Feuille de notes
24 février 2022 22:45

$$TOS = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$V(d) = V^+ e^{j\beta d} + V^- e^{-j\beta d}$$

$$= V^+ e^{j\beta d}\left(1 + |\Gamma_L| e^{j\angle\Gamma_L} e^{-j2\beta d}\right.$$

$$Z_L = Z_0\left(\frac{1+\Gamma_L}{1-\Gamma_L}\right) \qquad Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$\Rightarrow z_L = \dfrac{1+\Gamma_L}{1-\Gamma_L}$, si $\Gamma_L \in \mathbb{R}^+$

↳ $TOS = z_L$

$$\Delta d_{max} = \Delta d_{min} = \frac{\lambda}{2}$$

Puissance moyenne : $p = \frac{1}{2} Re\{V I^*\}$

$$\Gamma(d) = \Gamma_L e^{-2\gamma d}$$

$$\Gamma(d) = \frac{Z(d) - Z_0}{Z(d) + Z_0} \qquad Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$$

• Matrice ABCD :

$$\begin{bmatrix} V(d) \\ I(d) \end{bmatrix} = \begin{bmatrix} \cos\beta d & j Z_0 \sin\beta d \\ \frac{j}{Z_0}\sin\beta d & \cos\beta d \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}$$ SANS PERTES

$$Z(d) = \frac{Z_L \cos\beta d + j Z_0 \sin\beta d}{j\frac{Z_L}{Z_0}\sin\beta d + \cos\beta d} = Z_0 \frac{(Z_L + jZ_0 \tan\beta d)}{j Z_L \tan\beta d + Z_0}$$

$$z(d) = \frac{z_L + j\tan\beta d}{j z_L \tan\beta d + 1}$$

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

$$\begin{bmatrix} V(d) \\ I(d) \end{bmatrix} = \begin{bmatrix} \cosh\gamma d & Z_0 \sinh\gamma d \\ \frac{1}{Z_0}\sinh\gamma d & \cosh\gamma d \end{bmatrix} \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}$$ AVEC PERTES

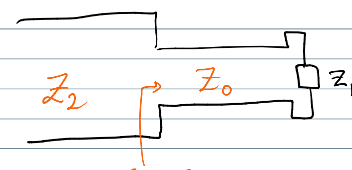
$$Z(d) = \frac{Z_L \cosh\gamma d + Z_0 \sinh\gamma d}{\frac{Z_L}{Z_0}\sinh\gamma d + \cosh\gamma d}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \beta = \omega\sqrt{LC} \qquad \alpha = \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right)$$

$$Z(d) = \frac{Z_0 Z_L + j Z_0 \tan\beta d}{j Z_L \tan\beta d + Z_0}$$

$$\Gamma(d) = \Gamma_L e^{-j2\beta d}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$




But : $Z_{in} = Z_2$

$$Z_0 = \sqrt{Z_1 Z_2}$$

$$\Gamma_V = \frac{R_L - Z_0}{R_L + Z_0}$$

$$\Gamma_I = \frac{G_L - Y_0}{G_L + Y_0} = -\Gamma_V$$

$$1 + \Gamma = T$$

Condition de Heaviside

$$RC = LG$$

$$\alpha = \sqrt{RG}$$

$$c = \frac{1}{\sqrt{LC}} = \lambda f = \frac{\omega}{\beta}$$

$$\beta = \omega\sqrt{LC}$$

$\alpha = \dfrac{\sqrt{LC}}{2}\left(\dfrac{R}{L} + \dfrac{G}{C}\right) = 0$ si pas pertes

$\xrightarrow{+s}$ $\xleftarrow{-s}$

$$V(s) = A e^{-\gamma s} + B e^{\gamma s}$$

$$= A e^{-\alpha s} e^{+j\beta s}$$

# ONDES

$\bar{\mathcal{E}}, \bar{\mathcal{H}}, \bar{\mathcal{J}}$ : vecteurs réels dépendent de $x, y, z$ et $t$ en général

$\bar{E}_c, \bar{H}_c, \bar{J}_c$ : vecteurs complexes

e.g. $\bar{\mathcal{E}} = Re\{\bar{E}_c\}$

$\bar{E}, \bar{H}, \bar{J}$ : vecteurs phaseurs

e.g. $\bar{E}_c = \bar{E}\, e^{j\omega t}$

$\underline{\gamma}, \underline{\varepsilon}$ : constantes complexes, scalaires

formules utiles :

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{\beta} = \lambda f$$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ F/m

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$c = 3 \cdot 10^{8}$ m/s

$$\beta = \|\bar{k}\| = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

$$\bar{H} = \bar{k} \times \bar{E} \qquad P_{moy} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\underline{\varepsilon} = \varepsilon_0\left(\varepsilon_R - j\frac{\sigma_0}{\omega\varepsilon_0}\right)$$

$$\underline{\bar{k}} = -j\underline{\gamma}\,\hat{n}$$

$$P_{moy} = \frac{\|\bar{E}\|^2}{2\eta} = \frac{\eta\|\bar{H}\|^2}{2}$$

$$\bar{P}_{moy} = \hat{n}\,\frac{|A_x|^2 + |A_y|^2 + |A_z|^2}{2\eta}$$

$$\bar{E} = \eta\,\bar{H}$$

$$\bar{E} = \bar{A} e^{-j(-j\gamma\hat{n}\cdot\bar{R})} + \bar{B} e^{+j(-j\gamma\hat{n}\cdot\bar{R})}$$
$$= \bar{A}\, e^{-\alpha\hat{n}\cdot\bar{R}} e^{-j\beta\hat{n}\cdot\bar{R}} + \bar{B}\, e^{+\alpha\hat{n}\cdot\bar{R}} e^{+j\beta\hat{n}\cdot\bar{R}}$$

