m}$$

c) $f = 2{,}45 \text{ GHz} < f_c$

$$f_c^{m,n} = \frac{u_{TEM}}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = 2{,}45 \text{ GHz}$$

$$u_{TEM} = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{\sqrt{\varepsilon_R}}$$

il faut $\varepsilon \nearrow$

augmenter ε_R

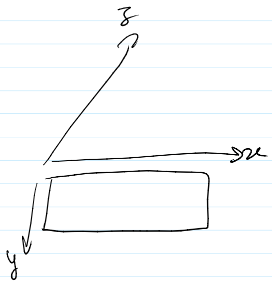
$$\frac{c}{\sqrt{\varepsilon_R}\, 2a} = 2{,}45 \cdot 10^9$$

$$\frac{c}{\sqrt{\varepsilon_R}\,2a} = 2{,}45.10^9$$

$$\Rightarrow \quad \varepsilon_R = \left(\frac{c}{2{,}45.10^9 \times 2a}\right)^2$$

$$\varepsilon_R \geq 1{,}04$$

d)



$$H_z = C\left[A\cos\left(\frac{\pi}{a}x\right) + B\sin\left(\frac{\pi}{a}x\right)\right]$$

$$k_x = \frac{\pi}{a}$$

$$k_y = 0$$

$$H_z = H_0 \cos\left(\frac{\pi}{a}x\right) e^{-jk_z z}$$

$$\cancel{E_x = \frac{-j\omega\mu}{k_r^2}\frac{\partial H_z}{\partial y}}$$

$$\cancel{= 0 \text{ pour tout } x}$$

pour placer une fente, il faut

$$H_z = 0$$

$$\Rightarrow \quad \cos\left(\frac{\pi}{a}x\right) = 0$$

$$\Rightarrow \frac{\cancel{\pi}}{a}x = \frac{\cancel{\pi}}{2} + k\cancel{\pi}, \; k \in \mathbb{Z}$$

$$\Rightarrow \quad x = \frac{a}{2} + ka, \; k \in \mathbb{Z}$$

$$0 < a < 6 \text{ cm}$$

$$\lambda = \frac{a}{2} = 3 \text{cm}$$

Chapitre 5 :

$\bar{R}$: point d'observation

$\bar{R'}$: coordonnées d'un point sur l'antenne

$R = \|\bar{R} - \bar{R'}\|$: Distance entre point sur l'antenne et observateur

Dipôle élémentaire :

$$d\bar{H} = \hat{\phi} \frac{-I\,dz}{4\pi} \beta^2 \left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} \right] \sin\theta e^{-j\beta R}$$

$$d\bar{E} = -\eta \frac{I\,dz\,\beta^2}{4\pi} \times \left\{ \hat{R} \cos\theta \left[\frac{2}{(j\beta R)^2} + \frac{2}{(j\beta R)^3} \right] + \hat{\theta} \sin\theta \left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] \right\} e^{-j\beta R}$$

→ si dipôle n'est pas situé à l'origine, R devient $\|\bar{R} - \bar{R'}\|$

Directivité : Source isotrope

Densité de puissance moyenne

$$P_R^{iso} = \frac{P_{Ray}}{4\pi R^2}$$

Directivité : Source non-isotrope

Densité de puissance moyenne :

$$P_R^{antenne} = \frac{P_{Ray}}{4\pi R^2} D(\theta, \phi)$$

avec $D(\theta, \phi) = \dfrac{P_R^{antenne}(\theta, \phi)}{P_R^{iso}}$

D est la directivité

Valeurs typiques de D :

$D_{max} > 1$

↳ D indique capacité à former faisceau étroit

Source isotrope : $D_{max} = 1 = 0$ dBi

Dipôle élémentaire : $D_{max} = 1{,}5 = 1{,}76$ dBi

Réflecteur parabolique : $D_{max} = 25$ à 50 dB.

dBi : décibels par rapport à source isotrope

$$P_{Ray} = \int_0^{2\pi}\int_0^{\pi} P_R^{antenne}(\theta,\phi)\, R^2 \sin\theta \, d\theta d\phi$$

exemple : si $P_R^{antenne}(\theta,\phi) = \dfrac{\|E(\theta,\phi)\|^2}{2\eta} = \begin{cases} \dfrac{a}{R^2} & \text{si } \frac{\pi}{2} \leq \theta \leq 0{,}8\pi \\ 0 & \text{sinon} \end{cases}$

$$\Rightarrow P_{Ray} = \int_0^{2\pi}\int_{\pi/2}^{0{,}8\pi} \frac{a}{R^2} R^2 \sin\theta \, d\theta d\phi = 2\pi a(-\cos 0{,}8\pi)$$

$$\Rightarrow P_R^{iso} = \frac{P_{Ray}}{4\pi R^2} = \frac{-2\pi a \cos 0{,}8\pi}{4\pi R^2}$$

$$\Rightarrow D(\theta,\phi) = \frac{P_R^{antenne}}{P_R^{iso}} = \begin{cases} (a/R)/P_R^{iso} & \text{si } \frac{\pi}{2} \leq \theta \leq 0{,}8\pi \\ 0 & \text{sinon} \end{cases}$$

