Feuille de notes
24 février 2022 22:45

$$TOS = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$
$$= V^+ e^{j\beta d}\left(1 + |\Gamma_L| e^{j\angle\Gamma_L} e^{-j2\beta d}\right)$$

$$Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) \qquad Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$\Rightarrow z_L = \frac{1+\Gamma_L}{1-\Gamma_L}$, si $\Gamma_L \in \mathbb{R}^+$
↳ $TOS = z_L$

$$\Delta d_{max} = \Delta d_{min} = \frac{\lambda}{2}$$

Puissance moyenne: $p = \frac{1}{2} Re\{V I^*\}$

$$\Gamma(d) = \Gamma_L e^{-2\gamma d}$$

$$\Gamma(d) = \frac{Z(d)-Z_0}{Z(d)+Z_0} \qquad Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$$

• Matrice ABCD :

$$\begin{bmatrix} V(d) \\ I(d) \end{bmatrix} = \begin{bmatrix} \cos\beta d & j Z_0 \sin\beta d \\ \frac{j}{Z_0}\sin\beta d & \cos\beta d \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} \quad \text{SANS PERTES}$$

$$Z(d) = \frac{Z_L \cos\beta d + j Z_0 \sin\beta d}{j\frac{Z_L}{Z_0}\sin\beta d + \cos\beta d} = \frac{Z_0\left(Z_L + jZ_0 \tan\beta d\right)}{j Z_L \tan\beta d + Z_0}$$

$$z(d) = \frac{z_L + j\tan\beta d}{j z_L \tan\beta d + 1}$$

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

$$\begin{bmatrix} V(d) \\ I(d) \end{bmatrix} = \begin{bmatrix} \cosh\gamma d & Z_0 \sinh\gamma d \\ \frac{1}{Z_0}\sinh\gamma d & \cosh\gamma d \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} \quad \text{AVEC PERTES}$$

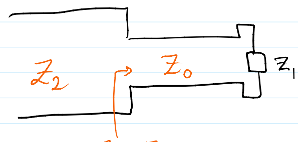
$$Z(d) = \frac{Z_L \cosh\gamma d + Z_0 \sinh\gamma d}{\frac{Z_L}{Z_0}\sinh\gamma d + \cosh\gamma d}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \beta = \omega\sqrt{LC} \qquad \alpha = \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right)$$

$$Z(d) = \frac{Z_0 Z_L + j Z_0 \tan\beta d}{j Z_L \tan\beta d + Z_0}$$

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$



But : $Z_{in} = Z_2$

$$Z_0 = \sqrt{Z_1 Z_2}$$

$$\Gamma_V = \frac{R_L - Z_0}{R_L + Z_0}$$

$$\Gamma_I = \frac{G_L - Y_0}{G_L + Y_0} = -\Gamma_V$$

$$1 + \Gamma = T$$

Condition de Heaviside

$$RC = LG$$
$$\alpha = \sqrt{RG}$$
$$c = \frac{1}{\sqrt{LC}} = \lambda f = \frac{\omega}{\beta}$$
$$\beta = \omega\sqrt{LC}$$
$$\alpha = \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right) = 0 \text{ si pas pertes}$$

$\xrightarrow{+s}$ $\xleftarrow{-s}$

$$V(s) = A e^{-\gamma s} + B e^{\gamma s}$$
$$= A e^{-\alpha s} e^{+j\beta s}$$

ONDES

$\bar{\mathcal{E}}, \bar{\mathcal{H}}, \bar{\mathcal{J}}$: vecteurs réels dépendent de x, y, z et t en général

$\bar{E}_c, \bar{H}_c, \bar{J}_c$: vecteurs complexes

$\overline{E}_c, \overline{H}_c, \overline{J}_c$: vecteurs complexes

e.g. $\overline{\mathcal{E}} = Re\{\overline{E}_c\}$

$\overline{E}, \overline{H}, \overline{J}$: vecteurs phaseurs

e.g. $\overline{E}_c = \overline{E}\, e^{j\omega t}$

$\underline{\gamma}, \underline{\varepsilon}$: constantes complexes, scalaires

formules utiles :

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{\beta} = \lambda f \quad , \quad \varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m$$

$$\mu_0 = 4\pi \cdot 10^{-7}\ H/m$$

$$c = 3 \cdot 10^{8}\ m/s$$

$$\beta = \|\overline{k}\| = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

$$\overline{H} = \overline{k} \times \overline{E} \qquad P_{moy} = \frac{1}{2} Re\{\overline{E} \times \overline{H}^*\}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\underline{\varepsilon} = \varepsilon_0 (\varepsilon_r - j \frac{\sigma_e}{\omega \varepsilon_0})$$

$$\overline{k} = -j \underline{\gamma}\, \hat{n}$$

$$P_{moy} = \frac{\|\overline{E}\|^2}{2\eta} = \frac{\eta \|\overline{H}\|^2}{2}$$

$$\overline{P}_{moy} = \hat{n} \frac{|A_x|^2 + |A_y|^2 + |A_z|^2}{2\eta}$$

$$\overline{E} = \eta\, \overline{H}$$

$$\overline{E} = \overline{A} e^{-j(-j\underline{\gamma}\hat{n}\cdot\overline{R})} + \overline{B} e^{+j(-j\underline{\gamma}\hat{n}\cdot\overline{R})}$$

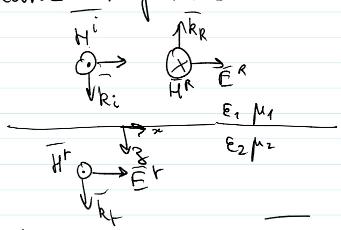
$$= \overline{A}\, e^{-\alpha \hat{n}\cdot\overline{R}} e^{-j\beta\hat{n}\cdot\overline{R}} + \overline{B}\, e^{+\alpha \hat{n}\cdot\overline{R}} e^{+j\beta\hat{n}\cdot\overline{R}}$$

Bon conducteur : $p = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma_e}{\omega \varepsilon_r \varepsilon_0} > 100$

$$\sin x = \cos\left(x - \frac{\pi}{2}\right) \qquad \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\alpha = \beta = \sqrt{\frac{\sigma \omega \mu}{2}}$$

Plane uniforme incidence normale : pol linéaire



$$\overline{H} = \frac{\overline{k} \times \overline{E}}{\omega \mu}$$

$$\begin{cases} P^i = P^i_z = \overline{P}^i \cdot \hat{z} = \frac{1}{2} Re\left\{\frac{E^i_x E^{i*}_x}{\eta_1^*}\right\} \cdot \hat{z} = \frac{1}{2} Re\left\{\frac{1}{\eta_1^*}\right\} |E^i_x|^2_{z=0} \\ P^R = P^R_z = P^R \cdot (-\hat{z}) = \frac{1}{2} Re\left\{\frac{1}{\eta_1^*}\right\} |E^R_x|^2_{z=0} \\ P^t = P^t_z = P^t \cdot (\hat{z}) = \frac{1}{2} Re\left\{\frac{1}{\eta_2^*}\right\} |E^t_x|^2_{z=0} \end{cases}$$

- $P^i = P^t + P^R \Rightarrow P^t = P^i - P^R \Rightarrow |T|^2 Re\left\{\frac{1}{\eta_2^*}\right\} = Re\left\{\frac{1}{\eta_1^*}\right\}(1 - |\Gamma|^2)$

- $P^i = P^t + P^r \Rightarrow P^t = P^i - P^R \Rightarrow |T|^2 \, Re\left\{\frac{1}{\eta_2^*}\right\} = Re\left\{\frac{1}{\eta_1^*}\right\}(1-|\Gamma|^2)$

$$\begin{cases} \overline{E^i} = \hat{x}\, E^i_x \\ \overline{E^R} = \hat{x}\, E^R_x \\ \overline{E^t} = \hat{x}\, E^t_x \end{cases} \quad \begin{cases} \overline{k_i} = \hat{n}_i \beta_1 = \hat{z}\beta_1 & ; \ \beta_1^2 = \omega^2 \mu_1 \varepsilon_1 \\ \overline{k_R} = \hat{n}_R \cdot \beta_1 \\ \overline{k_t} = \hat{n}_t \beta_2 = \hat{z}\beta_2 & ; \ \beta_2^2 = \omega^2 \mu_2 \varepsilon_2 \end{cases}$$

$$\Rightarrow \begin{cases} \overline{H^i} = \dfrac{\overline{k_i} \times \overline{E^i}}{\omega \mu_1 \varepsilon_1} = \dfrac{\beta_1 E^i_x \ \hat{z} \times \hat{x}}{\omega \mu_1} = \hat{y}\, E^i_x / \eta_1 \\ \overline{H^R} = \dfrac{\overline{k_R} \times \overline{E^R}}{\omega \mu_1 \varepsilon_1} = \dfrac{\beta_1 (-\hat{z}) \times \hat{x}\, E^R_x}{\omega \mu_1} = -\hat{y}\, E^R_x / \eta_1 \\ \overline{H^t} = \dfrac{\overline{k_t} \times \overline{E^t}}{\omega \mu_2 \varepsilon_2} = \dfrac{\beta_2 (\hat{z}) \times \hat{x}\, E^t_x}{\omega \mu_1} = \hat{y}\, E^t_x / \eta_2 \end{cases}$$

$$\overline{E^i_{tan}} + \overline{E^R_{tan}} = \overline{E^t_{tan}}$$

$$\Rightarrow E^i_x(z=0) \cdot e^{-j\overline{k_i}\cdot\overline{R}}\,\hat{x} + E^R_x(z=0)\cdot e^{-j\overline{k_R}\cdot\overline{R}}\,\hat{x} = E^t_x(z=0)\cdot e^{-j\overline{k_t}\cdot\overline{R}}\,\hat{x}$$