Bon conducteur : $p = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma_e}{\omega\varepsilon_R\varepsilon_0} > 100$

$$\sin x = \cos\left(x - \frac{\pi}{2}\right) \qquad \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\alpha = \beta = \sqrt{\frac{\sigma\omega\mu}{2}}$$

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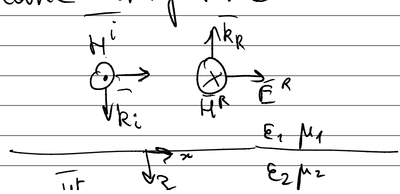
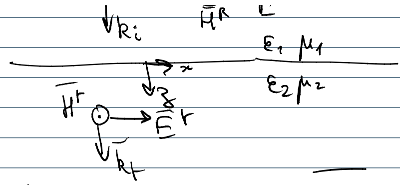
## Plane uniforme incidence normale : pol. linéaire




$$\bar{H} = \frac{\bar{k} \times \bar{E}}{\omega\mu}$$




$$H = \frac{\cdots}{\omega \mu}$$

$$\begin{cases} \overline{P}^i = P^i_z = \overline{P}^i \cdot \hat{z} = \frac{1}{2}\,\mathrm{Re}\left\{ \frac{E_x^i\, E_x^{i*}}{\eta_1^*} \right\} \hat{z}\cdot\hat{z} = \frac{1}{2}\,\mathrm{Re}\left\{\frac{1}{\eta_1^*}\right\} \left|E_x^i\right|^2_{z=0} \\ \overline{P}^R = P^R_z = \overline{P}^R \cdot (-\hat{z}) = \frac{1}{2}\,\mathrm{Re}\left\{\frac{1}{\eta_1^*}\right\} \left|E_x^R\right|^2_{z=0} \\ \overline{P}^t = P^t_z = \overline{P}^t \cdot (\hat{z}) = \frac{1}{2}\,\mathrm{Re}\left\{\frac{1}{\eta_2^*}\right\} \left|E_x^t\right|^2_{z=0} \end{cases}$$

- $P^i = P^t + P^R \Rightarrow P^t = P^i - P^R \Rightarrow |T|^2\,\mathrm{Re}\left\{\frac{1}{\eta_2^*}\right\} = \mathrm{Re}\left\{\frac{1}{\eta_1^*}\right\}\left(1 - |\Gamma|^2\right)$

$$\begin{cases} \overline{E}^i = \hat{x}\, E_x^i \\ \overline{E}^R = \hat{x}\, E_x^R \\ \overline{E}^t = \hat{x}\, E_x^t \end{cases} \qquad \begin{cases} \overline{k}_i = \hat{n}_i \beta_1 = \hat{z}\beta_1 & ;\ \beta_1^2 = \omega^2 \mu_1 \varepsilon_1 \\ \overline{k}_R = \hat{n}_R \cdot \beta_1 & \\ \overline{k}_t = \hat{n}_t \beta_2 = \hat{z}\beta_2 & ;\ \beta_2^2 = \omega^2 \mu_2 \varepsilon_2 \end{cases}$$

$$\Rightarrow \begin{cases} \overline{H}^i = \frac{\overline{k}_i \times \overline{E}^i}{\omega \mu_1 \varepsilon_1} = \frac{\beta_1 E_x^i\ \hat{z}\times\hat{x}}{\omega\mu_1} = \hat{y}\, E_x^i / \eta_1 \\ \overline{H}^R = \frac{\overline{k}_R \times \overline{E}^R}{\omega\mu_1\varepsilon_1} = \frac{\beta_1 (-\hat{z}) \times \hat{x}\, E_x^R}{\omega\mu_1} = -\hat{y}\, E_x^R / \eta_1 \\ \overline{H}^t = \frac{\overline{k}_t \times \overline{E}^t}{\omega\mu_2\varepsilon_2} = \frac{\beta_2 (\hat{z}) \times \hat{x}\, E_x^t}{\omega\mu_1} = \hat{y}\, E_x^t / \eta_2 \end{cases}$$

$$\overline{E}^i_{tan} + \overline{E}^R_{tan} = \overline{E}^t_{tan}$$

$$\Rightarrow E_x^i(z=0)\cdot e^{-j\overline{k}_i\cdot\overline{R}}\,\hat{x} + E_x^R(z=0)\cdot e^{-j\overline{k}_R\cdot\overline{R}}\,\hat{x} = E_x^t(z=0)\cdot e^{-j\overline{k}_t\cdot\overline{R}}\,\hat{x}$$

$\rightarrow$ À l'interface $R = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

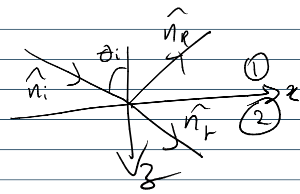
$$\Rightarrow \boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \qquad \boxed{T = 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1}}$$

$$Z \approx \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon_0 \varepsilon_R \left(1 + j \frac{\sigma}{\omega \varepsilon_0 \varepsilon_R}\right)}}$$

$$|Z| = \sqrt{\frac{\omega\mu}{\cdots}}$$

$$|Z| = \sqrt{\frac{\omega\mu}{\sigma}}$$

# Onde plane incidence oblique




$$\hat{n}_i = \begin{bmatrix} \sin\theta_i \\ 0 \\ \cos\theta_i \end{bmatrix} \rightarrow \overline{n_{i,\tan}} = \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{E^i} = \bar{A}\, e^{-j\beta_1 \hat{n}_i \cdot \bar{R}}$$

$$\overline{E^R} = \bar{B}\, e^{-j\beta_1 \hat{n}_R \cdot \bar{R}}$$

$$\overline{E^r} = \bar{C}\, e^{-j\beta_2 \hat{n}_r \cdot \bar{R}} \quad \text{ou } \bar{C}\, e^{-\gamma_2 \cdot \bar{R}}$$

$$\boxed{\begin{gathered}\bar{\gamma} = (\alpha + j\beta)\cdot \hat{n} \\ e^{-\bar{\gamma}\cdot\bar{R}} = e^{-\alpha\, \hat{n}\cdot\bar{R}}\, e^{-j\beta\, \hat{n}\cdot\bar{R}}\end{gathered}}$$

