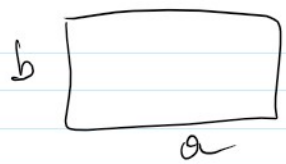
Devoir 2 Q2

26 mars 2022 20:17

a) $f = 1{,}2\, f_c^{10} \Rightarrow f_c^{10} = \frac{f}{1{,}2}$



$b = 1{,}52 \cdot 10^{-3}\ m$

$TE_{10} \rightarrow$ mode fondamental

$$f_c^{mn} = \frac{u_{TEM}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$m = 1 \quad n = 0$

$u_{TEM} = 3 \cdot 10^8\ m/s$

$f = 28 \cdot 10^9\ Hz$

$$f_c^{10} = \frac{u_{TEM}}{2a}$$

$$\Rightarrow \quad a = \frac{u_{TEM}}{2 f_c^{10}} = \frac{1{,}2 u_{TEM}}{2 f}$$

$$\boxed{a = 6{,}4\ mm}$$

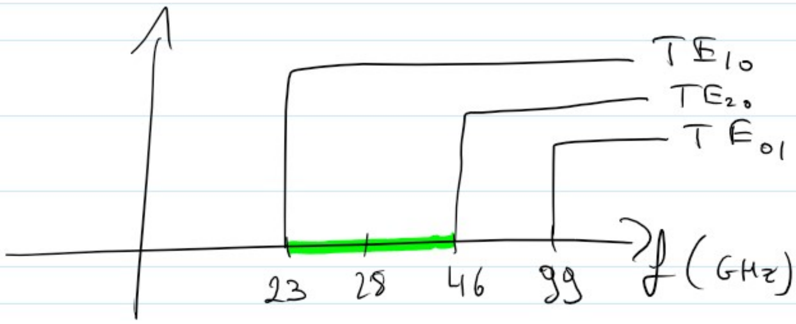
$$f_c^{10} = \frac{f}{1{,}2} = 23 \cdot 10^9\ Hz$$

$$f_c^{01} = 99\ GHz$$

$f_c = 99$ GHz

$f_c^{20} = 46$ GHz

$f_c^{11} > 100$ GHz



$\Delta f = f_c^{20} - f_c^{10} = 46\text{ GHz} - 23\text{ GHz}$

$= 23$ GHz

Entre 23 GHz et 46 GHz

b) $TE_{10} \Rightarrow$

$\tan\delta = 0{,}001 = \dfrac{\sigma_e}{\omega \varepsilon_0 \varepsilon_R}$

$\Rightarrow \sigma_e = \omega \varepsilon_0 \varepsilon_R \tan\delta$

$\sigma_e = 4{,}7 . 10^{-3}$ Siemens/m

$k_z = \sqrt{\omega^2 \mu_0 \underline{\varepsilon} - k_t^2}$

$k_t = \dfrac{\pi}{a}$

$\Rightarrow k_z = \pm\sqrt{\omega^2 \mu_0 \varepsilon_0 \left(\varepsilon_R - j\dfrac{\sigma_e}{\omega\varepsilon_0}\right) - \left(\dfrac{\pi}{a}\right)^2}$

$= \pm(891 - j\,0{,}58)$

Supposons que la propagation se fait en direction $+z$

$$\bar{E}, \bar{H} \propto e^{-jk_z z} = e^{-j(891-j0,58)z}$$

$$= e^{-j891} \, e^{-0,58z}$$

On a une exponentielle décroissante

$\Rightarrow$ Atténuation en se propageant.

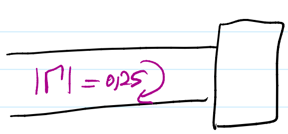
Donc prendre la direction de propagation $+z$ est le bon choix.

$$\Rightarrow k_z = 891 - j0,58$$

$$\text{et } e^{-\alpha_z z} = e^{-0,58z}$$

$$\Rightarrow \alpha_z = 0,58 \text{ neper/m}$$

c)



$$\Delta d_{min} = \frac{\lambda_g}{2}$$

$$\lambda_g^{10} = \frac{\lambda}{1-\left(\frac{f_c}{f}\right)^2}$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{c}{f}$$

$$f_c = f_c^{10} = 23\ GHz$$

$$f = 28\ GHz$$

$$\Rightarrow \lambda_g^{10} = 0{,}0351$$

$$\Rightarrow \Delta d_{min} = 17{,}5\ mm$$

no $|\Gamma|$?

d)

TE_{10} et $P^{inc} = 1W$

$\rightarrow k_t = \frac{\pi}{a}$

$\rightarrow E_z = 0$

$\rightarrow H_z = H_0 \cos\left(\frac{\pi x}{a}\right) e^{-jk_z z}$

$$E_x = -j \frac{\omega\mu}{k_t^2} \frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = 0$$

$$E_y = j \frac{\omega\mu}{k_t^2} \frac{\partial H_z}{\partial x}$$

$$= -j H_0 \frac{\omega \mu_0}{(\pi/a)^2} (\pi/a) \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z}$$

$$\Rightarrow E_y = -j H_0 \frac{\omega \mu_0}{\ldots} \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z}$$

