Numerical Simulation-Assignment 2

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Behavioral Economics - Assignment 1

Consider the following two-person p-Beauty Contest game. Both players simultaneously choose a number between 0 and 100. Both players have their own target value: p1 and p2, respectively. The player whose number is closest to pi times the average of the two numbers wins a fixed price (e.g., 10 Eurocent). Suppose p1 = 0.7 and p2 = 1.5.

a) Call the numbers that the players choose x1 and x2, respectively. Write down the inequality that needs to hold for player 1 to win the price

Answer:

$$\left| x_1 - 0.7\frac{(x_1 + x_2)}{2}) \right| < \left| x_2 - 1.5\frac{(x_1 + x_2)}{2} \right|$$

The left hand side of the inequality is the difference between the number that player 1 chooses and p1 times the average of the two numbers. The right hand side of the inequality is the same but for player 2....

b) Suppose that player 1 is a level-1 player (who thinks that player 2 is level-0). What number would player 1 choose?

Answer:

$$\begin{aligned}
0.7\frac{(50 + x_1)}{2} &= x_1 \\
\frac{(35 + 0.7x_1)}{2} &= x_1 \\
35 + 0.7x_1 &= 2x_1 \\
35 &= 1.3x_1 \\
x_1 &= \frac{35}{1.3} \\
x_1 &\approx 26.92 \\
\lceil x_1 \rceil &= 27.00
\end{aligned}$$

c) Suppose that player 2 is a level-1 player (who thinks that player 1 is level-0). What number would player 2 choose?

Answer:

$$
\begin{aligned}
1.5\frac{(50+x_2)}{2} &= x_2 \\
\frac{(75+1.5x_2)}{2} &= x_2 \\
75+1.5x_2 &= 2x_2 \\
75 &= 0.5x_2 \\
x_2 &= \frac{75}{0.5} \\
x_2 &= 150
\end{aligned}
$$

However, since the maximum number he can choose, we will choose 100 in all probability.

d) Suppose that player 1 is a level-2 player (who thinks that player 2 is level-1). What number would player 1 choose?

Answer:

If player 1 is a level-2 player, he will think that player 2 is level-1 player. This means that he expects that player 2 thinks player 1 is a level-0 player. So he expects player 2 to play 100.

$$
\begin{aligned}
0.7\frac{(100+x1)}{2} &= x_1 \\
\frac{(70+0.7x1)}{2} &= x_1 \\
70+0.7x_1 &= 2x_1 \\
70 &= 1.3x_1 \\
x_1 &= \frac{70}{1.3} \\
x_1 &\approx 53.85 \\
\lceil x_1 \rceil &= 54.00
\end{aligned}
$$

therefore the player will choose 54

e) Suppose that player 2 is a level-2 player (who thinks that player 1 is level-1). What number would player 2 choose?

Answer:

If player 2 is a level-2 player, he will think that player 1 is level-1 player. This means that he expects that player 1 thinks player 2 is a level-0 player. So he expects player 1 to play 27.

$$
\begin{aligned}
1.5\frac{(27 + x_2)}{2} &= x_2 \\
\frac{(40.5 + 1.5x_2)}{2} &= x_2 \\
40.5 + 1.5x_2 &= 2x_2 \\
40.5 &= 0.5x_2 \\
x_2 &= \frac{40.5}{0.5} \\
x_2 &= 81
\end{aligned}
$$

f) Suppose that player 1 is a level C_2 believing that player 2 is a mixture of level-0 and level-1 with $\tau = 2$. What number would player 1 choose?

Answer:

For this question, we will use the Cognitive Hierarchy model. The distribution of levels is assumed to follow a Poisson distribution, meaning that:

$$
\begin{aligned}
f(k) &= \frac{e^{-t}t^k}{k!} \\
Prob(level - k) &= f(k) \\
Prob(level - 0) &= \frac{e^{-2}2^0}{0!} \approx 0.135 \\
Prob(level - 1) &= \frac{e^{-2}2^1}{1!} \approx 0.271 \\
Prob(level - 2) &= \frac{e^{-2}2^2}{2!} \approx 0.271
\end{aligned}
$$

Type	**Choice**	**Type**	**Fraction** $f(k)$	**Beliefs** C_1	**Beliefs** C_2	**Choice**
L_0	50	C_0	$0,135$	1	$0,33$	50
L_1	100	C_1	0.271	-	0.67	27
L_2	81	C_2	0.271	-	-	45

Table 1: Results Table

If player 1 is a level C-1 player, he expects that player 2 is a level-0 player. This means that he thinks that player 2 will choose 50 with probability 1. In this case, player 1 will choose 27 (see question (b)).

If player 1 is a level C-2 player, he expects that player 2 is a level-0 player with probability 0.33

$$\frac{0.135}{0.135 + 0.271} \approx 0.33 \tag{1}$$

and a level-1 player with probability 0.67

$$\frac{0.271}{0.135 + 0.271} \approx 0.67 \tag{2}$$

So he thinks that player 2 will play, from (1) and (2):

$$(0.3350) + (0.67100) \approx 83.5$$

Therefore player 2 will choose 84.

Therefore player 1 will choose:

$$\begin{aligned} 0.7\frac{(84 + x_1)}{2} &= x_1 \\ \frac{(58.8 + 0.7x_1)}{2} &= x_1 \\ 58.8 + 0.7x_1 &= 2x_1 \\ 58.8 &= 1.3x_1 \\ x_1 &= \frac{58.8}{1.3} \\ x_1 &\approx 45.23 \\ \lceil x_1 \rceil &= 45 \end{aligned}$$

Therefore player 1 will choose 45.

Type	**Choice**	**Type**	**Fraction** $f(k)$	**Beliefs** C_1	**Beliefs** C_2	**Choice**
L_0	50	C_0	$0,135$	1	$0,33$	50
L_1	27	C_1	0.271	-	0.67	100
L_2	54	C_2	0.271	-	-	100

Table 2: Results Table

g) Suppose that player 2 is a level C-2 believing that player 1 is a mixture of level-0 and level-1 with $\tau = 2$. What number would player 2 choose?

Answer:

If player 2 is a level C-1 player, he expects that player 1 is a level-0 player. This means that he thinks that player 1 will choose 50 with probability 1. In this case, player 1 will choose 100 (see question (c)).

If player 2 is a level C-2 player, he expects that player 1 is a level-0 player with probability 0.33

$$\frac{0.135}{(0.135 + 0.271)} \approx 0.33 \tag{3}$$

and a level-1 player with probability 0.67

$$\frac{0.271}{0.135 + 0.271} \approx 0.67 \tag{4}$$

So he thinks that player 2 will play, from (3) and (4):

$$(0.3350) + (0.6727) \approx 34.59$$

Roughly, he will choose 35.

Therefore player 2 will choose:

$$
\begin{aligned}
1.5\frac{(35+x_2)}{2} &= x_2 \\
\frac{52.5+1.5x_2}{2} &= x_2 \\
52.5+1.5x_2 &= 2x_2 \\
52.5 &= 0.5x_2 \\
x_2 &= \frac{52.5}{0.5} \\
x_2 &= 105
\end{aligned}
$$

Therefore player 2 will choose 100 since that is the maximum.

h) What is the Nash equilibrium for the game?

Answer:

Solving the inequality in (a) (it means that $x_1 > .042x_2$), we can say that for every number that player 1 chooses between 1 to 42, player 2 has a best response for it (up to 100). However, since player 2 cannot best respond to numbers higher than 42, if player 1 plays 43, every other number player 2 selects become trivial. So NE will be $b_1 > 43$ and $b_2 =$ any response.Then no player has an incentive to deviate.