Pour $\hat{n}_i$, $\hat{n}_R$, $\hat{n}_r$, on a $\rightarrow$ $\hat{n} = 1\overline{n_{\tan}} + \hat{z}\, n_z$

à l'interface $z = 0$

De même, $\bar{R} = 1\overline{R_{\tan}} + z\hat{z}$

avec $\overline{R_{\tan}} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

À $z=0$, on a $\overline{E^i_{\tan}}(x,y,0) + \overline{E^R_{\tan}}(x,y,0) = \overline{E^r_{\tan}}(x,y,0)$

De même pour $H^{i,R,r}_{\tan}$

Composantes du champs à différents endroits

$$f(x,y,z)\,\hat{x} + g(x,y,z)\,\hat{y} + h(x,y,z)\,\hat{z}$$

$$g_{\tan} = f(x,y,z)\,\hat{x} + g(x,y,z)\,\hat{y} + 0\,\hat{z}$$

$$\tan_{\substack{\text{à frontière} \\ z=0}} = f(x,y,0)\,\hat{x} + g(x,y,0)\,\hat{y} + 0\,\hat{z}$$

i.e. : $\overline{E^i_{\tan}} = \begin{bmatrix} cte_x \\ cte_y \end{bmatrix} e^{-j\beta_1(\overline{R_{\tan}} \cdot \overline{n_{i,\tan}} + z\, n_{i,z})}$

i.e. : $\overline{E}^i_{tan} = \begin{bmatrix} \\ cte_y \\ 0 \end{bmatrix} e$

$= -\overline{\gamma_{tan}} \cdot \overline{R_{tan}}$

↳ De même pour $\overline{E}^r$ et $\overline{E}^R$

$\Rightarrow \overline{E}^i_{tan}\Big|_{z=0} = \begin{bmatrix} cte_x \\ cte_y \\ 0 \end{bmatrix} e^{-j\beta_1 \overline{R_{tan}} \cdot \overline{n_{i,tan}}}$

Condition frontière : $\overline{E}^i_{tan}\Big|_{z=0} + \overline{E}^R_{tan}\Big|_{z=0} = \overline{E}^r_{tan}\Big|_{z=0}$

$\Rightarrow \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_i e^{-j\beta_1 \overline{R_{tan}} \cdot \overline{n_{i,tan}}} + \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_R e^{-j\beta_1 \overline{R_{tan}} \cdot \overline{n_{R,tan}}} = \begin{bmatrix} cte \\ cte \\ 0 \end{bmatrix}_r e^{-j\beta_2 \overline{R_{tan}} \cdot \overline{n_{r,tan}}}$

$\Delta \quad \beta_1 \overline{R_{tan}} \cdot \overline{n_{i,tan}} = \beta_1 \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} \beta_1 \sin\theta_i \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}}$

dépendance du terme $E^i_{tan}|_{z=0}$ par rapport à $x$ et $y$

↳ équation satisfaite partout sur interface

⇒ les termes doivent avoir la même dépendance par rapport à $x$ et $y$ (i.e. la même combinaison de $x$ et $y$)

$\Rightarrow \boxed{\beta_1 \overline{n_{i,tan}} = \beta \overline{n_{R,tan}} = \beta \overline{n_{r,tan}}}$

**Reflexion**

$\Rightarrow \overline{n_{i,tan}} = \overline{n_{R,tan}}$

$\Rightarrow \begin{bmatrix} \sin\theta_i \\ 0 \end{bmatrix} = \overline{n_{R,tan}}$

**Transmission**

$\beta_1 \overline{n_{i,tan}} = \beta_2 \overline{n_{r,tan}}$

$\begin{bmatrix} \frac{\beta_1}{\beta_2} \sin\theta_i \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \sin\theta_i \\ 0 \\ 0 \end{bmatrix} = \overline{n_{R,\tan}}$$

$$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ n_{R,z} \end{bmatrix}$$

or $\|\hat{n}_R\| = 1$

$$\Rightarrow \sin^2\theta_i + n_{R,z}^2 = 1$$

$$\Rightarrow n_{R,z} = \pm\cos\theta_i$$

$$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ \pm\cos\theta_i \end{bmatrix} \text{ et } \overline{R} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\|\overline{E}^R\| \propto e^{-\overline{\gamma_1}\cdot\overline{R}} = e^{-(\alpha_1+j\beta_1)\hat{n}_R\cdot\overline{R}} = e^{-\alpha_1\hat{n}_R\cdot\overline{R}}\, e^{-j\beta_1\hat{n}_R\cdot\overline{R}}$$

$$= e^{-\alpha_1(x\sin\theta_i \pm z\cos\theta_i)}\, e^{-j\beta_1(x\sin\theta_i \pm z\cos\theta_i)}$$

$$= e^{-\alpha_1 x\sin\theta_i}\, e^{\mp\alpha_1 z\cos\theta_i}\, e^{-j\beta_1 x\sin\theta_i}\, e^{\mp j\beta_1 z\cos\theta_i}$$

