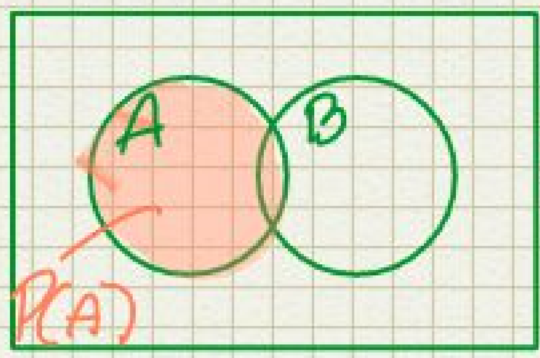
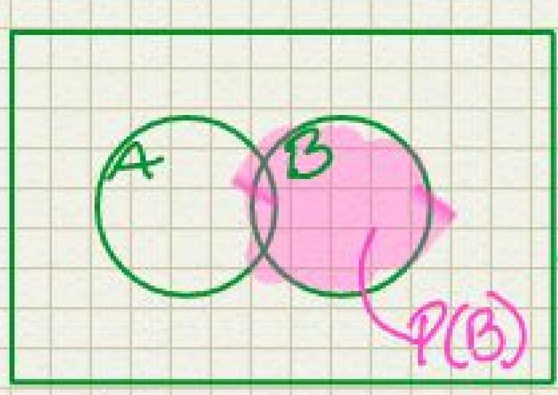
Consider influenza epidemics for two parent heterosexual families. Suppose that the probability is 15% that at least one of the parents has contracted the disease. The probability that the parent has contracted influenza is 10% while that the mother contracted the disease is 9%. What is the probability that the both contracted influenza expressed as a whole number percentage?



ONLY FATHER

A = FATHER

B = MOTHER



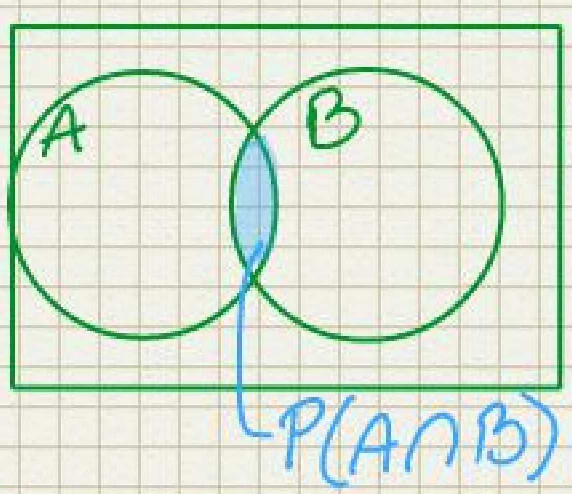
ONLY MOTHER

$P(A) = 0.1$

$P(B) = 0.09$

$P(A \cup B) = 0.15$

$P(A \cap B) = ?$

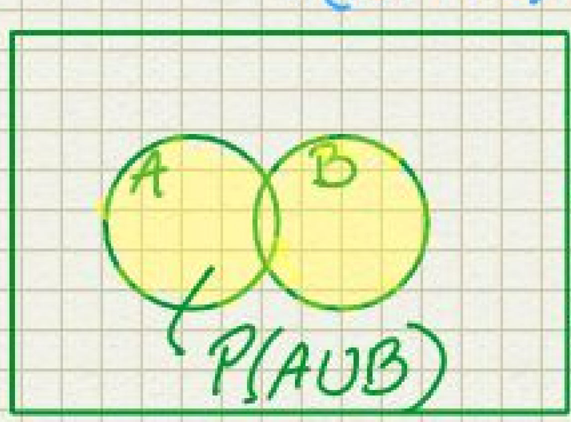


BOTH

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$P(A \cap B) = 0.1 + 0.09 - 0.15$$

$$P(A \cap B) = 0.04$$

$$\boxed{P(A \cap B) = 4\%}$$



ONE, THE OTHER OR BOTH

AT LEAST ONE

Consider influenza epidemics for two parent heterosexual families. Suppose that the probability is 17% that at least one of the parents has contracted the disease. The probability that the father has contracted influenza is 12% while the probability that both the mother and father have contracted the disease is 6%. What is the probability that the mother has contracted influenza?

