Tournament design: A review from an operational research perspective

Karel Devriesere[*1], László Csató[2,3], and Dries Goossens[1,4]

[1] *Faculty of Economics and Business Administration, Ghent University, Tweekerkenstraat 2, 9000 Ghent, Belgium*

[2] *Institute for Computer Science and Control (SZTAKI), Hungarian Research Network (HUN-REN), Hungary*

[3] *Corvinus University of Budapest, Hungary*

[4] *Core lab CVAMO, FlandersMake@UGent, Belgium*

Abstract

Every sport needs rules. Tournament design refers to the rules that determine how a tournament, a series of games between a number of competitors, is organized. This study aims to provide an overview of the tournament design literature from the perspective of operational research. Three important design criteria are discussed: efficacy, fairness, and attractiveness. Our survey classifies the papers discussing these properties according to the main components of tournament design: format, seeding, draw, scheduling, and ranking. We also outline several open questions and promising directions for future research.

Keywords: OR in sports; ranking; scheduling; seeding; tournament design

1 Introduction

Practicing sports is an important part of the life of many people all over the world as many health, psychological, and social benefits are associated with physical activity. Watching and following sports is even more popular than playing it. About 3 billion people watched the 2020 Summer Olympic Games (Statista, 2023), while no less than 1.5 billion people tuned in for the 2022 FIFA World Cup final (FIFA, 2023b). Even tournament design aspects can be popular: the draw for the 2021/22 UEFA Champions League Round of 16 has attracted more than 1 million views on the YouTube channel of UEFA (Boczoń and Wilson, 2023). Hence, "*designing an optimal contest is both a matter of significant financial concern for the organizers, participating individuals, and teams, and a matter of consuming personal interest for millions of fans*" (Szymanski, 2003, p. 1137).

Every tournament is governed by a set of rules that determine the set of contestants, the format, the schedule, the ranking, and the prize allocation. Deciding on these rules is what we call *tournament design*. In the following, the contestants will be called players or teams interchangeably. The set of matches to be played is given by the tournament *format*. *Seeding* is typically used to guarantee that a strong contestant is not eliminated at the early stages of the tournament merely because it plays against an even stronger opponent. Subject to these and other constraints, the contestants can be allocated randomly, through a so-called *draw*. At first sight, the *schedule*, the order and timing of the matches, seems a purely operational issue (involving travel distance, referee and venue availability, etc.). However, it may have powerful effects on the fairness and the outcome of the tournament. *Ranking* aims to order the teams according to their performance in the tournament. Although performance can be directly measured in some sports (e.g. by finishing times), most sports allow only to rank the contestants

---

by paired comparisons, games or matches. *Prize allocation* determines how prizes (e.g. promotion, relegation, qualification, broadcasting revenue) will be awarded to the contestants.

Given the variety of tournament designs, the question arises on what basis they can be compared. We identify four different criteria that appear frequently in the literature on tournament design: 1) Efficacy; 2) Fairness; 3) Attractiveness; and 4) Strategy-proofness. *Efficacy* is defined as the ability of a tournament to reveal the "true" (but hidden) ranking of participants. In contrast, *fairness* has no uniform definition (see Section 2.3.2 and Baumer (2024)); in our paper, it mainly refers to the principle of equal treatment. *Attractiveness* also has multiple interpretations, but they are always connected to the excitement that the tournament generates. Finally, all participants should have an incentive to exert full effort and play according to their true strength under any possible scenario in the tournament. That is, the tournament design needs to be *strategy-proof*.

Our focus on these criteria may suggest that sports federations can freely choose an optimal tournament design from their own perspective, i.e., a design showing an appropriate balance of theoretical properties. In practice, however, traditions and the nature of the sport may play an even larger role in this decision process. For instance, European football leagues are usually played in a round-robin format (Section 2.1.1), while tennis competitions are organized as a seeded knockout tournament (Section 2.1.2). Similarly, the Swiss system (Section 2.1.3) is mainly used in chess. Sports differ in how much time a player needs to recover between two matches: for instance, baseball teams play almost daily, while World Rugby recommends that at least four clear recovery days are provided after each match. The number of rest days required may depend on the frequency of changes allowed by the rules, too; ice hockey teams can play every second day mainly because they often change lines. Some sports plan their season in order avoid overlap with other popular sports (Pagels, 2018), while travel costs may force tournaments to be organized in a short time span on one location (Nurmi et al., 2014). Hence, any "good" tournament design depends on the particular sport, and operational research should take these constraints into account to improve decision making. However, in this paper – apart from a number of specific examples – we will maintain a more generic view.

Tournament design is a research field at the interface between operational research and (sports) economics. Operational research mainly contributes to tournament design by optimizing design choices over certain criteria and aiding decision-making. In sports economics, the main issue is typically how the tournament design may affect the behavior and efforts of the players, through theoretical or empirical research (Medcalfe, 2024; Palacios-Huerta, 2023). There is a clear synergy between these two angles: sports economics research can bring new criteria and conditions to be taken into account in optimization (or debunk myths that traditionally played a role in tournament design), and validate the decisions proposed by operational research.

In this paper, we assume an operational research perspective. Therefore, prize allocation will be discussed only marginally, as this is, in our view, related to sports economics rather than to operational research. We refer the reader to Moldovanu and Sela (2001), Bergantinos and Moreno-Ternero (2020), and Dietzenbacher and Kondratev (2023) for excellent discussions on this topic. Furthermore, even though this is certainly one of the most important theoretical properties of tournament design, we have decided *not* to review strategy-proofness here for the same reason. In addition, including this issue would have substantially increased the length of our survey, and a recent book deals with the problem of incentives from an operational research perspective (Csató, 2021c). Analogously, the effect of tournament design on competitive balance, consumer demand, or market structure will be mostly ignored; we refer the interested reader to a seminal economics paper on these aspects (Szymanski, 2003). Nevertheless, we always try to emphasize practical applications by connecting the topics to real-world tournaments. As such, this survey will hopefully be useful for sports economists, too, and advance collaboration between the two domains. Sports administrators and tournament organizers can also benefit from such an up-to-date review of the tournament design literature.

Previous surveys on operational research and tournament design have so far mainly focused on scheduling, and, in particular, its operational aspects, see Rasmussen and Trick (2008), Kendall et al. (2010), Ribeiro (2012), Van Bulck et al. (2020), and Ribeiro et al. (2023b). Self-contained textbooks about sports scheduling are Briskorn

(2008) and Ribeiro et al. (2023a). There are also good – albeit somewhat outdated – overviews of the operational research analysis of sports rules (Wright, 2009, 2014). Kendall and Lenten (2017) consider a number of examples from sports where rule changes have had unintended consequences. The recent survey of Lenten and Kendall (2022) collects academic works that proposed rule changes in sports. We add to the existing surveys by considering a broader set of papers and covering a wider set of design issues and criteria, motivated by recent contributions of operational research in these areas. On the other hand, we offer less by excluding many studies that do not explicitly deal with tournament design but focus on, for example, individual games.

The rest of the paper is organized as follows. Section 2 provides the necessary background and terminology on tournament formats, ranking, and the three design criteria presented above. Investigations of tournament formats, seeding policies, draw rules, scheduling, and ranking methods with respect to these criteria are discussed in Sections 3–7. Finally, Section 8 concludes and lists several open questions identified by us.

2 Basic terminology and background

Tournament design uses several, not always well-defined, notions. This section attempts to summarize and clarify them in order to prepare and structure the following discussion. In particular, Section 2.1 presents tournament formats, Section 2.2 overviews ranking, and Section 2.3 defines three criteria that play a crucial role in the evaluation of tournament designs.

2.1 Tournament formats

The format of a tournament indicates the set of matches to be played, without identifying the particular teams. Three fundamental types of tournament formats exist: knockout, round robin, and Swiss system tournaments. They are presented in Sections 2.1.1, 2.1.2, and 2.1.3, respectively. More complex tournament formats can be formed by combining these formats, as briefly discussed in Section 2.1.4.

2.1.1 Round robin

One of the basic competition formats is the *round robin tournament* (Harary and Moser, 1966). In a k-round robin (kRR) tournament, each participant faces each other k times. Most domestic football leagues are kRR tournaments (Lasek and Gagolewski, 2018); for example, the competitions in the top five leagues (England, France, Germany, Italy, Spain) are organized as a 2RR. Since 1RR is typically called "single round robin" and 2RR is called "double round robin", we will use the notations SRR and DRR, respectively, for these formats.

If not all rounds of a round robin tournament are played, by design (especially if there are many contestants), or because the tournament is aborted (see Section 7.2.1), we call it an *incomplete round robin tournament*. While this format dates back to at least Cochran (1971), who discussed it in the context of bridge, it got more attention through the qualifying phase of the 2019/20 CONCACAF Nations League, which was contested by 34 teams in a single group, playing just 4 matches each (two home and two away). UEFA adopted an incomplete round robin structure for its club competitions from the 2024/25 season: the group stage is replaced by an incomplete round robin phase with 36 clubs that play against six or eight different opponents (UEFA, 2023). Incomplete round robin tournaments can also emerge from unevenly combining round robin tournaments between a subset of the opponents (e.g. teams in the same division) and bipartite round robin tournaments, where each team plays against each team from another group (e.g. teams in different conferences, or interleague play) as is common in North American sports.

2.1.2 Knockout

Another traditional competition format is the *knockout tournament*. In contrast to round robin tournaments, here a player generally faces only a subset of the other players. In a single knockout (SKO) tournament, players are
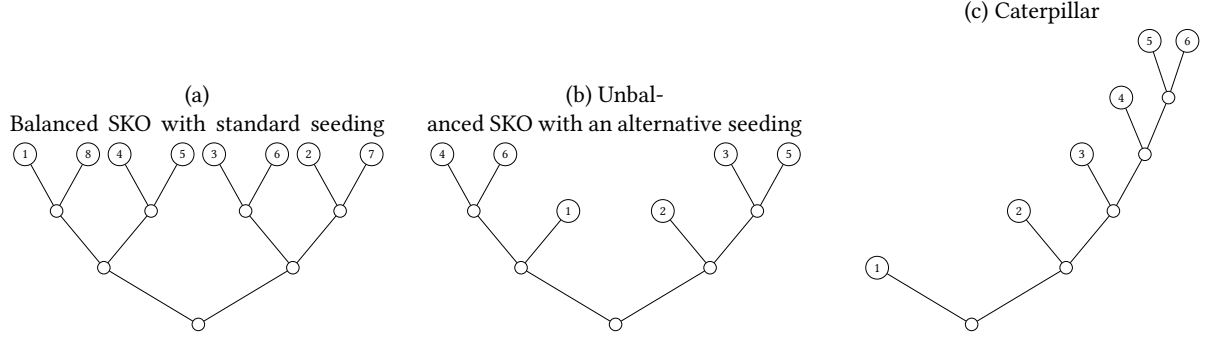
Figure 1: Different types of knockout tournaments

immediately eliminated if they lose.

In a double knockout (DKO) tournament, one loss is allowed in the tournament. This format is organized as two separate SKOs: the first SKO is called the main tournament, and the second SKO is called the consolation tournament. A player losing in the main tournament is moved to the consolation tournament. However, another loss in the consolation tournament eliminates the player. In essence, a player needs to lose twice in order to be knocked out. If the winner of the main round loses the final match against the winner of the consolation tournament, a rematch is played between them. The final phase of the Russian football cup is a double knockout tournament since the 2022/23 season. A variant of this structure is used, for example, in judo, where the loser of the final in the main tournament fights against the winner of the consolation tournament for the second place. Thus, the winner will always have no losses, the silver medalist will have one loss, and the bronze medalist will have two losses.

In some knockout tournaments, the two players facing each other play more than one match. For instance, in the NBA playoffs, the opponents play as a "best-of-7": the team that first reaches four wins advances to the next round. If multiple knockout tournaments are played by the same players, the SKO is said to be repeated (David, 1959) or serial (Lambers et al., 2024). The results of all subtournaments are aggregated to determine the final winner.

Knockout tournaments are characterized by their *seeding* policy, which is typically used to avoid matches between the strongest contestants at the beginning of the competition. For instance, assume that eight participants are ranked from one to eight, with 1 being the strongest player, 2 being the second strongest player, etc. The *standard* seeding for eight players is shown in Figure 1.a. Notation-wise, we use brackets to represent the structure of the knockout tree, e.g. ((a,b),(c,d)) denotes that the winners of matches (a,b) and (c,d) face each other in the subsequent round. Thus, the standard seeding for eight players can be written as (((1,8),(4,5)),((2,7),(3,6))). The standard seeding ensures that in round $r = 1, \ldots, n$ of a tournament with 2^n players, the sum of ranks in every match of round r is $2^{n-r+1} + 1$ if always the favorite wins. The tree diagram that represents the series of games to be played in a knockout tournament is called the *bracket*. If the seeding is done randomly, the knockout is said to be *unseeded* or random; the study of random KO structures dates back to at least Narayana and Zidek (1969).

In a *balanced* SKO, each player should win the same number of matches in order to win the tournament. This is not the case for an *unbalanced* SKO: in Figure 1.b, players 1 and 2 enter the tournament later than the other players and can claim the tournament victory by winning one game less. The most unbalanced case is shown in Figure 1.c, which is known as a "caterpillar". Unbalanced knockout tournaments are organized in a variety of sports, for instance, in baseball, boxing, bowling, and squash. A prominent example is the qualification for UEFA club competitions (Csató, 2022c).

4

2.1.3 Swiss system

The *Swiss system* is a non-eliminating tournament format where a predetermined number of rounds is played. Contrary to the round robin format, it contains fewer rounds than the $n - 1$ required to meet all players in a tournament with n players. Its key characteristic is that opponents are determined dynamically. In the first round, the matches depend on an exogenous ranking of the players. In all subsequent rounds, opponents are matched based on several criteria, but the primary goal is typically to play against an opponent having approximately the same number of points (Sauer et al., 2024). The Swiss system was first used at a chess tournament in Zürich in 1895, but its variants have recently been applied to schedule other sports competitions, including e-sports tournaments (Dong et al., 2023).

2.1.4 Multi-stage and hybrid tournaments

Tournaments can be played in multiple stages. Typically, only a subset of the players is allowed to proceed to the next stage, based on their performance in the previous stage. If the tournament formats differ in at least two stages, the tournament is commonly called *hybrid*. The FIFA World Cup, with a SRR group stage followed by a knockout stage for the best teams, is a well-known example.

2.2 Ranking

The ultimate aim of organizing a tournament is to rank the contestants. Section 2.2.1 introduces ranking in an individual tournament, Section 2.2.2 deals with the aggregation of rankings, and Section 2.2.3 gives a concise overview of global rankings, which rank the players on the basis of several tournaments.

2.2.1 Tournament ranking and tie-breaking

A tournament ranking gives an order of the teams based on their achievements in the tournament. In some sports, performance can be directly measured by distances or finishing times. In the case of paired comparisons, the score for each possible match outcome (win or loss, in some cases tie) needs to be decided. In football, for instance, a win is typically worth 3 points, a draw 1 point, and a loss 0 points, which is the (3,1,0) scoring/ranking system.

While various sophisticated ranking methods exist, the most widely used approach is to sum these scores. Naturally, this method does not take the quality of a win into account, i.e. matches played against a strong and a weak opponent yield the same number of points. This can be a problem especially in incomplete round robin and Swiss system tournaments, as will be discussed in Sections 7.2.1 and 7.2.3, respectively.

Summing the scores resembles counting the number of opponents a player has beaten, which is the only ranking rule satisfying three appealing axioms (Rubinstein, 1980): anonymity (the ranking method does not depend on the identity of the players), positive responsiveness (a loss does not result in a better rank than a win), and autonomous relative ranking (the relative ranking of two players is not influenced by a match where neither of them is involved in). Henriet (1985) extends this characterization by allowing for ties, which are worth half of a win. While this method is used in some team sports where ties are allowed such as handball, the (3,1,0) scoring system has no theoretical background according to our knowledge.

Another well-known property from social choice theory is *inversion* (Chebotarev and Shamis, 1998): the ranking is reversed if all results are reversed. Consider a SRR with four players i, j, k, ℓ where i won against j and ℓ, j won against ℓ, while the three matches of k resulted in a draw. This example shows that the (3,1,0) scoring system does not satisfy inversion. Indeed, using the notation $(=, >)$ for the pairwise majority relation based on the aggregated score, the ranking according to the (3,1,0) method is $i > j > k > \ell$, and the ranking after all results are reversed is $\ell > j > k > i$. Naturally, the (2,1,0) scoring system satisfies inversion; in our example, the original ranking is $i > j = k > \ell$, and the ranking after all results are reversed is $\ell > k = j > i$.

Often a complete ranking of the players is desired. Therefore, ranking methods contain various tie-breaking rules to rank players with the same score. In round robin tournaments, the most popular criteria are the number of matches won, head-to-head results, and superior goal difference (Csató, 2021c, Chapter 1.3). The effects of choosing a specific hierarchy of tie-breaking rules are often non-negligible; for example, they influence the probability of *heteronomous* relative ranking, i.e. the relative ranking of two teams depends on the outcome of a match in which neither team was involved (Berker, 2014).

Knockout tournaments also require tie-breaking rules in some sports such as football and handball. Tied matches are usually followed by extra time, and the final tie-breaking is usually a penalty shootout since the 1970s (Palacios-Huerta, 2014). UEFA introduced the away goals rule in the 1965/66 European Cup Winners' Cup for two-legged clashes. According to this principle, if both teams score the same number of goals in the two games, the team that scores more goals away will advance.

The majority of tie-breaking rules used in Swiss-system tournaments are based on the idea that players playing against stronger opponents should be preferred. One of the most extensively used methods is *Buchholz*, which is essentially the arithmetic mean of the scores obtained by the opponents. Freixas (2022) suggests using a weighted average of the opponents' scores where the weights are assigned according to the binomial distribution.