L'onde réfléchit se propage

en $z < 0 \Rightarrow \alpha_1 z\cos\theta_i < 0$

$\Rightarrow -\alpha_1 z\cos\theta_i > 0$

il faut choisir $\ominus\cos\theta_i$

$$\Rightarrow \hat{n}_R = \begin{bmatrix} \sin\theta_i \\ 0 \\ -\cos\theta_i \end{bmatrix}$$

$$\Rightarrow \boxed{\theta_i = \theta_R}$$

pour tout angle d'incidence entre 0 et 90°

$$\beta_1\, \overline{n_{i,\tan}} = \beta_2\, \overline{n_{r,\tan}}$$

$$\Rightarrow \begin{bmatrix} \beta_1\sin\theta_i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \beta_2 n_{r,x} \\ \beta_2 n_{r,y} \\ 0 \end{bmatrix}$$

$$\Rightarrow \beta_1\sin\theta_i = \beta_2 n_{r,x}$$

$$\Rightarrow n_{r,x} = \frac{\beta_1}{\beta_2}\sin\theta_i$$

et $n_{r,y} = 0$

$$\Rightarrow \hat{n}_r = \begin{bmatrix} \frac{\beta_1}{\beta_2}\sin\theta_i \\ 0 \\ n_{r,z} \end{bmatrix}$$

or $\|\hat{n}_r\| = 1$

$$\Rightarrow n_{r,z}^2 + \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i = 1$$

$$\Rightarrow n_{r,z} = \pm\sqrt{1 - \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}$$

on choisit $\oplus$ car $z > 0$ pour le rayon transmit

$$\Rightarrow \hat{n}_r = \begin{bmatrix} \frac{\beta_1}{\beta_2}\sin\theta_i \\ 0 \\ \sqrt{1-\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)^2} \end{bmatrix}$$

or $\hat{n}_r = \begin{bmatrix} \sin\theta_r \\ 0 \\ \cos\theta_r \end{bmatrix}$

$$\Rightarrow \sin\theta_r = \frac{\beta_1}{\beta_2}\sin\theta_i$$

$$\Rightarrow \boxed{N_2\sin\theta_r = N_1\sin\theta_i}$$

avec $N = \sqrt{\mu_R\varepsilon_R}$

Si on a des pertes dans milieu de transmission :

$$\varepsilon_2 = \varepsilon_0\,\varepsilon_{R_2}\left(1 - j\frac{\sigma}{\omega\,\varepsilon_0\,\varepsilon_{R_2}}\right)$$

$$\overline{\gamma_r} = \begin{bmatrix} \gamma_{x,r} \\ 0 \\ \gamma_{z,r} \end{bmatrix} = \begin{bmatrix} \beta_1\sin\theta_i \\ 0 \\ \gamma_{z,r} \end{bmatrix}$$

$$\left[ \gamma_{z,t} \right] \quad \left[ \gamma_{z,t} \right]$$

On sait que $\overline{\gamma_t} \cdot \overline{\gamma_t} = -\omega^2 \mu_2 \underline{\varepsilon_2}$

$$\Rightarrow \gamma_{x,t}^2 + \gamma_{z,t}^2 = -\omega^2 \mu_2 \underline{\varepsilon_2}$$

$$\Rightarrow \gamma_{z,t} = \pm \sqrt{-\omega^2 \mu_2 \underline{\varepsilon_2} - \gamma_x^2}$$

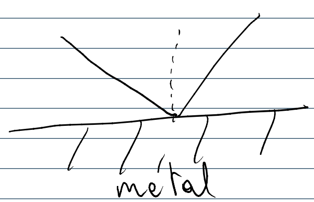
$$= \alpha_{z,t} + j \beta_{z,t}$$

il faut calculer $\gamma_{z,t}$ et choisir la racine $\alpha_{z,t} > 0$ pour atténuation car $\overline{E^t} = \overline{Cte}\, e^{-\gamma_{x,t} x} e^{-\alpha_{z,t} z} e^{-j\beta_{z,t} z}$

→ Si le milieu de transmission est un bon conducteur, alors :

$$\gamma_{z,t} \approx \sqrt{-\omega^2 \mu_2 \left(-j\frac{\sigma}{\omega}\right)} = \sqrt{\frac{\sigma \omega \mu_2}{2}}(1+j)$$

$$\Rightarrow \alpha_z = \beta_z = \sqrt{\frac{\sigma \omega \mu}{2}} \gg \beta_x$$




L'onde propagée selon $\hat{z}$ pratiquement

$$\overline{\gamma_t} = \overline{\alpha_t} + \overline{\beta_t}$$

$$\overline{\alpha_t} = \begin{bmatrix} 0 \\ 0 \\ \alpha_z \end{bmatrix}$$

$$\overline{\beta_t} = \begin{bmatrix} \beta_x \\ 0 \\ \beta_z \end{bmatrix}$$

$\hat{/\!/}$ et $\hat{\perp}$ : Caractéristiques

• $\dfrac{E_\perp^R}{E_\perp^i} = \Gamma_\perp = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$ , $\dfrac{E_\perp^t}{E_\perp^i} = T_\perp = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

et $1 + \Gamma_\perp = T_\perp$ $\qquad \eta_2 \cos\theta_i = \eta_1 \cos\theta_t + 1$

$\perp \Rightarrow$ TE

et $1 + \Gamma_\perp = T_\perp$ $\qquad \eta_2 \cos\ldots$

$\perp \Rightarrow$ TE

- $$\frac{E^R_{/\!/}}{E^i_{/\!/}} = \Gamma_{/\!/} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \ , \quad \frac{E^t_{/\!/}}{E^i_{/\!/}} = T_{/\!/} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\Gamma_{/\!/} = 0 \quad \text{si} \quad \boxed{\tan\theta_i = \frac{N_2}{N_1}}$$