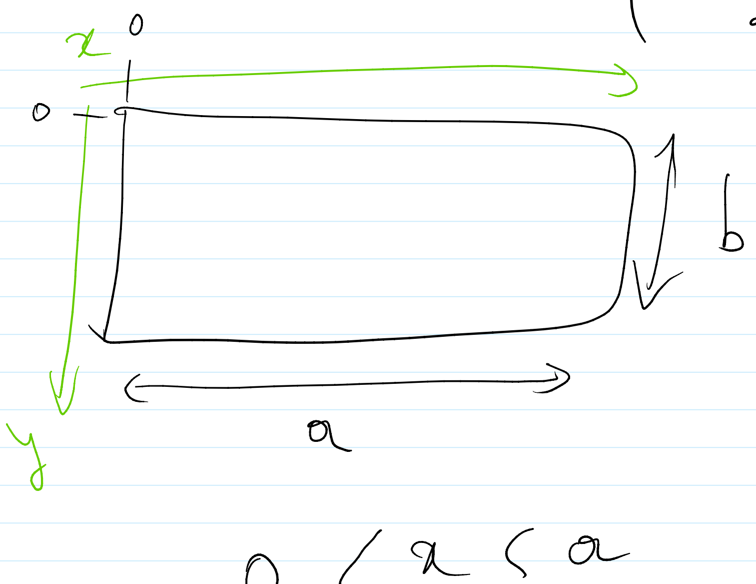
$$\Rightarrow E_y = -j H_0 \frac{\omega \mu_0}{\pi/a} \sin\left(\frac{\pi}{a}x\right) e^{-j\alpha z}$$

$$|E_y| = \frac{H_0 \omega \mu}{\pi/a} \times \sin\left(\frac{\pi}{a}x\right)$$

$$\Rightarrow |E_y|_{max} \Leftrightarrow \sin\left(\frac{\pi}{a}x\right) = \pm 1$$

$$\Rightarrow \frac{\pi}{a}x = \begin{cases} \frac{\pi}{2} + 2k\pi, & k \in \mathbb{Z} \\ -\frac{\pi}{2} + 2k\pi, & k \in \mathbb{Z} \end{cases}$$

$$\Rightarrow x = \begin{cases} \frac{a}{2} + 2ka, & k \in \mathbb{Z} \\ -\frac{a}{2} + 2ka, & k \in \mathbb{Z} \end{cases}$$



$$0 < x < a$$

$$\Rightarrow x = \frac{a}{2} = 3{,}2 \text{ mm}$$

Le champ $\bar{E}$ est maximal en $x = 3{,}2$ mm

$$|\bar{E}| = |E_y| = \frac{H_0 \omega \mu_0}{\ldots}$$

$$|\bar{E}|_{max} = |E_y|_{max} = \frac{H_0 \omega \mu_0}{\pi/a}$$

$$P = \frac{1}{4}|H_0|^2 \eta\, a\, b \left(\frac{f}{f_c}\right)^2 \sqrt{1-\left(\frac{f_c}{f}\right)^2} = 1\,W$$

$$\Rightarrow |H_0| = \sqrt{\frac{4 \times P}{\eta\, ab \left(\frac{f}{f_c}\right)^2 \sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

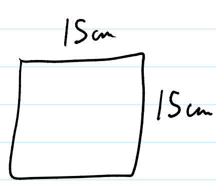
$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_R}} = 217{,}6\ \Omega$$

$\varepsilon_R = 3$

$$\Rightarrow |H_0| = 48{,}62$$

$$\Rightarrow |\bar{E}|_{max} = \frac{H_0 \omega \mu_0}{\pi/a} \simeq 22.10^3\ V/m$$

e)



$$u_g = \frac{u_{TEM}^2}{u_p} = \frac{u_{TEM}^2}{\underline{u_{TEM}}} = u_{TEM}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$u_p \qquad \frac{u_{TEM}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad \overline{f}$$

$$\Rightarrow u_g^{mn} = u_{TEM}\sqrt{1-\left(\frac{f_c^{mn}}{f}\right)^2}$$

$$\left(f_c^{mn}\right)^2 = \left(\frac{u_{TEM}}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\right)^2 = \frac{u_{TEM}^2}{4}\frac{m^2+n^2}{a^2}$$

$$\Rightarrow = u_{TEM}\sqrt{1-\frac{u_{TEM}^2(m^2+n^2)}{4a^2f^2}}$$

$$= u_{TEM}\frac{\sqrt{4a^2f^2-u_{TEM}^2(m^2+n^2)}}{2af}$$

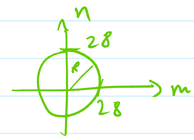
$$4a^2f^2 - u_{TEM}^2(m^2+n^2) \geq 0$$

$$\Rightarrow 4a^2f^2 \geq u_{TEM}^2(m^2+n^2)$$

$$784 = \frac{4a^2f^2}{u_{TEM}^2} \geq m^2+n^2$$

$$m^2+n^2 \leq 784$$

$$m^2+n^2 \leq 28^2$$

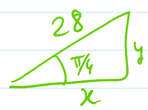


plusieurs valeurs de m et n maximisent la vitesse de phase.

Ainsi : $m = 28$ et $n = 0$

ou $n = 28$ et $m = 0$

ou



$$28\frac{\sqrt{2}}{2} = x = y \approx 19{,}8$$