2.2.2 Aggregation of rankings

Some tournaments consist of individual contests, which have their own rankings. Examples include cycling (e.g. Tour de France), motorsport racing (e.g. Formula One), or winter sports (e.g. Biathlon World Cup). Here, a score is typically associated with each position in each contest, and they are added to obtain the final tournament ranking. A similar aggregation of rankings can be favourable even if the official rule is based on the "objective" finishing times: Ausloos (2024) argues for ranking teams in cycling races by the cumulative sums of the places of their riders instead of their finishing times. However, the aggregation can lead to different paradoxes; Kaiser (2019) presents some of them from Formula One. Since rank aggregation is an extensively studied problem in social choice theory, we do not go into details; a recent summary is provided by Kondratev et al. (2023).

2.2.3 Global rankings

Ranking can also refer to the global ranking of players in a particular discipline. In that case, the ranking is often based on a rating, which quantifies their strength relative to each other. Since players in this case typically did not face equally strong opponents, the rating should account for the strength of their opponents. The widely used rating method developed by Arpad Elo, a Hungarian-born physics professor, does exactly that (Elo, 1978; Aldous, 2017). After each match, the rating of a player is updated by the formula

$$r_{new} = r_{old} + K(S - \mu), \tag{1}$$

where K is a sports-specific parameter reflecting the maximum possible adjustment per game, S is the outcome of the match (typically 1 for a win and 0 for a loss), and μ represents the expected outcome. The latter is determined by the ratings of players r_i and r_j, as well as the scaling parameter s according to the following formula:

$$\mu = \frac{1}{1 + 10^{(r_j - r_i)/s}}. \tag{2}$$

Naturally, the Elo method can be parameterized in different ways depending on the value of K, the definition of match outcome S, or the adjustment of r_i due to the potential home advantage. Gomes de Pinho Zanco et al. (2024) present a comprehensive stochastic analysis of the Elo algorithm in round robin tournaments, and investigate the effect of parameters K and μ on its performance to obtain design guidelines.

FIFA, for instance, after having received serious criticism in the academic literature (Lasek et al., 2016; Cea et al., 2020; Csató, 2021c; Kaminski, 2022), adopted the Elo method for their world ranking (FIFA, 2018). The popular-

ity of Elo ratings is shown by the fact that the FIFA world ranking has an alternative, the World Football Elo Ratings (https://www.eloratings.net/), that takes home advantage and goal difference into account as suggested by Szczecinski and Roatis (2022). Analogously, the performance of football clubs is measured by at least two independent websites, http://clubelo.com/ and http://elofootball.com/. Nonetheless, some global rankings do not take the strength of opponents into account. For example, UEFA club coefficients are based on awarding points for wins and reaching certain stages in previous UEFA club competitions (Dagaev and Rudyak, 2019).

There are many sports-specific ranking rules and rating systems. Stefani (2011) lists the official rating systems of 159 sports federations, while Langville and Meyer (2012) provide an overview of the mathematics of various ranking methods. An overview of ranking systems in football can be found in Van Eetvelde and Ley (2019).

2.3 Design criteria

Sections 2.3.1–2.3.3 shortly discuss the three main criteria that have been identified to evaluate a tournament design: efficacy, fairness, and attractiveness.

2.3.1 Efficacy

The ability of a tournament to rank contestants according to their true strength is called *efficacy* (Lasek and Gagolewski, 2018; Sziklai et al., 2022). Efficacy with respect to only the best player is also known as *effectivity* (Glenn, 1960) or *predictive power* (Ryvkin and Ortmann, 2008; Vu and Shoham, 2011).

By far the most popular way of evaluating these properties is to use simulation: all matches of the tournament are simulated and the observed ranking is compared with the assumed true ranking of players. By aggregating the results over many simulation runs (i.e. Monte Carlo simulation), meaningful insights can be derived about how efficacious the tournament is. Efficacy is mostly studied from the perspective of the tournament format and the seeding rule.

2.3.2 Fairness

In contrast to efficacy, there is no uniform definition of fairness as there exists no universally agreed system of ethics (Baumer, 2024; Palacios-Huerta, 2023). Still, we have identified two main principles of fairness. The first one is based on the idea that, *a priori*, a player should not be disadvantaged compared to any weaker player. This means that, for every player, the probability of winning the tournament should be higher than that of any weaker player. This concept is also commonly called *order preservation* (Schwenk, 2000; Groh et al., 2008; Vu and Shoham, 2011; Prince et al., 2013; Karpov, 2016; Placek, 2023).

The second approach of fairness takes on a completely different perspective. It is based on the principle of treating all players equally and tries to eliminate any favoritism towards particular players. For instance, if players are allocated to groups based on a draw, the procedure is said to be fair if all allocations are equally likely. Analogously, fairness in scheduling is mostly related to this principle by balancing some objectives (the number of rest days, the distances to be traveled, the number of consecutive home or away games, etc.) over the teams. Equal treatment of all players is sometimes weakened to equal treatment of equally strong players (Guyon, 2018b; Arlegi and Dimitrov, 2020; Csató, 2022a).

Note that the first fairness principle is closely related to efficacy. In fact, the main reason why an organizer would want to adhere to this principle is to guarantee efficacy. Hence, to reduce the overlap between the discussion of our design criteria, fairness will mean the principle of equal treatment in the following. Therefore, papers studying fairness from the perspective of order preservation will be presented in the sections dealing with efficacy.

2.3.3 Attractiveness

The attractiveness of a match can be defined in multiple ways: by its quality, competitive intensity, and importance. The quality of a match played by players i and j with their strengths r_i and r_j, respectively, can be quantified as $r_i + r_j$, while its competitive intensity can be defined as $-|r_i - r_j|$ (Csató, 2020a; Dagaev and Rudyak, 2019; Dagaev and Suzdaltsev, 2018). Both measures seem to be linked with higher attendance rates (Forrest and Simmons, 2002; García and Rodríguez, 2002) and TV audiences (Schreyer et al., 2018).

On the other hand, the importance of a match is usually defined with respect to a certain prize. Jennett (1984) suggested a simple measure to estimate the uncertainty of winning the championship *ex post*. Let k be the total number of points that turned out to be necessary to win the championship. The importance of a match is the inverse of the minimum number of matches required to achieve k points. This metric was later used by Borland and Lye (1992) and Dobson and Goddard (1992) to investigate attendance rates. A decade later, Schilling (1994) introduced probably the importance measure that is probably most well-known. The Schilling importance $_iS(X)_{t,t+k}$ of match $t + k$ at time period t for team i is defined as:

$$_iS(X)_{t,t+k} = Pr(_iX|_iW_{t+k}, H_t) - Pr(_iX|_iL_{t+k}, H_t), \tag{3}$$

where $Pr(_iX|_iW_{t+k}(L_{t+k}), H_t)$ is the probability that team i wins prize X (e.g. avoiding relegation) if i wins (loses) the match $t + k$, given the history H_t of all matches played before t. Lahvička (2015b) extends Schilling's measure to account for ties.

Many other ways have been suggested to quantify match importance. Goossens et al. (2012) take into account the number of teams that are still competing for a given prize. An integer program is used to determine the highest and lowest rank any team can still obtain at the end of the season when a certain match is played. Similar formulations are proposed by Ribeiro and Urrutia (2005); Raack et al. (2014); Gotzes and Hoppmann (2022). Geenens (2014) measures the decisiveness of a match based on the expected entropy (i.e. uncertainty). Corona et al. (2017) use an entropy-based measure to identify the most decisive matches in the 2014 FIFA World Cup and the 2015 Copa América. Goller and Heiniger (2024) extend the entropy measure by considering the difference between the reward probability distributions derived from the possible outcomes of a single event. Csató et al. (2024b) introduce a classification scheme to determine weakly (where the final rank of one team is known, regardless of the match outcome) and strongly (where the match outcome does not influence the ranking of both teams) stakeless matches.

There is a wide consensus in the literature that still being in contention for a prize positively affects attendance (García and Rodríguez, 2009; Pawlowski and Nalbantis, 2015; Losak and Halpin, 2024). In particular, the tightness of the contention seems to control the size of this effect (Krautmann et al., 2011). Furthermore, Buraimo et al. (2022) show that match importance is the key driver for the size of TV audiences in the case of English football games instead of the much more researched outcome uncertainty (see the so-called outcome uncertainty hypothesis (Rottenberg, 1956), and Schreyer and Ansari (2022) and van der Burg (2023) for recent overviews).

While importance is usually measured at the level of individual games, suspense refers to uncertainty in the outcome of the tournament. Although suspense can be related to any outcome, such as relegation or qualification for international competitions, it is typically interpreted within the context of the tournament winner. Suspense is an important measure of attractiveness as it is positively related to both stadium attendance (Pawlowski and Nalbantis, 2015) and TV audience (Bizzozero et al., 2016; Buraimo et al., 2020, 2022).

3 Tournament format

The comparison of tournament formats has received much attention in operational research since determining the format is probably the first task of the contest organizer. Even if the number of games that can be played is strictly limited, usually several options remain available. For example, Guyon (2020) uncovers a serious drawback of the original format of the 2026 FIFA World Cup and lists 7 alternative options. Rennó-Costa (2023) presents another one based on double elimination, while Guajardo and Krumer (2024) develop three further formats. Choosing the best from such a set of feasible designs clearly requires their evaluation with respect to a number of criteria.

3.1 Efficacy

At its inception, the tournament design literature was mainly concerned with the probability of the best player winning under a given tournament format, also called effectivity. Pioneering work was done by David (1959), who used combinatorial techniques to compute exact winning probabilities for a SKO and SRR. To that end, a series of pairwise win probabilities needs to be assumed; π_{ij} is the probability that i beats j, which are collected in matrix Π. David (1959) assumes a cumulative normal distribution $\Phi(\cdot)$ for the pairwise win probabilities, i.e. $\pi_{ij} = \Phi\{\frac{1}{\sigma}(r_i - r_j)\}$, with σ being the variance term and r_i being the "true" rating of player i. Glenn (1960) and Searls (1963) apply a similar approach to evaluate tournament structures, although they assume an arbitrary matrix Π. All three papers compute the probabilities that other players finish in a certain position as well. Although limited to tournaments with 8 players, the authors find SKO to be the least effective format. Double and serial knockout formats are more effective, in particular, if matches are played as a best of three. For random knockout tournaments, Knuth and Lossers (1987) compute the chance of player i winning the tournament, assuming that it wins against player $i+1$ with probability p and always defeats player j if $j > i+1$. Adler et al. (2017) derive an upper and lower bound on the probability for of winning the tournament for all players, if player i wins against player j with probability $v_i/(v_i + v_j)$, where the values $v_1, \ldots, v_n$ are exogenously given.

As the exact computation of these expressions quickly becomes intractable, the standard approach in the literature is to perform Monte Carlo simulation. Using this approach, several papers conclude that round robin tournaments are typically more efficacious than knockout tournaments (Appleton, 1995; McGarry and Schutz, 1997; Ryvkin and Ortmann, 2008; Scarf et al., 2009; Ryvkin, 2010; Scarf and Yusof, 2011; Csató, 2021b). Furthermore, the efficacy of a SKO increases by extending it towards a DKO (McGarry and Schutz, 1997; Stanton and Williams, 2013). Efficacy can usually be improved by increasing the number of matches (Lasek and Gagolewski, 2018) but the relationship is not necessarily monotonic for complex hybrid tournaments (Csató, 2021b). According to earlier studies (Appleton, 1995; Dinh et al., 2020), the Swiss system is rather weak in identifying the true ranking of players. However, Sziklai et al. (2022) find that the Swiss system outperforms other formats with the same number of games. To the best of our knowledge, the efficacy of incomplete round robin tournaments has not been examined yet.

Lahvička (2015a) studies the effect of the additional playoff stage on the efficacy of a DRR in the Czech ice hockey league. The average probability that the strongest team becomes a champion is found to decrease from 48% to 39%. The reduction is even higher if the strongest team is more dominant, and is robust with respect to relaxing the assumptions of constant team strengths and the lack of strategic behavior. This is an important message to administrators: uncertainty can be increased by organizing an extra knockout phase after a round robin tournament.

The balancedness of a knockout tournament also affects its efficacy. Balanced knockout tournaments with eight or more players cannot satisfy simultaneously that a) stronger players are favored over weaker players; and b) equally strong players are treated equally (Vu and Shoham, 2011; Arlegi and Dimitrov, 2020). In particular, pairwise probability matrices can be found to show the incompatibility of these two axioms, see Kulhanek and Ponomarenko (2020) and Arlegi (2022). Stronger players can be favored more explicitly by an

unbalanced knockout tree. In the most extreme case of a caterpillar SKO (Figure 1.c), a stronger player can always be favored over a weaker player (Vu and Shoham, 2011). Arlegi and Dimitrov (2020) generalize this finding: any unbalanced tree is verified to be efficacious if it satisfies the following two properties: a) it does not contain any *antler* (a special substructure, see Arlegi and Dimitrov (2020)); and b) the seeding rule places weaker players to higher levels in the knockout tree than stronger players.

A structured overview of the papers that either compute or simulate winning probabilities is presented in Tables 1. The formats considered are given in the second column. The third column shows the pairwise win probabilities π_{ij} that were used to compare the formats. Using Poisson distribution or arbitrary win probabilities turns out to be the most popular approaches. Next, the best and worst formats of all formats studied are reported. Finally, the last column indicates the number of players in the tournaments compared.

As can be seen from Table 1, almost all studies use different statistical models, each with its limitations. This raises questions about whether the results can be attributed to the tournament format or to the model assumptions. However, as noted by Appleton (1995), these simulation studies primarily aim to compare different tournament formats, and not to correctly estimate the chance of a player winning any particular tournament. Still, it would be interesting to see how the efficacy of tournament formats is influenced by the assumptions that are made about the strengths of the teams.

Finally, the effect of the number of participants (n) on efficacy is still a blind spot. Muller et al. (2023) use simulations to analyze how expanding the NCAA college football championship from four to eight or 12 teams affects the probability that the championship tournament includes and is won by the true best team. Further research in this direction would be particularly welcome since there exist many examples of well-known tournaments that have seen a change in the number of participants: the FIFA World Cup is organized with 32 teams in 2022 but 48 in 2026, the UEFA European Championship was contested by 16 teams in 2012 but by 24 in 2016, and the World Men's Handball Championship expanded from 24 to 32 teams between 2017 and 2019.

3.2 Fairness

The principle of equal treatment is discussed separately for the main tournament formats.

3.2.1 Round robin tournaments

In a round robin tournament, no player is a priori advantaged over any other player since each player faces each other player the same number of times. In the case of a kRR with odd k and matches played at the venue of the home team, a possible element of unfairness can be that different teams enjoy their home advantage against different teams. This happened for instance in the UEFA Cup between 2004 and 2009, when the group stage had groups of 5 playing SRR. Additionally, if the number of teams is even, half of the teams have one more home game than the remaining teams. This is an issue, for example, in the Faroe Islands and Hungarian domestic football league, which are 3RR tournaments.

In an incomplete round robin tournament, teams do not face each opponent the same number of times, which conflicts with the principle of equal treatment. One solution can be to determine each team's opponents such that the total strength of the opponents is the same over all teams. Fronček et al. (2006) and Fronček (2007) give conditions for the existence of this so-called *fair incomplete tournament*. On the other hand, Fronček (2013) designs an incomplete round robin tournament where all teams have *a priori* roughly the same chance of winning since the highest-ranked teams face the toughest opponents, and weaker teams play against weaker opponents; he calls this a handicap incomplete tournament.

Sporadically, unforeseen circumstances (e.g. COVID-19) force a round robin tournament to be incomplete. Hassanzadeh et al. (2024) focus on the problem of finishing a suspended league where there are not enough rounds left to complete all matches. They propose a data-driven model that exploits predictive and prescriptive

Table 1: Studies on the efficacy of tournament formats

a Papers focusing on efficacy

Study	Format	π_{ij}	Best	Worst	n
McGarry and Schutz (1997)	SKO, SKO$_S$, SKO$_A$, DKO, DKO$_S$, SRR, SRR+SKO, SRR+SRR	Π^a	DKO$_S$, SRR, SKO$_S$	SKO$_a$	8
Scarf et al. (2009)	DRR, DKO, DKO$_S$, SKO$_S$, SRR+DKO, SRR+DKO$_S$, DRR+DKO, DRR+DKO$_S$, DRR+SKO$_S$	Poisson($\cdot$)	DRR	SKO	32
Scarf and Yusof (2011)	23 formats	Poisson($\cdot$)	SRR	SKO	32
Stanton and Williams (2013)	SKO, DKO	CRM, $\frac{1}{2} + \frac{j-i}{2(N-1)}$	DKO	SKO	$8-10^{24}$
Lasek and Gagolewsk (2018)	SRR, multistage RR	Poisson($\cdot$)	3RR + (SRR/SRR)	SRR	12, 16, 18
Dinh et al. (2020)	SKO, DKO, SKO$_S$, SRR, Swiss, Reaper T	NS	SRR	SKO	8
Csató (2020a)	Three hybrid formats: D(8+6), D(4×7), D(4×6)[1]	$\frac{(57-r_i)^k}{(57-r_i)^k+(57-r_j)^k}$	D(8+6)	D(4 × 6)	24, 28
Csató (2021b)	Four hybrid formats: SRR, SKO, G64, G66, G64[2]	$\frac{1}{1+[(r_i+24)/(r_j+24)]}$	G66	G64	24
Sziklai et al. (2022)	SRR, DRR, SKO, SKO-3, DKO, SRR+SKO, Swiss	$\frac{1}{1+10^{(r_j-r_i)/400}}$	Swiss	SKO	32

b Papers focusing on effectivity

Study	Format	π_{ij}	Best	Worst	n
David (1959)	SRR, SKO, serial SKO	$\Phi\{(r_i - r_j)/\sigma\}$	serial SKO	NS	3-8
Glenn (1960)	SRR, SKO, SKO-3, serial SKO, DKO	Π^a	SKO-3	SKO	4
Searls (1963)	SKO, SKO-3, SKO+SKO, SKO-3+SKO-3	Π^a	SKO-3+SKO-3	SKO	8
Appleton (1995)	SKO, SKO$_S$, DKO, DKO$_S$, SKO-3, SRR, DRR, SRR-SKO, Swiss	$\Phi\{k(r_i - r_j)\}$	DKO$_S$, DRR	SKO	8, 16
Ryvkin and Ortmann (2008)	SKO, SRR	pdf[3]	SRR	SKO	NS
Ryvkin (2010)	SKO, SRR	pdf[3]	SRR	SKO	NS

NS: not specified

kRR+xKO stands for a hybrid format where kRR is followed by xKO.