Brewster

Ne veut pas forcément dire $T_{/\!/} = 1$ car impédance interface non nulle

Reflexion totale arrive pour

$$\Rightarrow \quad \sin\theta_t = \frac{N_1}{N_2}\sin\theta_i > 1$$

$$\rightarrow \cos\theta_t = \pm\sqrt{1 - \sin^2\theta_t}$$

$$= \pm j \underbrace{\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}}_{\in \mathbb{R}}$$

on sait que $\overline{E}^t = \overline{Cte}\, e^{-j\beta_2 \sin\theta_t x}\, e^{-j\beta_2 \cos\theta_t z}$

$$\Rightarrow E^t \propto e^{-j\beta_2 \cos\theta_t z}$$

$z > 0$

$$\Rightarrow E^t \propto e^{\pm \beta_2 \left(\sqrt{\frac{N_1^2}{N_2^2}\sin\theta_i - 1}\right) z}$$

il faut donc

$$\cos\theta_t < 0$$

$$\Rightarrow \cos\theta_t = -j\sqrt{\frac{N_1^2}{N_2^2}\sin^2\theta_i - 1}$$

# Chapitre 4 : Guides d'onde

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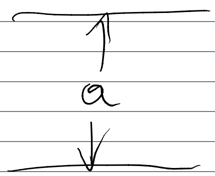



$a < \frac{\lambda}{2} \Rightarrow$ déphasage d'une même onde entre 2 point de son trajet dans le guide d'onde.

$a \geq \frac{\lambda}{2}$ condition nécessaire à propagation de 1 onde sans déphasage (vérifié pour mode TE)

mode TE $\longrightarrow$ $E_\perp$ et $H_{//}$

mode TM $\longrightarrow$ $E_{//}$ et $H_\perp$

$$\bar{E} = \bar{C}\, e^{-\bar{\gamma} \cdot \bar{R}}$$

$$k_t^2 + k_z^2 = \beta^2$$

$$\text{Rot}\,\bar{H} = \begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \begin{bmatrix} j\omega\varepsilon E_x \\ j\omega\varepsilon E_y \\ j\omega\varepsilon E_z \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{\partial H_z}{\partial y} + jk_z H_y \\ -jk_z H_x - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \begin{bmatrix} j\omega\varepsilon E_x \\ j\omega\varepsilon E_y \\ j\omega\varepsilon E_z \end{bmatrix}$$

$$\begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = j\omega\varepsilon E_x & (1) \\ -jk_z E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y & (2) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = \frac{\cancel{j}\,\omega\varepsilon}{-\cancel{j}k_z}\left(-j\omega\mu H_y + \frac{\partial E_z}{\partial x}\right) \\ E_x = \dfrac{\left(-j\omega\mu H_y + \frac{\partial E_z}{\partial x}\right)}{-jk_z} \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\partial H_z}{\partial y} + jk_z H_y = \frac{\omega\varepsilon}{k_z} j\omega\mu H_y - \frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x} \end{cases}$$

$$\Rightarrow H_y\left(jk_z - j\frac{\beta^2}{k_z}\right) = -\frac{\partial H_z}{\partial y} - \frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow -jH_y\left(k_t^2\right) = -k_z\frac{\partial H_z}{\partial y} - \omega\varepsilon\frac{\partial E_z}{\partial x}$$

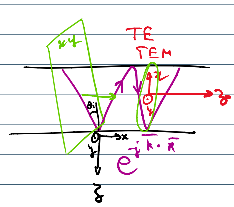
$$\Rightarrow H_y = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial y} - j\frac{\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow \Bigg\{ \quad H_y = \frac{-\frac{\omega\varepsilon}{k_z}\frac{\partial E_z}{\partial x} - \frac{\partial H_z}{\partial y}}{jk_z - \frac{j\omega^2\varepsilon\mu}{k_z}} = \frac{-\omega\varepsilon\frac{\partial E_z}{\partial x} - k_z\frac{\partial H_z}{\partial y}}{-jk_t^2} = \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x} - j\frac{k_z}{k_t^2}\frac{\partial H_z}{\partial y}$$

$$k_t^2 = \omega^2\mu\varepsilon - k_z^2$$

$E_z = 0$

$$\bar{E} = \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}$$



On obtient 6 équations impliquant les composantes des champs E et H.

| | |
|---|---|
| $\frac{\partial H_z}{\partial y} + jk_z H_y = j\omega\varepsilon E_x$ A | $\frac{\partial E_z}{\partial y} + jk_z E_y = -j\omega\mu H_x$ B |
| $-jk_z H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$ B | $-jk_z E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$ A |
| $\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$ | $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$ |

Avec paire A : $H_z = 0$ et $E_z = 0$ avec paire B :

$$E_x = \frac{-j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial y} - \frac{jk_z}{k_t^2}\frac{\partial E_z}{\partial x} \quad (1)$$

$$H_y = \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x} - \frac{jk_z}{k_t^2}\frac{\partial H_z}{\partial y} \quad (2)$$

$$E_y = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial x} - \frac{jk_z}{k_t^2}\frac{\partial E_z}{\partial y} \quad (3)$$

$$H_x = \frac{j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial y} - \frac{jkz}{k_t^2}\frac{\partial H_z}{\partial x} \quad (4)$$

| Modes Transverses Électriques ou TE $(E_z = 0,\ H_z \neq 0)$ | Modes Transverses Magnétiques ou TM $(H_z = 0,\ E_z \neq 0)$ |
|---|---|
| $E_x = \frac{-j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial y}$ | $E_x = \frac{-jk_z}{k_t^2}\frac{\partial E_z}{\partial x}$ |
| $H_y = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial y}$ | $H_y \ \frac{-j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial x}$ |
| $E_y = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial x}$ | $E_y = \frac{-jk_z}{k_t^2}\frac{\partial E_z}{\partial y}$ |
| $H_x = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial x}$ | $H_x = \frac{j\omega\varepsilon}{k_t^2}\frac{\partial E_z}{\partial y}$ |

$$\underbrace{k_x^2 + k_y^2}_{k_t^2} + k_z^2 = \beta^2 = \omega^2\mu\varepsilon$$

