Presentation on Behavioral Economics Assignment

Numerical Simulation

Konstantinos Pananas

MSc Economics
Tilburg University

Assignment 2

Assignment Description

The purpose of this assignment is to derive the Nash Equilibria for a two-person P-Beauty Contest. The ρ factors (Target Values) were given as follows:

$$\rho_1 = 0.7$$

$$\rho_2 = 1.5$$

Let the the numbers that the players choose be x_1 and x_2, respectively.

Question (a)

Write down the inequality that needs to hold for player 1 to win the price.

Answer:

$$\left| x_1 - 0.7\frac{(x_1 + x_2)}{2}) \right| < \left| x_2 - 1.5\frac{(x_1 + x_2)}{2} \right|$$

Level-k Thinking

Basic Idea

Player forms a belief about the behavior of others and responds optimally to that belief

A Level-1 player will believe that other players are Level-0

- Level - 0 players chooses non-strategically or randomly
- Level -1 players responds to that choice

A level-2 player will believe that other player are Level-1 and he optimally responds to that

Remark

A Level-k+1 believes that the other player is Level-k (optimally responding to a Level-k−1 player, and will optimally respond to that).

Question (b)

Suppose that player 1 is a level-1 player (who thinks that player 2 is level-0). What number would player 1 choose?

Answer:

$$
\begin{aligned}
0.7\frac{(50+x_1)}{2} &= x_1 \\
\frac{(35+0.7x_1)}{2} &= x_1 \\
35+0.7x_1 &= 2x_1 \\
35 &= 1.3x_1 \\
x_1 &= \frac{35}{1.3} \\
x_1 &\approx 26.92 \\
\lceil x_1 \rceil &= 27.00
\end{aligned}
$$

Question (c)

Suppose that player 2 is a level-1 player (who thinks that player 1 is level-0). What number would player 2 choose?

Answer:

$$\begin{aligned}
1.5\frac{(50+x_2)}{2} &= x_2\\
\frac{(75+1.5x_2)}{2} &= x_2\\
75+1.5x_2 &= 2x_2\\
75 &= 0.5x_2\\
x_2 &= \frac{75}{0.5}\\
x_2 &= 150
\end{aligned}$$

Question (c)

Remark

However, since the maximum number that player 2 can choose is 100 and $x_2 > 100$, he will choose 100.

Question (d)

Suppose that player 1 is a level-2 player (who thinks that player 2 is level-1). What number would player 1 choose?

Answer:

If player 1 is a level-2 player, he will think that player 2 is level-1 player. This means that he expects that player 2 thinks player 1 is a level-0 player. So he expects player 2 to play 100.

Question (d)

$$
\begin{aligned}
0.7\frac{(100+x1)}{2} &= x_1 \\
\frac{(70+0.7x1)}{2} &= x_1 \\
70+0.7x_1 &= 2x_1 \\
70 &= 1.3x_1 \\
x_1 &= \frac{70}{1.3} \\
x_1 &\approx 53.85 \\
\lceil x_1 \rceil &= 54.00
\end{aligned}
$$

Therefore the player will choose 54.

Question (e)

Suppose that player 2 is a level-2 player (who thinks that player 1 is level-1). What number would player 2 choose?

Answer:

If player 2 is a level-2 player, he will think that player 1 is level-1 player. This means that he expects that player 1 thinks player 2 is a level-0 player. So he expects player 1 to play 27.

Question (e)

$$
\begin{aligned}
1.5\frac{(27 + x_2)}{2} &= x_2 \\
\frac{(40.5 + 1.5x_2)}{2} &= x_2 \\
40.5 + 1.5x_2 &= 2x_2 \\
40.5 &= 0.5x_2 \\
x_2 &= \frac{40.5}{0.5} \\
x_2 &= 81
\end{aligned}
$$

Cognitive Hierarchy Model

In the Cognitive Heirarchy Model, a Level-k player believes that others are a mix of lower levels: Level-0,..., Level-$k-1$.
The distribution of levels is assumed to follow a Poison Distribution:

Remark

$$f(k) = \frac{e^{-t}t^k}{k!}$$
$$Prob(level - k) = f(k)$$

Question (f)

Suppose that player 1 is a level C_2 believing that player 2 is a mixture of level-0 and level-1 with $\tau = 2$. What number would player 1 choose?