The subscript S as in SKO$_{S(A)}$ indicates that the SKO is seeded according to the standard (an alternative) seeding.

xKO-y: best of y knockout format

[1] D(8+6): 2 groups of 8, 2 groups of 6; D(g, n): g groups of n teams, all followed by a knockout stage.

[2] Gxy: stage 1 with groups of x teams and stage 2 with groups of y teams, followed by a knockout stage.

[3] Due to the notational complexity of probability density functions (pdf), we refer to the paper itself for the formulas for π_{ij}.

analytics to select a subset of originally scheduled games, in a way that the resulting ranking is similar to the ranking that would have resulted had the season been completed fully.

3.2.2 Knockout tournaments

Fairness is an important criterion in serial knockout tournaments such as the format that has recently been adopted by the Professional Darts Corporation. According to Lambers et al. (2024), each pair of players should potentially meet equally often in each round. By making a connection to Galois fields, they construct a stable tournament with $n - 1$ subtournaments for any number of players n that is a power of two.

Contrary to a serial SKO, the four prestigious Grand Slam tennis tournaments are SKOs played independently from each other. However, since the seeding only aims to avoid matches between top players in earlier rounds, a weak player sometimes has to face strong players over-abundantly. Therefore, Della Croce et al. (2022) develop an integer quadratic program together with a heuristic to maximize the diversification of pairings in the first round. Furthermore, repeated pairings over the tournaments are limited as much as possible.

3.2.3 Swiss system tournaments

In these tournaments, the pairing for a round depends on the results of the previous rounds (see Section 2.1.3). Therefore, the tournament format can be described only together with the pairing rules that determine the set of matches to be played in the next round. The main goals are as follows: (1) players with (approximately) equal scores should play against each other; (2) each player alternately has the advantage (e.g. play white in chess or play at home in other sports) to the extent possible; and (3) no player is allowed to face the same opponent twice.

Ólafsson (1990) reveals how the pairing can be implemented by solving a weighted matching problem, which converts the pairing rules into penalty points. Similarly, Biró et al. (2017) show that the more complex matching procedures as described by the World Chess Federation (FIDE) can be calculated in polynomial time with maximum weight matching. Kujansuu et al. (1999) suggest using the well-known stable roommates algorithm to determine the pairs in a Swiss system tournament such that each player has a preference list on the other players, where the preferences are constructed to reflect the criteria above. Sauer et al. (2024) present an innovative pairing rule by computing maximum weight matchings in a carefully designed graph. On the basis of extensive simulations, the proposed procedure yields fairer pairings, and produces a final ranking that reflects the players' true strengths better than the FIDE pairing system.

3.3 Attractiveness

In some tournament formats, unimportant matches are difficult or even impossible to avoid. Faella and Sauro (2021) define a match as irrelevant if its outcome cannot change the ranking, and prove that kRR tournaments with more than five players, a fixed schedule and ranking by the win-loss method may contain irrelevant matches. Scarf and Shi (2008) calculate the Schilling importance and conclude that 52% of the matches in the second half of the 2004/05 English Premier League season did not have any effect on which team would win the league. Hence, although RRs are typically quite efficacious, they are also characterized by a high proportion of unimportant matches. According to Scarf et al. (2009), on average 18% of all matches are unimportant in a DRR.

Goossens et al. (2012) simulate match results based on historical data to compare four different competition formats considered for the Belgian football league. The 2RR format used at that time scored rather poorly; indeed, the tournament format of the league was reformed not much later. Csató (2020a) compares the expected quality and competitive balance of alternative tournament formats for the EHF Men's Handball Champions League. Similarly to Goossens et al. (2012), formats other than the traditional one are revealed to increase the attractiveness of the tournament.

Stakeless games (see Section 2.3.3) are detrimental not only due to problems with incentives but they are also probably much less interesting for the audience. The FIFA World Cup format has been extensively studied from this perspective (Chater et al., 2021; Guyon, 2020; Stronka, 2024). The novel incomplete round robin format of the UEFA Champions League (Section 2.1.1) is found to substantially decrease the probability of stakeless matches

(Gyimesi, 2024).

According to Section 3.1, the tournament outcome is generally less uncertain if more games are played. Thus, SKO provides more suspense than other formats, which can explain why many competitions include post-season playoffs (Bojke, 2007). The number of players qualifying for the playoffs also has an impact on suspense. Olson and Stone (2014) study the 2014/15 NCAA playoffs that featured only the two strongest teams from the regular season. Suspense is measured at any time period as the expected change in the teams' probabilities of being champion, summed across ranked teams. A playoff format with more than two teams is found to decrease overall suspense, because the gain in suspense in the playoffs resulting from playing more games does not counter the drop in suspense during the regular season.

In an incomplete round robin tournament, the number of matches between two teams is also a decision variable. Lenor et al. (2016) and Kyngäs et al. (2017) discuss how more popular matches (e.g. between local rivals) are selected to occur more frequently in the Australian Football League in order to boost attendance. Lenor et al. (2016) claim they could – at least as a short-term effect – increase attendance by 7% by programming more rivalry games at the expense of the randomly selected matches.

The number of participating teams is also an important consideration with respect to the attractiveness of the tournament format. In football, the practice seems undecided on this matter: of the 25 European football leagues reported in Goossens and Spieksma (2012b) for the 2008/09 season, 8 countries have increased their number of participants by the 2023/24 season, while 5 decreased it. A structural change happened in the basketball EuroLeague in the 2016/17 season, as the number of participating teams has been reduced from 24 to 16, while the number of games per team has increased. Di Mattia and Krumer (2023) find that this reform decreased attendance by about 9 percent per game, predominantly in the first half of the new DRR with 16 teams where the matches are probably perceived as less important.

4   Seeding

Seeding is typically used to avoid the top-ranked teams eliminating each other in the earliest rounds. Hence, round robin and Swiss-system tournaments do not require seeding but knockout tournaments, as a stand-alone tournament or as part of a multi-stage tournament, call for seeding. Naturally, the seeding policy may affect our three design criteria.

4.1   Efficacy

Seeding can directly enhance the ability of a tournament to select the best players as the winners. However, a misaligned seeding policy might easily favor a weaker player compared to a stronger one.

4.1.1   Single knockout tournaments

Perhaps the most popular seeding method is the standard seeding, see Figure 1.a. For four players, the seeding ((1,4),(2,3)) is generally considered to be the most efficacious option as the probability of winning will be a monotonic function of the rank of players (Horen and Riezman, 1985; Groh et al., 2008). For a SKO with eight players and $\pi_{ij} = r_i/(r_i + r_j)$, the standard seeding (((1,8),(4,5)),((2,7),(3,6))) is not the most efficacious: the second strongest player has a higher probability to advance to the finals than the strongest player (Prince et al., 2013). The reason is that the top player always plays against a player ranked 4 or 5 in the semifinals, while the second best player has a positive probability of playing against the player ranked 6, which increases its chance of reaching the final. Therefore, the seedings (((1,8),(2,7)),((3,6),(4,5))) and (((1,8)(3,6)),((2,7)(4,5))) can be more efficacious than the standard seeding (Prince et al., 2013). However, Horen and Riezman (1985) unexpectedly find that up to eight different seedings can maximize the probability of the best player winning the tournament. All eight

seedings are characterized by player 1 being matched with either player 7 or 8 in the first round, and players 2, 3 and 4 being in the other side of the bracket as 1.

Baumann et al. (2010) observed empirically that the 10th and 11th seeds typically progress further in the playoffs of the NCAA basketball tournament compared to the 8th and 9th seeds. Although the 8th and 9th seeds have a higher probability of winning in the first round, they almost certainly play against the first seed in the second round. In contrast, the 10th and 11th seeds have a more difficult first round but will meet the 2nd or 3rd seed in the next round. Hence, the advantage in the second round seems to outweigh the disadvantage in the first round. This also shows why fairness is sometimes related to efficacy.

Intuitively, random seeding can easily make a stronger player's winning probability smaller than that of a weaker player, since the stronger players might be unequally distributed over both sides of the bracket (Israel, 1981). This is especially true for an unbalanced KO. However, if the pairwise winning probabilities of players are *strongly stochastic transitive*, namely, Π is such that $i < j$ implies $\pi_{ik} > \pi_{jk}$ for all k and $\pi_{kk} = 1/2$, Chen and Hwang (1988) prove that balanced SKOs do not suffer from this anomaly. The condition of strong stochastic transitivity is crucial since the result does not hold for an arbitrary winning probability matrix (Arlegi and Dimitrov, 2020). Therefore, Vu and Shoham (2011) argue that efficacy can be guaranteed only if the seeding is allowed to vary according to the win probabilities, while Hwang (1982) advocates to use reseeding, i.e. seeding (re)done in every round depending on the relative ranks of the players in that round.

Hennessy and Glickman (2016) also investigate the optimal seeding that maximizes the probability of the best player winning the SKO but only partial information is assumed to be known about the strengths of the players. For 16 players, the optimal bracket places the seven weakest players on the side of player 1. Naturally, a different utility function can lead to another optimal seeding. For example, the seeding that maximizes the probability of the best two players meeting in the final is ((((1,16),(13,14)),((6,7),(8,9))),(((2,15),(11,12)),((3,10),(4,5)))). Since it is computationally prohibitive to obtain the exact values for 16 players, Monte Carlo simulation is used together with simulated annealing to estimate the utility functions. Hennessy and Glickman (2016) show that the most efficacious design depends on the subset of players that must be ranked according to their true strength.

Karpov (2016) proposes the so-called equal gap seeding, which is (((1,5)(3,7)),((2,6)(4,8))) for eight players. Here, the gap between the strength of players is balanced over all matches. The equal gap seeding can be characterized by a set of reasonable axioms and is the unique seeding that maximizes the probability that the strongest participant is the winner, the strongest two participants are the finalists, etc. under the assumptions of the paper.

Karpov (2018) formulates desirable properties for seeding in generalized knockout tournaments where one match involves three or four players. Both the standard and the equal gap seeding are extended to this case and justified by a set of axioms.

A structured overview of the papers analyzing seeding rules with respect to efficacy are provided in Table 2.

4.1.2 Multi-stage tournaments

In the playoffs for the 2020 UEFA European Football Championship, 16 teams were divided into four *parallel* SKOs (paths) with four teams each based on the results of the 2018/19 UEFA Nations League. Csató (2020b) defines a partial ranking on the set of these SKOs. The rules intend to favor UEFA Nations League group winners, creating appropriate incentives in this tournament. Still, they do not exclude a group winner playing in a more difficult playoff path than a non-group winner of the same league, which is not efficacious. The problem can be solved in an illuminating way: if a group winner envies the position of a non-group winner of its league, then its envy is justified and the group winner can request an exchange of the two teams. The play-offs in the European section of the 2022 FIFA World Cup qualification and the qualification for the 2024 UEFA European Championship suffer from the same theoretical shortcoming.

Seeding systems used in practice have also been extensively investigated with respect to efficacy. Dagaev and Rudyak (2019) and Corona et al. (2019) both analyze a reform in the seeding of the UEFA Champions League group stage

Table 2: Studies on the efficacy of seeding policies

a Papers focusing on efficacy

Study	Best	Worst	n
Hwang (1982)	Reseeding	Standard	arbitrary
McGarry and Schutz (1997)	(((1,8),(5,4)),((2,7),(6,3)))	(((1,8),(6,7)),((2,3),(4,5)))	8
Groh et al. (2008)	((1,4),(2,3))	Non-seeded	4
Scarf and Yusof (2011)	Reseeding	Random	32
Karpov (2016)	Equal gap	Standard	arbitrary

b Papers focusing on effectivity

Study	Best	Worst	n
Searls (1963)	(((1,8),(6,7)),((2,3),(4,5)))	(((1,3),(2,8)),((4,7),(5,6)))	8
Horen and Riezman (1985)	(((1,7/8), (5/6, 6,7)),((2, 3/4), (x,x)))	Non-seeded	4, 8
Hennessy and Glickman (2016)	((((1,16),(14,15)),((10,11),(12,13))), (((2,3)(4,5)),((6,7)(8,9))))	Random	16

by Monte Carlo simulation. The changed policy, in use from season 2015/16 to 2023/24, placed the champions of the strongest national associations in the first pot rather than the highest-ranked clubs. While this has only marginally worsened efficacy, the impact on individual clubs has been more powerful, sometimes modifying the probability of qualification for the Round of 16 by 10-15 percentage points.

On the other hand, the empirical study of Engist et al. (2021) has arrived at a remarkable conclusion based on the results of the UEFA Champions League and UEFA Europa League until 2019/20. In particular, even though a better seed implies weaker opponents on average in the group stage, it does not lead to a higher probability of qualifying for the knockout stage. In other words, seeding itself does not affect efficiency and efficacy. Consequently, it would be especially welcome to carry out similar regression discontinuity studies for other sports and tournaments.

Under the standard seeding, players should theoretically exert full effort in the group stage since this will lead to easier opponents at the beginning of the knockout stage. Nonetheless, the conventional bracket suffers from several weaknesses: the role of luck seems to be excessive and players have an incentive to deliberately lose in some situations (Hall and Liu, 2024). Therefore, it has been suggested to allow higher-ranked players to choose their opponent sequentially in each round (Guyon, 2022). This mechanism can increase the probability that the strongest contestant wins and the two strongest contestants play the final, as well as reduce shirking (i.e. deliberately losing a game in order to obtain an easier path through the tournament) (Hall and Liu, 2024).

A variant of the "choose your opponent" policy is used in skiing sprint elimination tournaments, however, the athletes seem to make suboptimal decisions (Lunander and Karlsson, 2023). The International Central European Hockey League also uses this rule to some extent. In particular, the teams ranked seventh to tenth after the regular (4RR) season play pre-playoffs where the seventh-placed team can choose whether to play the ninth or tenth-placed team. Analogously, the top three teams sequentially select their opponent in the quarterfinals, but the fourth-ranked team is not available to choose. Finally, the host Egypt had the right to assign themselves to a group of their choice in the 2021 World Men's Handball Championship (IHF, 2020).

4.2   Fairness

Equal treatment of equals is closely related to efficacy in knockout tournaments according to Section 4.1, so this topic will only shortly be discussed here. On the other hand, it poses a great challenge for certain hybrid tournaments.

4.2.1 Single knockout tournaments

While a balanced SKO may violate the efficacy principle of stronger players having a higher probability of winning the tournament, the seeding ((1,4)(2,3)) guarantees that equally strong players are treated equally in knockout tournaments with four players (Arlegi and Dimitrov, 2020).

Schwenk (2000) defines a SKO fair if the probability of winning the tournament is proportional to the inherent strength for each player. However, the author adds that this condition should not be guaranteed by receiving a favorable schedule, a property called *favoritism minimized*. In order to satisfy this requirement, the so-called *cohort randomized seeding* is suggested that involves random seeding and contains k cohorts for 2^k players. The first cohort consists of the two "best" players, i.e. $C_1 = \{1, 2\}$. Then, $C_i = \{2^{i-1} + 1, 2^i\}$ for all $2 \leq i \leq k$. The seeding of the SKO is designated by the numbers $1, \ldots, k$. A random draw determines the assignment of players to the cohorts. This method improves efficacy and still adheres to the desirable properties of knockout tournaments.

4.2.2 Multi-stage tournaments

To create appropriate incentives to perform in the group stage, the subsequent knockout stage typically matches group winners with teams that performed worse in the group stage. However, this natural structure can be achieved only if the number of groups is a power of two, which might be prohibited by time constraints. A prominent example is given by a tournament with 24 teams, with a group stage played in six groups of four teams. In football, three FIFA World Cups (1986, 1990, 1994), the UEFA European Championship since 2016, as well as the 2015 and 2019 FIFA Women's World Cups follow this format. The six group winners and the six runners-up qualify for the Round of 16, together with the four best third-placed teams. Thus, some group winners should be matched with a runner-up, and some with a third-placed team in the Round 16.

The traditional design, used until 2016 and depicted in Figure 2 suffers from severe shortcomings (Guyon, 2018b; Csató, 2021c). Indeed, the groups are not treated equally; the winner and runner-up of Group A (E) enjoy a much easier (tougher) road to the final compared to the other groups, which violates equal treatment of all players (especially if the host is automatically assigned to Group A). Furthermore, the policy used to assign the four third-placed teams to the matches R1–R8 in the Round of 16 is unfair, as the probability of ending up in a harsh quarter (i.e. where one meets a group winner in the quarterfinals) is highly unbalanced over the groups. Eventually, the 2019 FIFA Women's World Cup has exchanged some matches in the knockout bracket (Csató, 2021c) to mitigate these issues, and UEFA has implemented the recommendation of Guyon (2018b) in the 2020 and 2024 UEFA European Championships.

However, this issue remains relevant due to the recent decision of FIFA (FIFA, 2023a) to play the 2026 FIFA World Cup with 48 teams, in 12 groups of 4, essentially "doubling" the previous tournaments with 24 teams. Thus, similar to UEFA, FIFA can benefit from the proposal of Guyon (2018b) for future World Cups.

4.3 Attractiveness

There is an obvious trade-off between efficacy and suspense in knockout and multi-stage tournaments: suspense is heavily reduced in the earlier rounds by deliberately matching the strongest and the weakest teams (Ely et al., 2015). Karpov (2016) compares the equal gap seeding (Section 4.1.1) with the standard seeding and shows that the former results in a higher competitive intensity. Dagaev and Suzdaltsev (2018) consider seedings in a SKO with eight players concerning the quality and competitive intensity of the matches. Perhaps unsurprisingly, the seeding (((1,2),(3,4)),((5,6),(7,8))) is optimal for competitive intensity, and the seeding (((1,8),(2,7)),((3,6),(4,5))) maximizes the overall quality of the tournament.

