

Consider the following pmf:

$p \leftarrow 0.1, 0.2, 0.3, 0.4$

$x \leftarrow 2, 3, 4, 5$

What is the variance expressed to one decimal place?

$$\text{Var}(X) = E(X^2) - (E[X])^2$$

$$E[X] = 2(0.1) + 3(0.2) + 4(0.3) + 5(0.4) = 4$$

$$E[X^2] = 2^2(0.1) + 3^2(0.2) + 4^2(0.3) + 5^2(0.4) = 17$$

$$\text{Var}(X) = 17 - 4^2 = 1$$

$$\text{Var}(X) = 1.0$$

A random variable takes the value -4 with probability 0.2 and 1 with probability 0.8. What is the variance of this random variable?

$$\text{Var}(X) = E[(X - \mu)^2] = E[X^2] - (E[X])^2$$

$$E[X] = (-4)(0.2) + (1)(0.8) = -0.8 + 0.8 = 0$$

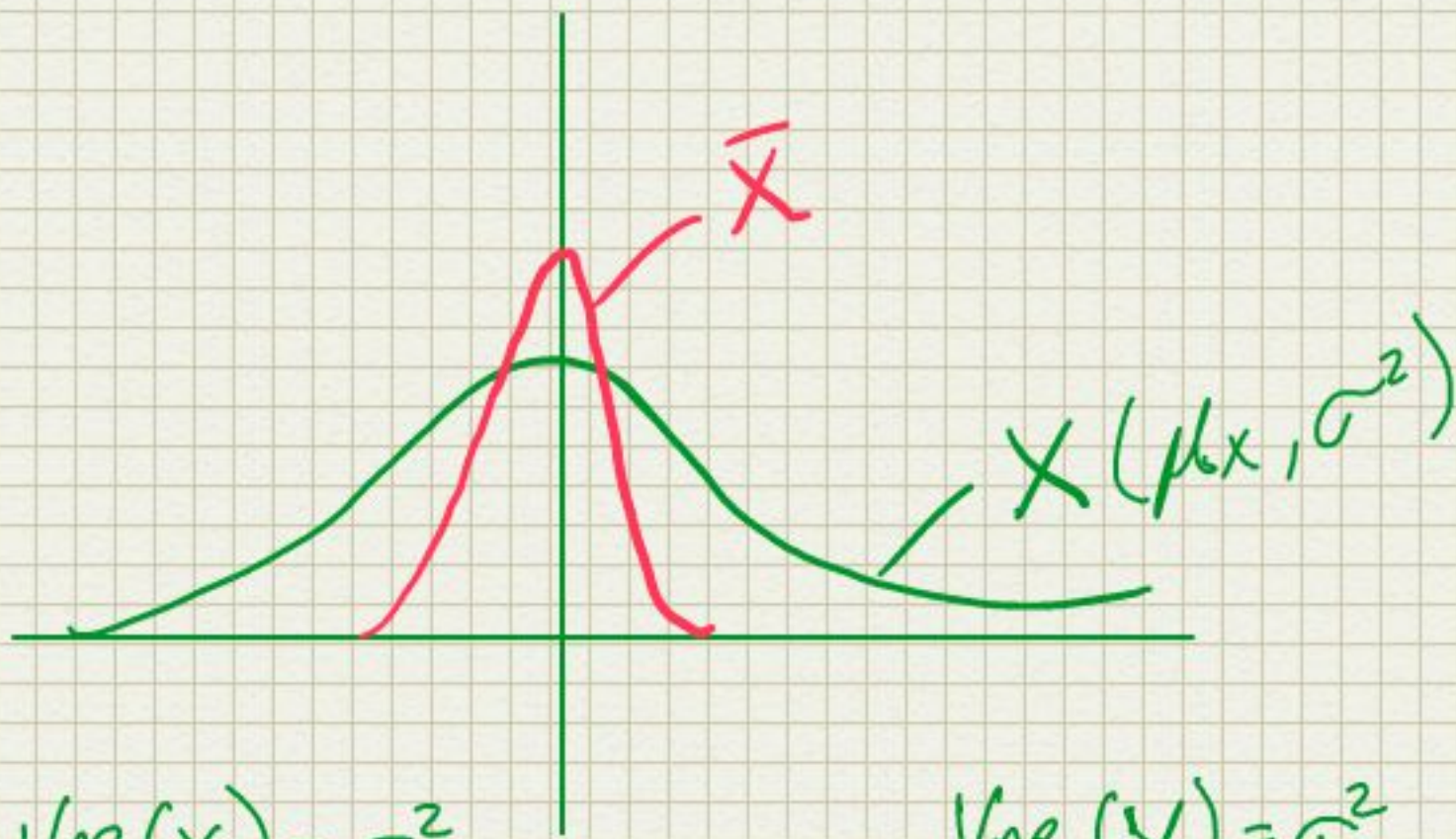
$$E[X] = 0$$

$$\text{Var}(X) = E[X^2] - 0 = E[X^2]$$

$$E[X^2] = (-4)^2(0.2) + (1)^2(0.8) = 16(0.2) + 0.8 = 4$$

$$\text{Var}(X) = 4$$

If \bar{X} and \bar{Y} are comprised of n iid random variables arising from distributions having means μ_x and μ_y , respectively and common variance σ^2 what is the variance $\bar{X} - \bar{Y}$?