$P(A) = 0.12$

$P(B) = ?$

$P(A \cup B) = 0.17$ AT LEAST ONE

$P(A \cap B) = 0.06$ BOTH

A = FATHER

B = MOTHER

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

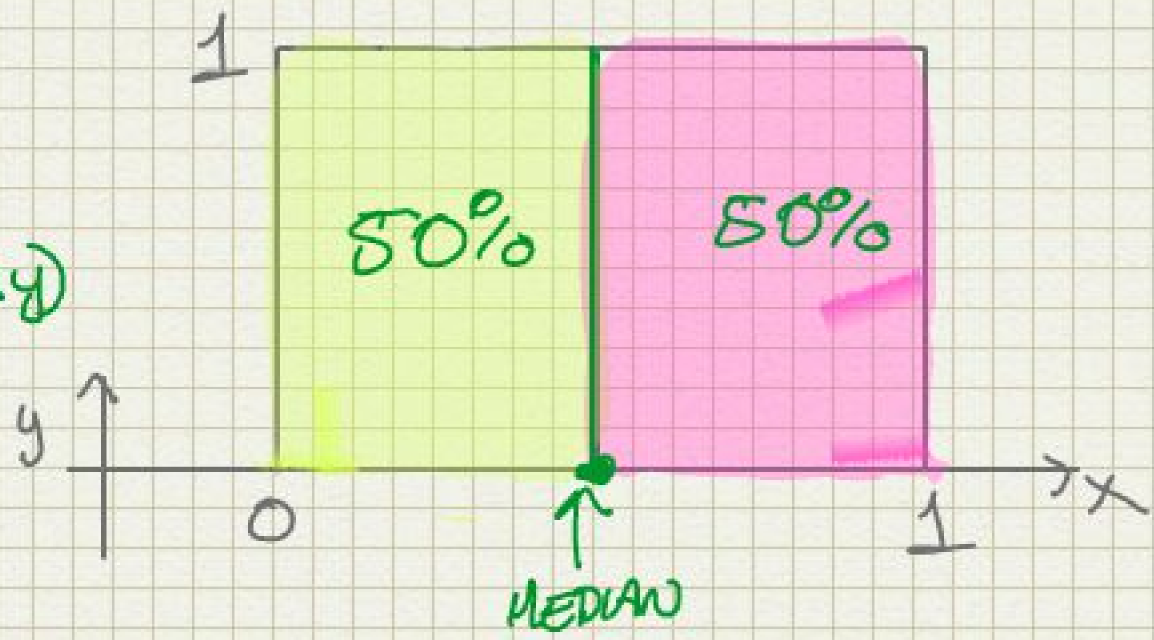
$$P(B) = P(A \cup B) - P(A) + P(A \cap B)$$

$$P(B) = 0.17 - 0.12 + 0.06$$

$$P(B) = 0.11$$

$$\boxed{P(B) = 11\%}$$

A random variable X, is uniform, a box from 0 to 1 height 1. (So that it's density is $f(x) = 1$ for $0 \leq x \leq 1$) What is it's median expressed to two decimal places?



MEDIAN: POINT SO THAT 50% OF DATA (OR DENSITY) LIES BELOW IT

MEDIAN

$$\int_0^{x_m} f(x)\,dx = 0.5$$

$$\int_0^{x_m} 1\,dx = 0.5$$

$$x \Big|_0^{x_m} = 0.5$$

$$x_m - 0 = 0.5$$

$$\boxed{x_m = 0.5}$$

You are playing a game with a friend where you flip a coin and if it comes up heads you give her X dollars and if it comes up tails she gives you Y dollars. The odds that the coin is heads in d. What is your expected earnings?