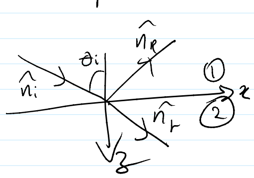
$\rightarrow$ À l'interface $R = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

$$\Rightarrow \boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \qquad \boxed{T = 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

$$Z \approx \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon_0 \varepsilon_R \left(1 + j\frac{\sigma}{\omega \varepsilon_0 \varepsilon_R}\right)}}$$

$$|Z| = \sqrt{\frac{\omega \mu}{\sigma}}$$

Onde plane incidence oblique



$$\hat{n}_i = \begin{bmatrix} \sin\theta_i \\ 0 \\ \cos\theta_i \end{bmatrix} \rightarrow \overline{n_{i,tan}} = \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{E^i} = \overline{A}\, e^{-j\beta_1 \hat{n}_i \cdot \overline{R}}$$

$$\overline{E^R} = \overline{B}\, e^{-j\beta_1 \hat{n}_R \cdot \overline{R}}$$

$$\overline{E^t} = \overline{C}\, e^{-j\beta_2 \hat{n}_t \cdot \overline{R}} \quad \text{ou} \quad \overline{C}\, e^{-\overline{\gamma_2}\cdot\overline{R}}$$

$$\boxed{\begin{gathered} \overline{\gamma} = (\alpha + j\beta)\cdot\hat{n} \\ e^{-\overline{\gamma}\cdot\overline{R}} = e^{-\alpha\,\hat{n}\cdot\overline{R}}\, e^{-j\beta\,\hat{n}\cdot\overline{R}} \end{gathered}}$$

Pour $\hat{n}_i, \hat{n}_R, \hat{n}_r$, on a → $\hat{n} = 1\overline{n}_{tan} + \hat{z}\, n_z$
à l'interface $z=0$

De même, $\overline{R} = 1\overline{R}_{tan} + z\hat{z}$
avec $\overline{R}_{tan} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

À $z=0$, on a $\overline{E}^i_{tan}(x,y,0) + \overline{E}^R_{tan}(x,y,0) = \overline{E}^r_{tan}(x,y,0)$
De même pour $\overline{H}^{i,R,r}_{tan}$

Composantes du champs à différents endroits

$$f(x,y,z)\,\hat{x} + g(x,y,z)\,\hat{y} + h(x,y,z)\,\hat{z}$$

$$\mathcal{G}_{tan} = f(x,y,z)\,\hat{x} + g(x,y,z)\,\hat{y} + 0\,\hat{z}$$

$$\tan_{\text{à frontière } z=0} = f(x,y,0)\,\hat{x} + g(x,y,0)\,\hat{y} + 0\,\hat{z}$$

i.e. : $\overline{E}^i_{tan} = \begin{bmatrix} cte_x \\ cte_y \\ 0 \end{bmatrix} e^{-j\beta_1\left(\overline{R_{tan}}\cdot\overline{n_{i,tan}} + z\, n_{i,z}\right)}$

$$\underbrace{\quad}_{= -\overline{\gamma_{tan}}\cdot\overline{R_{tan}}}$$

↳ De même pour $\overline{E}^r$ et $\overline{E}^R$

$$\Rightarrow \overline{E}^i_{tan}\Big|_{z=0} = \begin{bmatrix} cte_x \\ cte_y \\ 0 \end{bmatrix} e^{-j\beta_1\overline{R_{tan}}\cdot\overline{n_{i,tan}}}$$

Condition frontière : $\overline{E}^i_{tan}\big|_{z=0} + \overline{E}^R_{tan}\big|_{z=0} = \overline{E}^r_{tan}\big|_{z=0}$

$$\Rightarrow \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_i e^{-j\beta_1\overline{R_{tan}}\cdot\overline{n_{i,tan}}} + \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_R e^{-j\beta_1 R_{tan}\cdot n_{R,tan}} = \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_r e^{-j\beta_2 R_{tan}\cdot n_{r,tan}}$$

$$\Rightarrow \beta_1\overline{R_{tan}}\cdot\overline{n_{i,tan}} = \beta_1\begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \beta_1\sin\theta_i \\ 0 \\ 0 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}}_{\text{dépendance du terme } E_{tan}|_{z=0} \text{ par rapport à } x \text{ et } y}$$

↳ équation satisfaite partout sur interface

⇒ les termes doivent avoir la même dépendance par rapport à x et y (i.e. la même combinaison de x et y)

dépendance par rapport à x et y
(i.e. la même combinaison de x et y)

$$\Rightarrow \boxed{\beta_1 \overline{n_{i,tan}} = \beta \overline{n_{R,tan}} = \beta \overline{n_{r,tan}}}$$

Reflexion

$\Rightarrow n_{i,tan} = n_{R,tan}$

$$\Rightarrow \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix} = \overline{n_{R,tan}}$$

$$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ n_{R,z} \end{bmatrix}$$

or $\|\hat{n}_R\| = 1$

$\Rightarrow \sin^2\theta_i + n_{R,z}^2 = 1$

$\Rightarrow n_{R,z} = \pm\cos\theta_i$

$$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ \pm\cos\theta_i \end{bmatrix} \quad \text{et} \quad \overline{R} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\|\overline{E}^R\| \alpha\, e^{-\overline{\gamma_1}\cdot\overline{R}} = e^{-(\alpha_1 + j\beta_1)\hat{n}_R\cdot\overline{R}} = e^{-\alpha_1\hat{n}_R\cdot\overline{R}}\, e^{-j\beta_1\hat{n}_R\cdot\overline{R}}$$

$$= e^{-\alpha_1(x\sin\theta_i \pm z\cos\theta_i)}\, e^{-j\beta_1(x\sin\theta_i \pm z\cos\theta_i)}$$

$$= e^{-\alpha_1 x\sin\theta_i}\, e^{\mp\alpha_1 z\cos\theta_i}\, e^{-j\beta_1 x\sin\theta_i}\, e^{\mp j\beta_1 z\cos\theta_i}$$

L'onde réflechit se propage
en $z < 0 \Rightarrow \alpha_1 z\cos\theta_i < 0$
$\Rightarrow -\alpha_1 z\cos\theta_i > 0$
il faut choisir $-\cos\theta_i$

$$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ -\cos\theta_i \end{bmatrix}$$

$\Rightarrow \boxed{\theta_i = \theta_R}$
pour tout angle d'incidence entre 0 et 90°

Transmission

$$\beta_1 \overline{n_{i,tan}} = \beta_2 \overline{n_{r,tan}}$$

$$\Rightarrow \begin{bmatrix} \beta_1 \sin\theta_i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \beta_2\, n_{r,x} \\ \beta_2\, n_{r,y} \\ 0 \end{bmatrix}$$

$\Rightarrow \beta_1 \sin\theta_i = \beta_2\, n_{r,x}$

$\Rightarrow n_{r,x} = \frac{\beta_1}{\beta_2}\sin\theta_i$

et $n_{r,y} = 0$

$$\Rightarrow \hat{n}_r = \begin{bmatrix} \frac{\beta_1}{\beta_2}\sin\theta_i \\ 0 \\ n_{r,z} \end{bmatrix}$$

or $\|\hat{n}_r\| = 1$

$\Rightarrow n_{r,z}^2 + \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i = 1$

$\Rightarrow n_{r,z} = \pm\left|\sqrt{1 - \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}\right|$

on choisit $+$ car $z > 0$
pour le rayon transmit

$$\Rightarrow \hat{n}_r = \begin{bmatrix} \frac{\beta_1}{\beta_2}\sin\theta_i \\ 0 \\ \sqrt{1 - \left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)^2} \end{bmatrix}$$

$$\text{or } \hat{n}_r = \begin{bmatrix} \sin\theta_r \\ 0 \\ \cos\theta_r \end{bmatrix}$$

$\Rightarrow \sin\theta_r = \frac{\beta_1}{\beta_2}\sin\theta_i$

$\Rightarrow \boxed{N_2 \sin\theta_r = N_1 \sin\theta_i}$

avec $N = \sqrt{\mu_r \varepsilon_r}$

Si on a des pertes dans milieu de transmission :

$$\varepsilon_2 = \varepsilon_0 \varepsilon_{R_2} \left(1 - j\frac{\sigma}{\omega \varepsilon_0 \varepsilon_{R_2}}\right)$$

$$\overline{\gamma_r} = \begin{bmatrix} \gamma_{x,r} \\ 0 \\ \gamma_{z,r} \end{bmatrix} = \begin{bmatrix} \beta_1 \sin\theta_i \\ 0 \\ \gamma_{z,r} \end{bmatrix}$$

on sait que $\overline{\gamma_r} \cdot \overline{\gamma_r} = -\omega^2 \mu_2 \varepsilon_2$

$$\Rightarrow \gamma_{x,r}^2 + \gamma_{z,r}^2 = -\omega^2 \mu_2 \varepsilon_2$$

$$\Rightarrow \gamma_{z,r} = \pm\sqrt{-\omega^2 \mu_2 \varepsilon_2 - \gamma_x^2}$$