## Guide Rectangulaire :

$\boxed{TE} \rightarrow E_z = 0$

$$H_z = X(x)\,Y(y)\,e^{-jk_z z} = [A\cos(k_x x) + B\sin k_x x][C\cos(k_y y) + D\sin(k_y y)]$$

en appliquant conditions frontières $E_{tan} = 0$

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}$$

$$k_x = \frac{m\pi}{a} \quad , \quad k_y = \frac{n\pi}{b}$$

et $H_z = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$ , $m$ et $n \in \mathbb{N}$

$H_0$ constante

$\boxed{TM} \rightarrow H_z = 0$

$$E_z = \underbrace{\left[A\cos(k_x x) + B\sin(k_x x)\right]}_{X(x)} \underbrace{\left[C\cos(k_y y) + D\sin(k_y y)\right]}_{Y(y)} \underbrace{e^{-jk_z z}}_{Z(z)}$$

Pour $\overline{E}_{tan} = 0$, on obtient avec guide rectangulaire :

$$k_x = \frac{m\pi}{a} \quad , \quad k_y = \frac{n\pi}{b}$$

$$E_z = E_0 \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) e^{-jk_z z}$$

avec $m, n \in \mathbb{N}$ et $E_0$ une constante

$\boxed{F_{coupure} :}$

$$k_x^2 + k_y^2 + k_z^2 = \beta^2$$

$$\Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2 = \beta^2$$

$$\Rightarrow k_z^2 = \beta^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$k_z^2 < 0 \Rightarrow k_z$ réel $\Rightarrow$ propagation

$k_z^2 > 0 \Rightarrow k_z$ imaginaire $\Rightarrow$

$$k_r = \frac{\omega_{coupure}}{u_{TEM}}$$

$$f > \frac{u_{TEM}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$u_{TEM} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{k_z} = \frac{c}{N}$$

$$u_{TEM} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{k_z} = \frac{c}{N}$$

$$k_r^2 + k_z^2 = \beta^2 = \omega^2 \mu\varepsilon$$

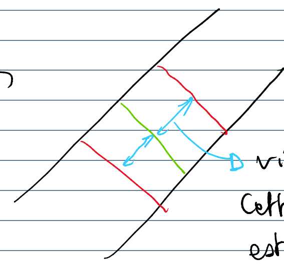
Terminologie : $k_z = \beta_g$ nombre d'onde guidé

$\beta_g = \dfrac{2\pi}{\lambda_g}$ longueur d'onde guidée

↳ période de l'onde le long de l'axe $z$

$u_p$ : vitesse de phase

$$u_p^{m,n} = \lambda_g f = \frac{\omega}{k_z} = \frac{\omega}{\frac{1}{u_{TEM}}\sqrt{\omega^2 - \omega_c^2}}$$



vitesse de cette zone est $u_p$

$$u_p^{m,n} = \frac{u_{TEM}}{\sqrt{1 - \left(f_c^{m,n}/f\right)^2}}$$

$$\beta_g^{m,n} = \beta_{libre}\sqrt{1 - \left(f_c^{m,n}/f\right)^2}$$

↳ avec $\beta_{libre} = \dfrac{\omega}{u_{TEM}}$

$$k_z = \beta\sqrt{1 - \frac{f_c^2}{f^2}} = \sqrt{\beta^2 - \frac{\beta^2 f_c^2}{f^2}} = \sqrt{\frac{\beta^2 f^2 - \beta^2 f_c^2}{f^2}} = \sqrt{\frac{\omega^2\mu\varepsilon f^2 - \omega^2\mu\varepsilon f_c^2}{f^2}} = \sqrt{\frac{\omega^2}{f^2 u_{TEM}^2}\left(f^2 - f_c^2\right)}$$

$$\left(\frac{\omega}{f}\right)^2 = (2\pi)^2$$

$$= \sqrt{\frac{(2\pi f)^2 - (2\pi f_c)^2}{u_{TEM}^2}} = \sqrt{\frac{\omega^2 - \omega_c^2}{u_{TEM}^2}}$$

$$\lambda_g^{m,n} = \frac{\lambda_{libre}}{\sqrt{1 - \left(f_c^{m,n}/f\right)^2}}$$

↳ avec $\lambda_{libre} = \dfrac{u_{TEM}}{f} = \dfrac{1}{\sqrt{\mu\varepsilon}\, f}$

$$u_g = \frac{d\omega}{dk_z} = u_{TEM}\sqrt{1 - \left(f_c/f\right)^2}$$

$$u_g \cdot u_p = u_{TEM}^2$$

$$\rightarrow P_{tot}^{TE} = \left(\frac{1}{4} \text{ ou } \frac{1}{8}\right)|H_0|^2 \eta\, ab \left(\frac{f}{f_c}\right)^2 \sqrt{1 - \left(f_c/f\right)^2}$$

$$\to P^{TE}_{tot} = \left(\frac{1}{4} \text{ ou } \frac{1}{8}\right)|H_0|^2 \eta\, ab \left(\frac{f}{f_c}\right)^2 \sqrt{1-(f_c/f)^2}$$

avec $\begin{cases} 1/4 \text{ si } m=0 \text{ ou } n=0 & (\text{pas les deux}) \\ 1/8 \text{ si } m\neq 0 \ \underline{\text{et}}\ n\neq 0 \end{cases}$

$$\to P^{TM}_{tot} = \frac{1}{8}\frac{1}{\eta}\left(\frac{f}{f_c}\right)^2 \sqrt{1-(f_c/f)^2}\,|E_0|^2 ab$$

$$\eta_g^{TE} = \left.\frac{\sqrt{|E_x|^2+|E_y|^2}}{\sqrt{|H_x|^2+|H_y|^2}}\right|_{\substack{x=a/2\\y=b/2}} = \frac{\eta_{libre}}{\sqrt{1-(f_c/f)^2}}$$