$$D_{max} = \max\left(\frac{\cancel{a}}{\cancel{R^2}} \times \frac{2\cancel{4\pi R^2}}{-\cancel{2\pi}\cancel{a}\cos 0{,}8\pi}\right)$$

$$= \frac{-2}{\cos 0{,}8\pi} = 2{,}47$$

ou $10\log_{10}(2{,}47) = 3{,}93$ dBi

Résistance de rayonnement :

$$P_{Ray} = \frac{1}{2} R_{Ray}|I|^2$$

avec $R_{Ray} = 80\left(\dfrac{\pi d_z}{\lambda}\right)^2$ ohms

Gain antenne isotrope :

isotrope $\Rightarrow \Gamma = 0$

ne dissipe pas de puissance

$\Rightarrow P_{Ray} = P_{Ransmis}$

isotrope $\Rightarrow D = 1$

Densité de puissance :

$$P_R^{isotrope} = P_R^{idéal} = \frac{P_T}{4\pi R^2}$$

Gain dans antenne :

$$P_{antenne} = (1 - |\Gamma|^2) P_{transmise}$$

puissance fournie

$$G_{\text{Realisé}} = \mathcal{E}_{Ray} (1 - |\Gamma|^2) D(\theta, \phi)$$

noté G

efficacité

EIRP

$$P^{antenne}(R) = \frac{P_{Transmise}\, G}{4\pi R^2}$$

Densité de puissance à une distance R

Transmission entre 2 antennes

↳ Friis

$$\frac{P_{Reçu_A}}{P_{Transmis_A}} = \frac{P_{Reçu_B}}{P_{Transmis_B}}$$

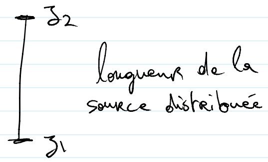
$$\boxed{P_{Recepteur} = \underbrace{P_T G_T}_{EIRP} \cdot G_{Reception} \left(\frac{\lambda}{4\pi R}\right)^2}$$

avec R distance entre les 2 antennes en mètres.

Superposition d'antennes :

$$dE_\theta = j\beta\eta\, I(z\hat{z}) \sin\theta \, \frac{e^{-j\beta R}}{4\pi R}\, dz$$

$$\Rightarrow E_\theta = \int_{z_1}^{z_2} dE_\theta$$



$$\Rightarrow E_\theta = \frac{j\beta\eta\, e^{-j\beta R}}{4\pi R} \sin\theta \int_{z_1}^{z_2} I(z)\, e^{j\beta z \cos\theta}\, dz \qquad \beta = \frac{2\pi}{\lambda}$$

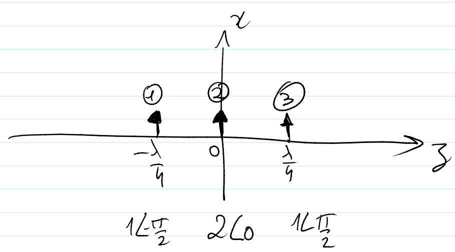
$$\text{avec } \hat{R} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \Rightarrow \hat{z}\cdot\hat{R} = \cos\theta$$

Approximation de N Rayons parallèles

Soit réseau de N éléments identiques situés à $\overline{R_n}$

$$\Rightarrow E = \frac{C^{te}}{R} e^{-j\beta R} \underbrace{g(\theta,\phi) \sum_{n=1}^{N} I_n\, e^{j\beta \overline{R_n}\cdot\hat{R}}}_{F(\theta,\phi) = g(\theta,\phi) \times AF(\theta,\phi)}$$

élement x Réseau



$$g(\theta,\phi) = \sin\theta_x$$

$$AF(\theta,\phi) = I_1 e^{j\beta \overline{R_1}\cdot\hat{R}} + I_2 e^{j\beta \overline{R_2}\cdot\hat{R}} + I_3 e^{j\beta \overline{R_3}\cdot\hat{R}}$$

$$\hat{R} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

$$\overline{R_1} = \begin{bmatrix} 0 \\ 0 \\ -\frac{\lambda}{4} \end{bmatrix} \quad \overline{R_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{R_3} = \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{4} \end{bmatrix}$$

$$AF(\theta,\phi) = I_1 e^{-j\beta\frac{\lambda}{4}\cos\theta} + I_2 + I_3 e^{j\beta\frac{\lambda}{4}\cos\theta}$$

$$\beta\frac{\lambda}{4} = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\Rightarrow AF(\theta,\phi) = 1 e^{-j\frac{\pi}{2}} e^{-j\frac{\pi}{2}\cos\theta} + 2e^{j0} + 1 e^{j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos\theta}$$

$$AF(\theta,\phi) = 1 e^{-j\frac{\pi}{2}(1+\cos\theta)} + 2e^{j0} + 1 e^{j\frac{\pi}{2}(1+\cos\theta)}$$

$$= 2\left(1 + \cos\left[\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right]\right)$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