Placek (2023) interprets competitive intensity similarly to Dagaev and Suzdaltsev (2018). Fairness is defined by minimizing the sum of the ranks of the winners in each match since, in a fair tournament, any player performs
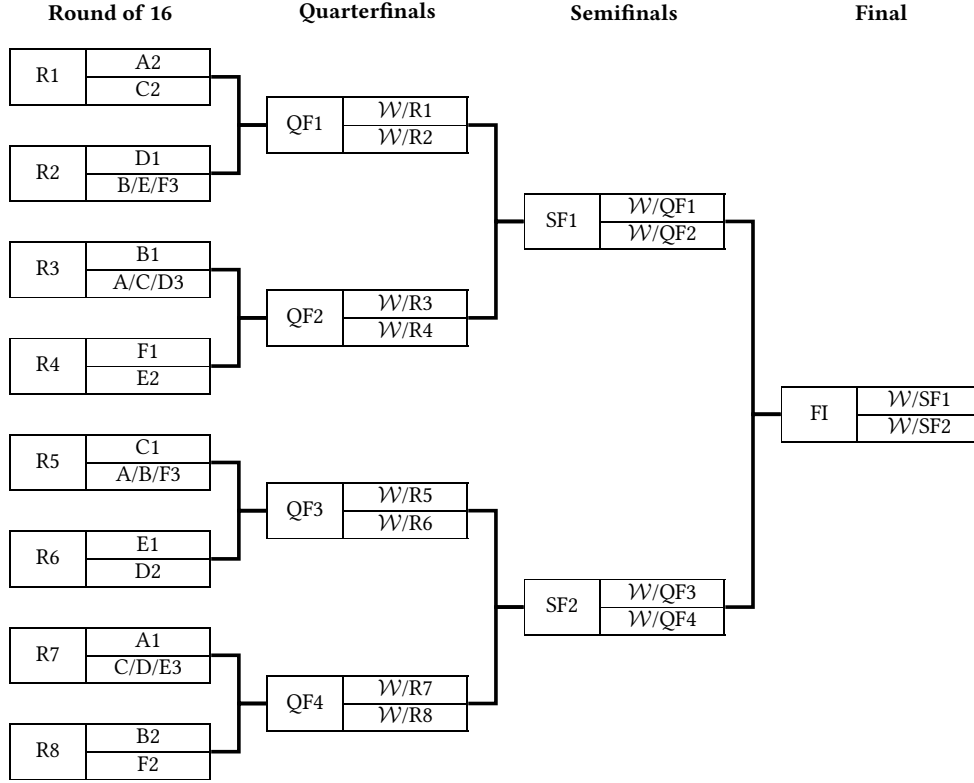
Figure 2: The traditional knockout bracket of football tournaments with 24 teams (Csató, 2021c, Figure 5.1)

Teams are on the right hand side, for instance, A2 is the runner-up of Group A, while B/E/F3 is the third-placed team from Group B, or Group E, or Group F, depending on the allocation rule of the third-placed teams.
$\mathcal{W}$ indicates the winner of the previous round.

at least as well as the lower-ranked players. An impossibility result is derived: under some reasonable conditions, a tournament cannot optimize both competitive intensity and fairness.

5 Draws

A draw refers to the allocation of teams to groups or knockout brackets, thus, it certainly affects efficacy (Section 5.1) and attractiveness (Section 5.3). Nonetheless, tournament organizers usually want to retain some randomness in this procedure in order to maximize public interest and carry out a draw as a live media show, which may have a price in fairness as presented in Section 5.2.

5.1 Efficacy

The FIFA World Cup draw has received a lot of criticism in the last decade for creating imbalanced groups, mainly due to the misaligned policy of guaranteeing the geographic diversity of the groups. Lapré and Palazzolo (2022) empirically assess competitive balance between the groups in the FIFA Women's World Cups from 1991 to 2019, while Lapré and Palazzolo (2023) repeat this analysis for the FIFA World Cups organized between 1954 and 2022. The groups of both tournaments are shown to exhibit substantial competitive imbalance.

Several solutions have been proposed in the literature to create groups with approximately equal strength subject to geographic constraints. Guyon (2015) suggests improving the draw procedure with an S-curve-type algorithm. Laliena and López (2019) develop another algorithm that outperforms the S-curve-type draw system of Guyon (2015). Both proposals involve a heuristic extraction of teams from pots. Laliena and López (2019) also provide a procedure for perfectly balanced groups, where the strength of each group coincides, by finding all

equal-sum partitions first and drawing one of them randomly.

The traditional solution to create k balanced groups is assigning the teams to k pots based on their strength such that teams of similar strength are in the same pot. Then, each group will contain a randomly drawn team from each pot. FIFA has introduced this procedure from the 2018 FIFA World Cup (Guyon, 2018a). However, a necessary precondition of balancedness is the appropriate classification of the teams into the pots. This cannot be guaranteed if the underlying ranking – such as the previous FIFA World Ranking – is inaccurate, which will be shortly discussed in Section 7.1.

Sometimes the identity of the participants is not known at the time of the draw. For example, the three winners of the playoffs have automatically been assigned to the weakest Pot 4 in the 2022 FIFA World Cup draw. This seeding policy has not balanced the strengths of the groups to the extent possible: a better alternative would have been to assign the placeholders according to the highest-ranked potential winner, similar to the rule used in the UEFA Champions League qualification (Csató, 2023b).

Entry to a tournament may be determined by geography. The most prominent example is the FIFA World Cup, where the six continental confederations of FIFA are responsible for their qualifying tournaments. However, the current system is based neither on ensuring the participation of the best teams in the world, nor does it fairly allocate slots by the number of teams per confederation or any other reasonable metric (Stone and Rod, 2016). Hence, exploiting the opportunity offered by the increase in the number of teams in the FIFA World Cup from 2026, Krumer and Moreno-Ternero (2023) explore the allocation of additional slots among continental confederations with the standard tools of fair allocation. Csató et al. (2024a) extend the methodology of the FIFA World Ranking to compare the performance of FIFA confederations for a merit-based allocation of qualifying slots among them. Csató (2023d) investigates the fairness of the 2018 FIFA World Cup qualification to verify that the qualifying probabilities of national teams differ substantially between continents and do not reflect the Elo ratings. For example, the move of Australia from the Oceanian to the Asian zone has increased its probability of playing in the 2018 FIFA World Cup by about 65%. A partial remedy is proposed by fixing the draw of the inter-continental playoffs.

Balancedness can also be achieved through optimization. Cea et al. (2020) provide a mixed integer linear program to balance the strength of the groups and satisfy all geographical constraints. The model is applied for the 2014 FIFA World Cup, and is shown to create groups with more balanced relative strengths compared to the original draw. Melkonian (2021) gives integer programming formulations for organizing recreational doubles tennis competitions such that for every player, the average ranking of partners in all matches is as close as possible to the average ranking of the opponents in all matches. Nevertheless, an optimization model can have multiple optimal solutions, although generating all of them may be computationally challenging. In this case, rather than simply letting the solver pick one, one should fairly select an optimal solution, which may again involve a lottery (see e.g. Demeulemeester et al. (2025)). For fairness and transparency reasons, however, most organizers stick to a draw.

5.2 Fairness

Dividing teams from k pots into k groups such that each group contains exactly one team from each pot can be done easily and spectacularly, by drawing the teams sequentially in a random order. However, the problem becomes more complicated if some assignments are invalid; for example, there are geographic restrictions in the FIBA Basketball World Cup draw to maximize the number of inter-continental matches. In such a constrained draw, fairness has a straightforward definition: each feasible allocation should be equally likely, namely, the draw should be uniformly distributed. Otherwise, teams playing against stronger opponents in expected terms may feel cheated.

This property has been violated in the 1990 (Jones, 1990), 2006 (Rathgeber and Rathgeber, 2007), and 2014 (Guyon, 2015) FIFA World Cup draws because the pre-assignment of certain teams to certain groups was com-

bined with the so-called group-skipping policy (i.e. if a draw restriction prevents the team drawn to be assigned to the first empty slot in a group, the next group is considered, and so on, until a group is found where the team can be assigned to) to ensure geographical diversity.

A similar problem emerged in the UEFA Champions League, where the Round of 16 draw was implemented with the following constraints between the 2003/04 and 2023/24 seasons: (a) the eight group-winners have to play against the eight runners-up; (b) teams from the same group cannot play against each other; (c) teams from the same national association cannot play against each other. UEFA used a computer-assisted but transparent draw procedure to enforce the imposed constraints. A probability calculator for the 2023/24 season is available at https://julienguyon.github.io/UEFA-draws/, and the underlying mathematics – essentially, Hall's marriage theorem – is explained by e.g. Wallace and Haigh (2013); Guyon and Meunier (2023). The mechanism used by UEFA is remarkably close to achieving the fairest possible outcome (Boczoń and Wilson, 2023). Nonetheless, while the distortions are small in terms of differences in pairwise match probabilities, in almost every season there exist teams with an expected loss of over 10 thousand euros in prize money due to the bias of the draw procedure (Klößner and Becker, 2013).

The UEFA draw procedure has inspired Guyon (2014) to suggest three sequential mechanisms for the FIFA World Cup draw. A variant of one recommended mechanism has been adopted for both the 2018 (Guyon, 2018a) and 2022 (Csató, 2023b) FIFA World Cup draws, and has also been used in the 2019 and 2023 FIBA Basketball World Cups. However, it is also non-uniformly distributed, and the biases are non-negligible, sometimes exceeding one percentage point in the probability of being the group winner or the runner-up (Csató, 2024b). Therefore, Roberts and Rosenthal (2024) propose some fair (uniformly distributed) draw methods that use balls and bowls to remain suitable for a televised event. These solutions can be tried for both the 2018 and 2022 FIFA World Cups at http://probability.ca/fdraw/.

A similar draw mechanism is used for the draw of the league phase of the UEFA Champions League and UEFA Europa League, introduced in the 2024/25 season, which are organized as an incomplete round robin tournament with the following constraints: (a) each of the 36 teams plays eight matches, one home and one away match against two different teams from each of the four pots; (b) teams from the same national association cannot play against each other; (c) no team is allowed to play against more than two teams from the same national association; and (d) the matches should be played over eight matchdays. A simulator of the draw procedure is available at https://julienguyon.pythonanywhere.com/uefa_app/ and the problem is discussed by Guyon et al. (2024).

Draw constraints may threaten fairness even if the draw is uniformly distributed. For instance, consider a SKO with three strong players 1, 2, 3, and one weak player 4. Assume that players 1 and 2 cannot be drawn against each other in the semi-final due to a draw constraint. Then, players 1 and 2 will meet the weak player 4 with a probability of 50%, but this is impossible for player 3, which violates the principle that equally strong players are treated equally. Csató (2022a) attempts to uncover the effects of draw restrictions on knockout tournaments. Nonetheless, both the quest for fairer draw procedures and the evaluation of existing mechanisms are far from completed.

5.3 Attractiveness

A disadvantage of striving for groups of equal strength is that it may result in several matches with low competitive intensity and low uncertainty of outcome. Therefore, some tournaments have been intentionally designed with imbalanced groups. The 2024 European Water Polo Championships divided the teams into two divisions based on their past performance. Division 1 (2) contained the first (second) eight teams that played in two groups of four teams each. The group winners and runners-up in Division 1 advanced to the quarterfinals, while the third- and fourth-placed teams had to play against the group winners and runners-up in Division 2 in a playoff round to qualify for the quarterfinals. A similar design was used in the EHF (men's handball) Champions League between the 2015/16 and 2019/20 seasons. According to the results of Csató (2020a), this format can lead to more

exciting matches without sacrificing fairness. The UEFA Nations League, introduced in 2018, follows a similar idea, too: the national teams are grouped into four leagues of different strengths with a promotion and relegation system (Csató, 2022b; Scelles et al., 2024).

Balanced groups reduce the competitive intensity of the group stage in order to increase the quality and competitive intensity in the next stage. As far as we know, it has never been considered whether this trade-off is the best solution with respect to the overall quality and competitive intensity of the tournament. According to Olson and Stone (2014), although organizing the playoffs with more players can increase suspense in the playoffs, the total suspense over the whole tournament actually decreases. It is nevertheless desirable if the most attractive matches are played near the end, culminating in an exciting final match.

Draw constraints can also be used to increase attractiveness. For example, FIFA maximizes the number of inter-continental games during the group stage of the FIFA World Cup to make it a unique tournament with matches never seen in continental championships. Csató (2022b) shows that restrictions in the group draw can decrease the probability of matches that are potentially vulnerable to manipulation, and the suggested framework may be used to avoid any kind of matches specified by the organizer. However, according to Section 5.2, the usual draw mechanisms cannot guarantee uniform distribution in the presence of draw constraints. A thorough analysis of this trade-off poses an interesting direction for future research.

6   Scheduling

The schedule of a tournament designates the order of the matches. Typically, the schedule is subject to many different constraints from various stakeholders that should be satisfied (Ribeiro and Urrutia, 2012; Rasmussen and Trick, 2008; Durán, 2021). Some constraints may be *soft*: they can be violated but this comes with a penalty, which allows trade-offs between different desiderata. Developing a schedule for a single round robin tournament without any constraint is easy (de Werra, 1980). However, the problem becomes $\mathcal{NP}$-hard as soon as some costs need to be minimized (Briskorn et al., 2010).

6.1   Efficacy

Compared to other design criteria, the scheduling literature is much less concerned about efficacy. Nevertheless, the schedule may have an impact on the efficacy of the tournament. Several empirical papers investigate whether the order of the matches in two-legged clashes in knockout tournaments matters, but the results are mixed (e.g. Flores et al. (2015); Amez et al. (2020)). Jost (2024) identifies the conditions under which a team prefers to play first home, first away, or is indifferent, depending on the size of home advantage, the relative strengths of the two teams, and the tie-breaking rule.

In a theoretical work, Krumer et al. (2017) consider a single round robin tournament with three symmetric players and one prize. The player that has no game in round 2 is found to have the highest expected payoff. In the same tournament with two prizes, the sequence of the matches does not affect the winning probabilities if and only if the second prize is valued at half of the first prize (Laica et al., 2021). In a single round robin tournament with four players where no games are played simultaneously, there is a *first-mover advantage*: the player who competes first in rounds 1 and 2 enjoys a significant advantage (Krumer et al., 2017). These theoretical predictions are confirmed by historical match outcomes from the FIFA World Cup and Olympic wrestling tournaments (Krumer and Lechner, 2017).

Even if a round robin tournament is efficacious at the end of the season, there are still concerns whether intermediate rankings reflect the true strength of the opponents (see also Section 7.2.1). A crucial issue is whether each team has played (approximately) the same number of games. The *games played difference index* (Suksompong, 2016) is the minimum integer p such that at any moment, the difference in the number of games played between any pair of teams is at most p. An integer program minimizing this measure is proposed by

Van Bulck and Goossens (2020). Goossens and Spieksma (2012b) call a schedule *g-ranking-balanced* if the difference between the number of home games played by any two teams up to each round is at most g. Ideally, the sequence of the strengths for the opponents of each team is also as balanced as possible over the tournament. Theoretical results about such schedules are given by Briskorn (2009) and Briskorn and Knust (2010). Briskorn and Drexl (2009) propose an integer program to distribute the matches of each team against teams of a specific strength group as evenly as possible.

6.2 Fairness

Fairness is an important concept in sports scheduling, reflected by the objective functions of many studies on constructing sports schedules. Van Bulck et al. (2020) mention travel distance, breaks, and the carry-over effect as possible objectives (besides the more general costs and soft constraints); in this section, we also discuss rest times. Typically, no schedule makes every team happy. Therefore, while most of the literature deals with minimizing the global dissatisfaction with the schedule (i.e. criterion efficiency), we particularly focus on how this aversion is balanced over the teams (i.e. distribution equity). We refer to Goossens et al. (2020) for a discussion of the trade-off between both angles in the context of sports scheduling.

Travel distance   Balanced travel times are particularly important if teams do not face each opponent an equal number of times at home and away (e.g. in a kRR tournament with an odd k), or if a team can make away trips, traveling directly from the venue of one opponent to the venue of the next without returning home – as in the well-known traveling tournament problem, see e.g. Ribeiro and Urrutia (2007); Irnich (2010).

Surprisingly, the relatively large literature on the traveling tournament problem has focused exclusively on minimizing total travel distance, ignoring the matter of balancing the travel distance over the teams. Durán et al. (2021) balance the travel distances of the teams in six Argentinean youth football divisions that play a SRR. Their solutions have been effectively implemented since 2018. Osicka and Guajardo (2023) adopt a cooperative game theoretical framework to allocate the travel distances among the teams according to different notions of fairness. They also study the trade-off between minimizing total travel and maximizing fairness.

Breaks   A *break* occurs if a team plays two consecutive games at home or away, which is considered undesirable. Consecutive home games have a negative impact on stadium attendance (Forrest and Simmons, 2006), and consecutive away games can result in substantial revenue loss (Durán et al., 2017). Breaks are especially deemed unfair at the beginning and the end of the season (Goossens and Spieksma, 2012b). The group stage of the UEFA Champions League has, for instance, been played without breaks in the first or last rounds for any team.

The definition of a break may depend on other characteristics of the tournament. For example, the qualifications for the FIFA World Cup are usually played in double rounds: the national teams play two matches over a week, followed by a longer period without any matches. Then the focus ought to be only on breaks in these double rounds. In particular, away breaks in double rounds are worth avoiding because they might imply a long period without home games and can mean long travels within just a few days. Therefore, Durán et al. (2017) develop an integer programming model for scheduling the South American Qualifiers for the FIFA World Cup that minimizes the total number of away breaks within double rounds. Their proposal was unanimously approved and has been used since the qualifiers for the 2018 World Cup. Goossens and Spieksma (2011) even consider breaks between non-consecutive rounds, motivated by the idea of balancing the number of midweek home games over the teams (see Krumer and Lechner (2018) and Goller and Krumer (2020) for empirical evidence that midweek games are different from the usual weekend games).