$$\eta_g^{TM} = \eta_{libre}\sqrt{1-(f_c/f)^2}$$

$$\bar{J}_s = \hat{n}\times\bar{H}$$

## Guides circulaires

TE

$$H_z = J_m(k_r\rho)\left[C\cos m\phi + D\sin m\phi\right]e^{-jk_z z} \quad \underline{\text{et}} \quad J_m'(k_r a)=0$$

$$E_\rho = \frac{-j\omega\mu}{k_r^2}\frac{J_m(k_r\rho)}{\rho}\left[-C\sin m\phi + D\cos m\phi\right]e^{-jk_z z}$$

$$E_\phi = \frac{j\omega\mu}{k_r^2}\frac{\partial H_z}{\partial\rho} = \frac{j\omega\mu}{k_r^2}J_m'(k_r\rho)\left[C\cos m\phi + D\sin m\phi\right]$$

$$E_\rho = -\frac{j\omega\mu}{k_t^2}\frac{1}{\rho}\frac{\partial H_z}{\partial\varphi}$$

$$= \frac{-j\omega\mu m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$C\cos(m(\phi-\phi_0))$

$$E_\varphi = \frac{j\omega\mu}{k_t^2}\frac{\partial H_z}{\partial\rho} = \frac{j\omega\mu}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$H_\rho = \frac{-jk_z}{k_t^2}\frac{\partial H_z}{\partial\rho} = \frac{-jk_z}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$H_\varphi = \frac{-jk_z}{k_t^2}\frac{1}{\rho}\frac{\partial Hz}{\partial\varphi} = \frac{-jk_z m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

TM

$$E_z = J_m(k_t\rho)\left[C\cos m\phi + D\sin m\phi\right]e^{-jk_z z} \quad \text{et } J_m(k_t a) = 0$$

ii) Modes TM ($E_z \neq 0$, $H_z = 0$)

$$\nabla^2 E_z + \beta^2 E_z = 0$$

$$\boxed{E_z = J_m(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}}$$

$$E_\rho = \frac{-jk_z}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

$$E_\varphi = \frac{-jk_z m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$$H_\rho = \frac{j\omega\varepsilon m}{k_t^2}\frac{J_m(k_t\rho)}{\rho}\left[-C\sin m\varphi + D\cos m\varphi\right]e^{-jk_z z}$$

$$H_\varphi = \frac{-j\omega\varepsilon}{k_t}J_m'(k_t\rho)\left[C\cos m\varphi + D\sin m\varphi\right]e^{-jk_z z}$$

conditions aux frontières : $\left.\begin{array}{l} E_z(\rho=a)=0 \\ E_\varphi(\rho=a)=0 \end{array}\right\} \rightarrow \boxed{J_m(k_t a)=0}$

On a donc que $k_t a$ est un zéro de $J_m(X)$ i.e. $k_t^{mn} a = X_{mn}$.

# Dérivées de Bessel

Zéros $X_{mn}'$ de la dérivée $J_{mn}'(X_{mn}')$ ($n$ = 1, 2, 3, …) de la fonction de Bessel $J_m(X)$
(Tiré de C. Balanis, Advanced Engineering Electromagnetics, John Wiley)

| | m=0 | m=1 | m=2 | m=3 | m=4 | m=5 | m=6 | m=7 | m=8 | m=9 | m=10 | m=11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n=1 | 3.8318 | 1.8412 | 3.0542 | 4.2012 | 5.3175 | 6.4155 | 7.5013 | 8.5777 | 9.6474 | 10.7114 | 11.7708 | 12.8264 |
| n=2 | 7.0156 | 5.3315 | 6.7062 | 8.0153 | 9.2824 | 10.5199 | 11.7349 | 12.9324 | 14.1155 | 15.2867 | 16.4479 | 17.6003 |
| n=3 | 10.1735 | 8.5363 | 9.9695 | 11.3459 | 12.6819 | 13.9872 | 15.2682 | 16.5294 | 17.7740 | 19.0046 | 20.2230 | 21.4309 |
| n=4 | 13.3237 | 11.7060 | 13.1704 | 14.5859 | 15.9641 | 17.3129 | 18.6375 | 19.9419 | 21.2291 | 22.5014 | 23.7607 | 25.0085 |
| n=5 | 16.4706 | 14.8636 | 16.3475 | 17.7888 | 19.1960 | 20.5755 | 21.9317 | 23.2681 | 24.5872 | 25.8913 | 27.1820 | 28.4609 |

# Fonction de Bessel

Zeros $X_{mn}$ de la function de Bessel $J_m(X_{mn})$ (n = 1, 2, 3, ….). Tiré de C. Balanis, Advanced Engineering Electromagnetics, John Wiley.

| | m=0 | m=1 | m=2 | m=3 | m=4 | m=5 | m=6 | m=7 | m=8 | m=9 | m=10 | m=11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n=1 | 2.4049 | 3.8318 | 5.1357 | 6.3802 | 7.5884 | 8.7715 | 9.9361 | 11.0864 | 12.2251 | 13.3543 | 14.4755 | 12.8264 |
| n=2 | 5.5201 | 7.0156 | 8.4173 | 9.7610 | 11.0647 | 12.3386 | 13.5893 | 14.8213 | 16.0378 | 17.2412 | 18.4335 | 19.6160 |
| n=3 | 8.6537 | 10.1735 | 11.6199 | 13.0152 | 14.3726 | 15.7002 | 17.0038 | 18.2876 | 19.5545 | 20.8071 | 22.0470 | 23.2759 |
| n=4 | 11.7915 | 13.3237 | 14.7960 | 16.2235 | 17.6160 | 18.9801 | 20.3208 | 21.6415 | 22.9452 | 24.2339 | 25.5095 | 26.7733 |
| n=5 | 14.9309 | 16.4706 | 17.9598 | 19.4094 | 20.8269 | 22.2178 | 23.5861 | 24.9349 | 26.2668 | 27.5838 | 28.8874 | 80.1791 |

Exemple d'application numérique : Soit un guide rempli d'air ayant un rayon de 5 cm. Quelle est la fréquence de coupure du mode $TM_{44}$?