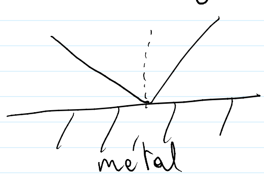
$$= \alpha_{z,r} + j\beta_{z,r}$$

il faut calculer $\gamma_{z,r}$ et choisir la racine $\alpha_{z,r} > 0$ pour atténuation car $\overline{E^r} = \overline{Cte}\, e^{-\gamma_{x,r} x} e^{-\alpha_{z,r} z} e^{-j\beta_{z,r} z}$

→ si le milieu de transmission est un bon conducteur, alors :

$$\gamma_{z,r} \approx \sqrt{-\omega^2 \mu_2 \left(-j\frac{\sigma}{\omega}\right)} = \sqrt{\frac{\sigma \omega \mu_2}{2}}(1+j)$$

$$\Rightarrow \alpha_z = \beta_z = \sqrt{\frac{\sigma \omega \mu}{2}} \gg \beta_x$$



↳ onde propagée selon $\hat{z}$ pratiquement

$$\overline{\gamma_r} = \overline{\alpha_r} + \overline{\beta_r}$$

$$\overline{\alpha_r} = \begin{bmatrix} 0 \\ 0 \\ \alpha_z \end{bmatrix}$$

$$\overline{\beta_r} = \begin{bmatrix} \beta_x \\ 0 \\ \beta_z \end{bmatrix}$$

$\hat{/\!/}$ et $\hat{\perp}$: Caractéristiques

- $\dfrac{E_\perp^R}{E_\perp^i} = \Gamma_\perp = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$, $\dfrac{E_\perp^t}{E_\perp^i} = T_\perp = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

et $1 + \Gamma_\perp = T_\perp$ $\qquad \eta_2 \cos\theta_i = \eta_1 \cos\theta_t + 1$

$\perp \Rightarrow$ TE

- $\dfrac{E_{/\!/}^R}{\ } = \Gamma \quad \eta_2 \cos\theta_t - \eta_1 \cos\theta_i \qquad \dfrac{E_{/\!/}^t}{\ } = T = \dfrac{2\eta_2 \cos\theta_i}{\ }$

- $$\frac{E^R_{//}}{E^i_{//}} = \Gamma_{//} = \frac{\eta_2 \cos\theta_r - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_r + \eta_1 \cos\theta_i} , \quad \frac{E^r_{//}}{E^i_{//}} = T_{//} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_r + \eta_1 \cos\theta_i}$$

$$\Gamma_{//} = 0 \quad \text{si} \quad \underbrace{\boxed{\tan\theta_i = \frac{N_2}{N_1}}}_{\text{Brewster}}$$

Ne veut pas forcément dire $T_{//} = 1$ car impédance interface non nulle

Réflexion totale arrive pour

$$\Rightarrow \sin\theta_r = \frac{N_1}{N_2} \sin\theta_i > 1$$

$$\rightarrow \cos\theta_r = \pm\sqrt{1 - \sin^2\theta_r}$$

$$= \pm j \underbrace{\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}}_{\in \mathbb{R}}$$

on sait que $\overline{E}^r = \overline{Cte}\, e^{-j\beta_2 \sin\theta_r x} e^{-j\beta_2 \cos\theta_r z}$

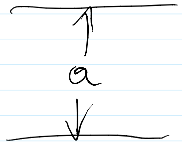
$$\Rightarrow E^r \propto e^{-j\beta_2 \cos\theta_r z}$$

$$\overset{z>0}{\Rightarrow} E^r \propto e^{\pm \beta_2 \left(\sqrt{\frac{N_1^2}{N_2^2}\sin\theta_i - 1}\right) z}$$

il faut donc $\cos\theta_r < 0$

$$\Rightarrow \cos\theta_r = -j\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}$$

Chapitre 4 : Guides d'onde



$a < \frac{\lambda}{2} \Rightarrow$ déphasage d'une même onde entre 2 point de son trajet dans le guide d'onde.

$a \geq \frac{\lambda}{2}$ condition nécessaire à

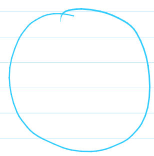
$a \geq \frac{\lambda}{2}$ condition nécessaire à propagation de 1 onde sans déphasage (vérifié pour mode TE)

mode TE $\longrightarrow$ $E_\perp$ et $H_{//}$

mode TM $\longrightarrow$ $E_{//}$ et $H_\perp$

$$\overline{E} = \overline{C}\, e^{-\overline{\gamma} \cdot \overline{R}}$$

$$k_r^2 + k_z^2 = \beta^2$$



$$\text{Rot}\, \overline{H} = \begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \begin{bmatrix} j\omega\varepsilon E_x \\ j\omega\varepsilon E_y \\ j\omega\varepsilon E_z \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} \frac{\partial H_z}{\partial y} + jk_z H_y \\ -jk_z H_x - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \begin{bmatrix} j\omega\varepsilon E_x \\ j\omega\varepsilon E_y \\ j\omega\varepsilon E_z \end{bmatrix}$$

$$\begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = j\omega\varepsilon E_x & (1) \\ -jk_z E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y & (2) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = \frac{\cancel{j}\omega\varepsilon}{-\cancel{j}k_z}\left(-j\omega\mu H_y + \frac{\partial E_z}{\partial x}\right) \\ E_x = \frac{\left(-j\omega\mu H_y + \frac{\partial E_z}{\partial x}\right)}{-jk_z} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = \frac{\omega\varepsilon}{k_z} j\omega\mu H_y - \frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x} \end{cases}$$

$-jk_z$

$$\Rightarrow \begin{cases} \dfrac{\partial H_z}{\partial y} + jk_z H_y = \dfrac{\omega\varepsilon}{k_z} j\omega\mu H_y - \dfrac{\omega\varepsilon}{k_z}\dfrac{\partial E_z}{\partial x} \end{cases}$$

$$\hookrightarrow H_y\left(jk_z - j\frac{\beta^2}{k_z}\right) = -\frac{\partial H_z}{\partial y} - \frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x}$$

$$\hookrightarrow \Rightarrow -jH_y\left(k_t^2\right) = -k_z\frac{\partial H_z}{\partial y} - \omega\varepsilon\frac{\partial E_z}{\partial x}$$

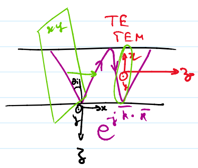
$$\Rightarrow H_y = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial y} - j\frac{\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow \Big\{ \quad H_y = \frac{-\frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x} - \frac{\partial H_z}{\partial y}}{jk_z - \frac{j\omega^2\varepsilon\mu}{k_z}} = \frac{-\omega\varepsilon\frac{\partial E_z}{\partial x} - k_z\frac{\partial H_z}{\partial y}}{-jk_t^2} = \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x} - j\frac{k_z}{k_t^2}\frac{\partial H_z}{\partial y}$$

$$k_t^2 = \omega^2\mu\varepsilon - k_z^2$$

$E_z = 0$

$$E = \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}$$



On obtient 6 équations impliquant les composantes des champs E et H.

$-\partial$

$\dfrac{\partial H_z}{\partial y} + jk_z H_y = j\omega\varepsilon E_x$ A	$\dfrac{\partial E_z}{\partial y} + jk_z E_y = -j\omega\mu H_x$ B
$-jk_z H_x - \dfrac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$ B	$-jk_z E_x - \dfrac{\partial E_z}{\partial x} = -j\omega\mu H_y$ A
$\dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$	$\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -j\omega\mu H_z$

Avec paire A : $H_z = 0$ et $E_z = 0$ avec paire B :

$$E_x = \frac{-j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial y} - \frac{jk_z}{k_t^2}\frac{\partial E_z}{\partial x} \quad (1)$$

$$E_y = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial x} - \frac{jk_z}{k_t^2}\frac{\partial E_z}{\partial y} \quad (3)$$

$$H_y = \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x} - \frac{jk_z}{k_t^2}\frac{\partial H_z}{\partial y} \quad (2)$$

$$H_x = \frac{j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial y} - \frac{jkz}{k_t^2}\frac{\partial H_z}{\partial x} \quad (4)$$

Modes Transverses Électriques ou TE $(E_z = 0,\ H_z \neq 0)$	Modes Transverses Magnétiques ou TM $(H_z = 0,\ E_z \neq 0)$
$E_x = \frac{-j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial y}$	$E_x = \frac{-jk_z}{k_t^2}\frac{\partial E_z}{\partial x}$
$H_y = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial y}$	$H_y \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x}$
$E_y = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial x}$	$E_y = \frac{-jk_z}{k_t^2}\frac{\partial E_z}{\partial y}$
$H_x = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial x}$	$H_x = \frac{j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial y}$

$$\underbrace{k_x^2 + k_y^2}_{k_t^2} + k_z^2 = \beta^2 = w^2\mu\varepsilon$$

Guide Rectangulaire :

$\boxed{TE} \rightarrow E_z = 0$

$$H_z = X(x)\,Y(y)\,e^{-jk_z z} = \left[A\cos(k_x x) + B\sin k_x x\right]\left[C\cos(k_y y) + D\sin(k_y y)\right]$$

en appliquant conditions frontières $\overline{E}_{tan} = 0$

$$k_x = \frac{m\pi}{a} \quad , \quad k_y = \frac{n\pi}{b}$$

et $H_z = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-jk_z z}$, m et $n \in \mathbb{N}$