Several studies attempt to minimize the total number of breaks, both from a practical and theoretical point of view (see e.g. Elf et al. (2003); Rasmussen and Trick (2007); van't Hof et al. (2010); Ribeiro and Urrutia (2012); Tanaka and Miyashiro (2016); Van Bulck and Goossens (2020); Ribeiro et al. (2023b)). Finding an equitable sched-

ule with a minimal number of breaks such that breaks are equally distributed over all teams, has also received some attention. For a SRR with n teams, an equitable schedule where each team has one break exists (de Werra, 1980). Fronček and Meszka (2005) show that a unique schedule without breaks exists if one extra round is used (and hence each team has one round without a match). Miyashiro and Matsui (2005) propose a polynomial-time algorithm to find an equitable schedule for a given timetable of a round-robin tournament.

The carry-over effect   Player i is said to give a *carry-over effect* to player j if another player k plays against j immediately after playing against i (Russell, 1980). It is considered unfair if a player systematically gives a carry-over effect to the same player. Take, for example, the TATA Steel Chess Championship 2020. The winner of the tournament, *Fabiano Caruana*, was ranked second in the world at that time. Remarkably, Caruana played in 10 of the 13 rounds against someone who had played in the previous round against *Magnus Carlsen*, the number one player in the world at that time. According to Lambers et al. (2020), this schedule gave Caruana an unfair advantage, letting him play often against opponents who were "exhausted" or "depressed" after (losing) their match against Carlsen. On the other hand, Goossens and Spieksma (2012a) find no evidence that carry-over has a significant impact on the results of Belgian professional football teams. Nevertheless, substantial effort has been devoted to balancing the carry-over effects as evenly as possible over all players (e.g. Russell (1980); Anderson (1999); Trick (2000); Henz et al. (2004); Miyashiro and Matsui (2006); Kidd (2010); Guedes and Ribeiro (2011)).

Rest time differences   Shorter rest times are related to both increased injury rates (Dupont et al., 2010; Bengtsson et al., 2013) and higher levels of fatigue. This introduces an element of unfairness if a team with a fully rested squad plays against a team having a short rest time (Scoppa, 2015). Indeed, Bowman et al. (2022) find that in the NBA, back-to-back games are the single most significant factor affecting team success. An easy way of reducing the differences in rest times between teams is to limit the total number of games per week for each team (Knust, 2010). Other approaches are to directly minimize the rest time differences between teams (Atan and Çavdaroğlu, 2018; Çavdaroğlu and Atan, 2020; Durán et al., 2022; Tuffaha et al., 2023), or to minimize an aggregated rest time penalty (ARTP) which gives a higher weight to shorter rest times (Van Bulck and Goossens, 2020). Based on the latter, Goossens et al. (2020) propose an integer program formulation and a bi-criteria evolutionary algorithm to generate a set of so-called equitably-efficient schedules. A schedule is equitably-efficient if no schedule exists with a smaller ARTP for one team, or with a smaller ARTP for a worse-off team at the expense of a better-off team, without worsening the ARTP of the other teams compared to this schedule. They apply and visualize this for a non-professional indoor football league, allowing practitioners to make a well-informed trade-off.

Unsurprisingly, there exist unavoidable trade-offs between these fairness criteria. The distance minimization and the break maximization problems are equivalent if the distance between every pair of teams is equal (Urrutia and Ribeiro, 2006). Craig et al. (2009) focus on the trade-off between minimizing travel distance for teams, avoiding sequences of consecutive home or away games, and ensuring equity in rest times. For a SRR with n teams, the minimal number of breaks is $n - 2$, which is achieved (for instance) by the *canonical* schedule. However, this schedule also yields maximally unbalanced carry-over effects for an even number of teams (Lambrechts et al., 2018). Günneç and Demir (2019), Goossens et al. (2020), and Çavdaroğlu and Atan (2022) discuss the trade-off between minimizing breaks and balancing the carry-over effect. Obviously, there are also trade-offs between fairness and organizational or financial criteria (see e.g. Goossens and Spieksma (2009); Ribeiro and Urrutia (2012)).

6.3   Attractiveness

In the sports scheduling literature, basically two approaches exist to deal with attractiveness. One approach assumes that the set of popular matches is known or given by a broadcaster, possibly inspired by studies like

Wang et al. (2018); Krumer (2020); Jakee et al. (2022). The schedule should then assign as many of these games as possible to commercially interesting timeslots, and avoid any overlap between them. This approach has been reported in several practical applications, from e.g. Australian football (Kyngäs et al., 2017), football (Goossens and Spieksma, 2009; Ribeiro and Urrutia, 2012), and volleyball (Cocchi et al., 2018). Matsumura and Hasuike (2022) estimate the impact of scheduling decisions on TV audiences for the Japanese professional baseball league by multiple regression analysis to develop a schedule that maximizes the expected number of spectators. Domestic football competitions occasionally shut down for some days due to periods reserved for international matches played by national teams. According to Pérez (2024), home wins have a lower probability after such an interruption, which leads to greater uncertainty of outcome. Consequently, the set of matches played immediately after "FIFA reserved dates" needs to be chosen strategically.

The other, much less attempted approach tries to find a schedule that is as attractive as possible given a prediction of match results, or robust with respect to a subset of possible outcomes. Yi (2021) proposes an implementer-adversary approach to dynamically develop round robin schedules that maximize suspense, measured by how late in the season the winner is decided. The algorithm works by generating a set of scenarios (realizations of match outcomes) based on historical data from betting odds. The implementer problem finds a schedule that reveals the winner as late as possible, given the information about match outcomes. Next, the adversary problem finds a new (realistic) scenario under which the generated schedule is the worst possible, and this new scenario is added to the set of scenarios. The algorithm continues until the implementer and adversary solutions converge.

We see at least two avenues for future research on improving the attractiveness of tournament schedules. The first is to include match importance measures (see Section 2.3.3) in order to obtain a schedule that maximizes the total importance of matches over the season. This can be easily done in a SRR with three or four teams; papers focusing on stakeless matches usually recommend an optimal (Chater et al., 2021; Guyon, 2020) schedule. Using complete enumeration, Csató et al. (2024b) have managed to solve this problem in a DRR with four teams and some scheduling constraints. Given the complexity of these measures, such an endeavor will be challenging computationally for larger tournaments. The second avenue focuses on dynamic scheduling, which means that the schedule is not computed and announced fully at the beginning of the season, but is released gradually during the season. Except for recent applications in the NFL (Bouzarth et al., 2020), in the FIFA World Cup (Stronka, 2024), and in e-sports (Dong et al., 2023), dynamic scheduling has not received much attention in the literature. Yet, it may have a great potential to improve the attractiveness of tournaments, as recent information about the strength and ranking of players can be taken into account in order to maximize the importance of the remaining matches.

7 Ranking methods

The theoretical properties of ranking procedures have been widely discussed in the literature. González-Díaz et al. (2014) provide a comprehensive axiomatic analysis of ranking methods for general tournaments where the players may play an arbitrary number of matches against each other. Applying maximum likelihood to the (Zermelo–)Bradley–Terry model (Zermelo, 1929; Bradley and Terry, 1952), and the generalized row sum, a parametric family of ranking methods introduced by Chebotarev (1994), turn out to be the most appealing by satisfying all reasonable axioms that are compatible with each other.

7.1 Efficacy

While efficacy in the context of tournament formats and seeding rules refers to the ability to rank the players according to their true (but hidden) strength, the efficacy of a ranking method usually refers to its *predictive power*, the ability of a ranking to accurately predict future matches. McHale and Davies (2007) test a previous formula of the FIFA World Ranking from this perspective. Lasek et al. (2013) compare the predictive power of several

Table 3: Papers on the efficacy of ranking methods

	Application	Best	Worst
Govan (2008)	NFL NCAAF NCAAB	Massey	Keener
Lasek et al. (2013)	National football teams	Elo (eloratings.net)	Network based
Gásquez and Royuela (2016)	National football teams	Elo (eloratings.net)	(previous) FIFA World Ranking
Chartier et al. (2011)	NFL	Massey, Colley	PageRank
Barrow et al. (2013)	MLB MBA NCAAB NCAAF	— Keener — Least squares	PageRank Win percentage — —
Dabadghao and Vaziri (2022)	NBA NFL NHL	$(1,\alpha)$ $(1,\alpha)$ $(1,\alpha)$	Elo Win-loss Win-loss
Csató (2024a)	UEFA Champions League	Elo (clubelo.com)	UEFA club coefficient

methods for ranking national football teams. The Elo rating is found to be a better indicator of success than the old FIFA World Ranking, a result that is reinforced by Gásquez and Royuela (2016). Indeed, FIFA reformed the World Ranking in 2018 (FIFA, 2018). Nonetheless, the predictive power of the current FIFA World Ranking can still be considerably improved by incorporating home-field advantage and introducing a weighting by goal difference (Szczecinski and Roatis, 2022). Somewhat analogously, a variant of the Elo rating (Football Club Elo Ratings, http://clubelo.com/) robustly outperforms the UEFA club coefficient in terms of predictive accuracy for the UEFA Champions League (Csató, 2024a).

In round robin tournaments, the points scored in individual matches are usually summed for each team, and the teams are ordered based on the total number of points obtained (Section 2.2.1). Dabadghao and Vaziri (2022) call this the win-loss method, which is compared to other, more sophisticated ranking methods for the NBA, NHL, and NFL. Although the win-loss method is used officially in these leagues, it is the worst performing method for the NFL and NHL, and the second worst for the NBA. An extension of the Markov method, which is a pairwise comparison ranking method that uses Markov chains to rate and rank the competitors, turns out to be the most efficacious in each of the three cases. Govan (2008) and Barrow et al. (2013) also compare ranking methods in North American sports competitions. Tiwisina and Külpmann (2019) develop a statistical model that assumes only transitivity to describe the outcomes of sports matches, and applies it to evaluate ranking methods. Scoring systems (3,1,0) and (2,1,0) are found to be close to each other and competitive with the Elo rating in the German football league and NFL. Table 3 presents a concise overview of some of these results.

Standard scoring rules perform badly in settings where players play a different number of matches and some pairs of players do not play against each other, which is the case, for instance, in tennis. Radicchi (2011) shows that a network-based ranking method has better predictive power than other well-established methods. Bozóki et al. (2016) and Temesi et al. (2024) use the eigenvector and least squares methods to derive all-time rankings for men and women tennis players, respectively. Chao et al. (2018) use a similar methodology to rank players in Go.

7.2 Fairness

Vaziri et al. (2018) introduce three criteria for the fairness of a ranking: (a) the quality of a victory or defeat is taken into account (opponent strength); (b) a team always has a clear incentive to win a match to improve its rank (incentive compatibility); (c) a factor should not be considered if the teams have no direct control over it (independence of the schedule). Opponent strength is often not taken into account in round robin tournaments, but it can be important in other tournaments. Incentive compatibility, which is basically equivalent to the mono-

tonicity properties of González-Díaz et al. (2014), is not discussed in this survey (see Section 1). One might think that all reasonable ranking methods are independent of the schedule, but this is not the case. In particular, the Elo rating depends on the sequence of matches: if two teams play two games against each other such that one is won by both teams, then winning the second game leads to a higher rating (Csató et al., 2024a).

A possible interpretation of fairness is *self-consistency*, which requires assigning the same rank for players with equivalent results, while a player who performs better than another should be ranked strictly higher. This fundamental axiom is however incompatible with two other reasonable properties on the set of general tournaments where the players are allowed to play any number of matches against each other. The first is *independence of irrelevant matches* (Csató, 2019a): the relative ranking of two players should be independent of the outcome of any match played by other players. The second is *order preservation* (Csató, 2019b): if player i is not worse than player j in two tournaments where all players have played the same number of rounds, then their relative ranking should remain the same even when the two tournaments are aggregated.

However, these results do not imply that the axioms cannot be met simultaneously on a restricted set of tournaments. In particular, the three properties above are satisfied in a round robin tournament if the players are ranked according to the number of points scored, provided that a tie is worth half of a win. Note that both the maximum likelihood and the generalized row sum ranking methods are score consistent, they coincide with the ranking based on scores on the set of round robin tournaments (González-Díaz et al., 2014).

7.2.1 Round robin tournaments

Ranking in a round robin tournament usually does not necessitate taking the strength of opponents into account because any two players play against the same set of opponents (themselves disregarded). However, this does not hold in an incomplete tournament.

During the COVID-19 crisis, almost all sports competitions were suddenly shut down, and some of them never finished, accidentally turning them into incomplete round robin tournaments. Obviously, summing all the points obtained over the unfinished season does not result in a fair ranking, since the strength of opponents can substantially differ between the teams.

Thus, several ranking methods that take the strength of opponents into account have been proposed in order to construct a fair ranking. In particular, the method of Gorgi et al. (2023) measures the performance of the teams in the previous matches of the season and predicts the remaining non-played matches through a paired-comparison model. Van Eetvelde et al. (2023) develop a tool to calculate the probabilities of possible final rankings. Csató (2021a) identifies a set of desired axioms for ranking in these incomplete tournaments and verifies that the generalized row sum satisfies all of them.

7.2.2 Knockout tournaments

Bahamonde-Birke and Bahamonde-Birke (2023) find that in home-away clashes between two teams, the advantage that the team closing the series at home enjoys can be countered to some extent by valuing away goals over home goals in the event of a tie. However, UEFA decided to abolish this away goals rule in its competitions starting from the 2021/22 season. Note that this issue is also related to scheduling in knockout tournaments, see Section 6.1.

Among all topics of tournament design, penalty shootouts have received probably the most serious attention in the economic literature; highly influential empirical works are Apesteguia and Palacios-Huerta (2010), Kocher et al. (2012), Palacios-Huerta (2014), and Kassis et al. (2021). There are also several suggestions to improve the fairness of penalty shootouts as a tie-breaker by changing the standard shooting order, which are summarized in Csató and Petróczy (2022).

7.2.3 Swiss system tournaments

In a traditional Swiss system tournament, the players are paired using a set of complex rules, which basically aim to ensure that the opponents have a similar running score (see Section 3.2.3). Take two players i and j who finish with the same score such that i has scored most of its points at the beginning of the tournament, while j has scored most of its points at the end of the tournament. Consequently, the opponents of i have probably *higher* scores than i (it is unlikely that all of them show the same decline in performance as i) and the opponents of j have probably *lower* scores than j (it is unlikely that all of them show the same improvement in performance as j).

As we have seen in Section 2.2.1, the strength of opponents is considered by some tie-breaking rules; for instance, the Buchholz score adds the scores of all opponents. However, taking the strength of opponents into account only in breaking ties is obviously insufficient. In the situation above, player j might easily score marginally more points than player i by playing against much weaker opponents. Csató (2021c, Chapter 1.2) presents an example from the 2011 European Team Chess Championship Open tournament: Serbia scored 10 and France scored 9 points but their Buchholz scores were 152 and 179, respectively. Therefore, ranking Serbia above France can be easily debated. Csató (2013) and Csató (2017) have proposed novel methods, however, the optimal trade-off between the number of points scored and the strength of the schedule remains unknown.

7.3 Attractiveness

It is probably harmful to attendance in any tournament if the final position of a team is already secured, independently of the outcomes of the remaining games. Csató (2023c) uncovers that tie-breaking rules have a non-negligible role in this respect: the chance of a fixed position in the group ranking can be reduced by at least two and usually five percentage points in the last round of a DRR with four teams if goal difference and number of goals scored are the primary tie-breaking criteria instead of head-to-head results.

Garcia-del Barrio and Reade (2022) show that interest in Formula One decreases once the driver's champion is known. Although a natural solution would be to decrease the relative value of winning a race, this would increase the probability that a driver could win the competition without winning any races – as happened in the 1999 Motorcycle Grand Prix 125cc. Csató (2023a) evaluates point scoring systems for Formula One from this perspective, and finds that the current point scoring system provides a reasonable compromise between these two perils. Furthermore, it is competitive with the geometric scoring rules proposed by Kondratev et al. (2023) based on axiomatic arguments.

Until the 1998/99 season, the NHL gave 2 points for a win, 1 point for a draw, and 0 points for a loss. However, starting with the 1999/2000 season, the league awarded one point for a loss in overtime in order to decrease the frequency of overtime games ending in a tie. The new scoring system has achieved the outcome intended by league officials but the ratio of overtime games has also risen as the total reward has increased for games ending in overtime (Abrevaya, 2004).

Almost all round robin football leagues award a win by 3 points, a draw by 1 point, and a loss by 0 points. This method was introduced first in England in the 1981/82 season, and adopted by other tournaments mainly in the 1990s. Moschini (2010) develops a game-theoretic model that reflects the crucial strategic elements of a scoring system where the value of a win is a parameter. The empirical results are in line with the model implications: the (3,1,0) system leads to a significant increase in the expected number of goals and a decrease in the ratio of draws. However, according to Guedes and Machado (2002), a higher reward for victory induces the weaker team to play more defensively if the difference between the strengths of the two teams is sufficiently high. The authors also find empirical support for this hypothesis.

There are many more papers on this issue. However, since an overwhelming majority of them have been published in (sports) economics journals, they are not discussed here. The main message is probably that changing ranking rules in order to increase the attractiveness of the tournament may have unintended consequences.

Hence, consultation with the scientific community before rule changes would not be a futile exercise.

8 Conclusions and directions for further research

Our study has aimed to overview the tournament design literature from the perspective of operational research. A basic question of the field is the efficacy (i.e. the ability to rank all players according to their true strength) of tournament designs. In the case of knockout tournaments, seeding is usually found to have a significant effect on both measures, although some works challenge this result. Fairness is also an important aspect of tournament design but has no uniform definition. Many papers call a tournament fair if it favors stronger players, while other studies argue that players should be treated equally. The importance, suspense, and quality of matches are also crucial criteria for choosing a tournament design.