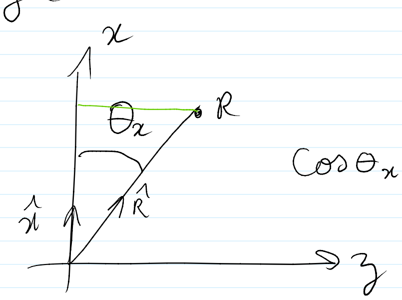
$$\cos\left(\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right) = -\sin\left(\frac{\pi}{2}\cos\theta\right)$$

$$\Rightarrow AF(\theta,\phi) = 2\left(1 - \sin\left(\frac{\pi}{2}\cos\theta\right)\right)$$

$$= 2 - 2\sin\left(\frac{\pi}{2}\cos\theta\right)$$

b)

$$g(\theta,\phi) = \sin\theta_x = \sqrt{1 - \cos^2\theta_x}$$



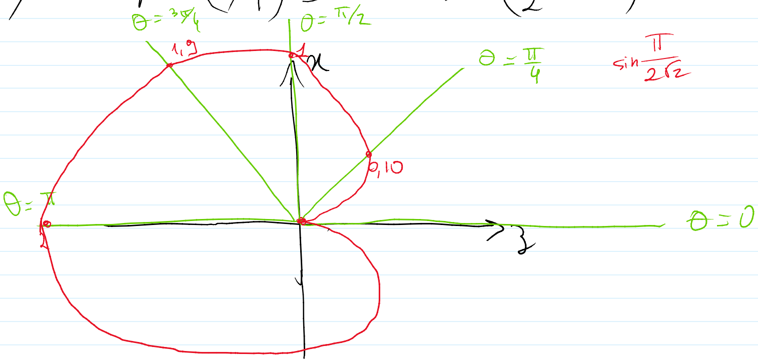
$$\cos\theta_x = \hat{x} \cdot \hat{R} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

$$\Rightarrow \cos\theta_x = \sin\theta\cos\phi$$

$$\Rightarrow g(\theta,\phi) = \sqrt{1 - \sin^2\theta\cos^2\phi}$$

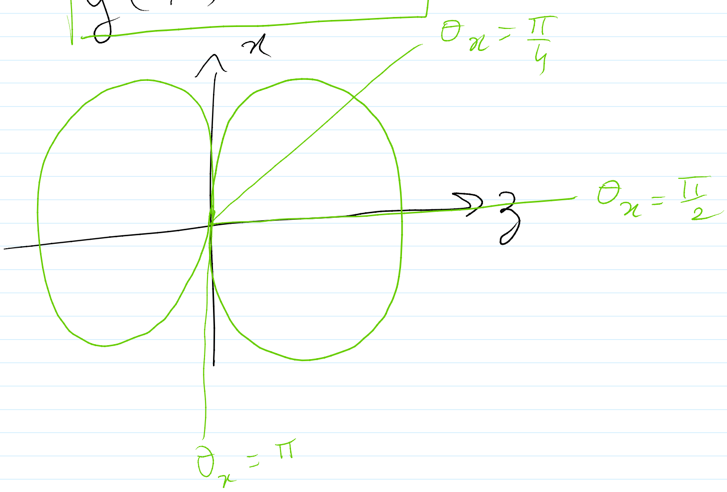
$$F(\theta,\phi) = g(\theta,\phi)\, AF(\theta,\phi)$$
$$= \sqrt{1-\sin^2\theta\cos^2\phi}\left(1-\sin\left(\frac{\pi}{2}\cos\theta\right)\right)$$

c) $AF(\theta,\phi) \equiv 1-\sin\left(\frac{\pi}{2}\cos\theta\right)$

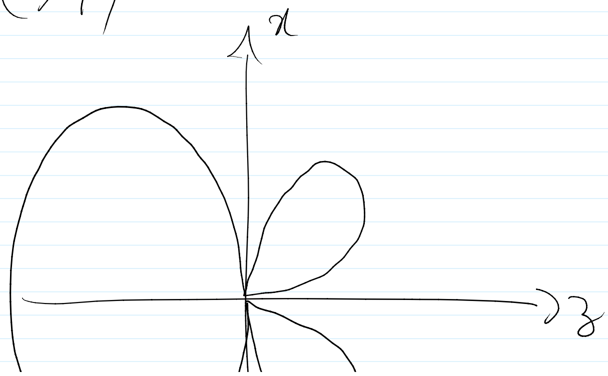


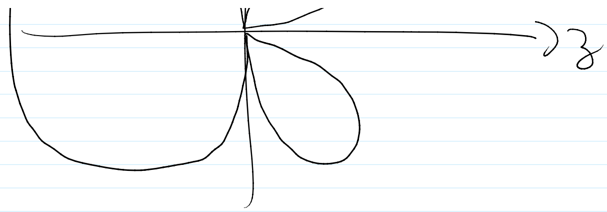
$\theta_z \neq \theta_x$

$$\boxed{g(\theta,\phi) = \sin\theta_x}$$



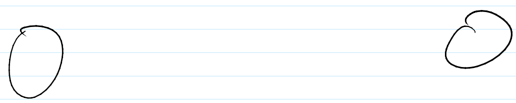
$F(\theta,\phi)$





d)

