Devoir 2 Q2
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Devoir 2
Question 2
ELE3500

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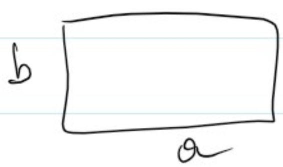
a) $f = 1{,}2\, f_c^{10} \Rightarrow f_c^{10} = \frac{f}{1{,}2}$




$b = 1{,}52 \cdot 10^{-3}\, m$

$TE_{10} \rightarrow$ mode fondamental

$$f_c^{mn} = \frac{u_{TEM}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$m = 1 \qquad n = 0$

$u_{TEM} = c / \sqrt{\varepsilon_R}$

$f = 28 \cdot 10^9\, Hz$

$$f_c^{10} = \frac{u_{TEM}}{2a}$$

$$\Rightarrow \quad a = \frac{u_{TEM}}{2 f_c^{10}} = \frac{1{,}2\, u_{TEM}}{2 f}$$

$$\boxed{a = 3{,}7 \quad mm}$$

✓

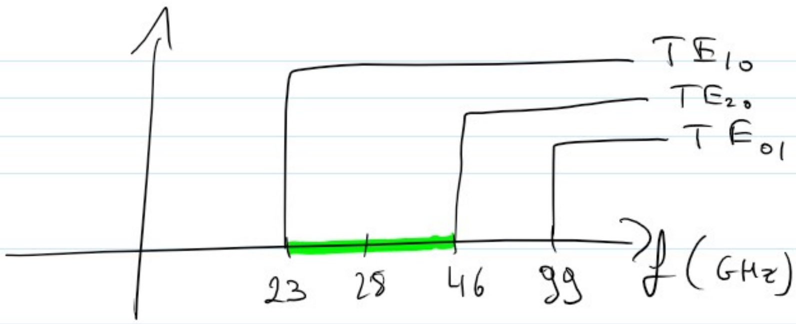
$$f_c^{10} = \frac{f}{1{,}2} = 23 \cdot 10^9\, Hz$$

$$f_c^{10} = \frac{f}{1{,}2} = 23 \cdot 10^9 \text{ Hz}$$

$$f_c^{01} = 99 \text{ GHz}$$

$$f_c^{20} = 46 \text{ GHz}$$

$$f_c^{11} > 100 \text{ GHz}$$




$$\Delta f = f_c^{20} - f_c^{10} = 46 \text{ GHz} - 23 \text{ GHz}$$

$$= 23 \text{ GHz}$$

Entre 23 GHz et 46 GHz ✓ 4

b) $TE_{10}$

$$\tan\delta = 0{,}001 = \frac{\sigma_e}{\omega \varepsilon_0 \varepsilon_R}$$

$$\Rightarrow \sigma_e = \omega \varepsilon_0 \varepsilon_R \tan\delta$$

$$\sigma_e = 4{,}7 \cdot 10^{-3} \text{ Siemens/m}$$

$$k_z = \sqrt{\omega^2 \mu_0 \underline{\varepsilon} - k_t^2}$$

$$k_t = \frac{\pi}{a}$$

$$\rightarrow k_z = \pm\sqrt{\omega^2 \mu_0 \varepsilon_0 \left(\varepsilon_R - j\frac{\sigma_e}{\omega \varepsilon_0}\right) - \left(\frac{\pi}{a}\right)^2}$$

$$\Rightarrow k_z = \pm\sqrt{\omega^2 \mu_0 \varepsilon_0 \left(\varepsilon_R - j\frac{\sigma_c}{\omega \varepsilon_0}\right) - \left(\frac{\pi}{a}\right)^2}$$

$$= \pm\left(562 - j\,0{,}91\right)$$

Supposons que la propagation se fait en direction $+z$

$$\bar{E}, \bar{H} \propto e^{-jk_z z} = e^{-j(562 - j0{,}91)z}$$

$$= e^{-j562} \underbrace{e^{-0{,}91 z}}$$

On a une exponentielle décroissante
$\rightarrow$ Atténuation en se propageant.
Donc prendre la direction de propagation $+z$ est le bon choix.