ODDS THAT YOU LOSE ON A GIVEN ROUND ARE GIVEN BY

$$\frac{p}{(1-p)} = d \rightarrow p = \frac{d}{1+d}$$

$$\text{Ernis} = -Xp + Y(1-p)$$
$$= -X\left(\frac{d}{1+d}\right) + Y\left(1 - \frac{d}{1+d}\right)$$
$$= -X\left(\frac{d}{1+d}\right) + Y\left(\frac{1+d-d}{1+d}\right)$$

$$\boxed{E = -X\left(\frac{d}{1+d}\right) + Y\left(\frac{1}{1+d}\right)}$$

You are playing a game with a friend where you flip a coin and if it comes up heads you give her X dollars and if it comes up tails she gives you Y dollars. The probability that the coin is heads is p (some number between 0 and 1.) What has to be true about X and Y to make so that both of your expected total earnings is 0. The game would then be called "fair".

$p \rightarrow P(\text{COIN IS HEADS})$

$1-p \rightarrow P(\text{COIN IS TAILS})$

$$E = -Xp + Y(1-p)$$

$$0 = -Xp + Y(1-p)$$

$$Xp = Y(1-p)$$

$$\boxed{\frac{p}{1-p} = \frac{Y}{X}}$$

A random variable takes the value -4 with probability 0.2 and 1 with probability 0.8 What is the variance of this random variable?