$$\text{Var}(X) = \sigma^2$$

$$\text{Var}(\bar{X}) = \sigma^2/n$$

$$\text{Var}(Y) = \sigma^2$$

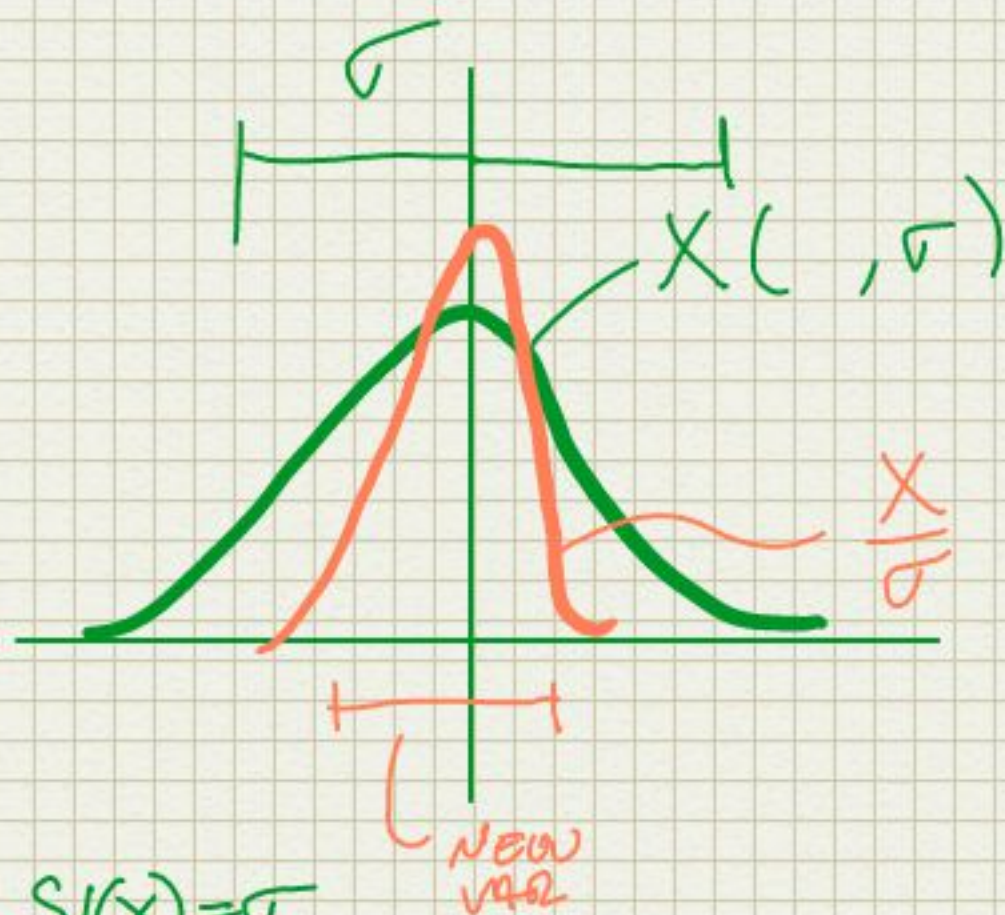
$$\text{Var}(\bar{Y}) = \sigma^2/n$$

$$\begin{aligned}\text{Var}(\bar{X} - \bar{Y}) &= \text{Var}(\bar{X} + (-\bar{Y})) = \text{Var}(\bar{X}) + \text{Var}(-\bar{Y}) \\ &= \text{Var}(\bar{X}) + (-1)^2 \text{Var}(\bar{Y}) \\ &= \text{Var}(\bar{X}) + \text{Var}(\bar{Y})\end{aligned}$$

$$\text{Var}(\bar{X} - \bar{Y}) = \frac{\sigma^2}{n} + \frac{\sigma^2}{n}$$

$$\boxed{\text{Var}(\bar{X} - \bar{Y}) = \frac{2\sigma^2}{n}}$$

Let X be a random variable having standard deviation σ . What can be said about X / σ ?



$$SD(X) = \sigma$$

$$VAR(X) = \sigma^2$$

$$VAR(aX) = a^2 VAR(X)$$

$$VAR\left(\frac{X}{\sigma}\right) = VAR\left(\frac{1}{\sigma} X\right) = \frac{1}{\sigma^2} VAR(X) = \frac{1}{\sigma^2} \sigma^2 = 1$$

$$VAR\left(\frac{X}{\sigma}\right) = 1$$