H_0 constante

$\boxed{TM} \rightarrow H_z = 0$

$$E_z = \underbrace{\left[A\cos(k_x x) + B\sin(k_x x)\right]}_{X(x)}\underbrace{\left[C\cos(k_y y) + D\sin(k_y y)\right]}_{Y(y)}\underbrace{e^{-jk_z z}}_{Z(z)}$$

Pour $\overline{E}_{tan} = 0$, on obtient avec guide rectangulaire :

$$k_x = \frac{m\pi}{a} \quad , \quad k_y = \frac{n\pi}{b}$$

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-jk_z z}$$

avec $m, n \in \mathbb{N}$ et E_0 une constante

$\boxed{F_{coupure}:}$

$$k_x^2 + k_y^2 + k_z^2 = \beta^2$$

Coupure

$$k_x^2 + k_y^2 + k_z^2 = \beta^2$$

$$\Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = \beta^2$$

$$\Rightarrow k_z^2 = \beta^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$k_z^2 < 0 \Rightarrow k_z$ réel $\Rightarrow$ propagation

$k_z^2 > 0 \Rightarrow k_z$ imaginaire $\Rightarrow$

$$k_r = \frac{\omega_{coupure}}{u_{TEM}}$$

$$f > \frac{u_{TEM}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u_{TEM} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{k_z} = \frac{c}{N}$$

$$k_r^2 + k_z^2 = \beta^2 = \omega^2\mu\varepsilon$$

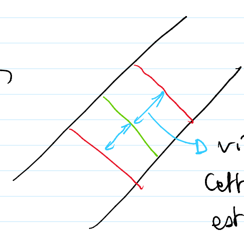
Terminologie : $k_z = \beta_g$ nombre d'onde guidé

$\beta_g = \frac{2\pi}{\lambda_g}$ longueur d'onde guidée

↳ période de l'onde le long de l'axe z

u_p : vitesse de phase

$$u_p^{m,n} = \lambda_g f = \frac{\omega}{k_z} = \frac{\omega}{\frac{1}{u_{TEM}}\sqrt{\omega^2 - \omega_c^2}}$$



→ vitesse de cette zone est u_p

$$u_p^{m,n} = \frac{u_{TEM}}{\sqrt{1 - \left(f_c^{m,n}/f\right)^2}}$$

$$\beta_g^{m,n} = \beta_{libre}\sqrt{1 - \left(f_c^{m,n}/f\right)^2}$$

↳ avec $\beta_{libre} = \frac{\omega}{u_{TEM}}$

$$\lambda_g^{m,n} = \frac{\lambda_{libre}}{\sqrt{1 - \left(f_c^{m,n}/f\right)^2}}$$

↳ avec $\lambda_{libre} = \frac{u_{TEM}}{f} = \frac{1}{\sqrt{\mu\varepsilon}\, f}$

$$k_z = \beta\sqrt{1 - \frac{f_c^2}{f^2}}$$
$$= \sqrt{\beta^2 - \frac{\beta^2 f_c^2}{f^2}}$$
$$= \sqrt{\frac{\beta^2 f^2 - \beta^2 f_c^2}{f^2}}$$
$$= \sqrt{\frac{\omega^2\mu\varepsilon f^2 - \omega^2\mu\varepsilon f_c^2}{f^2}}$$
$$= \sqrt{\frac{\omega^2}{f^2 u_{TEM}^2}\left(f^2 - f_c^2\right)}$$
$$\left(\frac{\omega}{f}\right)^2 = (2\pi)^2$$
$$= \sqrt{\frac{(2\pi f)^2 - (2\pi f_c)^2}{\ldots}}$$

$\rightarrow$ avec $\lambda_{libre} = \frac{\;}{f} = \frac{\;}{\sqrt{\mu\varepsilon}\, f}$

$$\left(\frac{\omega}{f}\right)^2 = (2\pi)^2$$

$$u_g = \frac{d\omega}{dk_z} = u_{TEM}\sqrt{1-\left(f_c/f\right)^2}$$

$$= \sqrt{\frac{(2\pi f)^2 - (2\pi f_c)^2}{u_{TEM}^2}}$$

$$= \sqrt{\frac{\omega^2 - \omega_c^2}{u_{TEM}^2}}$$

$$u_g \cdot u_p = u_{TEM}^2$$

$$\rightarrow P_{tot}^{TE} = \left(\frac{1}{4} \text{ ou } \frac{1}{8}\right)|H_0|^2 \eta\, ab \left(\frac{f}{f_c}\right)^2 \sqrt{1-\left(f_c/f\right)^2}$$

avec $\begin{cases} 1/4 \text{ si } m=0 \text{ ou } n=0 \quad (\text{pas les deux}) \\ 1/8 \text{ si } m\neq 0 \text{ } \underline{\text{et}} \text{ } n\neq 0 \end{cases}$

$$\rightarrow P_{tot}^{TM} = \frac{1}{8}\,\frac{1}{\eta}\left(\frac{f}{f_c}\right)^2\sqrt{1-\left(f_c/f\right)^2}\,|E_0|^2 ab$$

$$\left(\begin{array}{l} \eta_g^{TE} = \left.\dfrac{\sqrt{|E_x|^2+|E_y|^2}}{\sqrt{|H_x|^2+|H_y|^2}}\right|_{\substack{x=a/2 \\ y=b/2}} = \dfrac{\eta_{libre}}{\sqrt{1-\left(f_c/f\right)^2}} \\ \eta_g^{TM} = \eta_{libre}\sqrt{1-\left(f_c/f\right)^2} \end{array}\right)$$

$$\bar{J}_s = \hat{n} \times \bar{H}$$

$\boxed{\text{Guides circulaires}}$

TE

$$H_z = J_m(k_r\rho)\left[C\cos m\phi + D\sin m\phi\right]e^{-jk_z z} \quad \underline{\text{et}} \quad J_m'(k_r a) = 0$$

$$E_\rho = -\frac{j\omega\mu}{k_r^2}\,\frac{J_m(k_r\rho)}{\rho}\left[-C\sin m\phi + D\cos m\phi\right]e^{-jk_z z}$$

$$E_\phi = \frac{j\omega\mu}{k_r^2}\,\frac{\partial H_z}{\partial\rho} = \frac{j\omega\mu}{k_r^2}\,J_m'(k_r\rho)\left[C\cos m\phi + D\sin m\phi\right]$$

$$E_\rho = -\frac{j\omega\mu}{k_t^2}\frac{1}{\rho}\frac{\partial H_z}{\partial\varphi}$$

$$= \frac{-j\omega\mu m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$C\cos(m(\phi-\phi_0))$

$$E_\varphi = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial\rho} = \frac{j\omega\mu}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$H_\rho = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial\rho} = \frac{-jk_z}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$H_\varphi = \frac{-jk_z}{k_t^2}\frac{1}{\rho}\frac{\partial Hz}{\partial\varphi} = \frac{-jk_z m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

TM

$$E_z = J_m(k_t\rho)\left[C\cos m\phi + D\sin m\phi\right]e^{-jk_z z} \quad \text{et } J_m(k_t a) = 0$$

ii) Modes TM ($E_z \neq 0$, $H_z = 0$)

$$\nabla^2 E_z + \beta^2 E_z = 0$$

$$\boxed{E_z = J_m(k_t\,\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}}$$

$$E_\rho = \frac{-jk_z}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$E_\varphi = \frac{-jk_z m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$$H_\rho = \frac{j\omega\varepsilon m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$$H_\varphi = \frac{-j\omega\varepsilon}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

conditions aux frontières : $\left.\begin{array}{l} E_z(\rho=a)=0 \\ E_\varphi(\rho=a)=0 \end{array}\right\} \rightarrow \boxed{J_m(k_t a)=0}$

On a donc que $k_t a$ est un zéro de $J_m(X)$ i.e. $k_t^{mn} a = X_{mn}$.

Dérivées de Bessel

Zéros X_{mn}' de la dérivée $J_{mn}'(X_{mn}')$ ($n = 1, 2, 3, \ldots$) de la fonction de Bessel $J_m(X)$
(Tiré de C. Balanis, Advanced Engineering Electromagnetics, John Wiley)

	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$	$m=9$	$m=10$	$m=11$
$n=1$	3.8318	1.8412	3.0542	4.2012	5.3175	6.4155	7.5013	8.5777	9.6474	10.7114	11.7708	12.8264
$n=2$	7.0156	5.3315	6.7062	8.0153	9.2824	10.5199	11.7349	12.9324	14.1155	15.2867	16.4479	17.6003
$n=3$	10.1735	8.5363	9.9695	11.3459	12.6819	13.9872	15.2682	16.5294	17.7740	19.0046	20.2230	21.4309
$n=4$	13.3237	11.7060	13.1704	14.5859	15.9641	17.3129	18.6375	19.9419	21.2291	22.5014	23.7607	25.0085
$n=5$	16.4706	14.8636	16.3475	17.7888	19.1960	20.5755	21.9317	23.2681	24.5872	25.8913	27.1820	28.4609

Fonction de Bessel

Zeros X_{mn} de la function de Bessel $J_m(X_{mn})$ (n = 1, 2, 3, ….). Tiré de C. Balanis, Advanced Engineering Electromagnetics, John Wiley.

	m=0	m=1	m=2	m=3	m=4	m=5	m=6	m=7	m=8	m=9	m=10	m=11
n=1	2.4049	3.8318	5.1357	6.3802	7.5884	8.7715	9.9361	11.0864	12.2251	13.3543	14.4755	12.8264
n=2	5.5201	7.0156	8.4173	9.7610	11.0647	12.3386	13.5893	14.8213	16.0378	17.2412	18.4335	19.6160
n=3	8.6537	10.1735	11.6199	13.0152	14.3726	15.7002	17.0038	18.2876	19.5545	20.8071	22.0470	23.2759
n=4	11.7915	13.3237	14.7960	16.2235	17.6160	18.9801	20.3208	21.6415	22.9452	24.2339	25.5095	26.7733
n=5	14.9309	16.4706	17.9598	19.4094	20.8269	22.2178	23.5861	24.9349	26.2668	27.5838	28.8874	80.1791

Exemple d'application numérique : Soit un guide rempli d'air ayant un rayon de 5 cm. Quelle est la fréquence de coupure du mode TM_{44}?