$$VAR(X) = E[(X-\mu)^2] = E[X^2] - (E[X])^2$$

$$E[X] = (-4)(0.2) + (1)(0.8) = -0.8 + 0.8 = 0$$

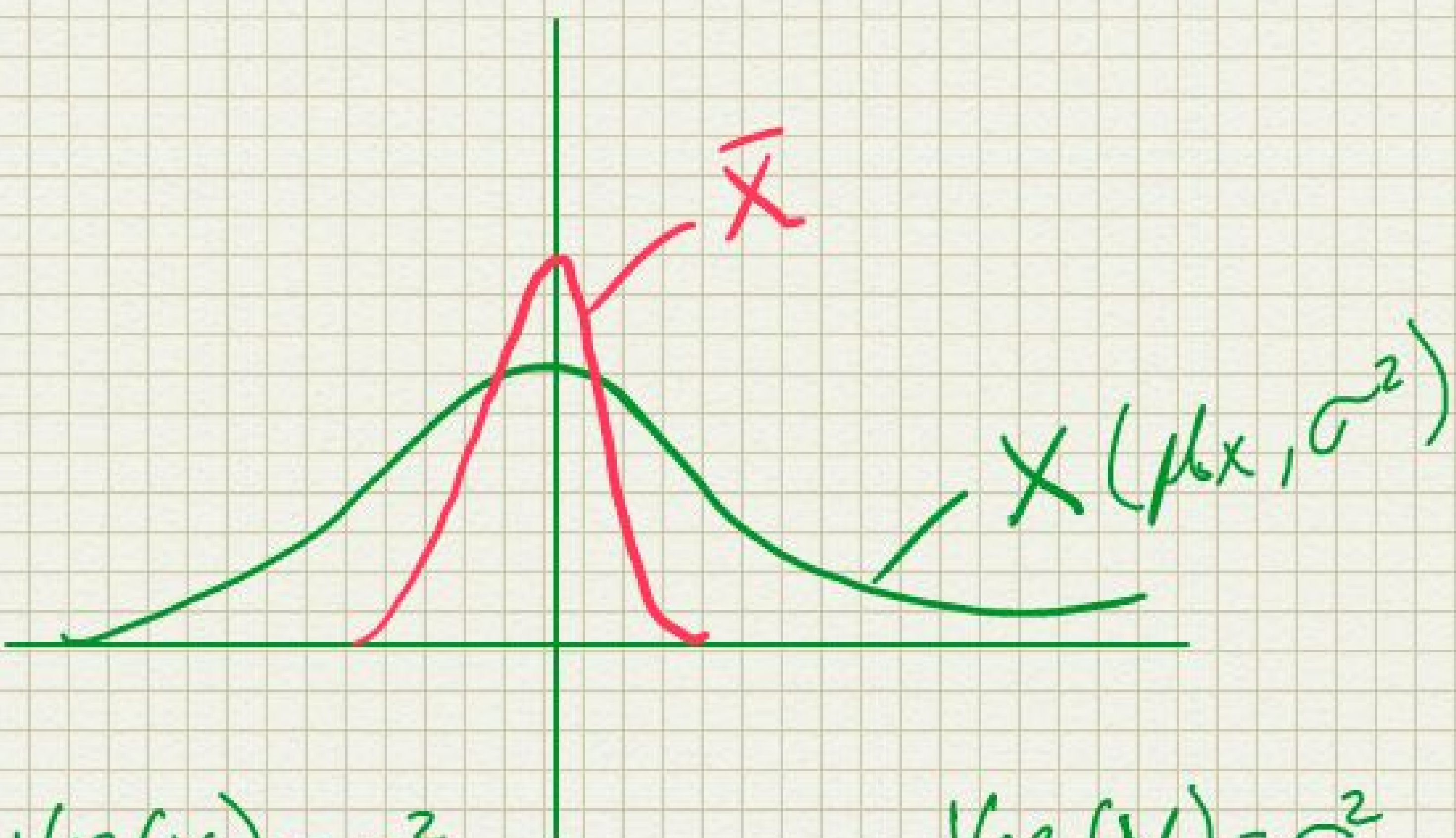
$$E[X] = 0$$

$$VAR(X) = E[X^2] - 0 = E[X^2]$$

$$E[X^2] = (-4)^2(0.2) + (1)^2(0.8) = 16(0.2) + 0.8$$
$$= 4$$

$$\boxed{VAR(X) = 4}$$

If $\overline{X}$ and $\overline{Y}$ are comprised of n iid random variables arising from distributions having means μ_x and μ_y, respectively and common variance σ^2 what is the variance $\overline{X} - \overline{Y}$?