Gain = 6 dBi



$$\frac{d}{\lambda} = 500$$

$$10 \log G_T = 6 \text{ dBi}$$

$$\Rightarrow \quad G_T = 10^{6/10}$$

$$\Rightarrow \quad G_R = 10^{6/10}$$

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R} \right)^2$$

$$R = 500\lambda$$

$$\Rightarrow \frac{P_R}{P_T} = \left(10^{6/10}\right)^2 \left(\frac{1}{4\pi \times 500} \right)^2$$

$$= 4{,}01 \cdot 10^{-7}$$

Question 3

$\Gamma K (\sin\theta)^5 \quad 0 \le \theta \le \pi/2$

$$P_R = \begin{cases} \frac{K}{R^2}(\cos\theta)^5 & 0 \leq \theta \leq \pi/2 \\ 0 & \frac{\pi}{2} < \theta \leq \pi \end{cases}$$

$$P_R^{iso} = \frac{P_{Ray}}{4\pi R^2}$$

$$P_{Ray} = \int_0^{2\pi}\int_0^{\pi/2} \frac{K}{\cancel{R^2}}(\cos\theta)^5 \cancel{R^2} \sin\theta \, d\theta \, d\phi$$

$$= 2\pi K \int_0^{\pi/2} (\cos\theta)^5 \sin\theta \, d\theta$$

$u = \cos\theta$

$\Rightarrow du = -\sin\theta \, d\theta$

$\Rightarrow -du = \sin\theta \, d\theta$

$\Rightarrow -\int u^5 du = -\frac{u^6}{6}$

$= -\frac{\cos^6\theta}{6}$

$$\Rightarrow P_{Ray} = -\frac{2\pi K}{6}\left[\cos^6\theta\right]_0^{\pi/2}$$

$$= -\frac{\pi}{3}K(0^6 - 1^6) = \frac{\pi K}{3}$$

$$P_R^{iso} = \frac{\pi K}{12\pi R^2} = \frac{K}{12R^2}$$

$$D(\theta,\phi) = \frac{P_R^{antenne}}{P_R^{iso}} = \begin{cases} 12\cos^5\theta & si \ 0 \leq \theta \leq \pi/2 \\ 0 & si \ \frac{\pi}{2} < \theta < \pi \end{cases}$$

$\Rightarrow D_{max} = 12\cos^5 0 = 1$ pour $\theta = 0$

b) $P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$

$R = 36\,000 \cdot 10^3$ m

$\lambda = \frac{c}{f}$ avec $c = 3.10^8 \, m/s$
$f = 20 \, GHz$

$P_T = 100W$

$G_T = 10^{45/10}$

$$P_R = \frac{P_T G}{4\pi R^2} = 0{,}184.10^{-9} \, W/m^2$$

$R = 36\,000.10^3 \, m$
$G = 10^{45/10}$
$P_T = 100W$

$$G = \varepsilon(1 - |\Gamma|^2) D(\theta, \phi)$$

$\varepsilon = 1$
$|\Gamma| = 0$

$\Rightarrow G = D(\theta, \phi) = 1{,}5 \sin^2\theta$

$\Rightarrow G_{max} = D_{max} = 1{,}5$

c) $$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

$$10 \log\left(\frac{P_R}{1mW}\right) = -104 \, dBm$$

$\Rightarrow P_R = 10^{-104/10} \times 10^{-3} = 3{,}98 . 10^{-14} \, W$

$P_T = 100W$
$G_T = 10^{45/10}$

$$G_R = \frac{P_R}{P_T G_T \left(\frac{\lambda}{4\pi R}\right)^2}$$

$\lambda = \frac{3.10^8}{20.10^9} = 0{,}015 \, m$
$R = 36 . 10^6 \, m$

$\Rightarrow G_R = 11{,}45$

d) $N = 10$

$$\sum^{10} I_n e^{j\beta \bar{R}_n \cdot \hat{R}}$$

$$\sum_{n=1}^{10} I_n e^{j\beta \bar{R}_n \cdot \hat{R}}$$

$$\bar{R}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{R} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

$\Rightarrow$ 1ère élément $\rightarrow I \angle 0$

$$\bar{R}_2 = \begin{bmatrix} 0 \\ 0 \\ 3\lambda/8 \end{bmatrix}$$

2ème $\rightarrow I \angle \Delta\phi + \frac{3\pi}{4}\cos\theta$

$$\bar{R}_3 = \begin{bmatrix} 0 \\ 0 \\ 6\lambda/8 \end{bmatrix} \qquad \Delta\phi + \frac{3\pi}{4} \left| \cos\theta_{max} \right. = 0$$

$\Rightarrow \Delta\phi = \frac{-3\pi}{4}$

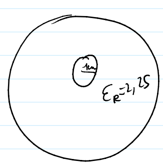
3ème $\rightarrow I \angle 2\Delta\phi + \frac{3}{2}\pi\cos\theta$