Unsurprisingly, round robin tournaments seem to be better at identifying the true ranking of the competitors, while knockout tournaments require fewer matchdays and do not contain unimportant matches. This may explain why popular sports competitions, like the FIFA World Cup, have adopted hybrid designs consisting of a round robin group stage followed by a single knockout tournament. However, the traditional group stage often becomes "boring", hence, there are recent attempts to create imbalanced groups or replace groups with an incomplete round robin tournament.

Based on the previous sections, several issues require further investigation by the operational research community. In particular, we think more research is needed in the following areas:

Optimizing the number of contestants What is the optimal number of contestants for a tournament in terms of efficacy and attractiveness? Changes in the number of contestants, increases as well as decreases, have regularly occurred during the recent decades in practice; however, these decisions seem to have rarely been based on scientific research.

Robustness of seeding policies Recently, Engist et al. (2021) have found no empirical evidence that seeding contributes to success in the UEFA club competitions. Since this result is in contrast to the common assumption of the operational research literature, more studies are needed for other tournaments and sports to decide whether it is only a statistical oddity or a more general finding. The robustness of the (optimal) seeding rule also requires further investigation since the true ranking of the players can only be approximated in practice.

Constrained draw If draw constraints exist, the currently used random draw procedures do not guarantee that all valid assignments are equally likely. How can we develop draw mechanisms that respect all constraints (e.g. geographic, scheduling), are (approximately) fair, but still offer a transparent implementation that is appealing to a TV audience? In particular, the issue is worth studying in the incomplete round robin format, which contains an added complexity: all teams should eventually be ordered in a single ranking, even though the teams have faced different sets of opponents (determined by the draw).

Imbalanced groups versus incomplete round robin tournaments It seems that organizers attempt to increase attractiveness and guarantee more matches between the top players by two different formats: deliberately creating imbalanced groups (Section 5.3) or replacing the group stage with a single incomplete round robin tournament (Section 2.1.1). It remains an open question to what extent can these formats improve our main design criteria. In particular, the novel incomplete round robin format of UEFA club competitions is foreseen to provide greater fairness, a more dynamic ranking, and a higher level of sporting interest (UEFA, 2023). However, these expectations have not been confirmed by a scientific analysis at the moment, except for a recent study focusing on competitive balance (Gyimesi, 2024). It would also be interesting to explore how a predetermined set of matches worsens efficacy compared to the Swiss system.

Organising incomplete round robin tournaments The incomplete round robin format of UEFA club competitions, introduced in the 2024/25 season, substantially increases the complexity of addressing the theoretical criteria discussed in our survey. The set of matches to be played depends on the seeding rules and is determined by a draw. However, it may lead to an infeasible schedule (if the chromatic index of the regular graph representing the matches exceeds the number of available match days), in other words, the matches-first approach should consider at least some scheduling constraints. On the other hand, the schedule-first approach, when the teams are assigned to the already chosen slots, is too rigid and may threaten the randomness of the draw (Guyon et al., 2024). Finally, the properties of the ranking method are also affected by the seeding regime and the set of matches to be played. Therefore, the organizer needs to simultaneously take all these aspects into account, and their optimal treatment seems to provide serious challenges for OR in the coming years.

Dynamic scheduling What flexibility remains to schedule the last half/quarter of the tournament, once the first part has been played (see Lambers et al. (2023) for some measures)? To what extent can extra information (e.g. on which teams are still running for which prizes) be taken into account? What is the impact of (partial) dynamic scheduling on efficacy, fairness, and attractiveness? To reduce practical challenges, the organizers could fix the home game days for each team at the start of the season, and fill in the opponents gradually during the season. How should each team's home dates be set in order to maximize planning flexibility for the later parts of the season? What is the trade-off between the ratio of matches that are scheduled dynamically and the performance of the tournament with respect to the main design criteria?

Multi-league scheduling The extant literature focuses mainly on scheduling a single league or tournament. How to deal with multiple tournaments that are played simultaneously and linked with each other, e.g. through shared infrastructure, or players taking part in more than one tournament (Davari et al., 2020; Li et al., 2023)? To what extent is scheduling linked with the grouping of teams into leagues (resulting from a draw, promotion/relegation, or optimization over e.g. travel distance)?

Optimizing global ranking systems How can the ranking methods of sports federations be optimized to predict future performance? Can they be improved by changing their methodology, or taking further factors into account? To what extent can other tournament design choices (format, schedule, draw, etc.) contribute to this accuracy? The issue is important especially because global rankings are widely used for seeding and the allocation of qualifying slots.

Tournament design is a research domain that has a strong link with practice. Practitioners and sports governing bodies will hopefully take an interest in this survey, too. In recent years, the results of academic research have frequently been adopted by sports administrators:

- FIFA has introduced a better ranking procedure for men's national teams in 2018 (FIFA, 2018) (Section 2.2.3);

- UEFA has modified the knockout bracket of the 2020 and 2024 UEFA European Championships to balance group advantages according to the recommendation of Guyon (2018b) (Section 4.2.2);

- FIFA has reformed the draw of the 2018 and 2022 FIFA World Cups inspired by Guyon (2014), see Guyon (2018a) (Section 5.2);

- The proposal of Durán et al. (2017) for a fairer schedule of the South American Qualifiers for the 2018 FIFA World Cup has been unanimously approved by the participants and is currently being used (Section 6.2).

Governing bodies in sports can also learn from each other. Both the FIBA Basketball World Cup and the FIFA World Cup aim to maximize geographic diversity in the group stage since maximizing intercontinental matches is useful both for attendance and ranking purposes (Guyon, 2015). On the other hand, the 2023 World Men's

Handball Championship contained a group with two of the three African teams, another group with two of the four South American teams, while the number of European teams varied between one and three in the eight groups.

The example of the 2022 FIFA World Cup draw warns that it is worth consulting with the wider scientific community before rule changes are implemented. On 22 March 2022, FIFA released the details of this draw procedure, which treated the three placeholders of the playoffs unfairly and threatened the balancedness of the groups as discussed in Section 5.1. Even though Csató (2023b) immediately realized the problem, it was clearly impossible to correct the mistake before the draw happened.

Consequently, since governing bodies in sports quite often change the design of their tournaments, usually driven by commercial motives, and some innovative ideas repeatedly emerge in practice, operational research in sports will certainly remain a growing research field for the coming decades.

Acknowledgements

References

Abrevaya, J. (2004). Fit to be tied: The incentive effects of overtime rules in professional hockey. *Journal of Sports Economics*, 5(3):292–306.

Adler, I., Cao, Y., Karp, R., Peköz, E. A., and Ross, S. M. (2017). Random knockout tournaments. *Operations Research*, 65(6):1589–1596.

Aldous, D. (2017). Elo ratings and the sports model: A neglected topic in applied probability? *Statistical Science*, 32(4):616–629.

Amez, S., Baert, S., Neyt, B., and Vandemaele, M. (2020). No evidence for second leg home advantage in recent seasons of European soccer cups. *Applied Economics Letters*, 27(2):156–160.

Anderson, I. (1999). Balancing carry-over effects in tournaments. In Quinn, K., Webb, B., Rowley, C., and Holroyd, F. C., editors, *Combinatorial Designs and Their Applications*, pages 1–16. Routledge, New York.

Apesteguia, J. and Palacios-Huerta, I. (2010). Psychological pressure in competitive environments: Evidence from a randomized natural experiment. *American Economic Review*, 100(5):2548–2564.

Appleton, D. R. (1995). May the best man win? *Journal of the Royal Statistical Society: Series D (The Statistician)*, 44(4):529–538.

Arlegi, R. (2022). How can an elimination tournament favor a weaker player? *International Transactions in Operational Research*, 29(4):2250–2262.

Arlegi, R. and Dimitrov, D. (2020). Fair elimination-type competitions. *European Journal of Operational Research*, 287(2):528–535.

Atan, T. and Çavdaroğlu, B. (2018). Minimization of rest mismatches in round robin tournaments. *Computers & Operations Research*, 99:78–89.

Ausloos, M. (2024). Hierarchy selection: New team ranking indicators for cyclist multi-stage races. *European Journal of Operational Research*, 314(2):807–816.

Bahamonde-Birke, F. J. and Bahamonde-Birke, R. A. (2023). About the "away goals rule" in association football. Does scrapping the rule increase the fairness of the game? *Journal of Sports Economics*, 24(3):310–328.

Barrow, D., Drayer, I., Elliott, P., Gaut, G., and Osting, B. (2013). Ranking rankings: an empirical comparison of the predictive power of sports ranking methods. *Journal of Quantitative Analysis in Sports*, 9(2):187–202.

Baumann, R., Matheson, V. A., and Howe, C. A. (2010). Anomalies in tournament design: The madness of March Madness. *Journal of Quantitative Analysis in Sports*, 6(2).

Baumer, B. S. (2024). Editor's note: on fairness in sports analytics. *Journal of Quantitative Analysis in Sports*, 20(1):1–3.

Bengtsson, H., Ekstrand, J., and Hägglund, M. (2013). Muscle injury rates in professional football increase with fixture congestion: an 11-year follow-up of the UEFA Champions League injury study. *British Journal of Sports Medicine*, 47(12):743–747.

Bergantinos, G. and Moreno-Ternero, J. D. (2020). Sharing the revenues from broadcasting sport events. *Management Science*, 66(6):2417–2431.

Berker, Y. (2014). Tie-breaking in round-robin soccer tournaments and its influence on the autonomy of relative rankings: UEFA vs. FIFA regulations. *European Sport Management Quarterly*, 14(2):194–210.

Biró, P., Fleiner, T., and Palincza, R. P. (2017). Designing chess pairing mechanisms. In Frank, A., Recski, A., and Wiener, G., editors, *Proceedings of the 10th Japanese-Hungarian Symposium on Discrete Mathematics and Its Applications*, pages 77–86, Budapest. Budapesti Műszaki és Gazdaságtudományi Egyetem.

Bizzozero, P., Flepp, R., and Franck, E. (2016). The importance of suspense and surprise in entertainment demand: Evidence from Wimbledon. *Journal of Economic Behavior & Organization*, 130:47–63.

Boczoń, M. and Wilson, A. J. (2023). Goals, constraints, and transparently fair assignments: A field study of randomization design in the UEFA Champions League. *Management Science*, 69(6):3474–3491.

Bojke, C. (2007). The impact of post-season play-off systems on the attendance at regular season games. In Albert, J. and Koning, R. H., editors, *Statistical Thinking in Sports*, pages 179–202. CRC Press, Boca Raton, Florida.

Borland, J. and Lye, J. (1992). Attendance at Australian Rules football: A panel study. *Applied Economics*, 24(9):1053–1058.

Bouzarth, E. L., Cromer, A. W., Fravel, W. J., Grannan, B. C., and Hutson, K. R. (2020). Dynamically scheduling NFL games to reduce strength of schedule variability. *Journal of Sports Analytics*, 6(4):281–293.

Bowman, R. A., Harmon, O., and Ashman, T. (2022). Schedule inequity in the National Basketball Association. *Journal of Sports Analytics*, 9(1):61–76.

Bozóki, S., Csató, L., and Temesi, J. (2016). An application of incomplete pairwise comparison matrices for ranking top tennis players. *European Journal of Operational Research*, 248(1):211–218.

Bradley, R. A. and Terry, M. E. (1952). Rank analysis of incomplete block designs: I. The method of paired comparisons. *Biometrika*, 39(3/4):324–345.

Briskorn, D., , Drexl, A., and Spieksma, F. (2010). Round robin tournaments and three index assignments. *4OR*, 8(4):365–374.

Briskorn, D. (2008). *Sports Leagues Scheduling: Models, Combinatorial Properties, and Optimization Algorithms*. Springer, Berlin, Heidelberg, Germany.

Briskorn, D. (2009). Combinatorial properties of strength groups in round robin tournaments. *European Journal of Operational Research*, 192(3):744–754.

Briskorn, D. and Drexl, A. (2009). IP models for round robin tournaments. *Computers & Operations Research*, 36(3):837–852.

Briskorn, D. and Knust, S. (2010). Constructing fair sports league schedules with regard to strength groups. *Discrete Applied Mathematics*, 158(2):123–135.

Buraimo, B., Forrest, D., McHale, I. G., and Tena, J. (2022). Armchair fans: Modelling audience size for televised football matches. *European Journal of Operational Research*, 298(2):644–655.

Buraimo, B., Forrest, D., McHale, I. G., and Tena, J. D. (2020). Unscripted drama: Soccer audience response to suspense, surprise, and shock. *Economic Inquiry*, 58(2):881–896.

Çavdaroğlu, B. and Atan, T. (2020). Determining matchdays in sports league schedules to minimize rest differences. *Operations Research Letters*, 48(3):209–216.

Çavdaroğlu, B. and Atan, T. (2022). Integrated break and carryover effect minimization. *Journal of Scheduling*, 25(6):705–719.

Cea, S., Durán, G., Guajardo, M., Sauré, D., Siebert, J., and Zamorano, G. (2020). An analytics approach to the FIFA ranking procedure and the World Cup final draw. *Annals of Operations Research*, 286(1-2):119–146.

Chao, X., Kou, G., Li, T., and Peng, Y. (2018). Jie Ke versus AlphaGo: A ranking approach using decision making method for large-scale data with incomplete information. *European Journal of Operational Research*, 265(1):239–247.

Chartier, T. P., Kreutzer, E., Langville, A. N., and Pedings, K. E. (2011). Sensitivity and stability of ranking vectors. *SIAM Journal on Scientific Computing*, 33(3):1077–1102.

Chater, M., Arrondel, L., Gayant, J.-P., and Laslier, J.-F. (2021). Fixing match-fixing: Optimal schedules to promote competitiveness. *European Journal of Operational Research*, 294(2):673–683.

Chebotarev, P. Yu. (1994). Aggregation of preferences by the generalized row sum method. *Mathematical Social Sciences*, 27(3):293–320.

Chebotarev, P. Yu. and Shamis, E. (1998). Characterizations of scoring methods for preference aggregation. *Annals of Operations Research*, 80:299–332.

Chen, R. and Hwang, F. (1988). Stronger players win more balanced knockout tournaments. *Graphs and Combinatorics*, 4(1):95–99.

Cocchi, G., Galligari, A., Nicolino, F. P., Piccialli, V., Schoen, F., and Sciandrone, M. (2018). Scheduling the Italian national volleyball tournament. *Interfaces*, 48(3):271–284.

Cochran, R. S. (1971). Designs with redeeming social aspects for evenings of social bridge. *The American Statistician*, 25(2):12–15.

Corona, F., Forrest, D., Tena Horrillo, J. de D., and Wiper, M. (2019). Bayesian forecasting of UEFA Champions League under alternative seeding regimes. *International Journal of Forecasting*, 35(2):722–732.

Corona, F., Tena Horrillo, J. de D., and Wiper, M. P. (2017). On the importance of the probabilistic model in identifying the most decisive games in a tournament. *Journal of Quantitative Analysis in Sports*, 13(1):11–23.

Craig, S., While, L., and Barone, L. (2009). Scheduling for the National Hockey League using a multi-objective evolutionary algorithm. In Nicholson, A. and Li, X., editors, *AI 2009: Advances in Artificial Intelligence*, pages 381–390, Berlin, Heidelberg, Germany. Springer.

Csató, L. (2013). Ranking by pairwise comparisons for Swiss-system tournaments. *Central European Journal of Operations Research*, 21(4):783–803.

Csató, L. (2017). On the ranking of a Swiss system chess team tournament. *Annals of Operations Research*, 254(1-2):17–36.

Csató, L. (2019a). An impossibility theorem for paired comparisons. *Central European Journal of Operations Research*, 27(2):497–514.

Csató, L. (2019b). Some impossibilities of ranking in generalized tournaments. *International Game Theory Review*, 21(1):1940002.

Csató, L. (2020a). Optimal tournament design: Lessons from the men's handball Champions League. *Journal of Sports Economics*, 21(8):848–868.

Csató, L. (2020b). Two issues of the UEFA Euro 2020 qualifying play-offs. *International Journal of Sport Policy and Politics*, 12(3):471–484.

Csató, L. (2021a). Coronavirus and sports leagues: obtaining a fair ranking when the season cannot resume. *IMA Journal of Management Mathematics*, 32(4):547–560.

Csató, L. (2021b). A simulation comparison of tournament designs for the World Men's Handball Championships. *International Transactions in Operational Research*, 28(5):2377–2401.

Csató, L. (2021c). *Tournament Design: How Operations Research Can Improve Sports Rules*. Palgrave Pivots in Sports Economics. Palgrave Macmillan, Cham, Switzerland.

Csató, L. (2022a). The effects of draw restrictions on knockout tournaments. *Journal of Quantitative Analysis in Sports*, 18(4):227–239.

Csató, L. (2022b). Quantifying incentive (in)compatibility: A case study from sports. *European Journal of Operational Research*, 302(2):717–726.

Csató, L. (2022c). UEFA against the champions? An evaluation of the recent reform of the Champions League qualification. *Journal of Sports Economics*, 23(8):991–1016.

Csató, L. (2023a). A comparative study of scoring systems by simulations. *Journal of Sports Economics*, 24(4):526–545.

Csató, L. (2023b). Group draw with unknown qualified teams: A lesson from the 2022 FIFA World Cup. *International Journal of Sports Science & Coaching*, 18(2):539–551.

Csató, L. (2023c). How to avoid uncompetitive games? The importance of tie-breaking rules. *European Journal of Operational Research*, 307(3):1260–1269.

Csató, L. (2023d). Quantifying the unfairness of the 2018 FIFA World Cup qualification. *International Journal of Sports Science & Coaching*, 18(1):183–196.

Csató, L. (2024a). Club coefficients in the UEFA Champions League: Time for shift to an Elo-based formula. *International Journal of Performance Analysis in Sport*, 24(2):119–134.

Csató, L. (2024b). The fairness of the group draw for the FIFA World Cup. *International Journal of Sports Science & Coaching*, accepted. DOI: 10.48550/arXiv.2103.11353.