$m = 4$, $n = 4$ $\qquad X_{mn} = 17.616 = k_t a$

$k_t = 17.616/5\text{cm} = 352.32\ \text{m}^{-1}$

$\omega_c = ck_t = 3\times10^8 \times 352.32$

On trouve, $f_c^{44} = \omega_c/(2\pi) = 16.82$ GHz

Dans guide circulaire, mode TE_{11} dominant !

Guide coaxial

$$\bar{E} = \frac{V_2 - V_1}{\ln b/a}\,\frac{1}{\rho}\,e^{-j\beta z}\,\hat{\rho}$$

mode TEM dominant

Impédance caractéristique

$$Z_0 = \frac{\eta}{2\pi}\ln(b/a)$$

en posant $Z_0 = \sqrt{\frac{L}{C}}$ et $u_{TEM} = \frac{1}{\sqrt{LC}}$

on peut extraire L et C

- Extraction de α, atténuation dûe au métal

$\llcorner$

$$\alpha_{TEM} = \frac{R_s}{2\eta}\,\rho \qquad R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$$

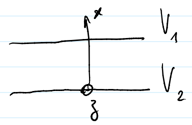
$$\alpha_{TE_{10}} = \frac{R_s\left[1 + \frac{2b}{a}\left(f_c/f\right)^2\right]}{b\,\eta\sqrt{1-(f_c/f)^2}}$$

$$P_z(z) = P_z(0)\,e^{-2\alpha z}$$

Conditions TEM:

1. 2 conducteurs
2. milieu homogène
3. $\nabla\cdot\bar{E} = 0 \Rightarrow \nabla_t^2 V = 0$

Plaques parallèles :



Conditions frontières : $V(x=0) = V_2$
$V(x=d) = V_1$

on a $\nabla_t^2 V = 0$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

puisque $\frac{\partial(\cdot)}{\partial y} = 0$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = 0$$

$$\Rightarrow V = Ax + B$$

$$\Rightarrow \begin{cases} x = 0 \Rightarrow B = V_2 \\ x = d \Rightarrow A = \frac{V_1 - V_2}{d} \end{cases}$$

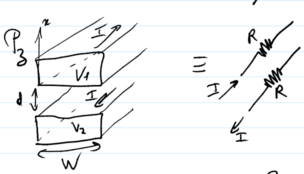
$$\Rightarrow V = \frac{V_1 - V_2}{d} x + V_2$$

$$\Rightarrow \bar{E} = -\nabla_t V e^{-j\beta z} = -\frac{\partial V}{\partial x} \hat{x} e^{-j\beta z}$$

$$\Rightarrow \bar{E} = \frac{V_2 - V_1}{d} e^{-j\beta z} \hat{x}$$

$$\vec{H} = \frac{\nabla \times \bar{E}}{-j\omega\mu} = \begin{bmatrix} 0 \\ H_y \\ 0 \end{bmatrix}$$

avec $H_y = \frac{1}{\eta} \frac{V_2 - V_1}{d} e^{-j\beta z} = \frac{E_x}{\eta}$



$$E_x = \frac{V_2 - V_1}{d} e^{-j\beta z}$$

$$H_y = \frac{E_x}{\eta}$$

$$P_z = \frac{1}{2} \mathrm{Re}\{E_x H_y^*\} \times d \times W$$

$$= \frac{|E_x|^2}{2\eta} W \times d$$

ΔP_z = $-$ puissance dissipée

$= -\frac{1}{2} R|I|^2 \times 2$

avec :

$R = \frac{R_s \times \Delta z}{W}$ et $|I| = \|\bar{J}_s\| \cdot w$

$= \|\hat{n} \times \bar{H}\| \cdot W$

$= |H_y| \cdot w$

$= E_x / \eta$

$$\Rightarrow \Delta P_z = -\sqrt{\frac{\omega\mu}{2\sigma}} \frac{\Delta z}{W} \frac{W^2 |E_x|^2}{\eta^2}$$

$$\Rightarrow \frac{\Delta P_z}{\Delta z} = -\sqrt{\frac{\omega\mu}{2\sigma}} \frac{W}{\eta} \underbrace{\frac{|E_x|^2}{\eta}}_{= \frac{2 P_z}{W \cdot d}}$$

$$\Rightarrow \frac{dP_z}{dz} = -\sqrt{\frac{\omega\mu}{2\sigma}} \frac{W}{\eta} \frac{2 P_z}{W d} = -\frac{2 R_s}{\eta d} P_z$$

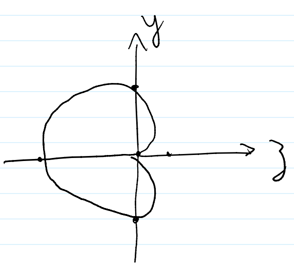
or $\frac{dP_z}{dz} = -2\alpha P_z$

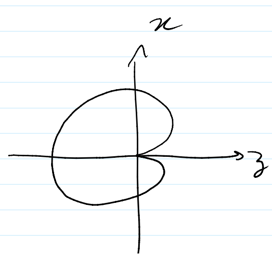
$$\Rightarrow \alpha = \frac{R_s}{d\eta}$$

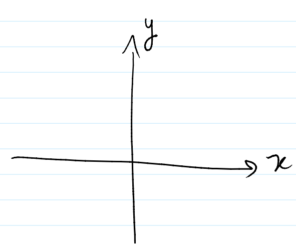
Pour guide coaxial :

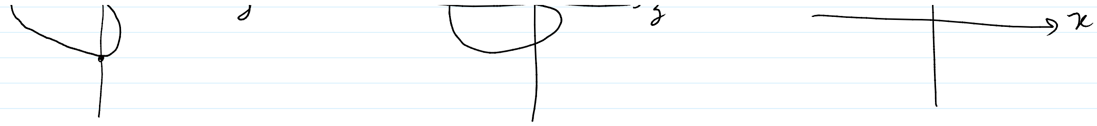
$$\alpha = \frac{R_s\left(\frac{1}{a} + \frac{1}{b}\right)}{4\pi Z_0}$$

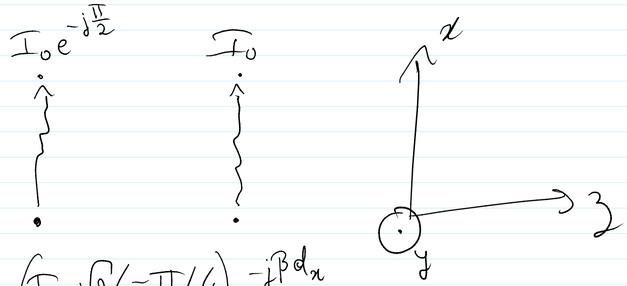
avec $Z_0 = \frac{\eta}{2\pi} \ln(b/a)$



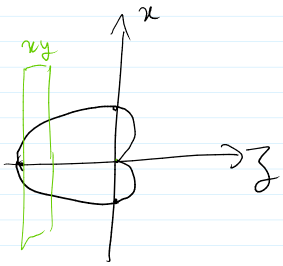


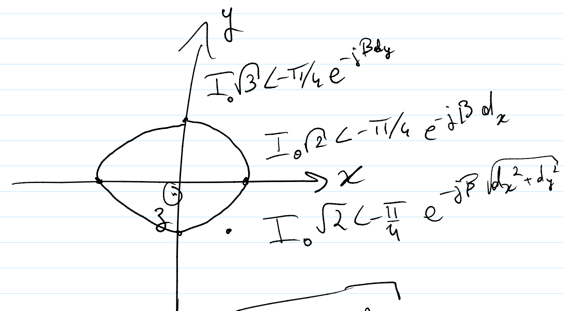






$$AF = \Sigma = \left(I_0 \sqrt{2} \angle -\pi/4\right) e^{-j\beta d_x}$$



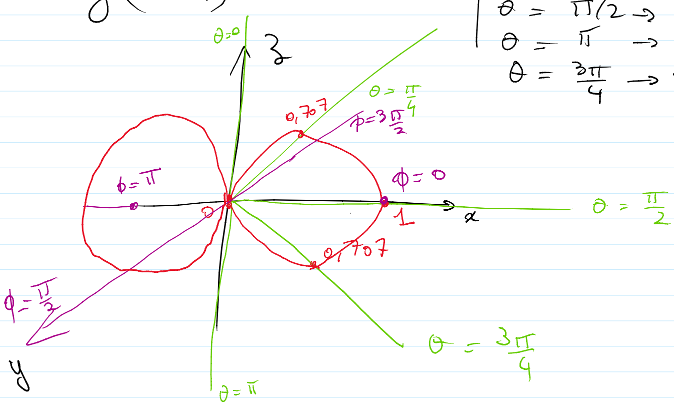


$$d_{xy} = \sqrt{d_x^2 + d_y^2}$$

$$g(\theta,\phi) = \sin\theta$$

$\theta = 0 \rightarrow \sin\theta = 0$
$\theta = \pi/4 \rightarrow \sin\theta = 0{,}707$
$\theta = \pi/2 \rightarrow \sin\theta = 1$
$\theta = \pi \rightarrow \sin\theta = 0$
$\theta = \frac{3\pi}{4} \rightarrow \sin\theta = 0{,}707$