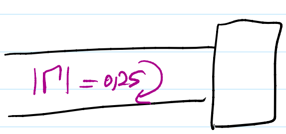
$$\Rightarrow k_z = 562 - j\,0{,}91$$

et $e^{-\alpha_z z} = e^{-0{,}91 z}$

$$\Rightarrow \alpha_z = 0{,}91 \text{ neper/m}$$

✓ 4

c)




$$\Delta d_{min} = \frac{\lambda_g}{2}$$

$$\lambda_g^{10} = \frac{\lambda}{1-\left(\frac{f_c}{f}\right)^2}$$

$$\lambda = \frac{c}{f}$$

$$f_c = f_c^{10} = 23\ GHz$$

$$f = 28\ GHz$$

$$\Rightarrow \lambda_g^{10} = 0{,}0112$$

$$\Rightarrow \Delta d_{min} = 5{,}6\ mm$$ ✓ 4

d)

$TE_{10}$ et $P^{inc} = 1W$

$\rightarrow k_r = \frac{\pi}{a}$

$\rightarrow E_z = 0$

$\rightarrow H_z = H_0 \cos\left(\frac{\pi x}{a}\right) e^{-jk_z z}$

$$E_x = -j\frac{\omega\mu}{k_r^2} \frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = 0$$

$$E_y = j\frac{\omega\mu}{k_r^2} \frac{\partial H_z}{\partial x}$$

$$= -j H_0 \frac{\omega \mu_0}{} (\pi/a) \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z}$$

$$= -j H_0 \frac{\omega \mu_0}{(\pi/a)^2} (\pi/a) \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z}$$

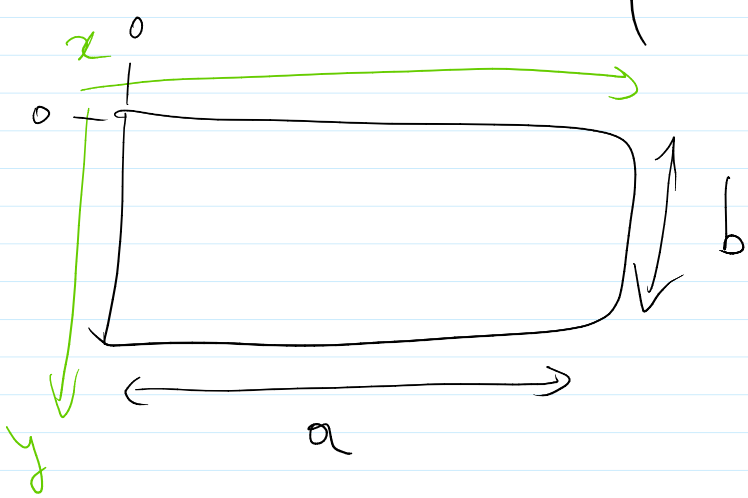
$$\Rightarrow E_y = -j H_0 \frac{\omega \mu_0}{\pi/a} \sin\left(\frac{\pi}{a} x\right) e^{-jk_z z}$$

$$|E_y| = \frac{H_0 \omega \mu}{\pi/a} \times \sin\left(\frac{\pi}{a} x\right)$$

$$\Rightarrow |E_y|_{max} \Leftrightarrow \sin\left(\frac{\pi}{a} x\right) = \pm 1$$

$$\Rightarrow \frac{\pi}{a} x = \begin{cases} \frac{\pi}{2} + 2k\pi, & k \in \mathbb{Z} \\ -\frac{\pi}{2} + 2k\pi, & k \in \mathbb{Z} \end{cases}$$

$$\Rightarrow x = \begin{cases} \frac{a}{2} + 2ka, & k \in \mathbb{Z} \\ -\frac{a}{2} + 2ka, & k \in \mathbb{Z} \end{cases}$$