4ème $\rightarrow I \angle 3\Delta\phi + 3\pi\cos\theta$

5ème $\rightarrow I \angle 4\Delta\phi + 6\pi\cos\theta$

6ème $\rightarrow I \angle 5\Delta\phi + 12\pi\cos\theta$

Question 1 H18 :



a) $Z_0 = 75\ \Omega$

$$Z_0 = \frac{\eta}{2\pi} \ln\frac{b}{a}$$

$$e^{\frac{2\pi Z_0}{\eta}} a = b$$

$\Rightarrow b = 6,5\,cm$

b)

$$P = \frac{{V^+_{max}}^2}{2Z_0}$$

$\Rightarrow V^+_{max} = \sqrt{2Z_0 P}$

$$P = \frac{\quad}{2Z_0}$$

$$\Rightarrow V^+_{max} = \sqrt{2 Z_0 P}$$

$$\Rightarrow E_{max} = \frac{V^+_{max}}{\ln b/a} \times \frac{1}{a} = \frac{\sqrt{2 Z_0 P}}{\ln b/a} \times \frac{1}{a}$$

$$= 4627 \ V/m$$

c/ $TE_{11} \rightarrow$ Tableau

$$f_c = \frac{u_{TEM}}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= 10{,}1 \ GHz$$

$$k_r = \frac{\omega_c}{u_{TEM}}$$

TE_{11} $\quad k_r a = 1{,}8412$

$$k_r = \frac{1{,}8412}{a} = \frac{2\pi f_c}{u_{TEM}}$$

d) $\boxed{\begin{matrix} TE_{11} \\ TM_{01} \\ TE_{21} \end{matrix}}$ $\quad \frac{2\pi f_c^{21}}{u_{TEM}} = k_r = \frac{3{,}0542}{b}$

$$f_c^{21} =$$

Question 2 :

$f = 2{,}45 \ GHz$

$a = 7{,}21 \ cm$

$b = 3{,}4 \ cm$

$$P(z) = P(0) \, e^{-2\alpha z}$$

$$P(z) = P(0)\, e^{-2\alpha z}$$

$$\Rightarrow 10 \log e^{-2\alpha z} = -30\,dB$$

$$\alpha_{TE_{10}} = \frac{R_s \left(1 + \frac{2b}{a}\left(f_c/f\right)^2\right)}{\quad}$$

$$k_z = \sqrt{w^2 \mu \varepsilon - k_r^2}$$

$$k_r^2 = k_x^2 + k_y^2$$

$$k_r^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow k_z = j\sqrt{-w^2 \mu_0 \varepsilon_0 + \left(\frac{\pi}{a}\right)^2} \in \mathbb{R}$$

$$k_z < 0$$

$$k_z = -j\ 3{,}14 . 10^3 = -j\alpha$$

$$\Rightarrow \alpha = 3{,}14 . 10^3$$

$$e^{-jk_z z} = e^{-j(-j\alpha)z}$$

$$\frac{P(z)}{P(0)} = e^{-2\alpha z}$$

$$10 \log\left(e^{-2\alpha z}\right) = -30\,dB$$

$$\Rightarrow \quad z = \ln\left(10^{-30/10}\right) \times \frac{1}{-2\alpha}$$

$$\Rightarrow z = 1{,}1\,mm$$

b) f_c

c) $\sigma = 1{,}3 . 10^6\ S/m$

$$P^i = 1000\,W$$

$$\Gamma = 0$$

$$P^i = P^r + P^R$$

$$P(z) = P(0)\, e^{-2\alpha z}$$

$$\Delta P = P(0) - P(z) = P(0)\left(1 - e^{-2\alpha z}\right)$$

$$R_s\left[1 + \frac{2b}{a}\left(\frac{f_c^{10}}{f}\right)^2\right]$$

$\Delta P = P(0) - P(z) = P(0)(1 - e$ $)$

$$\alpha_{TE_{10}} = \frac{R_s \left[1 + \frac{2b}{a}\left(\frac{f_c^{10}}{f}\right)^2\right]}{b\eta\sqrt{1-\left(\frac{f_c^{10}}{f}\right)^2}}$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\Rightarrow \alpha_{TE_{10}} = 0,02139$$

$$\Delta P = 1000 \times \left(1 - e^{-2\times 0,02139 \times 15.10^{-2}}\right)$$

$$= 6,4\ W$$

Question 3:

$$\mathcal{E} = 0,5$$

$$G = 1,5 \sin^2\theta$$

$$P_T = 10\ W$$

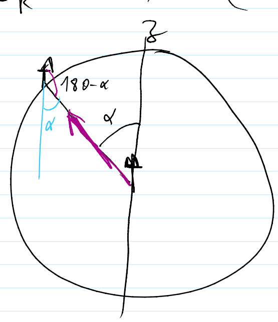
$$f = 300\ MHz$$

a) $$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

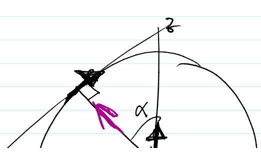
$P_T = 10W$ $\qquad$ $R = 100\,m$

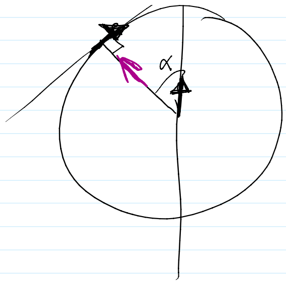
$G_T = \mathcal{E}\cdot 1,5 \sin^2\alpha$ $\qquad$ $\lambda = \frac{c}{f} = 1$