Csató, L., Kiss, L. M., and Szádoczki, Zs. (2024a). The allocation of FIFA World Cup slots based on the ranking of confederations. *Annals of Operations Research*, in press. DOI: 10.1007/s10479-024-06091-5.

Csató, L., Molontay, R., and Pintér, J. (2024b). Tournament schedules and incentives in a double round-robin tournament with four teams. *International Transactions in Operational Research*, 31(3):1486–1514.

Csató, L. and Petróczy, D. G. (2022). Fairness in penalty shootouts: Is it worth using dynamic sequences? *Journal of Sports Sciences*, 40(12):1392–1398.

Dabadghao, S. and Vaziri, B. (2022). The predictive power of popular sports ranking methods in the NFL, NBA, and NHL. *Operational Research*, 22(3):2767–2783.

Dagaev, D. and Rudyak, V. (2019). Seeding the UEFA Champions League participants: Evaluation of the reform. *Journal of Quantitative Analysis in Sports*, 15(2):129–140.

Dagaev, D. and Suzdaltsev, A. (2018). Competitive intensity and quality maximizing seedings in knock-out tournaments. *Journal of Combinatorial Optimization*, 35(1):170–188.

Davari, M., Goossens, D., Beliën, J., Lambers, R., and Spieksma, F. C. (2020). The multi-league sports scheduling problem, or how to schedule thousands of matches. *Operations Research Letters*, 48(2):180–187.

David, H. A. (1959). Tournaments and paired comparisons. *Biometrika*, 46(1/2):139–149.

de Werra, D. (1980). Geography, games and graphs. *Discrete Applied Mathematics*, 2(4):327–337.

Della Croce, F., Dragotto, G., and Scatamacchia, R. (2022). On fairness and diversification in WTA and ATP tennis tournaments generation. *Annals of Operations Research*, 316(2):1107–1119.

Demeulemeester, T., Goossens, D., Hermans, B., and Leus, R. (2025). Fair integer programming under dichotomous and cardinal preferences. *European Journal of Operational Research*, 320(3):465–478.

Di Mattia, A. and Krumer, A. (2023). Fewer teams, more games, larger attendance? Evidence from the structural change in basketball's Euroleague. *European Journal of Operational Research*, 309(1):359–370.

Dietzenbacher, B. J. and Kondratev, A. Y. (2023). Fair and consistent prize allocation in competitions. *Management Science*, 69(6):3474–3491.

Dinh, A. V. N., Bao, N. P. H., Khalid, M. N. A., and Iida, H. (2020). Simulating competitiveness and precision in a tournament structure: A reaper tournament system. *International Journal of Information Technology*, 12(1):1–18.

Dobson, S. M. and Goddard, J. A. (1992). The demand for standing and seated viewing accommodation in the English Football League. *Applied Economics*, 24(10):1155–1163.

Dong, Z.-L., Ribeiro, C. C., Xu, F., Zamora, A., Ma, Y., and Jing, K. (2023). Dynamic scheduling of e-sports tournaments. *Transportation Research Part E: Logistics and Transportation Review*, 169:102988.

Dupont, G., Nedelec, M., McCall, A., McCormack, D., Berthoin, S., and Wisløff, U. (2010). Effect of 2 soccer matches in a week on physical performance and injury rate. *American Journal of Sports Medicine*, 38(9):1752–1758.

Durán, G. (2021). Sports scheduling and other topics in sports analytics: a survey with special reference to Latin America. *Top*, 29(1):125–155.

Durán, G., Guajardo, M., and Sauré, D. (2017). Scheduling the South American Qualifiers to the 2018 FIFA World Cup by integer programming. *European Journal of Operational Research*, 262(3):1109–1115.

Durán, G., Guajardo, M., and Zamorano, G. (2022). Mathematical models for rescheduling Ecuador's 2020 professional football league season disrupted by COVID-19. *Society and Economy*, 44(4):360–377.

Durán, G. A., Guajardo, M., López, A. F., Marenco, J., and Zamorano, G. A. (2021). Scheduling multiple sports leagues with travel distance fairness: An application to Argentinean youth football. *INFORMS Journal on Applied Analytics*, 51(2):136–149.

Elf, M., Jünger, M., and Rinaldi, G. (2003). Minimizing breaks by maximizing cuts. *Operations Research Letters*, 31(5):343–349.

Elo, A. E. (1978). *The Rating of Chessplayers: Past and Present.* Arco, New York.

Ely, J., Frankel, A., and Kamenica, E. (2015). Suspense and surprise. *Journal of Political Economy*, 123(1):215–260.

Engist, O., Merkus, E., and Schafmeister, F. (2021). The effect of seeding on tournament outcomes: Evidence from a regression-discontinuity design. *Journal of Sports Economics*, 22(1):115–136.

Faella, M. and Sauro, L. (2021). Irrelevant matches in round-robin tournaments. *Autonomous Agents and Multi-Agent Systems*, 35(1):1–34.

FIFA (2018). Revision of the FIFA / Coca-Cola World Ranking. https://img.fifa.com/image/upload/edbm045h0udbwkqew35a.pdf.

FIFA (2023a). FIFA Council approves international match calendars. 14 March. https://www.fifa.com/about-fifa/organisation/fifa-council/media-releases/fifa-council-approves-international-match-calendars.

FIFA (2023b). One Month On: 5 billion engaged with the FIFA World Cup Qatar 2022. 18 January. https://www.fifa.com/tournaments/mens/worldcup/qatar2022/news/one-month-on-5-billion-engaged-with-the-fifa-world-cup-qatar-2022-

Flores, R., Forrest, D., de Pablo, C., and Tena Horrillo, J. de D. (2015). What is a good result in the first leg of a two-legged football match? *European Journal of Operational Research*, 247(2):641–647.

Forrest, D. and Simmons, R. (2002). Outcome uncertainty and attendance demand in sport: the case of English soccer. *Journal of the Royal Statistical Society Series D: The Statistician*, 51(2):229–241.

Forrest, D. and Simmons, R. (2006). New issues in attendance demand: The case of the English Football League. *Journal of Sports Economics*, 7(3):247–266.

Freixas, J. (2022). The decline of the Buchholz tiebreaker system: A preferable alternative. In Nguyen, N. T., Kowalczyk, R., Mercik, J., and Motylska-Kuźma, A., editors, *Transactions on Computational Collective Intelligence XXXVII*, pages 1–20. Springer, Berlin, Heidelberg, Germany.

Fronček, D. (2007). Fair incomplete tournaments with odd number of teams and large number of games. *Congressus Numerantium*, 187:83.

Fronček, D. (2013). Handicap distance antimagic graphs and incomplete tournaments. *AKCE International Journal of Graphs and Combinatorics*, 10(2):119–127.

Fronček, D., Kovár, P., and Kovárová, T. (2006). Fair incomplete tournaments. *Bulletin of the ICA*, 48(1):31–33.

Fronček, D. and Meszka, M. (2005). Round robin tournaments with one bye and no breaks in home-away patterns are unique. In Kendall, G., Burke, E. K., Petrovic, S., and Gendreau, M., editors, *Multidisciplinary Scheduling: Theory and Applications*, pages 331–340. Springer, Boston, Massachusetts.

García, J. and Rodríguez, P. (2002). The determinants of football match attendance revisited: Empirical evidence from the Spanish football league. *Journal of Sports Economics*, 3(1):18–38.

García, J. and Rodríguez, P. (2009). Sports attendance: A survey of the literature 1973-2007. *Rivista di Diritto ed Economia dello Sport*, 5(2):111–151.

Garcia-del Barrio, P. and Reade, J. J. (2022). Does certainty on the winner diminish the interest in sport competitions? The case of Formula One. *Empirical Economics*, 63(2):1059–1079.

Gásquez, R. and Royuela, V. (2016). The determinants of international football success: A panel data analysis of the Elo rating. *Social Science Quarterly*, 97(2):125–141.

Geenens, G. (2014). On the decisiveness of a game in a tournament. *European Journal of Operational Research*, 232(1):156–168.

Glenn, W. (1960). A comparison of the effectiveness of tournaments. *Biometrika*, 47(3/4):253–262.

Goller, D. and Heiniger, S. (2024). A general framework to quantify the event importance in multi-event contests. *Annals of Operations Research*, 341(1):71–93.

Goller, D. and Krumer, A. (2020). Let's meet as usual: Do games played on non-frequent days differ? Evidence from top European soccer leagues. *European Journal of Operational Research*, 286(2):740–754.

Gomes de Pinho Zanco, D., Szczecinski, L., Kuhn, E. V., and Seara, R. (2024). Stochastic analysis of the Elo rating algorithm in round-robin tournaments. *Digital Signal Processing*, 145:104313.

González-Díaz, J., Hendrickx, R., and Lohmann, E. (2014). Paired comparisons analysis: an axiomatic approach to ranking methods. *Social Choice and Welfare*, 42(1):139–169.

Goossens, D., Beliën, J., and Spieksma, F. C. R. (2012). Comparing league formats with respect to match importance in Belgian football. *Annals of Operations Research*, 194(1):223–240.

Goossens, D. and Spieksma, F. C. R. (2009). Scheduling the Belgian soccer league. *Interfaces*, 39(2):109–118.

Goossens, D. and Spieksma, F. C. R. (2012a). The carryover effect does not influence football results. *Journal of Sports Economics*, 13(3):288–305.

Goossens, D. and Spieksma, F. C. R. (2012b). Soccer schedules in Europe: an overview. *Journal of Scheduling*, 15(5):641–651.

Goossens, D., Yi, X., and Van Bulck, D. (2020). Fairness trade-offs in sports timetabling. In Ley, C. and Dominicy, Y., editors, *Science Meets Sports: When Statistics Are More Than Numbers*, pages 213–244. Cambridge Scholars Publishing, Newcastle upon Tyne, United Kingdom.

Goossens, D. R. and Spieksma, F. C. (2011). Breaks, cuts, and patterns. *Operations Research Letters*, 39(6):428–432.

Gorgi, P., Koopman, S. J., and Lit, R. (2023). Estimation of final standings in football competitions with a premature ending: The case of COVID-19. *AStA Advances in Statistical Analysis*, 107(1-2):233–250.

Gotzes, U. and Hoppmann, K. (2022). Bounding the final rank during a round robin tournament with integer programming. *Operational Research*, 22(1):123–131.

Govan, A. Y. (2008). *Ranking Theory with Application to Popular Sports.* PhD thesis, North Carolina State University.

Groh, C., Moldovanu, B., Sela, A., and Sunde, U. (2008). Optimal seedings in elimination tournaments. *Economic Theory*, 49(1):59–80.

Guajardo, M. and Krumer, A. (2024). Tournament design for a FIFA World Cup with 12 four-team groups: Every win matters. In Breuer, M. and Forrest, D., editors, *The Palgrave Handbook on the Economics of Manipulation in Sport*, pages 207–230. Palgrave Macmillan, Cham, Switzerland.

Guedes, A. C. B. and Ribeiro, C. C. (2011). A heuristic for minimizing weighted carry-over effects in round robin tournaments. *Journal of Scheduling*, 14(6):655–667.

Guedes, J. C. and Machado, F. S. (2002). Changing rewards in contests: Has the three-point rule brought more offense to soccer? *Empirical Economics*, 27(4):607–630.

Günneç, D. and Demir, E. (2019). Fair-fixture: Minimizing carry-over effects in football leagues. *Journal of Industrial & Management Optimization*, 15(4):1565.

Guyon, J. (2014). Rethinking the FIFA World Cup™ final draw. Manuscript. DOI: 10.2139/ssrn.2424376.

Guyon, J. (2015). Rethinking the FIFA World Cup™ final draw. *Journal of Quantitative Analysis in Sports*, 11(3):169–182.

Guyon, J. (2018a). Pourquoi la Coupe du monde est plus équitable cette année. *The Conversation*. 13 June. https://theconversation.com/pourquoi-la-coupe-du-monde-est-plus-equitable-cette-annee-97948.

Guyon, J. (2018b). What a fairer 24 team UEFA Euro could look like. *Journal of Sports Analytics*, 4(4):297–317.

Guyon, J. (2020). Risk of collusion: Will groups of 3 ruin the FIFA World Cup? *Journal of Sports Analytics*, 6(4):259–279.

Guyon, J. (2022). "Choose your opponent": A new knockout design for hybrid tournaments. *Journal of Sports Analytics*, 8(1):9–29.

Guyon, J. and Meunier, F. (2023). UEFA draws: probability calculator. 13 December. https://julienguyon.github.io/UEFA-draws/ExactProbabilities_ChampionsLeague_RoundOf16_text_website-1.pdf.

Guyon, J., Meunier, F., Ben Salem, A., Buchholtzer, T., and Tanré, M. (2024). The drawing of the teams in the future Champions League. Academic report. https://julienguyon.pythonanywhere.com/uefa_app/.

Gyimesi, A. (2024). Competitive balance in the post-2024 Champions League and the European Super League: A simulation study. *Journal of Sports Economics*, 25(6):707–734.

Hall, N. G. and Liu, Z. (2024). Opponent choice in tournaments: winning and shirking. *Journal of Quantitative Analysis in Sports*, 20(2):67–86.

Harary, F. and Moser, L. (1966). The theory of round robin tournaments. *The American Mathematical Monthly*, 73(3):231–246.

Hassanzadeh, A., Hosseini, M., and Turner, J. G. (2024). How to conclude a suspended sports league? *Manufacturing & Service Operations Management*, 26(5):1692–1711.

Hennessy, J. and Glickman, M. (2016). Bayesian optimal design of fixed knockout tournament brackets. *Journal of Quantitative Analysis in Sports*, 12(1):1–15.

Henriet, D. (1985). The Copeland choice function: An axiomatic characterization. *Social Choice and Welfare*, 2(1):49–63.

Henz, M., Müller, T., and Thiel, S. (2004). Global constraints for round robin tournament scheduling. *European Journal of Operational Research*, 153(1):92–101.

Horen, J. and Riezman, R. (1985). Comparing draws for single elimination tournaments. *Operations Research*, 33(2):249–262.

Hwang, F. K. (1982). New concepts in seeding knockout tournaments. *The American Mathematical Monthly*, 89(4):235–239.

IHF (2020). Egypt 2021 draw details confirmed. 25 August. https://www.ihf.info/media-center/news/egypt-2021-draw-details-confirmed.

Irnich, S. (2010). A new branch-and-price algorithm for the traveling tournament problem. *European Journal of Operational Research*, 204(2):218–228.

Israel, R. B. (1981). Stronger players need not win more knockout tournaments. *Journal of the American Statistical Association*, 76(376):950–951.

Jakee, K., Kenneally, M., and Dineen, D. (2022). Scheduling slots and league objectives: An empirical analysis of Australia's AFL. *The Quarterly Review of Economics and Finance*, 84:359–371.

Jennett, N. (1984). Attendances, uncertainty of outcome and policy in Scottish league football. *Scottish Journal of Political Economy*, 31(2):176–198.

Jones, M. C. (1990). The World Cup draw's flaws. *The Mathematical Gazette*, 74(470):335–338.

Jost, P.-J. (2024). First home or first away? Optimal ordering in two-legged ties. *Managerial and Decision Economics*, 45(1):54–69.

Kaiser, B. (2019). Strategy and paradoxes of Borda count in Formula 1 racing. *Decyzje*, 6(31):115–132.

Kaminski, M. M. (2022). How strong are soccer teams? The "host paradox" and other counterintuitive properties of FIFA's former ranking system. *Games*, 13(2):22.

Karpov, A. (2016). A new knockout tournament seeding method and its axiomatic justification. *Operations Research Letters*, 44(6):706–711.

Karpov, A. (2018). Generalized knockout tournament seedings. *International Journal of Computer Science in Sport*, 17(2):113–127.

Kassis, M., Schmidt, S. L., Schreyer, D., and Sutter, M. (2021). Psychological pressure and the right to determine the moves in dynamic tournaments – evidence from a natural field experiment. *Games and Economic Behavior*, 126:278–287.

Kendall, G., Knust, S., Ribeiro, C. C., and Urrutia, S. (2010). Scheduling in sports: An annotated bibliography. *Computers & Operations Research*, 37(1):1–19.

Kendall, G. and Lenten, L. J. (2017). When sports rules go awry. *European Journal of Operational Research*, 257(2):377–394.

Kidd, M. P. (2010). A tabu-search for minimising the carry-over effects value of a round-robin tournament. *ORiON*, 26(2):125–141.

Klößner, S. and Becker, M. (2013). Odd odds: The UEFA Champions League Round of 16 draw. *Journal of Quantitative Analysis in Sports*, 9(3):249–270.

Knust, S. (2010). Scheduling non-professional table-tennis leagues. *European Journal of Operational Research*, 200(2):358–367.

Knuth, D. E. and Lossers, O. P. (1987). A random knockout tournament. *SIAM Review*, 29(1):127–129.

Kocher, M. G., Lenz, M. V., and Sutter, M. (2012). Psychological pressure in competitive environments: New evidence from randomized natural experiments. *Management Science*, 58(8):1585–1591.

Kondratev, A. Y., Ianovski, E., and Nesterov, A. S. (2023). How should we score athletes and candidates: Geometric scoring rules. *Operations Research*, in press. DOI: 10.1287/opre.2023.2473.

Krautmann, A. C., Lee, Y. H., and Quinn, K. (2011). Playoff uncertainty and pennant races. *Journal of Sports Economics*, 12(5):495–514.

Krumer, A. (2020). Testing the effect of kick-off time in the UEFA Europa League. *European Sport Management Quarterly*, 20(2):225–238.

Krumer, A. and Lechner, M. (2017). First in first win: Evidence on schedule effects in round-robin tournaments in mega-events. *European Economic Review*, 100:412–427.

Krumer, A. and Lechner, M. (2018). Midweek effect on soccer performance: Evidence from the German Bundesliga. *Economic Inquiry*, 56(1):193–207.

Krumer, A., Megidish, R., and Sela, A. (2017). First-mover advantage in round-robin tournaments. *Social Choice and Welfare*, 48(3):633–658.