$VAR(X) = \sigma^2$

$VAR(\overline{X}) = \sigma^2/n$

$VAR(Y) = \sigma^2$

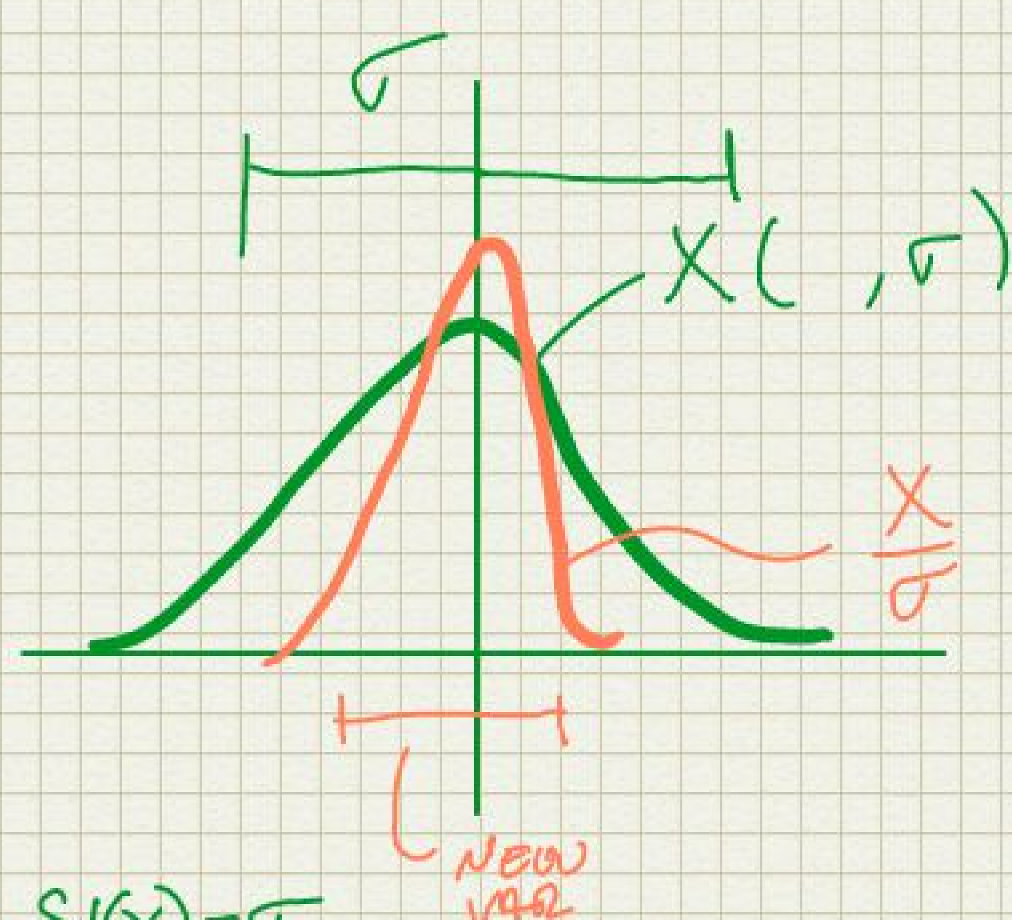
$VAR(\overline{y}) = \sigma^2/n$

$$VAR(\overline{X} - \overline{y}) = VAR(\overline{X} + (-\overline{y})) = VAR(\overline{X}) + VAR(-\overline{y})$$
$$= VAR(\overline{X}) + (-1)^2 VAR(\overline{y})$$
$$= VAR(\overline{X}) + VAR(\overline{y})$$

$$VAR(\overline{X} - \overline{y}) = \frac{\sigma^2}{n} + \frac{\sigma^2}{n}$$

$$\boxed{VAR(\overline{X} - \overline{y}) = \frac{2\sigma^2}{n}}$$

Let X be a random variable having standard deviation σ . What can be said about X / σ ?