$G_R = \mathcal{E}\cdot 1,5 \sin^2(180-\alpha)$



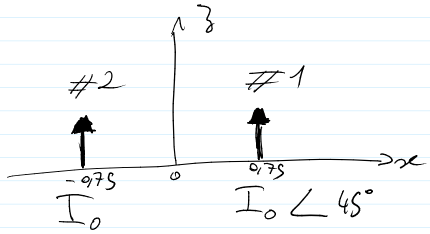
$$P_R = 8,9\ .10^{-7}\ W$$





$P_R = 1{,}78 \cdot 10^{-6}$ W

c)



$$AF = \underbrace{I_0 e^{j\frac{\pi}{4}} e^{j\beta\, 0{,}75 \sin\theta\cos\phi}}_{\#1} + \underbrace{I_0 e^{-j\beta\, 0{,}75 \sin\theta \cos\phi}}_{\#2}$$

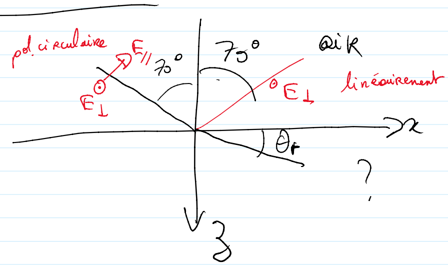
$$= I_0 e^{j\frac{\pi}{8}} \left(e^{j\frac{\pi}{8}} e^{j\beta 0{,}75 \sin\theta\cos\phi} + e^{-j\left(\frac{\pi}{8} + \beta 0{,}75 \sin\theta\cos\phi\right)}\right)$$

$$= 2 I_0 e^{j\frac{\pi}{8}} \cos\left(\frac{\pi}{8} + \beta \frac{3}{4} \sin\theta\cos\phi\right)$$

$$g(\theta,\phi) = \sin\theta$$

$$AF(\theta,\phi) \equiv \cos\left(\frac{\pi}{8} + \beta\frac{3}{4}\sin\theta\cos\phi\right)$$

Question 4 :



a) $P^i = 1$ W/m²

$$\Gamma_{//} = 0$$

$$\theta_i = \theta_B = 70°$$

$$\tan(\theta_B) = \frac{N_2}{N_1} = \frac{N_2}{n_1} = \frac{\sqrt{\varepsilon_{R2}}}{1}$$

$$\tan(\theta_B) = \frac{N_2}{N_1} = \frac{N_2}{N_{air}} = \frac{\sqrt{\varepsilon_{R_2}}}{1}$$

$$\varepsilon_{R_2} = \tan(70°)^2$$

$$\Rightarrow \varepsilon_{R_2} = 7{,}5$$

$$N_2 \sin\theta_t = N_1 \sin\theta_i$$

$$\theta_t = \sin^{-1}\left(\frac{N_1}{N_2}(\sin\theta_i)\right)$$

$$= 20{,}06°$$

b) $$T_{/\!/} = \frac{2\eta_2 \cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = 0{,}36 \in \mathbb{R}$$

$$T_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = 0{,}23 \in \mathbb{R}$$

$$\eta_2 = \frac{377}{\sqrt{\varepsilon_{R_2}}}$$

$\Rightarrow$ Polarisation elliptique

$$E_\perp^t = T_\perp E_\perp^i$$

$$\bar{P} = 1 = \frac{|E_\perp^i|^2 + |E_{/\!/}^i|^2}{2\eta_1}$$

$$\frac{|E_\perp^i|^2}{2\eta_1} = \frac{1}{2}$$

$$\Rightarrow |E_\perp^i| = 19{,}41 \text{ V/m}$$

$$= |E_{/\!/}^i|$$

$$\Rightarrow \angle E_\perp^i = \angle E_{/\!/}^i + 90°$$

en retard

$$\Rightarrow |E_\perp^i| = 19{,}41 \angle E_{/\!/}^i + 90°$$

$$\Rightarrow |E_{\perp}^{i}| = 19{,}41 \angle E_{//}^{i} + 90°$$

$$|E_{//}^{i}| = 19{,}41 \angle E_{//}^{i}$$

$$\Rightarrow |E_{\perp}^{r}| = 4{,}46 \angle E_{//}^{i} + 90°$$

$$|E_{//}^{t}| = 6{,}98 \angle E_{//}^{i}$$

$$\Rightarrow \frac{|E_{\perp}^{r}|}{|E_{//}^{t}|} = 0{,}64$$

c) $f = 10\,GHz$

$$\sigma = 500 \text{ S/m}$$

$$\bar{\gamma}_t = \begin{bmatrix} \gamma_{x,t} \\ 0 \\ \gamma_{z,t} \end{bmatrix} = \begin{bmatrix} \beta_1 \sin\theta_i \\ 0 \\ \gamma_{z,t} \end{bmatrix}$$