Answer:

$$
\begin{aligned}
Prob(level - 0) &= \frac{e^{-2}2^0}{0!} \approx 0.135 \\
Prob(level - 1) &= \frac{e^{-2}2^1}{1!} \approx 0.271 \\
Prob(level - 2) &= \frac{e^{-2}2^2}{2!} \approx 0.271
\end{aligned}
$$

Question (f)

Type	Choice	Type	Fraction $f(k)$	Beliefs C_1	Beliefs C_2	Choic
L_0	50	C_0	$0,135$	1	$0,33$	50
L_1	100	C_1	0.271	-	0.67	27
L_2	81	C_2	0.271	-	-	45

Table: Results Table I

Question (f)

If player 1 is a level C-1 player, he expects that player 2 is a level-0 player. This means that he thinks that player 2 will choose 50 with probability 1. In this case, player 1 will choose 27 (see question (b)).
If player 1 is a level C-2 player, he expects that player 2 is a level-0 player with probability 0.33

$$\frac{0.135}{0.135+0.271} \approx 0.33 \tag{1}$$

and a level-1 player with probability 0.67

$$\frac{0.271}{0.135+0.271} \approx 0.67 \tag{2}$$

So he thinks that player 2 will play, from (1) and (2):

$$(0.3350) + (0.67100) \approx 83.5$$

Therefore player 2 will choose 84.

Question (f)

Therefore player 1 will choose:

$$
\begin{aligned}
0.7\frac{(84 + x_1)}{2} &= x_1 \\
\frac{(58.8 + 0.7x_1)}{2} &= x_1 \\
58.8 + 0.7x_1 &= 2x_1 \\
58.8 &= 1.3x_1 \\
x_1 &= \frac{58.8}{1.3} \\
x_1 &\approx 45.23 \\
\lceil x_1 \rceil &= 45
\end{aligned}
$$

Therefore player 1 will choose 45.

Question (g)

Suppose that player 2 is a level C-2 believing that player 1 is a mixture of level-0 and level-1 with $\tau = 2$. What number would player 2 choose?

Question (g)

Answer:

Type	Choice	Type	Fraction $f(k)$	Beliefs C_1	Beliefs C_2	Choic
L_0	50	C_0	$0,135$	1	$0,33$	50
L_1	27	C_1	0.271	-	0.67	100
L_2	54	C_2	0.271	-	-	100

Table: Results Table II

Question (g)

If player 2 is a level C-1 player, he expects that player 1 is a level-0 player. This means that he thinks that player 1 will choose 50 with probability 1. In this case, player 1 will choose 100 (see question (c)).
If player 2 is a level C-2 player, he expects that player 1 is a level-0 player with probability 0.33

$$\frac{0.135}{(0.135 + 0.271)} \approx 0.33 \tag{3}$$

and a level-1 player with probability 0.67

$$\frac{0.271}{0.135 + 0.271} \approx 0.67 \tag{4}$$

So he thinks that player 2 will play, from (3) and (4):

$$(0.3350) + (0.6727) \approx 34.59$$

Roughly, he will choose 35.

Question (g)

Therefore player 2 will choose:

$$
\begin{aligned}
1.5\frac{(35+x_2)}{2} &= x_2 \\
\frac{52.5+1.5x_2}{2} &= x_2 \\
52.5+1.5x_2 &= 2x_2 \\
52.5 &= 0.5x_2 \\
x_2 &= \frac{52.5}{0.5} \\
x_2 &= 105
\end{aligned}
$$

Therefore player 2 will choose 100, since that is the maximum.

Question (h)

What is the Nash equilibrium for the game?

Answer:

Solving the inequality in (a) (it means that $x_1 > .042x_2$), we can say that for every number that player 1 chooses between 1 to 42, player 2 has a best response for it (up to 100). However, since player 2 cannot best respond to numbers higher than 42, if player 1 plays 43, every other number player 2 selects become trivial. So NE will be $b_1 > 43$ and $b_2 =$ any response.Then no player has an incentive to deviate.