$$0 < x < a$$

$$\Rightarrow x = \frac{a}{2} = 1{,}85 \text{ mm}$$

$$\Rightarrow \quad x = \frac{a}{2} = 1,85 \text{ mm}$$ ✓

Le champ $\bar{E}$ est maximal en $x = 1,85$ mm

$$|\bar{E}|_{max} = |E_y|_{max} = \frac{H_0 \omega \mu_0}{\pi / a}$$

$$P = \frac{1}{4} |H_0|^2 \eta \, a \, b \left(\frac{f}{f_c}\right)^2 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 1 \text{ W}$$

$$\Rightarrow |H_0| = \sqrt{\frac{4 \times P}{\eta \, ab \left(\frac{f}{f_c}\right)^2 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

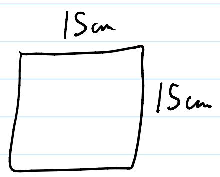
$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_R}} = 217,6 \ \Omega$$

$\varepsilon_R = 3$

$$\Rightarrow |H_0| = 63,99 \text{ A/m}$$

$$\Rightarrow |\bar{E}|_{max} = \frac{H_0 \omega \mu_0}{\pi / a} \simeq 16,7.10^3 \text{ V/m}$$ ✓ 4

e)



$$u_g = \frac{u_{TEM}^2}{u_p} = \frac{u_{TEM}^2}{\dfrac{u_{TEM}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}} = u_{TEM}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\Rightarrow u_g^{mn} = u_{TEM}\sqrt{1-\left(\frac{f_c^{mn}}{f}\right)^2}$$

$$\left(f_c^{mn}\right)^2 = \left(\frac{u_{TEM}}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\right)^2 = \frac{u_{TEM}^2}{4}\,\frac{m^2+n^2}{a^2}$$

$$\Rightarrow\ = u_{TEM}\sqrt{1-\frac{u_{TEM}^2\,(m^2+n^2)}{4\,a^2 f^2}}$$

$$= u_{TEM}\,\frac{\sqrt{4a^2f^2 - u_{TEM}^2\,(m^2+n^2)}}{2af}$$

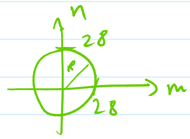
$$4a^2f^2 - u_{TEM}^2\,(m^2+n^2) \geq 0$$

$$\Rightarrow\quad 4a^2f^2 \geq u_{TEM}^2\,(m^2+n^2)$$

$$784 = \frac{4a^2f^2}{u_{TEM}^2} \geq m^2+n^2$$

$$m^2+n^2 \leq 784\ \checkmark$$

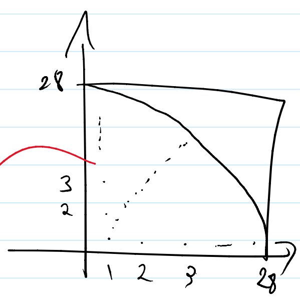
$m^2 + n^2 \leqslant 28^2$




plusieurs valeurs de m et n maximisent la vitesse de phase.

Avec l'aide de Matlab, on voit qu'on a une vitesse de groupe :

- maximale pour $m = 1$ et $n = 0 \longrightarrow$ mode $TE_{10}$
  et pour $m = 0$ et $n = 1 \longrightarrow$ mode $TE_{01}$
- minimale pour $m = 7$ et $n = 27$ et pour $m = 27$ et $n = 7$ $\rightarrow$ mode $TE_{7,27}$ ou $TM_{7,27}$ / mode $TE_{27,7}$ ou $TM_{27,7}$




D 639 points dans le disque

$\Rightarrow$ 639 modes TE possibles

et 585 modes TM possibles

Valeur + lente vitesse ?

3

Donc il s'agit d'un guide d'onde très mauvais car il permet beaucoup plus d'un mode.

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