$$\gamma_{z,t} = \pm \sqrt{-\omega^2 \mu_2 \varepsilon_2 - (\beta_1 \sin\theta_i)^2}$$

$$= \pm 6283 \angle 90{,}5°$$

$$= 54{,}83 - j\,6282{,}76$$

$$\gamma_{z,t} = j k_{z,t} = j(-6282{,}76 + j\,54{,}83)$$

$$\bar{k}_t = \begin{bmatrix} -j\,196{,}8 \\ 0 \\ -6282{,}76 + j\,54{,}83 \end{bmatrix}$$

Question 4 devoir 2 :

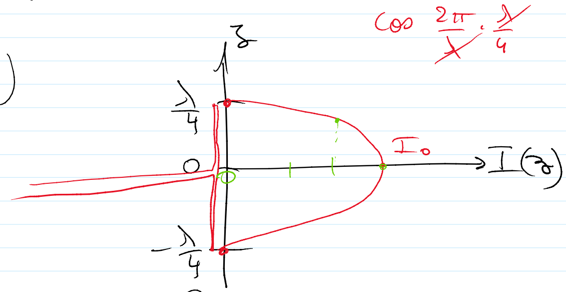
$$I(z) = \begin{cases} I_0 \cos(\beta z) & \text{si } |z| \leq \frac{\lambda}{4} \end{cases}$$

$$I(z) = \begin{cases} I_0 \cos(\beta z) & \text{si } |z| \leq \frac{\lambda}{4} \\ 0 & \text{si } |z| > \frac{\lambda}{4} \end{cases}$$

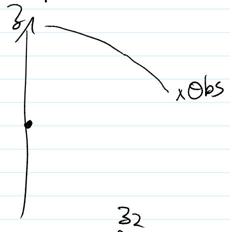
$f = 2{,}45$ GHz

$\cos \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{4}$

a)



b)



$$\hat{R} = \begin{bmatrix} \sin\theta \cos\phi \\ \sin\theta \sin\phi \\ \cos\theta \end{bmatrix}$$

$\bar{R}$: point sur l'antenne

$\Rightarrow \bar{R} = z\hat{z} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}$

$$E_\theta(R,\theta) = \int_{z_1}^{z_2} dE_\theta$$

$$\rightarrow \quad dE_\theta = j\beta\eta I(z\hat{z}) \frac{\sin\theta \, e^{-j\beta \bar{R}\cdot\hat{R}}}{4\pi R} dz$$

$$I(z\hat{z}) = \begin{cases} I_0 \cos(\beta z)\, \hat{z} & |z| \leq \frac{\lambda}{4} \\ 0\, \hat{z} & |z| > \frac{\lambda}{4} \end{cases}$$

$$E_\theta = \int_{-\lambda/4}^{\lambda/4} j\beta\eta I_0 \cos(\beta z) \sin\theta \frac{e^{-j\beta z\cos\theta}}{4\pi R} dz$$

$$= \frac{j\beta\eta I_0 \sin\theta}{4\pi R} \underbrace{\int_{-\lambda/4}^{\lambda/4} \cos(\beta z)\, e^{-j\beta z\cos\theta} dz}_{A}$$

$$A = \frac{1}{2}\int_{-\lambda/4}^{\lambda/4} \left(e^{j\beta z} + e^{-j\beta z}\right) e^{-j\beta z \cos\theta} dz$$

$$\pi = 2\int (\quad -\frac{\lambda}{4}$$

$$= \frac{1}{2}\int_{-\frac{\lambda}{4}}^{\lambda/4} e^{j\beta z(1-\cos\theta)} + e^{-j\beta z(1+\cos\theta)}\, dz$$

$$= \frac{1}{2}\left(\frac{e^{j\beta(1-\cos\theta)\frac{\lambda}{4}} - e^{-j\beta(1-\cos\theta)\frac{\lambda}{4}}}{j\beta(1-\cos\theta)} + \frac{e^{j\beta(1+\cos\theta)\frac{\lambda}{4}} - e^{-j\beta(1+\cos\theta)\frac{\lambda}{4}}}{j\beta(1+\cos\theta)} \right)$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2j}\sin\left(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right)}{\cancel{j}\beta(1-\cos\theta)} + \frac{\cancel{2j}\sin\left(\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right)}{\cancel{j}\beta(1+\cos\theta)} \right)$$

$$\sin(a-b) = \sin a\cos b - \cos a \sin b$$

$$\Rightarrow \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right) = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

$$\sin(a+b) = \sin a\cos b + \cos a\sin b$$

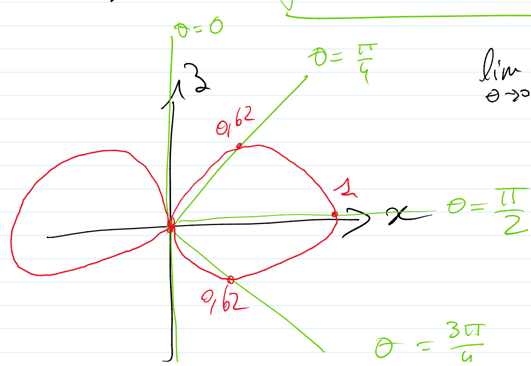
$$\Rightarrow \sin\left(\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right) = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

$$\Rightarrow A = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\beta(1-\cos\theta)} + \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\beta(1+\cos\theta)}$$