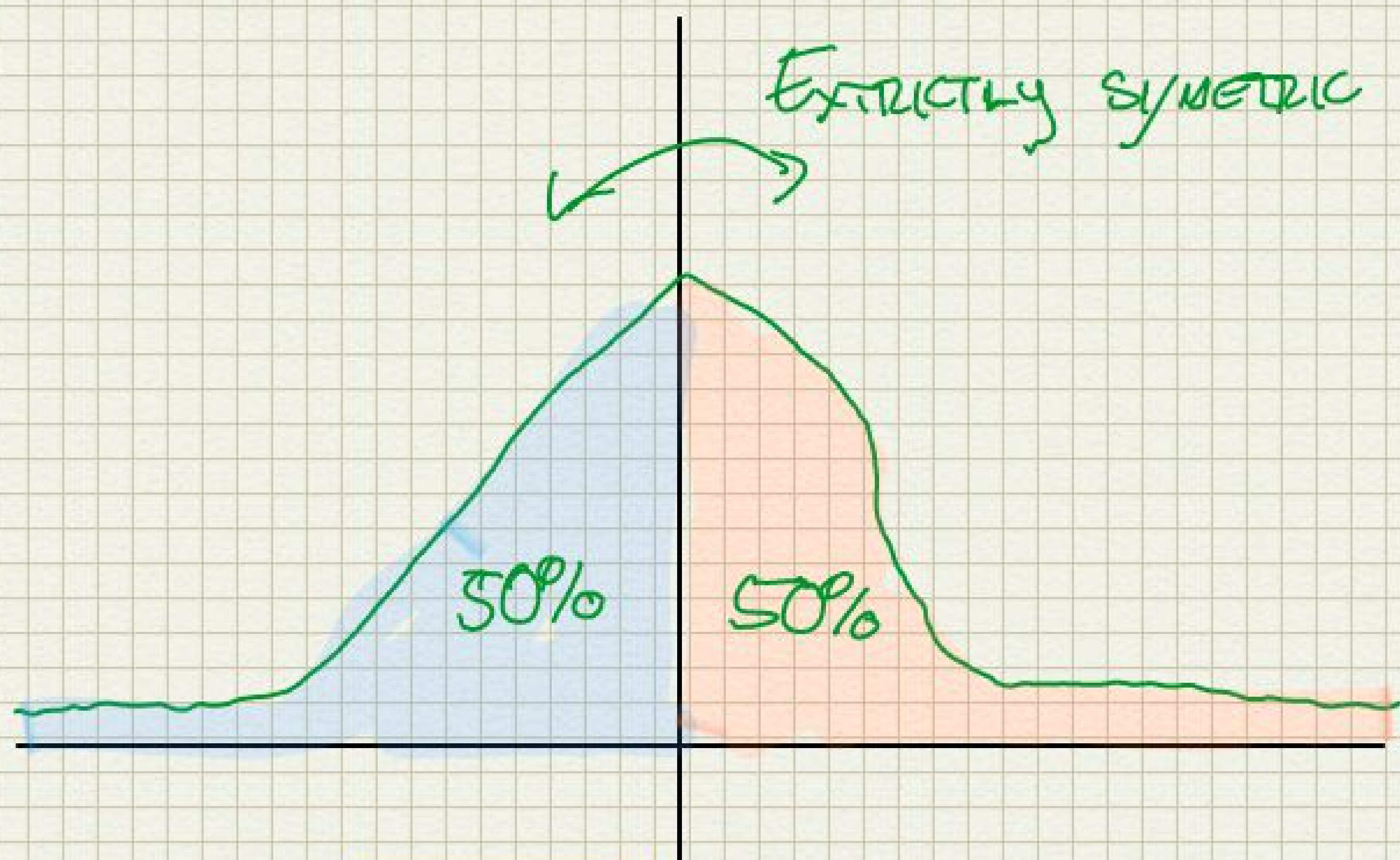
$Sd(X) = \sigma$

$VAR(X) = \sigma^2$ $\qquad$ $VAR(aX) = a^2 VAR(X)$

$$VAR\left(X/\sigma\right) = VAR\left(\frac{1}{\sigma} X\right) = \frac{1}{\sigma^2} VAR(X) = \frac{1}{\sigma^2} \sigma^2 = 1$$

$$\boxed{VAR\left(\frac{X}{\sigma}\right) = 1}$$

If a continuous density that never touches the horizontal axis is symmetric about zero, can we say that its associated median is zero?



$\therefore$ $\boxed{\text{YES}}$

Consider the following pmf:
p <- 0.1, 0.2, 0.3 0.4
x <- 2, 3, 4, 5
What is the variance expresed to one decimal place?

$$VAR(X) = E(X^2) - (E[X])^2$$

$$E[X] = 2(0.1) + 3(0.2) + 4(0.3) + 5(0.4) = 4$$

$$E[X^2] = 2^2(0.1) + 3^2(0.2) + 4^2(0.3) + 5^2(0.4) = 17$$

$$VAR(X) = 17 - 4^2 = \underline{1}$$

$$\boxed{VAR(X) = 1.0}$$

Consider the following pmf:
p <- 0.1, 0.2, 0.3, 0.4
x <- 1, 2, 3, 4
What is the mean?

$$E[x] = 1(0.1) + 2(0.2) + 3(0.3) + 4(0.4) = 3$$

$$\boxed{E[x] = \mu = 3}$$

A web site for home pregnancy tests cites the following: "When the subjects using the test were women who collected and tested their own samples, the overall sensitivity was 75%. Specificity was also low, in the range 52% to 75%." Assume the lower value for the specificity. Suppose a subject has a positive test and that 30% of women taking pregnancy tests are actually pregnant. What number is closest to the probability of pregnancy given the positive test?

SENSITIVITY $= P(+|D) = 0.75$

SPECIFICITY $= P(-|D^c) = 0.52$

SUBJECT $\rightarrow +$

$P(D) = 0.3$ PREVALENCE

$P(D|+) = ?$

USING BAYE'S RULE

$$P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|D^c)P(D^c)}$$

$$= \frac{P(+|D)P(D)}{P(+|D)P(D) + (1 - P(-|D^c))(1 - P(D))}$$

$$= \frac{0.75(0.3)}{0.75(0.3) + (1-0.52)(1-0.3)} = \frac{0.225}{0.561}$$

$P(D|+) = 0.4011$

$$\boxed{P(D|+) \approx 40\%}$$