$0 < \theta < \pi$



$$0 < \phi < 2\pi$$

En coordonnées sphériques :

$$\hat{R} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \qquad \hat{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

5.2 Antenne de type « dipôle élémentaire »

- Champs électriques et magnétiques du dipôle <u>valides pour toute distance r entre le dipôle et l'observateur:</u>

$$d\bar{H} = \hat{\varphi} \; -\frac{I\,dz}{4\pi}\beta^2\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2}\right]\sin\theta\, e^{-j\beta r}$$

$$d\bar{E} = -\eta\frac{I\,dz}{4\pi}\beta^2 \times \left\{\hat{r}\cos\theta\left[\frac{2}{(j\beta r)^2} + \frac{2}{(j\beta r)^3}\right] + \hat{\theta}\sin\theta\left[\frac{1}{j\beta r} + \frac{1}{(j\beta r)^2} + \frac{1}{(j\beta r)^3}\right]\right\} e^{-j\beta r}$$

- Si le dipôle n'est pas situé à l'origine on remplace r par R ($= \|\mathbf{r} - \mathbf{r}'\|$)

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5.2.1 Champs lointains

- Si on est loin de l'antenne r est très grand, alors $1/r \gg 1/r^2 \gg 1/r^3$, donc:

$$d\mathbf{E} \approx \hat{\boldsymbol{\theta}}\, j\eta\beta I dz \sin\theta \frac{e^{-j\beta r}}{4\pi r}$$

$$d\mathbf{H} \approx \hat{\boldsymbol{\varphi}}\, j\beta I dz \sin\theta \frac{e^{-j\beta r}}{4\pi r}$$

- On appelle les termes en $1/r$ « le champ lointain »
- Concept applicable à toutes les antennes

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$\boxed{\text{Final H2020}}$

Question 1 :

$E_\perp = 1\ V/m$

$E_{//} = 1\ V/m$

pol. linéaire

a)

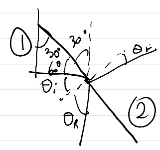
couche $\Gamma^r = \Gamma^i$

$|\Gamma| = 0$

$P^i_\perp = \frac{1}{2}\,\frac{1}{377} \times |E^i_\perp|^2$

$$P^r_\perp = P^i_\perp = \frac{1}{2} \operatorname{Re}\left\{\frac{1}{\eta_2^*}\right\} |E^r_\perp|^2$$

$$N_2 \sin\theta_r = N_1 \sin\theta_i$$



$$\theta_i = 30° = \theta_B$$

$$\tan\theta_B = \frac{N_2}{N_1} \qquad N_2 = \sqrt{1} = 1$$

$$\Rightarrow N_1 = \sqrt{\varepsilon_{R_1}} = \frac{N_2}{\tan(30°)} = \frac{1}{\tan 30°} = 1{,}73$$

$$\Rightarrow \varepsilon_{R_1} = \left(\frac{1}{\tan 30°}\right)^2 = 3$$

b)

$$P^r = P^i = \frac{1}{2} \frac{1}{\eta_{air}} \times \left(|E^i_\perp|^2 + |E^i_{//}|^2\right)$$

$$= \frac{1}{2} \frac{1}{\eta_{prisme}} \times \left(|E^r_\perp|^2 + |E^r_{//}|^2\right)$$

$$\frac{\eta_{prisme}}{\eta_{air}} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{1}{3}}}{\sqrt{\frac{\mu_0}{\varepsilon_0}}} = \frac{1}{\sqrt{3}}$$

$$\frac{\eta_{air}}{\eta_{prisme}} \left(2|E^r_\perp|^2\right) = 2|E^i_\perp|^2$$

η_{prisme}

$$\Rightarrow |E_\perp^t| = \sqrt{\frac{\eta_{prisme}}{\eta_{air}} |E_\perp^i|^2} = 0,76 \ V/m$$

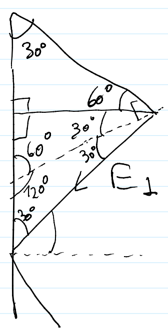
$$\Rightarrow |E_\parallel^r| = 0,76 \ V/m$$

c) d → // et ⊥

e → ⊥

Brewster

$\Gamma_\parallel = 0$



f → $E_\perp$

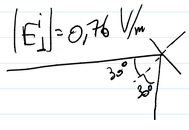
$$\sin\theta_r = \frac{N_1}{N_2}\sin\theta_i = 1$$

$$\frac{1}{\sqrt{3}}\sin 30° = 0,28 < 1$$

Donc $\theta_i \neq \theta_c$

d) $E_\parallel = 0$

$E_\perp = ?$



$$\Gamma_\perp = \eta_2 \cos 30° - \eta$$

$$\theta_r = \sin^{-1}\left(\frac{N_1}{N_2} \sin 30°\right) \qquad N_1 = \sqrt{3} \quad N_2 = 1$$

$$= 60°$$

$$\Gamma_\perp = \frac{377 \cos 30° - \frac{377}{\sqrt{3}} \cos 60°}{377 \cos 30° + \frac{377}{\sqrt{3}} \cos 60°}$$

$$= 0,5$$

$$\Rightarrow |E_\perp^R| = 0,5 \times 0,76$$
$$= 0,38 \quad V/m$$

e) $E_{//} = 0 \ V/m$

$E_\perp = ?$

$$T_\perp = ? = \frac{2\eta_{air} \cos 30°}{\eta_{air} \cos 30° + \eta_{prisme} \cos 60°} = 1,5$$

$$\theta_r = \sin^{-1}\left(\frac{N_1}{N_2} \sin 60°\right) > 1$$
$$N_1 = \sqrt{3} \quad N_2 = 1$$

$$\cos\theta_t = -j\sqrt{\frac{N_1^2}{N_2^2} \sin^2\theta_i - 1}$$
$$N_1 = \sqrt{3} \quad N_2 = 1$$

$$= -j\,1,12$$

$$\Rightarrow T_\perp = \frac{2 \times 377 \cos 60°}{377 \cos 60° + \frac{377}{\sqrt{3}}(-j\,1,12)}$$

$$= 1,22 \angle 52,2°$$

$$\Rightarrow |E_\perp^t| = 1,22 \times 0,38 = 0,4636$$

$$e^{-\beta_2 \sqrt{\frac{N_1^2}{N_2^2} \sin\theta_i - 1}\, z} \approx 8{,}9 . 10^{-4}$$

$$z = 1$$

$$\left| E_\perp^r \right|_{z=1} = \left| E_\perp^r \right| \times 8{,}9 . 10^{-4}$$
$$= 4{,}13 . 10^{-4} \ V/m$$

Question 2 : H20

$$f = 100 \text{ MHz}$$

$$Z = 100 \ \Omega$$

$$b = 1 . 10^{-2} \ m$$

$$\varepsilon_R = 2{,}25$$

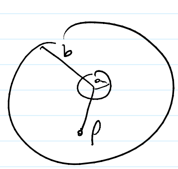
$$|E|_{max} = 10^6 \ V/m$$

a)
$$Z_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \left(e^{\frac{2\pi Z_0}{\eta}}\right)^{-1} \times b = a = 8{,}21 . 10^{-4} \ m$$

b)
$$P_{max} = \frac{|V^+_{max}|^2}{2 Z_0} = \frac{|I^+_{max}|^2 Z_0}{2}$$

$$P_{tot} = \int_a^b \int_0^{2\pi} \frac{\rho |I^+|^2 Z_0^2}{2 \eta \rho^2 (\ln b/a)^2} \, d\phi \, d\rho = \frac{|I^+|^2 Z_0}{2}$$



$$E_\rho = \frac{(V_2 - V_1)_{max}}{\ln b/a} \times \frac{1}{\rho}\Big|_{\rho = a}$$

$$E_{\rho_{max}} = \frac{(V_2 - V_1)_{max}}{\ln b/a} \times \frac{1}{\rho}\Big|_{\rho = a}$$

$$\Rightarrow (V_2 - V_1)_{max} = E_{\rho_{max}}\, a \ln \frac{b}{a}$$

$$E_{\rho_{max}} = 10^6 \text{ V/m}$$

$$\Rightarrow (V_2 - V_1)_{max} = 10^6\, a \ln \frac{b}{a}$$
$$= 2051 \text{ V}$$

$$\Rightarrow P_{max} = \frac{2051^2}{2 Z_0} = 21 \text{ kW}$$

c) $\sigma = 3{,}5 \cdot 10^7$ Sm

$\alpha = \cdot$

$\alpha_{TEM} = \frac{R_S}{d\eta}$ avec $R_S = \sqrt{\frac{\omega \mu}{2\sigma}}$