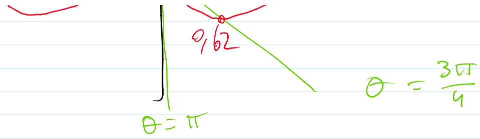
$$E_\theta = \frac{j\cancel{\beta} h I_0 \cancel{\sin\theta}}{4\pi R}\left[\frac{2\cancel{\beta}}{\beta^2 \sin^{\cancel{2}}\theta}\right]\cos\left(\frac{\pi}{2}\cos\theta\right)$$

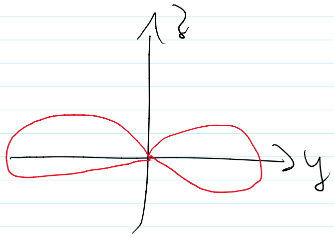
$$E_\theta = j\frac{I_0 h}{2\pi R}\boxed{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}}$$

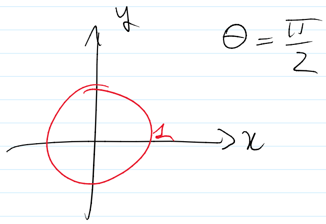
c)



$$\lim_{\theta\to 0} \frac{-\frac{\pi}{2}\sin\theta \cdot \sin\left(\frac{\pi}{2}\cos\theta\right)}{\cos\theta} = 0$$







d) $D(\theta, \phi)$

$$G = \varepsilon(1-|\Gamma|^2) D(\theta, \phi)$$

$\theta_T = \tan^{-1} \frac{100}{60} \simeq 60°$

$$D(\theta,\phi) \propto P_R^{antenne} \propto |E_\theta|^2 \propto \left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)^2$$

$$D_{max} = 1{,}64 = \text{Constante}\left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right)^2\Bigg|_{\theta=\frac{\pi}{2}}$$

$$\Rightarrow 1{,}64 = \text{constante}$$

Donc $$D(\theta,\phi) = \text{constante} \cdot \left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin(\theta)}\right)^2\Bigg|_{\theta = \theta_R \text{ ou } \theta_T}$$

$$\Rightarrow D(\theta,\phi)_T = 1{,}64 \times \frac{2}{3}$$

$$D(\theta,\phi)_R = 1{,}64 \times \frac{2}{3}$$

$$\Gamma = \frac{75-50}{75+50} =$$

$$D(\theta,\phi)_R = 1{,}64 \times \frac{2}{3}$$

$$\Rightarrow G_T = G_R = 0{,}9 \times \left(1 - |\Gamma|^2\right) \times 1{,}64 \times \frac{2}{3}$$

$$\Rightarrow G_T = G_R \approx 0{,}94464$$

$$P_T = 100\,mW$$

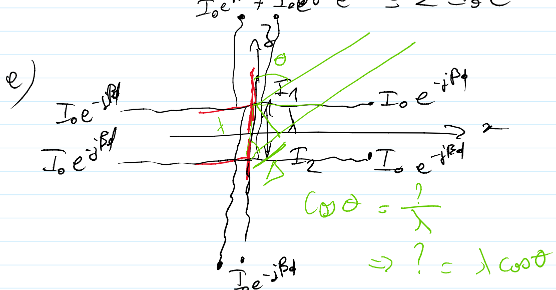
Ainsi

$$P_R = P_T\, G_T\, G_R \cdot \left(\frac{\lambda}{4\pi R}\right)^2$$

$$= 6{,}23 \cdot 10^{-10}\ W$$

$$I_0 e^{-j\beta d} + I_0 e^{-j\beta\lambda} e^{-j\beta d} = 2 I_0 e^{-j\beta d}$$

e)



$$+ I_0 e^{+j\beta\lambda} e^{-j\beta d}$$

$$e^{j2\pi} = e^{j0}$$

$$\Rightarrow 2 I_0 e^{-j\beta d}$$

$$\Delta = \lambda\cos\theta$$

$$I_0\, e^{-j2\pi\cos\theta} e^{-j\beta d} + I_0 e^{-j\beta d}$$

$$I_0 e^{-j\beta d}\left(1 + e^{-j\beta\Delta}\right)$$

$$\Delta = n\lambda$$

$$\Rightarrow \beta\Delta = 2\pi$$

$\Rightarrow$ maximum

$\Rightarrow$