$m = 4, n = 4$ $\quad$ $X_{mn} = 17.616 = k_t a$

$k_t = 17.616/5\text{cm} = 352.32\ \text{m}^{-1}$

$\omega_c = ck_t = 3\times10^8 \times 352.32$

On trouve, $f_c^{44} = \omega_c/(2\pi) = 16.82$ GHz

Dans guide circulaire, mode $TE_{11}$ dominant !

## $\boxed{\text{Guide coaxial}}$

$$\bar{E} = \frac{V_2 - V_1}{\ln b/a} \frac{1}{\rho} e^{-j\beta z} \hat{\rho}$$

mode TEM dominant

## Impédance caractéristique

$$Z_0 = \frac{\eta}{2\pi} \ln(b/a)$$

en posant $Z_0 = \sqrt{\frac{L}{C}}$ et $u_{TEM} = \frac{1}{\sqrt{LC}}$

on peut extraire L et C

- Extraction de $\alpha$, atténuation due au métal

L

$$\alpha_{TEM} = \frac{R_S}{d\eta} \quad \mu \qquad R_S = \sqrt{\frac{\omega\mu}{2\sigma}}$$

$$\alpha_{TE_{10}} = \frac{R_S\left[1 + \frac{2b}{a}\left(f_c/f\right)^2\right]}{b\,\eta\sqrt{1-\left(f_c/f\right)^2}}$$

$$\overline{\quad b\, h \sqrt{1-(f_c/f)^2}}$$

$$P_z(z) = P_z(0)\, e^{-2\alpha z}$$

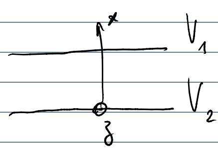
Conditions TEM :

① 2 conducteurs

② milieu homogène

③ $\nabla \cdot \bar{E} = 0 \Rightarrow \nabla_t^2 V = 0$

Plaques parallèles :




conditions frontières : $V(x=0) = V_2$
$V(x=d) = V_1$

on a $\nabla_t^2 V = 0$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

puisque $\frac{\partial(\cdot)}{\partial y} = 0$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = 0$$

$$\Rightarrow V = Ax + B$$

$$\Rightarrow \begin{cases} x = 0 \Rightarrow B = V_2 \\ x = d \Rightarrow A = \dfrac{V_1 - V_2}{d} \end{cases}$$

$$\Rightarrow V = \frac{V_1 - V_2}{d}\, x + V_2$$

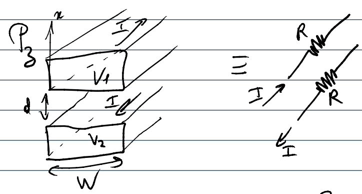
$$\Rightarrow V = \frac{V_1 - V_2}{d} x + V_2$$

$$\Rightarrow \bar{E} = -\nabla_t V e^{-j\beta z} = -\frac{\partial V}{\partial x} \hat{x} e^{-j\beta z}$$

$$\Rightarrow \bar{E} = \frac{V_2 - V_1}{d} e^{-j\beta z} \hat{x}$$

$$\vec{H} = \frac{\nabla \times \bar{E}}{-j\omega\mu} = \begin{bmatrix} 0 \\ H_y \\ 0 \end{bmatrix}$$

avec $H_y = \frac{1}{\eta} \frac{V_2 - V_1}{d} e^{-j\beta z} = \frac{E_x}{\eta}$




$$E_x = \frac{V_2 - V_1}{d} e^{-j\beta z}$$

$$H_y = \frac{E_x}{\eta}$$

$$P_z = \frac{1}{2} \mathrm{Re}\{E_x H_y^*\} \times d \times W$$

$$= \frac{|E_x|^2}{2\eta} W \times d$$

$\Delta P_z$ = − puissance dissipée

$$= -\frac{1}{2} R |I|^2 \times 2$$

avec :

$R = \frac{R_s \times \Delta z}{W}$ et $|I| = \|\bar{J}_s\| \cdot W$

$$= \|\hat{n} \times \bar{H}\| \cdot W$$

$$= |H_y| \cdot W$$

$$= E_x / \eta$$

$$\Rightarrow \Delta P_z = -\sqrt{\frac{\omega\mu}{2\sigma}} \frac{\Delta z}{W} W^2 \frac{|E_x|^2}{\eta^2}$$

$$\Rightarrow \frac{\Delta P_z}{\Delta z} = -\sqrt{\frac{\omega\mu}{2\sigma}} \frac{W}{\eta} \underbrace{\frac{|E_x|^2}{\eta}}$$

$$\Delta z \quad \sqrt{2\sigma} \quad \eta \quad \underbrace{\overline{\eta}}_{=\frac{2P_z}{w.d}}$$

$$\Rightarrow \frac{dP_z}{dz} = -\sqrt{\frac{\omega\mu}{2\sigma}} \, \frac{W}{\eta} \, \frac{2P_z}{Wd} = -\frac{2R_s}{\eta d} P_z$$

$$\text{or} \quad \frac{dP_z}{dz} = -2\alpha P_z$$

$$\Rightarrow \quad \alpha = \frac{R_s}{d\eta}$$

Pour guide coaxial :

$$\alpha = \frac{R_s\left(\frac{1}{a} + \frac{1}{b}\right)}{4\pi Z_0}$$

$$\text{avec} \quad Z_0 = \frac{\eta}{2\pi} \ln(b/a)$$