$d = b - a$

$\eta = \frac{377}{\sqrt{2{,}25}}$

$\omega = 2\pi \times 100 \cdot 10^6$ rad/s

$\mu = 4\pi \cdot 10^{-7}$ F/m

$\Rightarrow \alpha_{TEM} = 1{,}46 \cdot 10^{-3}$ neper/m

$\alpha = 3{,}53 \cdot 10^{-3} = \frac{R_s \left(\frac{1}{a} + \frac{1}{b}\right)}{4\pi Z_0}$

$R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$

d) mode TEM garanti

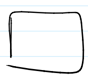
guide et non dispersif

a) mode TEM garanti

guide est non dispersif

TE_{10} $f > f_c = \frac{3.10^8}{2a}$

$\Rightarrow a > \frac{3.10^8}{2f} = 1,5\,m$

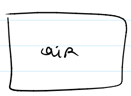


a

alors qu'ici $b = 1$ cm (b)

Question 3 :

$\sigma = 5,8 \cdot 10^7$



b = 3 cm

a = 6 cm

$\frac{u_{TEM}}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{0}{b}\right)^2} < f < 5\,GHz$

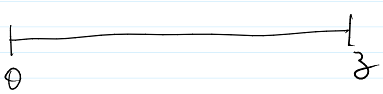
2,5 GHz

TE_{01}

TE_{20}

$\Delta f = 2,5\,GHz$

b) $f = 1,5 f_c = 3,75\,GHz$



$P(z) = P(0)\, e^{-2\alpha z}$

$\frac{P(z)}{P(0)} = e^{-2\alpha z}$

$$10 \log \frac{P(z)}{P(0)} = -0{,}5 \text{ dB}$$

$$\Rightarrow 10 \log e^{-2\alpha z} = -0{,}5$$

$$\Rightarrow z = \ln\left(10^{-0{,}5/10}\right) \times \left(\frac{1}{-2\alpha}\right)$$

$$\alpha_{TE_{10}} = \frac{R_S\left[1 + \frac{2b}{a}\left(\frac{f_c^{10}}{f}\right)^2\right]}{b\,\eta_{air}\sqrt{1-\left(\frac{f_c^{10}}{f}\right)^2}}$$

$$f_c = 2{,}5 \text{ GHz}$$
$$f = 3{,}75 \text{ GHz}$$

$$R_S = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\Rightarrow \alpha_{TE_{10}} = 2{,}73 \cdot 10^{-3} \text{ neper/m}$$

$$z = 21{,}03 \text{ m}$$

c) $f = 2{,}45 \text{ GHz} < f_c$

$$f_c^{m,n} = \frac{u_{TEM}}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = 2{,}45 \text{ GHz}$$

$$u_{TEM} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_R}}$$

il faut $\varepsilon \nearrow$

augmenter ε_R

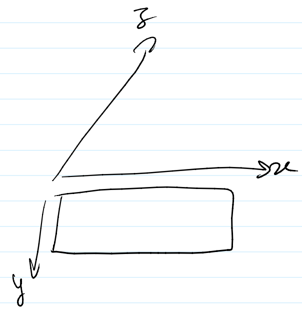
$$\underline{c} = 2{,}45 \cdot 10^9$$

$$\frac{c}{\sqrt{\varepsilon_R}\, 2a} = 2{,}45.10^9$$

$$\Rightarrow \quad \varepsilon_R = \left(\frac{c}{2{,}45.10^9 \times 2a} \right)^2$$

$$\varepsilon_R \geq 1{,}04$$

d)



$$H_z = C\left[A\cos\left(\frac{\pi}{a}x\right) + B\sin\left(\frac{\pi}{a}x\right)\right]$$

$$k_x = \frac{\pi}{a}$$

$$k_y = 0$$

$$H_z = H_0 \cos\left(\frac{\pi}{a}x\right) e^{-jk_z z}$$

$$\cancel{E_x = \frac{-j\omega\mu}{k_r^2} \frac{\partial H_z}{\partial y}}$$

$$\cancel{= 0 \text{ pour tout } x}$$

pour placer une fente, il faut

$$H_z = 0$$

$$\Rightarrow \cos\left(\frac{\pi}{a}x\right) = 0$$

$$\Rightarrow \frac{\cancel{\pi}}{a}x = \frac{\cancel{\pi}}{2} + k\cancel{\pi}, \; k \in \mathbb{Z}$$

$$\Rightarrow \quad x = \frac{a}{2} + ka, \; k \in \mathbb{Z}$$

$$0 < x < 6 \text{ cm}$$

$$x = \frac{a}{2} = 3\text{cm}$$

Chapitre 5 :

$\overline{R}$: point d'observation

$\overline{R'}$: coordonnées d'un point sur l'antenne

$R = \|\overline{R} - \overline{R'}\|$: Distance entre point sur l'antenne et observateur

Dipôle élémentaire :

$$d\overline{H} = \hat{\phi} \, \frac{-I \, dz}{4\pi} \beta^2 \left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} \right] \sin \theta e^{-j\beta R}$$

$$d\overline{E} = -\eta \frac{I \, dz \, \beta^2}{4\pi} \times \left\{ \hat{R} \cos\theta \left[\frac{2}{(j\beta R)^2} + \frac{2}{(j\beta R)^3} \right] + \hat{\theta} \sin\theta \left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] \right\} e^{-j\beta R}$$

→ si dipôle n'est pas situé à l'origine, R devient $\|\overline{R} - \overline{R'}\|$

Directivité : Source isotrope

Densité de puissance moyenne

$$P_R^{iso} = \frac{P_{Ray}}{4\pi R^2}$$

Directivité : Source non-isotrope

Densité de puissance moyenne :

$$P_R^{antenne} = \frac{P_{Ray}}{4\pi R^2} D(\theta, \phi)$$

$$\text{avec } D(\theta, \phi) = \frac{P_R^{antenne}(\theta, \phi)}{P_R^{iso}}$$

D est la directivité

Valeurs typiques de D :

$D_{max} > 1$

↳ D indique capacité à former faisceau étroit

Source isotrope : $D_{max} = 1 = 0$ dBi

Dipôle élémentaire : $D_{max} = 1{,}5 = 1{,}76$ dBi

Réflecteur parabolique : $D_{max} = 25$ à 50 dB.

dBi : decibels par rapport à source isotrope

$$P_{Ray} = \int_0^{2\pi} \int_0^{\pi} P_R^{antenne}(\theta, \phi) \, R^2 \sin\theta \, d\theta d\phi$$

exemple : si $P_R^{antenne}(\theta, \phi) = \frac{\| E(\theta, \phi) \|^2}{2\eta} = \begin{cases} \frac{a}{R^2} \text{ si } \frac{\pi}{2} \leq \theta \leq 0{,}8\pi \\ 0 \text{ sinon} \end{cases}$

$$\Rightarrow P_{Ray} = \int_0^{2\pi} \int_{\pi/2}^{0{,}8\pi} \frac{a}{R^2} R^2 \sin\theta \, d\theta d\phi = 2\pi a (-\cos 0{,}8\pi)$$

$$\Rightarrow P_R^{iso} = \frac{P_{Ray}}{4\pi R^2} = \frac{-2\pi a \cos 0{,}8\pi}{4\pi R^2}$$

$$\Rightarrow D(\theta, \phi) = \frac{P_R^{antenne}}{P_R^{iso}} = \begin{cases} (a/R) / P_R^{iso} & \text{si } \frac{\pi}{2} \leq \theta \leq 0{,}8\pi \\ 0 & \text{sinon} \end{cases}$$

$$D_{max} = \max\left(\frac{\cancel{a}}{\cancel{R^2}} \times \frac{2\cancel{4\pi}\cancel{R^2}}{-\cancel{2\pi}\cancel{a} \cos 0{,}8\pi} \right)$$

$$= \frac{-2}{\cos 0{,}8\pi} = 2{,}47$$

ou $10 \log_{10}(2{,}47) = 3{,}93$ dBi

Résistance de Rayonnement :

$$P_{Ray} = \frac{1}{2} R_{Ray} |I|^2$$

avec $R_{Ray} = 80 \left(\frac{\pi dz}{\lambda} \right)^2$ ohms

Gain antenne isotrope :

isotrope $\Rightarrow \Gamma = 0$

ne dissipe pas de puissance

$\Rightarrow P_{Ray} = P_{Ransmis}$

isotrope $\Rightarrow D = 1$

Densité de puissance :

$$P_R^{isotrope} = P_R^{idéal} = \frac{P_T}{4\pi R^2}$$

Gain dans antenne :

$$P_{antenne} = (1 - |\Gamma|^2) P_{transmise}$$

puissance fourni

$$Gain_{Realisé} = \mathcal{E}_{Ray} (1 - |\Gamma|^2) D(\theta, \phi)$$

noté G

efficacité

EIRP

$$P^{antenne}(R) = \frac{P_{Transmise} G}{4 \pi R^2}$$

Densité de puissance

|

Densité de puissance
à une distance R

Transmission entre 2 antennes

↳ Friis

$$\frac{P_{Reçu_A}}{P_{Transmis_A}} = \frac{P_{Reçu_B}}{P_{Transmis_B}}$$

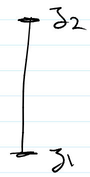
$$\boxed{P_{Recepteur} = \underbrace{P_T G_T}_{EIRP} \cdot G_{Reception} \left(\frac{\lambda}{4\pi R}\right)^2}$$

avec R distance entre les 2 antennes en mètres.