Krumer, A. and Moreno-Ternero, J. (2023). The allocation of additional slots for the FIFA World Cup. *Journal of Sports Economics*, 24(7):831–850.

Kujansuu, E., Lindberg, T., and Mäkinen, E. (1999). The stable roommates problem and chess tournament pairings. *Divulgaciones Matemáticas*, 7(1):19–28.

Kulhanek, T. and Ponomarenko, V. (2020). Surprises in knockout tournaments. *Mathematics Magazine*, 93(3):193–199.

Kyngäs, J., Nurmi, K., Kyngäs, N., Lilley, G., Salter, T., and Goossens, D. (2017). Scheduling the Australian Football League. *Journal of the Operational Research Society*, 68(8):973–982.

Lahvička, J. (2015a). The impact of playoffs on seasonal uncertainty in the Czech Ice Hockey Extraliga. *Journal of Sports Economics*, 16(7):784–801.

Lahvička, J. (2015b). Using Monte Carlo simulation to calculate match importance: The case of English Premier League. *Journal of Sports Economics*, 16(4):390–409.

Laica, C., Lauber, A., and Sahm, M. (2021). Sequential round-robin tournaments with multiple prizes. *Games and Economic Behavior*, 129:421–448.

Laliena, P. and López, F. J. (2019). Fair draws for group rounds in sport tournaments. *International Transactions in Operational Research*, 26(2):439–457.

Lambers, R., Goossens, D., and Spieksma, F. C. (2023). The flexibility of home away pattern sets. *Journal of Scheduling*, 26(5):413–423.

Lambers, R., Nederlof, J., and Spieksma, F. C. R. (2020). How the schedule in the TATA Steel Chess Championship forced Carlsen to help Caruana win. 27 April. https://www.networkpages.nl/how-the-schedule-in-the-tata-steel-chess-championship-forced-carlsen-to-help-caruana-win/.

Lambers, R., Pendavingh, R., and Spieksma, F. (2024). How to design a stable serial knockout competition. *Mathematics of Operations Research*, in press. DOI: 10.1287/moor.2022.0352.

Lambrechts, E., Ficker, A. M. C., Goossens, D. R., and Spieksma, F. C. R. (2018). Round-robin tournaments generated by the Circle Method have maximum carry-over. *Mathematical Programming*, 172(1-2):277–302.

Langville, A. N. and Meyer, C. D. (2012). *Who's #1?: The Science of Rating and Ranking*. Princeton University Press, Princeton, New Jersey.

Lapré, M. A. and Palazzolo, E. M. (2022). Quantifying the impact of imbalanced groups in FIFA Women's World Cup tournaments 1991–2019. *Journal of Quantitative Analysis in Sports*, 18(3):187–199.

Lapré, M. A. and Palazzolo, E. M. (2023). The evolution of seeding systems and the impact of imbalanced groups in FIFA Men's World Cup tournaments 1954–2022. *Journal of Quantitative Analysis in Sports*, 19(4):317–332.

Lasek, J. and Gagolewski, M. (2018). The efficacy of league formats in ranking teams. *Statistical Modelling*, 18(5-6):411–435.

Lasek, J., Szlávik, Z., and Bhulai, S. (2013). The predictive power of ranking systems in association football. *International Journal of Applied Pattern Recognition*, 1(1):27–46.

Lasek, J., Szlávik, Z., Gagolewski, M., and Bhulai, S. (2016). How to improve a team's position in the FIFA ranking? A simulation study. *Journal of Applied Statistics*, 43(7):1349–1368.

Lenor, S., Lenten, L., and McKenzie, J. (2016). Rivalry effects and unbalanced schedule optimisation in the Australian Football League. *Review of Industrial Organization*, 49(1):43–69.

Lenten, L. J. and Kendall, G. (2022). Scholarly sports: Influence of social science academe on sports rules and policy. *Journal of the Operational Research Society*, 73(12):2591–2601.

Li, M., Davari, M., and Goossens, D. (2023). Multi-league sports scheduling with different leagues sizes. *European Journal of Operational Research*, 307(1):313–327.

Losak, J. M. and Halpin, S. A. (2024). Does every game matter? A new perspective on the league standing effect in Major League Baseball. *Sports Economics Review*, 5:100028.

Lunander, A. and Karlsson, N. (2023). Choosing opponents in skiing sprint elimination tournaments. *Journal of Quantitative Analysis in Sports*, 19(3):205–221.

Matsumura, M. and Hasuike, T. (2022). Sports scheduling of professional baseball games for the purpose of increasing audiences. *International Journal of Japan Association for Management Systems*, 14(1):55–62.

McGarry, T. and Schutz, R. (1997). Efficacy of traditional sport tournament structures. *Journal of the Operational Research Society*, 48(1):65–74.

McHale, I. and Davies, S. (2007). Statistical analysis of the effectiveness of the FIFA World Rankings. In Albert, J. and Koning, R. H., editors, *Statistical Thinking in Sports*, pages 77–90. CRC Press, Boca Raton, Florida.

Medcalfe, S. (2024). Behavioral responses to sporting contest design: A review of the literature. *American Behavioral Scientist*, in press. DOI: 10.1177/00027642241235814.

Melkonian, V. (2021). Fair scheduling models for doubles group competitions. *American Journal of Operations Research*, 11(6):338–356.

Miyashiro, R. and Matsui, T. (2005). A polynomial-time algorithm to find an equitable home–away assignment. *Operations Research Letters*, 33(3):235–241.

Miyashiro, R. and Matsui, T. (2006). Minimizing the carry-over effects value in a round-robin tournament. In Burke, E. K. and Rudová, H., editors, *Proceedings of the 6th International Conference on the Practice and Theory of Automated Timetabling*, pages 460–463, Berlin, Heidelberg, Germany. Springer.

Moldovanu, B. and Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review*, 91(3):542–558.

Moschini, G. (2010). Incentives and outcomes in a strategic setting: The 3-points-for-a-win system in soccer. *Economic Inquiry*, 48(1):65–79.

Muller, G., Hood, S., and Sokol, J. (2023). Randomness, uncertainty, and the optimal college football championship tournament size. *Journal of Sports Analytics*, 9(1):1–37.

Narayana, T. V. and Zidek, J. (1969). Contributions to the theory of tournaments part I: The combinatorics of knock-out tournaments. *Cahiers du Bureau universitaire de recherche opérationnelle Série Recherche*, 13:3–18.

Nurmi, K., Goossens, D., and Kyngäs, J. (2014). Scheduling a triple round robin tournament with minitournaments for the Finnish national youth ice hockey league. *Journal of the Operational Research Society*, 65(11):1770–1779.

Ólafsson, S. (1990). Weighted matching in chess tournaments. *Journal of the Operational Research Society*, 41(1):17–24.

Olson, J. and Stone, D. F. (2014). Suspense-optimal college football play-offs. *Journal of Sports Economics*, 15(5):519–540.

Osicka, O. and Guajardo, M. (2023). Fair travel distances in tournament schedules: A cooperative game theory approach. *Sports Economics Review*, 2:100011.

Pagels, J. (2018). Competition between sports hurts TV ratings: How to shift league calendars to optimize viewership. *Journal of Sports Analytics*, 4(3):193–199.

Palacios-Huerta, I. (2014). *Beautiful Game Theory: How Soccer Can Help Economics*. Princeton University Press, Princeton, New Jersey.

Palacios-Huerta, I. (2023). The beautiful dataset. Manuscript. DOI: 10.2139/ssrn.4665889.

Pawlowski, T. and Nalbantis, G. (2015). Competition format, championship uncertainty and stadium attendance in European football – a small league perspective. *Applied Economics*, 47(38):4128–4139.

Pérez, L. (2024). On the design of international match calendar: the effect of "FIFA reserved dates" on European football matches' outcomes. *Journal of Quantitative Analysis in Sports*, 20(1):51–57.

Placek, P. C. (2023). The impossibility of a perfect tournament. *Entertainment Computing*, 45:100540.

Prince, M., Cole Smith, J., and Geunes, J. (2013). Designing fair 8-and 16-team knockout tournaments. *IMA Journal of Management Mathematics*, 24(3):321–336.

Raack, C., Raymond, A., Schlechte, T., and Werner, A. (2014). Standings in sports competitions using integer programming. *Journal of Quantitative Analysis in Sports*, 10(2):131–137.

Radicchi, F. (2011). Who is the best player ever? A complex network analysis of the history of professional tennis. *PloS One*, 6(2):e17249.

Rasmussen, R. V. and Trick, M. A. (2007). A Benders approach for the constrained minimum break problem. *European Journal of Operational Research*, 177(1):198–213.

Rasmussen, R. V. and Trick, M. A. (2008). Round robin scheduling – a survey. *European Journal of Operational Research*, 188(3):617–636.

Rathgeber, A. and Rathgeber, H. (2007). Why Germany was supposed to be drawn in the group of death and why it escaped. *Chance*, 20(2):22–24.

Rennó-Costa, C. (2023). A double-elimination format for a 48-team FIFA World Cup. Manuscript. DOI: 10.48550/arXiv.2301.03411.

Ribeiro, C. C. (2012). Sports scheduling: Problems and applications. *International Transactions in Operational Research*, 19(1-2):201–226.

Ribeiro, C. C. and Urrutia, S. (2005). An application of integer programming to playoff elimination in football championships. *International Transactions in Operational Research*, 12(4):375–386.

Ribeiro, C. C. and Urrutia, S. (2007). Heuristics for the mirrored traveling tournament problem. *European Journal of Operational Research*, 179(3):775–787.

Ribeiro, C. C. and Urrutia, S. (2012). Scheduling the Brazilian soccer tournament: Solution approach and practice. *Interfaces*, 42(3):260–272.

Ribeiro, C. C., Urrutia, S., and de Werra, D. (2023a). *Combinatorial Models for Scheduling Sports Tournaments*. EURO Advanced Tutorials on Operational Research. Springer, Cham, Switzerland.

Ribeiro, C. C., Urrutia, S., and de Werra, D. (2023b). A tutorial on graph models for scheduling round-robin sports tournaments. *International Transactions in Operational Research*, 30(6):3267–3295.

Roberts, G. O. and Rosenthal, J. S. (2024). Football group draw probabilities and corrections. *Canadian Journal of Statistics*, 52(3):659–677.

Rottenberg, S. (1956). The baseball players' labor market. *Journal of Political Economy*, 64(3):242–258.

Rubinstein, A. (1980). Ranking the participants in a tournament. *SIAM Journal on Applied Mathematics*, 38(1):108–111.

Russell, K. G. (1980). Balancing carry-over effects in round robin tournaments. *Biometrika*, 67(1):127–131.

Ryvkin, D. (2010). The selection efficiency of tournaments. *European Journal of Operational Research*, 206(3):667–675.

Ryvkin, D. and Ortmann, A. (2008). The predictive power of three prominent tournament formats. *Management Science*, 54(3):492–504.

Sauer, P., Cseh, Á., and Lenzner, P. (2024). Improving ranking quality and fairness in Swiss-system chess tournaments. *Journal of Quantitative Analysis in Sports*, 20(2):127–146.

Scarf, P., Yusof, M. M., and Bilbao, M. (2009). A numerical study of designs for sporting contests. *European Journal of Operational Research*, 198(1):190–198.

Scarf, P. A. and Shi, X. (2008). The importance of a match in a tournament. *Computers & Operations Research*, 35(7):2406–2418.

Scarf, P. A. and Yusof, M. M. (2011). A numerical study of tournament structure and seeding policy for the soccer world cup finals. *Statistica Neerlandica*, 65(1):43–57.

Scelles, N., François, A., and Valenti, M. (2024). Impact of the UEFA Nations League on competitive balance, competitive intensity, and fairness in European men's national team football. *International Journal of Sport Policy and Politics*, 27(3):267–284.

Schilling, M. F. (1994). The importance of a game. *Mathematics Magazine*, 67(4):282–288.

Schreyer, D. and Ansari, P. (2022). Stadium attendance demand research: A scoping review. *Journal of Sports Economics*, 23(6):749–788.

Schreyer, D., Schmidt, S. L., and Torgler, B. (2018). Game outcome uncertainty in the English Premier League: Do German fans care? *Journal of Sports Economics*, 19(5):625–644.

Schwenk, A. J. (2000). What is the correct way to seed a knockout tournament? *The American Mathematical Monthly*, 107(2):140–150.

Scoppa, V. (2015). Fatigue and team performance in soccer: Evidence from the FIFA World Cup and the UEFA European Championship. *Journal of Sports Economics*, 16(5):482–507.

Searls, D. T. (1963). On the probability of winning with different tournament procedures. *Journal of the American Statistical Association*, 58(304):1064–1081.

Stanton, I. and Williams, V. V. (2013). The structure, efficacy, and manipulation of double-elimination tournaments. *Journal of Quantitative Analysis in Sports*, 9(4):319–335.

Statista (2023). Olympic Summer Games total global TV audience in 2008 and 2020 (in billions). https://www.statista.com/statistics/280502/total-number-of-tv-viewers-of-olympic-summer-games-worldwide/#:~:text=The%202020%20Oly

Stefani, R. (2011). The methodology of officially recognized international sports rating systems. *Journal of Quantitative Analysis in Sports*, 7(4).

Stone, C. and Rod, M. (2016). Unfair play in World Cup qualification? An analysis of the 1998–2010 FIFA World Cup performances and the bias in the allocation of tournament berths. *Soccer & Society*, 17(1):40–57.

Stronka, W. (2024). Demonstration of the collusion risk mitigation effect of random tie-breaking and dynamic scheduling. *Sports Economics Review*, 5:100025.

Suksompong, W. (2016). Scheduling asynchronous round-robin tournaments. *Operations Research Letters*, 44(1):96–100.

Szczecinski, L. and Roatis, I.-I. (2022). FIFA ranking: Evaluation and path forward. *Journal of Sports Analytics*, 8(4):231–250.

Sziklai, B. R., Biró, P., and Csató, L. (2022). The efficacy of tournament designs. *Computers & Operations Research*, 144:105821.

Szymanski, S. (2003). The economic design of sporting contests. *Journal of Economic Literature*, 41(4):1137–1187.

Tanaka, T. and Miyashiro, R. (2016). Fair home-away tables in sports scheduling. *Journal of Advanced Mechanical Design, Systems, and Manufacturing*, 10(3):JAMDSM0037.

Temesi, J., Szádoczki, Zs., and Bozóki, S. (2024). Incomplete pairwise comparison matrices: Ranking top women tennis players. *Journal of the Operational Research Society*, 75(1):145–157.

Tiwisina, J. and Külpmann, P. (2019). Probabilistic transitivity in sports. *Computers & Operations Research*, 112:104765.

Trick, M. A. (2000). A schedule-then-break approach to sports timetabling. In Burke, E. and Erben, W., editors, *Practice and Theory of Automated Timetabling III*, pages 242–253. Springer, Berlin, Heidelberg, Germany.

Tuffaha, T., Çavdaroğlu, B., and Atan, T. (2023). Round-robin scheduling with regard to rest differences. *TOP*, 31(2):269–301.

UEFA (2023). 2023 UEFA Circular Letter No36: UEFA club competitions cycle 2024–27 ("Post 2024"). 10 July. https://www.uefa.com/insideuefa/documentlibrary/about-uefa/circular-letters/.

Urrutia, S. and Ribeiro, C. C. (2006). Maximizing breaks and bounding solutions to the mirrored traveling tournament problem. *Discrete Applied Mathematics*, 154(13):1932–1938.

Van Bulck, D. and Goossens, D. (2020). Handling fairness issues in time-relaxed tournaments with availability constraints. *Computers & Operations Research*, 115:104856.

Van Bulck, D., Goossens, D., Schönberger, J., and Guajardo, M. (2020). RobinX: A three-field classification and unified data format for round-robin sports timetabling. *European Journal of Operational Research*, 280(2):568–580.

van der Burg, T. (2023). Competitive balance and demand for European men's football: a review of the literature. *Managing Sport and Leisure*, in press. DOI: 10.1080/23750472.2023.2206815.

Van Eetvelde, H., Hvattum, L. M., and Ley, C. (2023). The Probabilistic Final Standing Calculator: a fair stochastic tool to handle abruptly stopped football seasons. *AStA Advances in Statistical Analysis*, 107(1-2):251–269.

Van Eetvelde, H. and Ley, C. (2019). Ranking methods in soccer. In Kenett, R. S., Longford, T. N., Piegorsch, W., and Ruggeri, F., editors, *Wiley StatsRef: Statistics Reference Online*, pages 1–9. Springer, Hoboken, New Jersey.

van't Hof, P., Post, G., and Briskorn, D. (2010). Constructing fair round robin tournaments with a minimum number of breaks. *Operations Research Letters*, 38(6):592–596.

Vaziri, B., Dabadghao, S., Yih, Y., and Morin, T. L. (2018). Properties of sports ranking methods. *Journal of the Operational Research Society*, 69(5):776–787.

Vu, T. and Shoham, Y. (2011). Fair seeding in knockout tournaments. *ACM Transactions on Intelligent Systems and Technology*, 3(1):1–17.

Wallace, M. and Haigh, J. (2013). Football and marriage – and the UEFA draw. *Significance*, 10(2):47–48.

Wang, C., Goossens, D., and Vandebroek, M. (2018). The impact of the soccer schedule on TV viewership and stadium attendance: Evidence from the Belgian Pro League. *Journal of Sports Economics*, 19(1):82–112.

Wright, M. (2009). 50 years of OR in sport. *Journal of the Operational Research Society*, 60(1):S161–S168.

Wright, M. (2014). OR analysis of sporting rules – A survey. *European Journal of Operational Research*, 232(1):1–8.

Yi, X. (2021). *Dealing with uncertainty in round robin sports scheduling*. PhD thesis, Ghent University.

Zermelo, E. (1929). Die Berechnung der Turnier-Ergebnisse als ein Maximumproblem der Wahrscheinlichkeitsrechnung. *Mathematische Zeitschrift*, 29:436–460.