Superposition d'antennes :

$$dE_\theta = j\beta\eta I(z\hat{z}) \sin\theta \frac{e^{-j\beta R}}{4\pi R} dz$$

$$\Rightarrow E_\theta = \int_{z_1}^{z_2} dE_\theta$$



longueur de la source distribuée

$$\Rightarrow E_\theta = \frac{j\beta\eta e^{-j\beta R}}{4\pi R} \sin\theta \int_{z_1}^{z_2} I(z) e^{j\beta z \cos\theta} dz$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\text{avec } \hat{R} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \Rightarrow \hat{z}\cdot\hat{R} = \cos\theta$$

Approximation de N Rayons parallèles

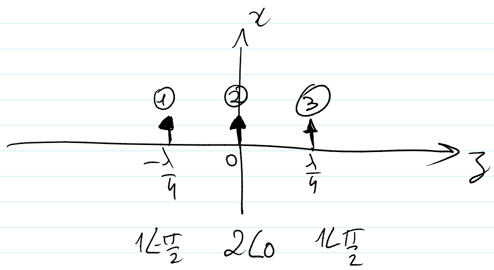
Soit réseau de N éléments identiques situés à $\overline{R_n}$

Soit réseau de N éléments identiques situés à $\overline{R_n}$

$$\Rightarrow E = \frac{Cte}{R} e^{-j\beta R} \underbrace{g(\theta,\phi) \sum_{n=1}^{N} I_n e^{j\beta \overline{R_n}\cdot\hat{R}}}_{}$$

$F(\theta,\phi) = g(\theta,\phi) \times AF(\theta,\phi)$

élement x réseau



$$g(\theta,\phi) = \sin\theta_x$$

$$AF(\theta,\phi) = I_1 e^{j\beta \overline{R_1}\cdot\hat{R}} + I_2 e^{j\beta \overline{R_2}\cdot\hat{R}} + I_3 e^{j\beta \overline{R_3}\cdot\hat{R}}$$

$$\hat{R} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

$$\overline{R_1} = \begin{bmatrix} 0 \\ 0 \\ -\frac{\lambda}{4} \end{bmatrix} \quad \overline{R_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{R_3} = \begin{bmatrix} 0 \\ 0 \\ \frac{\lambda}{4} \end{bmatrix}$$

$$AF(\theta,\phi) = I_1 e^{-j\beta\frac{\lambda}{4}\cos\theta} + I_2 + I_3 e^{j\beta\frac{\lambda}{4}\cos\theta}$$

$$\beta\frac{\lambda}{4} = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\Rightarrow AF(\theta,\phi) = 1 e^{-j\frac{\pi}{2}} e^{-j\frac{\pi}{2}\cos\theta} + 2e^{j0} + 1 e^{j\frac{\pi}{2}} e^{j\frac{\pi}{2}\cos\theta}$$

$$AF(\theta,\phi) = 1 e^{-j\frac{\pi}{2}(1+\cos\theta)} + 2e^{j0} + 1 e^{j\frac{\pi}{2}(1+\cos\theta)}$$

$$= 2\left(1 + \cos\left[\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right]\right)$$

$$AF(\theta,\phi) = 1 e^{j\ldots} + 2 e^{j\ldots} + 1 e^{j2\ldots}$$

$$= 2\left(1 + \cos\left[\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right]\right)$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

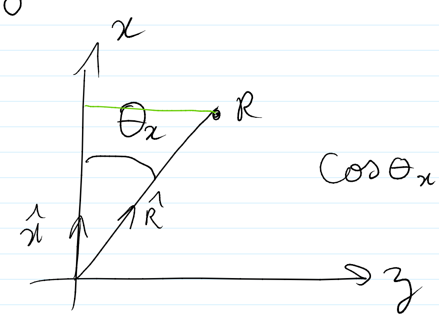
$$\cos\left(\frac{\pi}{2} + \frac{\pi}{2}\cos\theta\right) = \qquad - \sin\left(\frac{\pi}{2}\cos\theta\right)$$

$$\Rightarrow AF(\theta,\phi) = 2\left(1 - \sin\left(\frac{\pi}{2}\cos\theta\right)\right)$$

$$= 2 - 2\sin\left(\frac{\pi}{2}\cos\theta\right)$$

b)

$$g(\theta,\phi) = \sin\theta_x = \sqrt{1 - \cos^2\theta_x}$$



$$\cos\theta_x = \hat{x}\cdot\hat{R} = \begin{bmatrix}1\\0\\0\end{bmatrix} \cdot \begin{bmatrix}\sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{bmatrix}$$

$$\Rightarrow \cos\theta_x = \sin\theta\cos\phi$$

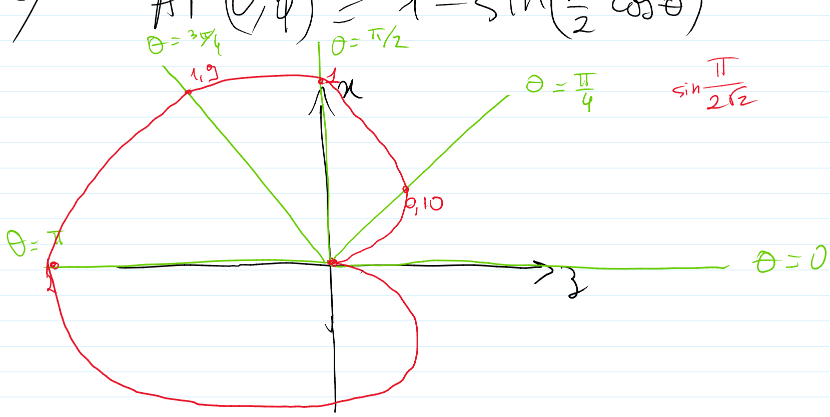
$$\Rightarrow g(\theta,\phi) = \sqrt{1 - \sin^2\theta\cos^2\phi}$$

$$F(\theta,\phi) = g(\theta,\phi)\, AF(\theta,\phi)$$

$$= \sqrt{1 - \sin^2\theta\cos^2\phi}\left(1 - \sin\left(\frac{\pi}{2}\cos\theta\right)\right)$$

c)

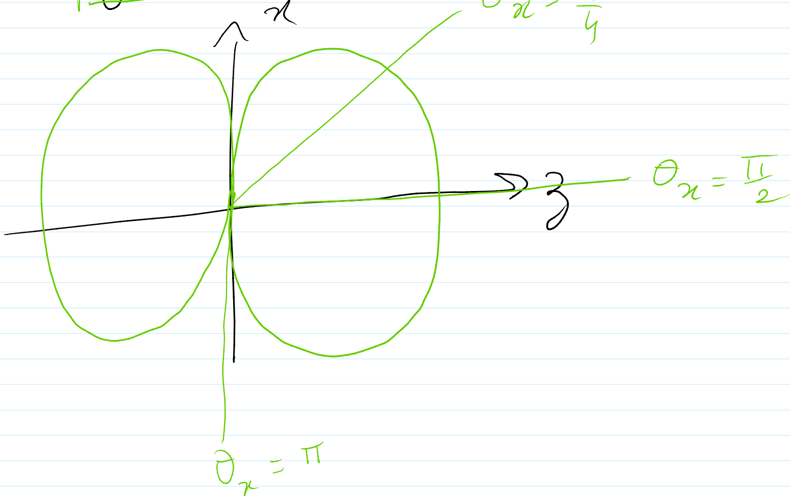
$$AF(\theta,\phi) \equiv 1 - \sin\left(\frac{\pi}{2}\cos\theta\right)$$



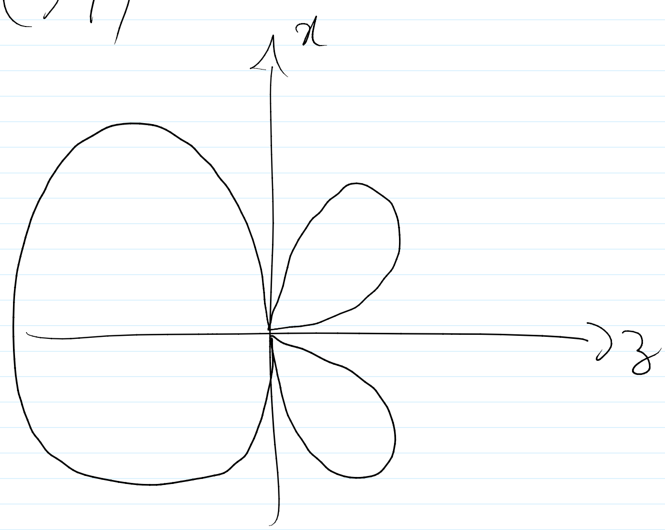
$\sin\frac{\pi}{2\sqrt{2}}$

$\theta_z \neq \theta_x$

$\boxed{g(\theta, \phi) = \sin\theta_x}$



$F(\theta, \phi)$



d)

Gain = 6 dBi

0 O

$$\frac{d}{\lambda} = 500$$

$$10 \log G_T = 6 \text{ dBi}$$

$$\Rightarrow \quad G_T = 10^{6/10}$$

$$\Rightarrow \quad G_R = 10^{6/10}$$

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi R} \right)^2$$

$$R = 500\lambda$$

$$\Rightarrow \quad \frac{P_R}{P_T} = \left(10^{6/10}\right)^2 \left(\frac{1}{4\pi \times 500} \right)^2$$

$$= 4{,}01 \cdot 10^{-7}$$